Mathematics and Statistics 1.9

Internally assessed
2 credits

AS 91034

Apply transformation geometry in solving problems

Note: To prepare for this Achievement Standard the student needs to have the usual equipment plus protractor, ruler, pencil and compasses.

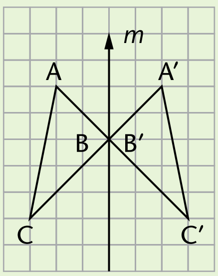
Transformations – reflection

A **transformation** is a change made to a figure.

- The original figure, called the **object**, is transformed to a new figure called the **image**.
- Vertices of images are often labelled using dashes on the original letters: A → A' and so on.
- Properties of a figure that remain unchanged by a transformation are called **invariant**.

Reflection

A **reflection** flips a figure over a **mirror line**, *m*.



Triangle A'B'C' is the image of triangle ABC under reflection in the mirror line, *m*. Note that ABC is named **clockwise** and A'B'C' is named **anticlockwise**.

Properties of reflection

For reflection (as illustrated in the example above) invariant properties are:

- Side lengths of the figure:
  e.g. AC = A'C'
- Angles (shape) of the figure:
  e.g. ∠ABC = ∠A'B'C'
- Area (size) of figure:
  e.g. area of Δ ABC = area of Δ A'B'C'.
- Points on the mirror line:
  e.g. B = B'

Note that the **sense** of the figure, i.e. the direction in which vertices are named, is changed by the reflection (see example above).

Finding the mirror line for a reflection

Suppose a figure and its image under a reflection are given. In order to describe the reflection fully, the mirror line needs to be located. This is done as follows:

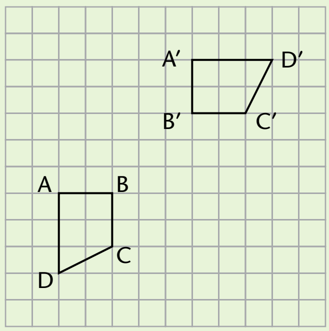
1. Join a point and its image under reflection (e.g., A to A').
2. Construct the perpendicular bisector of the line segment AA'.

The line constructed in step 2 is the mirror line.

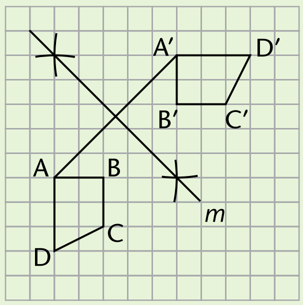
Example

Q. Trapezium ABCD is reflected to A'B'C'D'.

Find the mirror line.



A.



The mirror line *m* is found by constructing the perpendicular bisector of AA'.

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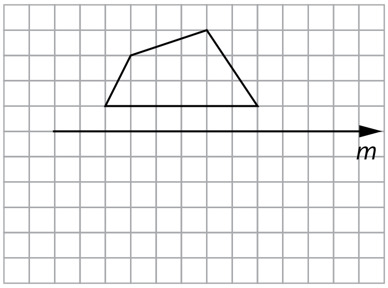
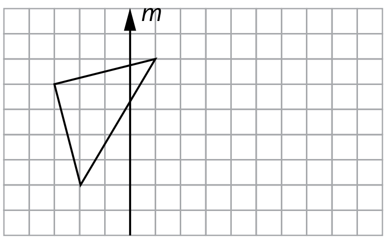
Exercise A: Reflection

1. Reflect the following figures in the mirror lines, *m*, shown.

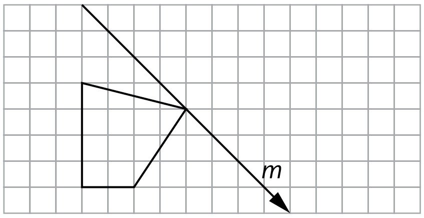
a.



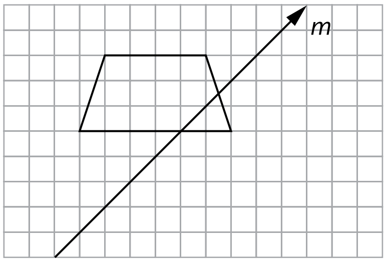
b.



c.

