Mathematics and Statistics 1.7

Internally assessed
3 credits

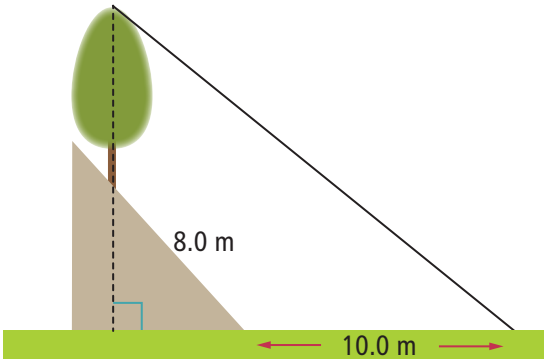
Applying right-angled triangles in solving measurement problems
Online practice assessment task

1. Inaccessible tree

A tree surveyor wants to work out the height of a tree that is growing partway up a sloping bank.

Getty Images – Paolo Viecelli / EyeEm

The diagram shows some measurements that are known.

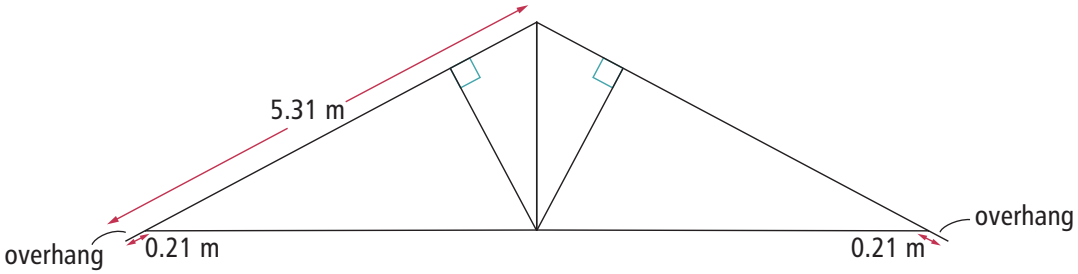


- The distance from the bottom of the bank to the base of the tree is 8 metres.
- The surveyor stands 10 metres from the bottom of the bank and looks up at the tree.

Explain a strategy for working out the height of the tree, without climbing the bank (which is very crumbly) or the tree.
Discuss the accuracy of your estimate of the height of the tree.

2. Brace yourself

The diagram shows a timber framework for a symmetrical roof. The outside section has an angle of elevation of 28° and is 5.31 metres long, including a 21-centimetre overhang. Three pieces of bracing timber are positioned as shown; one is perpendicular to the horizontal base section of the framework, and the two sloping braces are perpendicular to the sloping outside section of the framework.



Work out what length of timber is required for the whole framework.

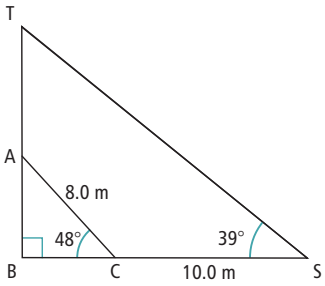
Discuss some practical aspects of your solution.

Solutions

1. Inaccessible tree

The surveyor can use an angle measuring device to measure the angle the cliff makes with horizontal ground (say 48º) and the angle of elevation of the top of the tree (say 39º).

The labelled diagram shows the measurements.



The strategy is to work out the length of AB and BC (using the right-angled triangle ABC), then work out SB and TB (in the right-angled triangle TSB). Finally AB is subtracted from TB to find the height of the tree.

In triangle ABC, working to 1 decimal place

$\sin 48^\circ = \frac{AB}{8}$ $\quad\quad$ $\cos 48^\circ = \frac{BC}{8}$

$AB = 8 \sin 48^\circ$ $\quad\quad$ $BC = 8 \cos 48^\circ$

$AB = 5.9$ metres $\quad\quad$ $BC = 5.4$ metres

So $SB = 15.4$ metres

In triangle TSB

$\tan 39^\circ = \frac{TB}{15.4}$

$TB = 15.4 \tan 39^\circ$

$TB = 12.5$ metres (1 d.p.)

So $TA = 12.5 - 5.9$

$= 6.6$ metres

The angles were measured to the nearest degree. The accuracy of the angle measurement would depend on the measurement device used (using a laser pointer mounted on a tripod may give more accurate angle measurements that this).

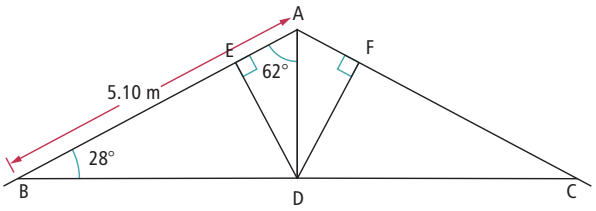
Distances were measured in metres to 1 decimal place, i.e. to the nearest 10 centimetres. The choice of 1 decimal place for the accuracy of the distances is reasonable as in outdoor contexts the exact point which is the position of the base of a bank is debatable; also the base of the tree can be defined to be the outer edge of the trunk, or estimated as the centre of the trunk. To take in such inaccuracies, measurement to the nearest 10 centimetres seems reasonable.

The answer has also been given to 1 decimal place of accuracy, which is consistent with the accuracy of the measurements of distance used in the calculations. However, because of the rounding involved in the calculations, an answer of between 6 and 7 metres would seem to be more realistic as an estimate of the height of the tree.

For example, if the angles were 47.5º and 39.5º then, using similar working to that given above, the height of the tree would be 6.8 m; and if the angles were 48.5 and 38.5 then the height of the tree would be 6.2 metres.

2. Brace yourself

In the diagram, first find AD and BD (using right-angled triangle ABD), then find ED (using right-angled triangle AED). $AB = 5.31 - 0.21 = 5.10$ m



In triangle ABD

$\sin 28^\circ = \frac{AD}{5.10}$ $\quad\quad$ $\cos 28^\circ = \frac{BD}{5.10}$

$AD = 5.10 \sin 28^\circ$ $\quad\quad$ $BD = 5.10 \cos 28^\circ$

$AD = 2.39$ metres $\quad\quad$ $BD = 4.50$ metres

In triangle AED, AD is the hypotenuse and $EAD = 62^\circ$ angle sum triangle ABD

$\sin 62^\circ = \frac{ED}{2.39}$

$ED = 2.39 \sin 62^\circ$

$ED = 2.11$ metres

Adding all lengths in cross-section (using symmetry of figure)

Total length of timber required $= 2(5.31 + 2.11 + 4.50) + 2.39$

$= 26.23$ metres

This answer is correct to the nearest centimetre (the same accuracy as the measurements in the figure) whereas builders usually measure to the nearest millimetre.

In practical terms the amount of wood required would be a bit more than this, as angled cuts would be required, and even for straight cuts, a certain amount of wood length loss would occur.