Mathematics and Statistics 1.3

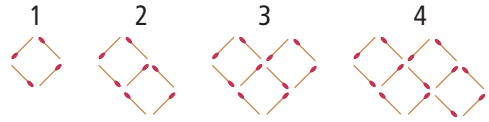
Externally assessed
4 credits

Investigate relationships between tables, equations and graphs

Online practice assessment task

Question 1

a. Jake uses matchsticks to make a sequence of joined diamond patterns, as shown.

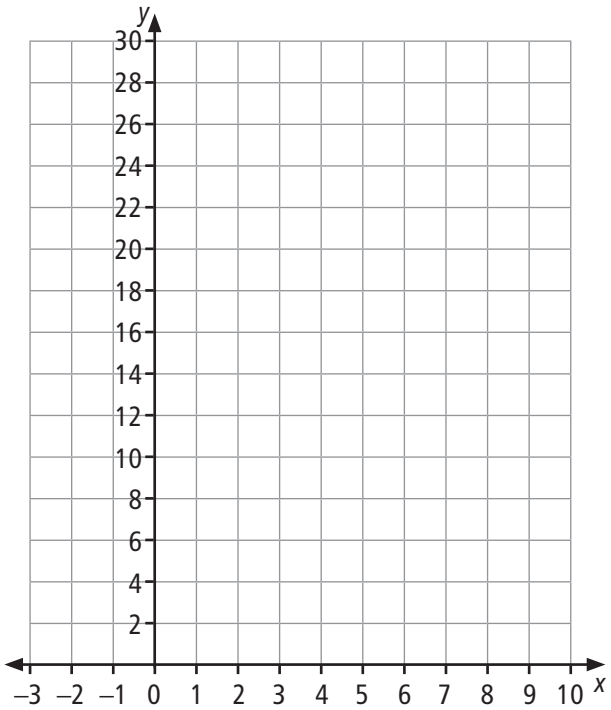


He draws up a table showing the number of matchsticks needed for each pattern.

Diamonds (*n*)	Matchsticks (*m*)
1	4
2	7
3	
4	
5	

i. Complete the table and work out the rule for the number of matchsticks *m* Jake needs for a pattern with *n* joined diamonds.

ii. On the grid below, sketch a graph showing the number of matchsticks required for up to the 8th pattern.



iii. Give the rule for the *total* number of matchsticks *T* that Jake would need if he continued the pattern up to *n* joined diamonds. Use this rule to work out the total number of matchsticks Jake would need to complete his first 12 patterns.

b. Sam makes a different sequence of diamond patterns. Sam also starts with 1 diamond, but he continues his pattern by adding diamonds as shown below.

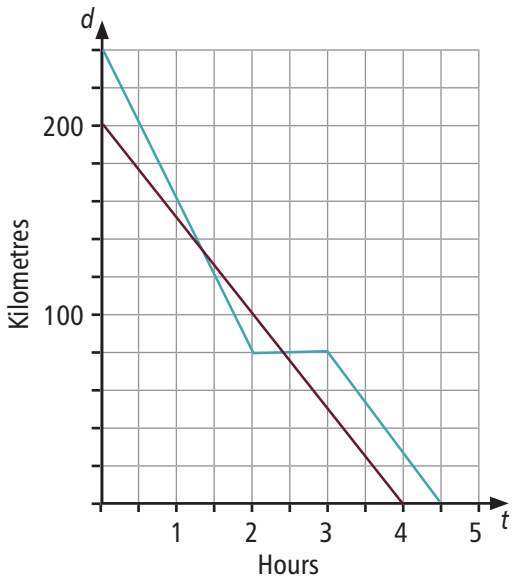
i. Give the rule for the number of matchsticks needed to make the *n*th diamond pattern in Sam's sequence.

ii. Use this rule to find out which of Sam's patterns would use 100 matchsticks.

iii. Describe how the graph for the number of matchsticks Sam uses would relate to Jake's graph.