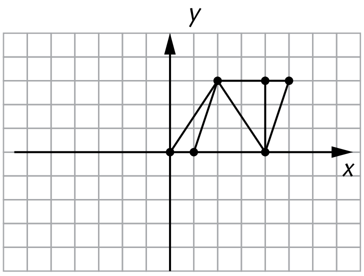


d.

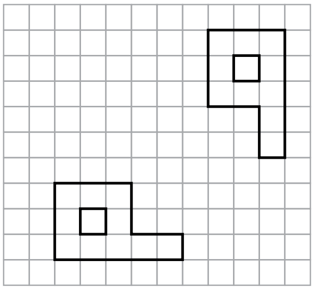


2. a. Reflect the figure below in the *x*-axis.

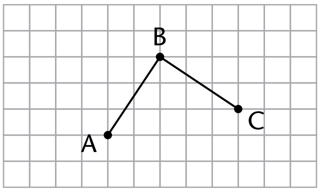
b. Reflect both the figure and its image in the *y*-axis.



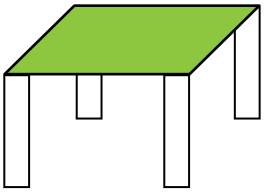
3. A figure and its image under reflection are shown. Construct the mirror line for this reflection.



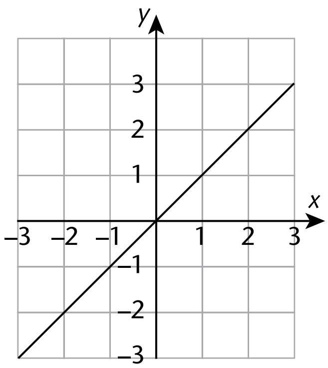
4. AB → CB under a reflection. Locate the mirror line for the reflection.



5. Reflect the table in the shaded face.



6. The straight line $y = x$ is drawn on the grid below.



a. The points with the following coordinates A(2,1), B(1,–2), C(–2,1), D(–2,–3) are reflected in the mirror line $y = x$, so that A→A’, B→B’, etc.

Write down the coordinates of their images.

i. A' ____

ii. B' ____

iii. C' ____

iv. D' ____

b. i. What do you notice about how the coordinates of the images relate to the original coordinates after reflection in the line $y = x$?

____

____

ii. Write down the coordinates of the image of the point E(10,–8) after reflection in the line $y = x$.

____

c. Draw the line $y = -x$ on the grid (this line passes through (1,–1), (–1,1) etc.

The points A(2,1), B(1,–2), C(–2,1), D(–2,–3) are reflected in the mirror line $y = -x$, so that A→A", etc.

Write down the coordinates of their images.

i. A'' ____

ii. B'' ____

iii. C'' ____

iv. D'' ____

d. i. What do you notice about how the coordinates of the images relate to the original coordinates after reflection in the line $y = -x$?

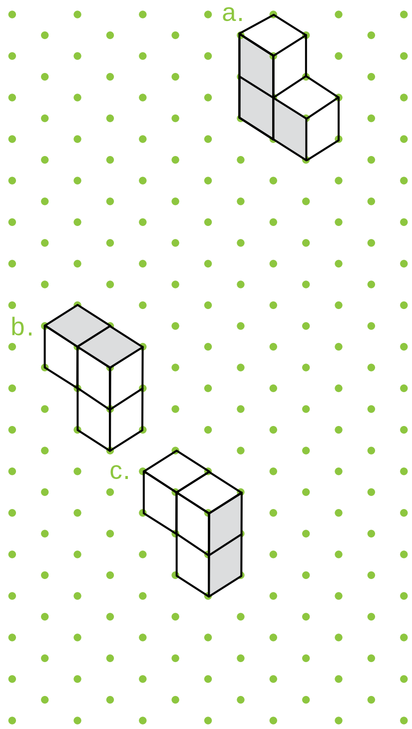
____

____

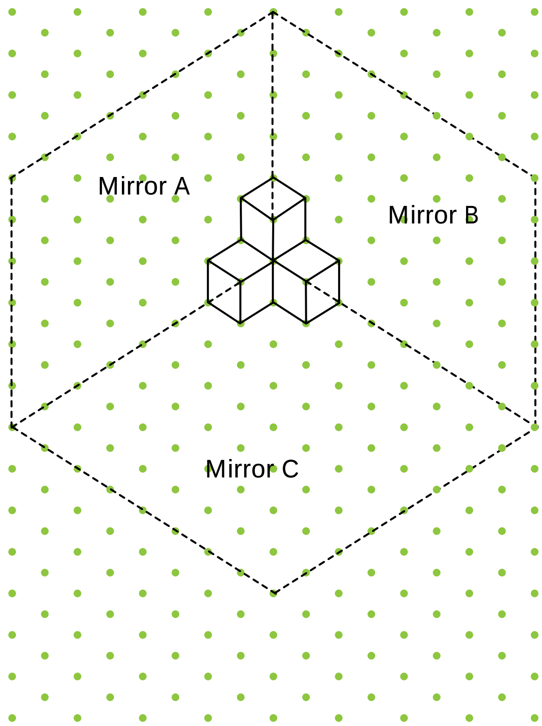
ii. Write down the coordinates of the image of the point E(10,–8) after reflection in the line y = –x.

____

7. On the isometric grid, draw the image of each shape after a reflection in the shaded face.



8. Reflect the solid made from four blocks in the mirror planes A, B, C (some blocks are not visible in the diagram).



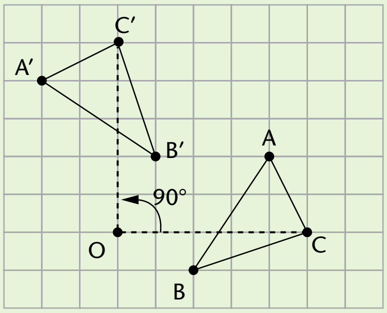
Rotation

A **rotation** turns a figure about a point called the **centre of rotation**, through a given **angle of rotation**.

- A positive angle indicates an *anticlockwise* rotation.
- A **full turn** is 360°, a **half turn** is 180° and a **quarter turn** is 90°.

Example

In the example below, triangle ABC is rotated to triangle A'B'C' through 90° about O.



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Note: Angles AOA', BOB' and COC' are all equal to 90°.

The **invariant** (unchanged) properties of a rotation are:

- size, shape and area – see example above
- centre of rotation is the only invariant point
- sense.

Finding the centre and angle of rotation

To find the centre of rotation:

1. join any point A to its image A' and construct the perpendicular bisector of AA' (this is a line which runs at right angles to AA' and cuts AA' at its midpoint)
2. repeat with another point B and its image B'.

The intersection of the two perpendicular bisectors is the centre, O, of the rotation.

To find the angle of rotation: measure the angle AOA' (or BOB', etc.).

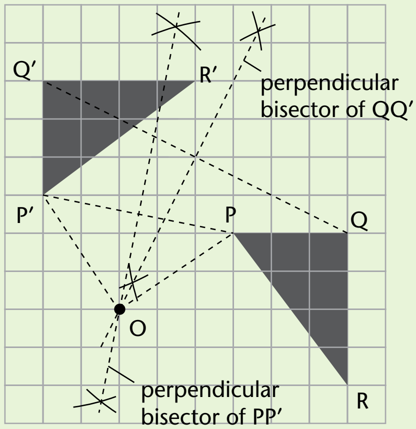
Note: An anticlockwise rotation is positive.

Example

Triangle PQR is rotated to triangle P'Q'R'.

To find the centre of the rotation, the perpendicular bisectors of PP' and QQ' are constructed. The centre is their point of intersection, marked O.

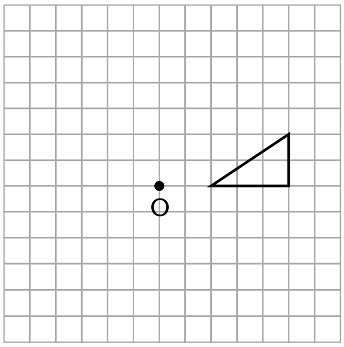
The angle of rotation is $\angle POP' = 90°$.