Question 2

The Abbot family and the Brown family drive to the same beach for their holidays. They both leave at 10 am, but the Abbot family takes a break during the drive to the beach. The graph shows the distance from the beach, d kilometres, plotted against time, t hours since leaving.



a. One line on the graph represents the Brown family's trip.

i. How far do the Browns live from the beach?

ii. At what time did the Brown family get to the beach?

iii. What is the equation of the line representing the Brown family's distance d from the beach at time t.

b. The Abbot family's graph shows that their trip had three sections.

i. At what speed do the Abbots drive before taking a break?

ii. What is the equation of the graph for this section of the trip?

c. The middle section of the graph shows the break that the Abbot family took on their way to the beach.

i. Between what two times was the break?

ii. What is the equation of the graph for this section of the trip?

d. Find the equation of the final section of the Abbots' graph.

e. The Chen family drives away from the beach at 10.30 am, driving at the same speed as the Brown family drives. Draw the graph of the Chen family's trip on the grid and work out when the Chen family and the Brown family are both the same distance from the beach.

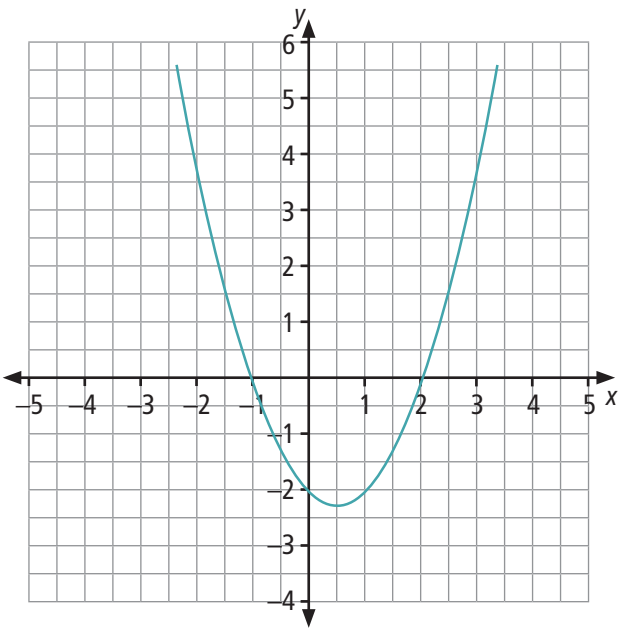
Question 3

a. For the parabola drawn below give:

i. the intercepts

ii. the equation of the curve

iii. the coordinates of the vertex



The parabola is reflected in the x-axis then translated 2 units right.

iv. What is the equation of the parabola now?

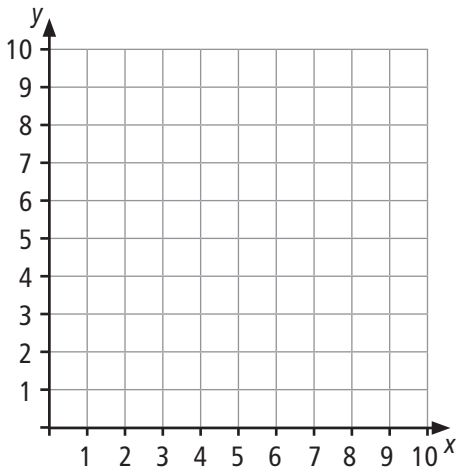
v. What is the y-intercept of the parabola now?

vi. What is the vertex of the parabola now?

b. Water from a fountain makes a parabolic arch which can be modelled by the function $h = (x + 1)(5 - x)$, where h metres is the height of the water x metres horizontally from the fountain head.

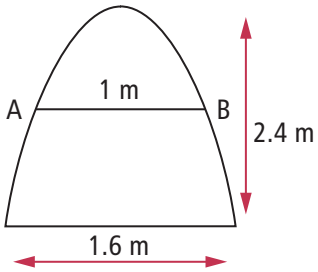
Getty Images – FedotovAnatoly

i. Draw the graph of the part of the function that models the water coming out of the fountain and hitting the ground.



ii. What is the maximum height the water from the fountain reaches?

c. Bobbits live in villages with doors in the shape of parabolas, as shown. Bobo the bobbit has a door which is 1.6 m wide and 2.4 m high. A 1-m-wide support brace AB is attached horizontally part-way up the door. Find the distance AB is below the top of the door.