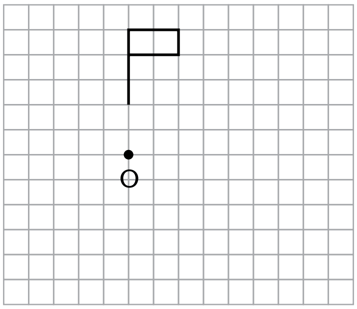
Exercise B: Rotation

1. Rotate the following figures, centre O, through the angles given.

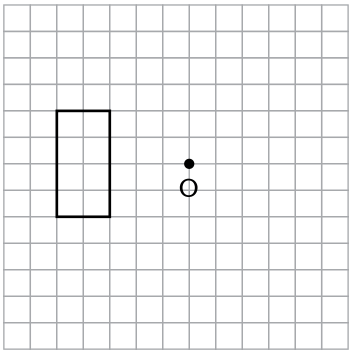
   a. 90°



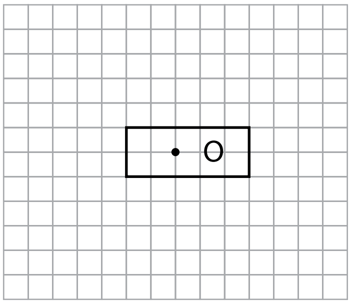
   b. 180°



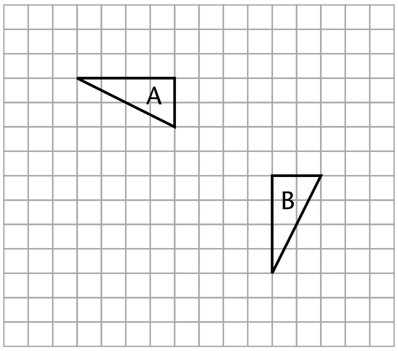
c. 270°



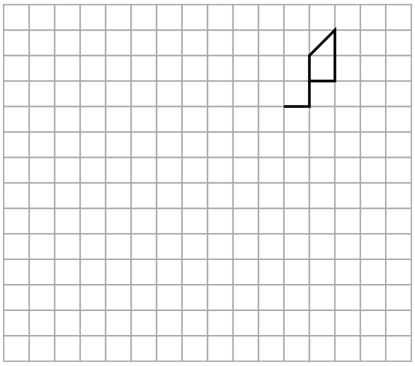
d. –90°



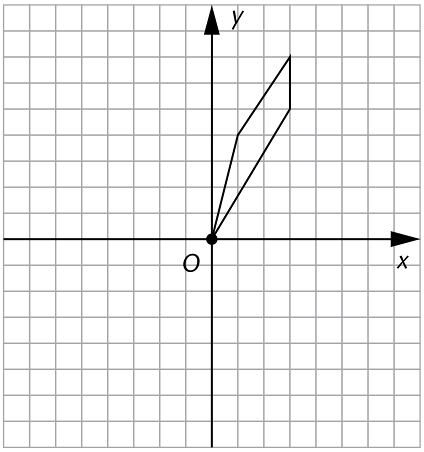
2. Figure A and its image, B, under rotation are shown. Find the centre and angle of rotation.



3. Add more lines to the figure below to create a shape of your own design. Rotate the shape through various angles, about a centre point of your choice, to create a design. Mark your chosen centre point clearly on the design.

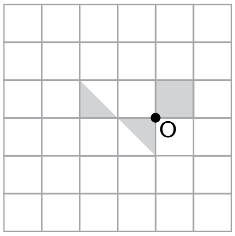


4. Rotate the given figure 90° centre O, then rotate the figure and its image, centre O through 180°.

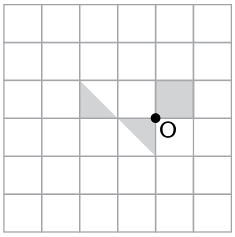


5. A shaded shape is shown on the grids below. Rotate the shaded shape about the point O through:

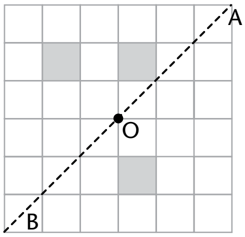
a. 90° clockwise



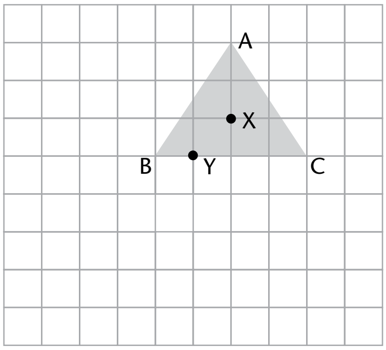
b. 45° anticlockwise



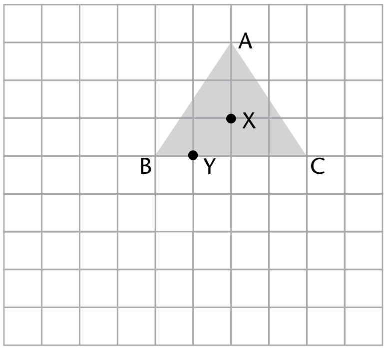
6. Draw the image of the shaded figure after a reflection in the mirror line AB followed by a rotation about the point O through an angle of −90°.



7. a. Rotate triangle ABC through 90° anticlockwise about the point X.

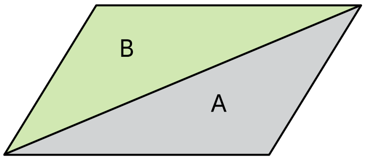


b. Rotate triangle ABC clockwise through 90° about the point Y.

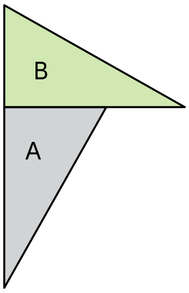


8. Describe fully the rotation that maps A onto B. For each figure: mark the centre of rotation, O, on each figure and give the angle.

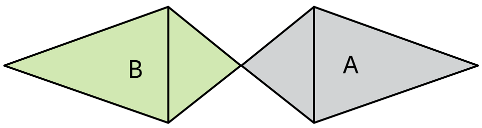
a.



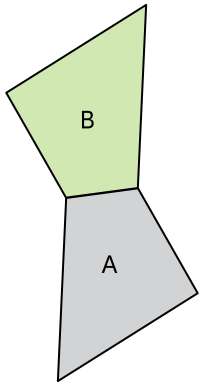
b.



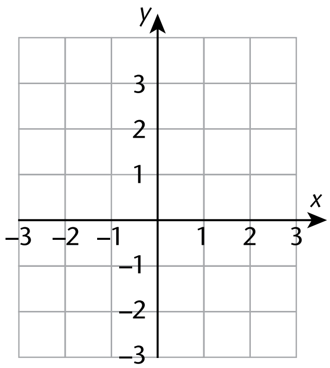
c.



d.



9. Find the coordinates of the images of the following points after the rotations described (you may wish to use the grid to plot points).



a. (2,1) rotate 90° anticlockwise about the point (0,0)

b. (–2,3) 180° about the point (0,0)

c. (–2,–1) 90° clockwise about the point (1,0)

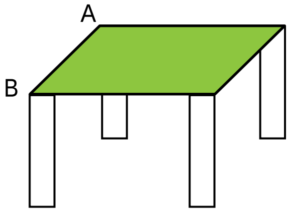
d. (3,–2) 270° anticlockwise about the point (2,–1)

e. (12,–8) 180° about the point (1,1)

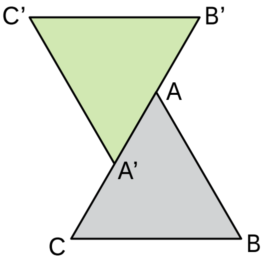
f. (a,b) 180° about the point (0,0)

g. (a,b) 90° about the point (0,0)

10. Rotate the table 90° anticlockwise about the axis AB.

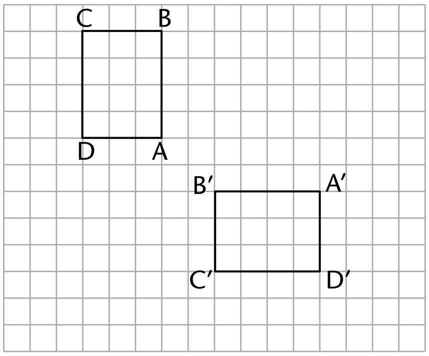


11. The figure shows triangle ABC which is rotated to its image triangle A'B'C'.

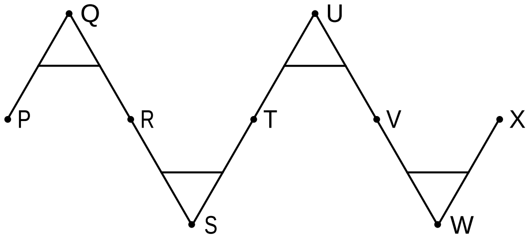


Find the angle of rotation and the point which is the centre of rotation.