Answers

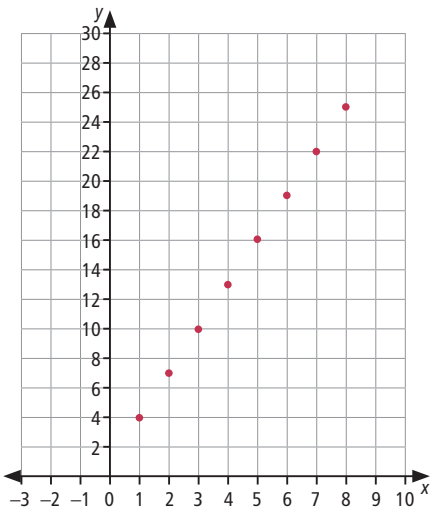
Question 1

a.

Diamonds (n)	Matchsticks (m)
1	4
2	7
3	10
4	13
5	16

i. $m = 3n + 1$

ii.



iii. $T = 1.5n^2 + 2.5n$; for 12 patterns need 246 matchsticks

b. i. $m = 4n$

ii. 25th pattern

iii. Same first point, but line of points is steeper after that.

Question 2

a. i. 200 km

ii. 2 pm

iii. $d = -50t + 200$

b. i. 80 km h^{-1}

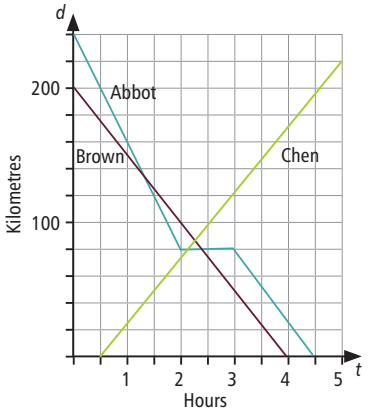
ii. $d = -80t + 240$

c. i. 12–1 p.m.

ii. $d = 80$

d. $d = -80t + 320$

e.



From the graph the lines intersect at 2.25.

Alternatively, the Chen family line has equation $d = 50t + c$

Substituting (0.5,0) gives $c = -25$, so the line is $d = 50t - 25$

Solving $50t - 25 = -50t + 200$ gives $100t = 225$, so $t = 2.25$

At 12.15 pm the Chen family and the Brown family are the same distance from the beach.

Question 3

a. i. (–1,0), (2,0), (0,–2)

ii. $y = (x + 1)(x - 2)$

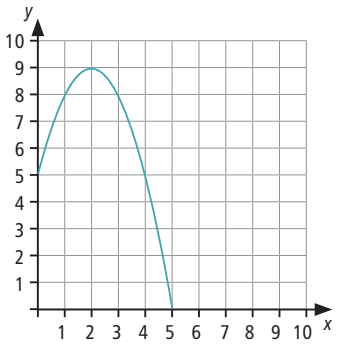
iii. (0.5,–2.25)

iv. $y = -(x - 1)(x - 4)$ or $y = -x^2 + 5x - 4$

v. (0,–4)

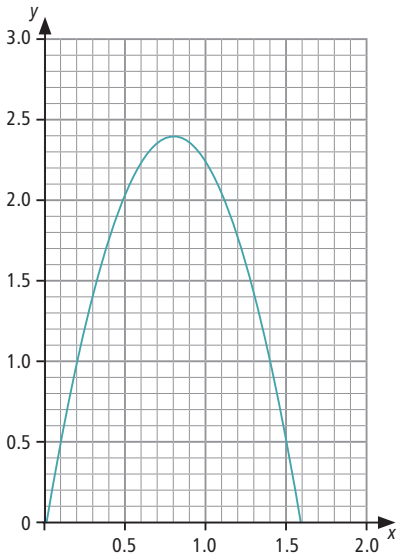
vi. (2.5,2.25)

b. i.



ii. 9 metres

c.



Equation of curve is $y = kx(x - 1.6)$

Substituting (0.8,2.4) gives $k = -3.75$

So equation of curve is $y = -3.75x(x - 1.6)$

When $x = 0.3$, $y = 1.4625$

this is $2.4 - 1.4625 = 0.9375$ m below the top of the door.