12. Find the centre and angle of rotation that maps ABCD onto A′B′C′D′.

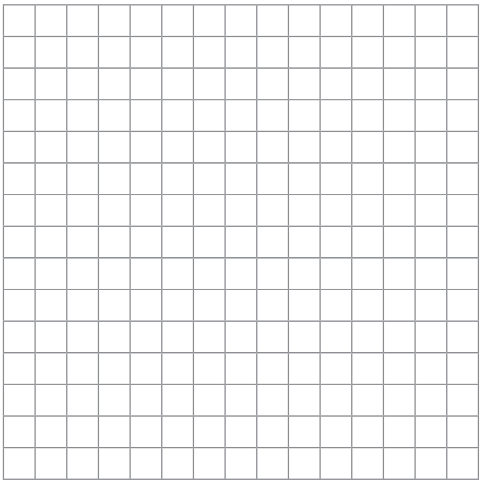


13.



Describe four rotations in this pattern of As. Points P to X are marked on the figure for reference.

14. a. Use your initials to create a design using rotation on the grid below.



b. Carefully describe the rotations in your design.

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Translation

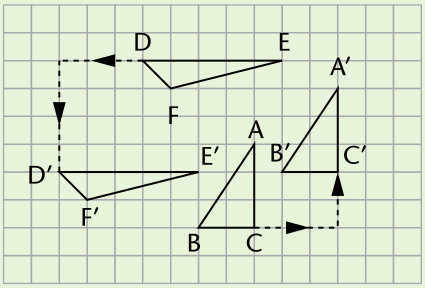
A **translation** slides a figure so that each point in the figure moves the same distance in the same direction. Translations can be described by:

- compass directions, or bearings
- horizontal and vertical components, often expressed as **vectors**.

For translation using vectors on graphs:

- positive **horizontal components** move right, negative move left
- positive **vertical components** move up, negative move down.

Example



1. Triangle ABC → A'B'C' under the translation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
2. Triangle DEF → D'E'F' under the translation $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$.

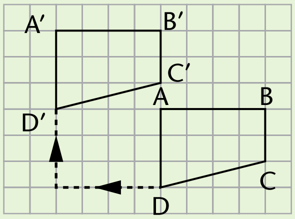
The **size**, **shape**, **area** and **sense** of a figure are invariant under translation.

Finding the translation vector

You may need to find the translation vector when a figure and its image under a translation are given. As all points move the same distance in the same direction, you need only identify the components of movement for any point P on the figure as it moves to its image point P'.

Example

Trapezium ABCD → A'B'C'D' under a translation.



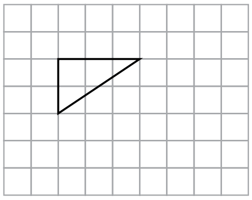
D moves 4 left and 3 up to reach D' (see arrows).

The translation vector is therefore $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

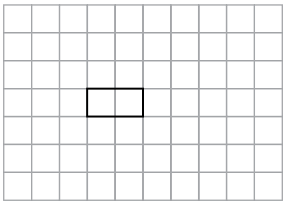
Exercise C: Translation

1. Translate each figure in the direction indicated by the given vector.

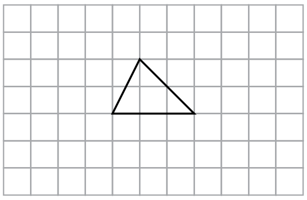
a. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$



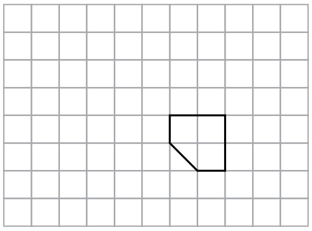
b. $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$



c. $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$

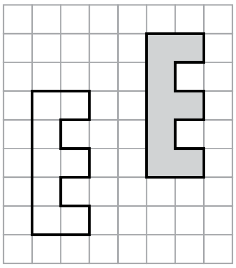


d. $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

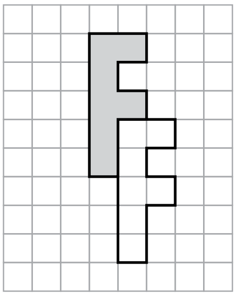


2. State the translation vector for each of the following translations. In each case the image is the figure shaded.

a.



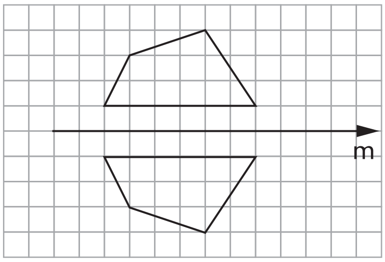
b.



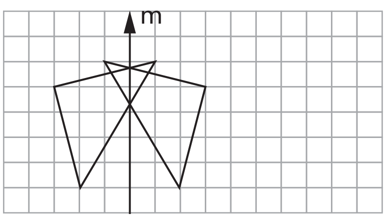
Answers

Exercise A: Reflection (page 2)

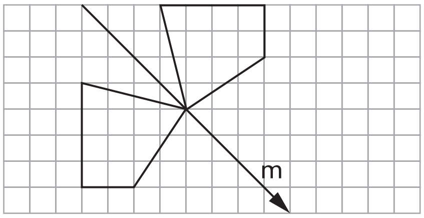
1. a.



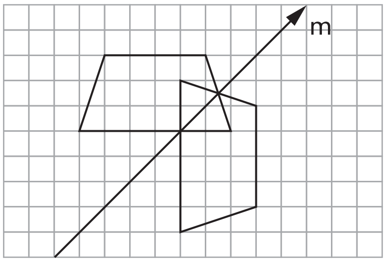
b.



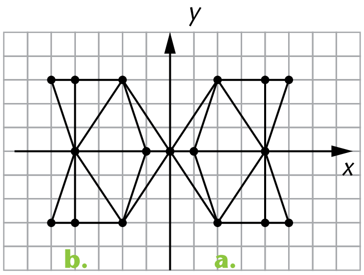
c.



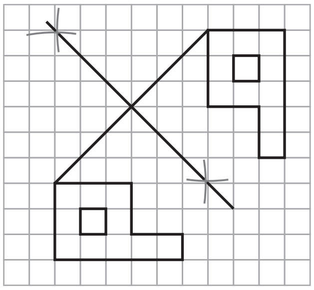
d.



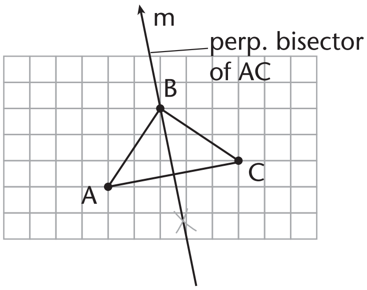
2.



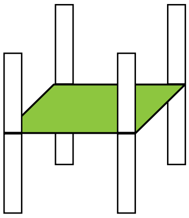
3.



4.



5.



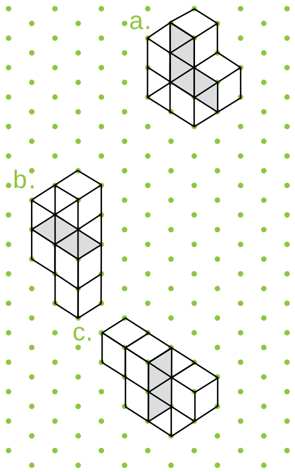
6. a. i. A'(1,2) ii. B'(–2,1)
iii. C'(1,–2) iv. D'(–3,–2)

b. i. Coordinates are reversed.
ii. (–8,10)

c. i. A"(–1,–2) ii. B"(2,–1)
iii. C"(–1,2) iv. D"(3,2)

d. i. Coordinates swap places and signs are reversed.
ii. (8,–10)

7.



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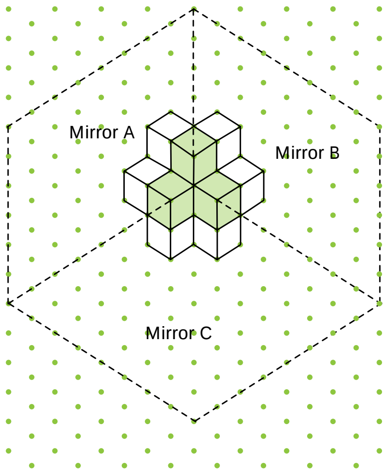
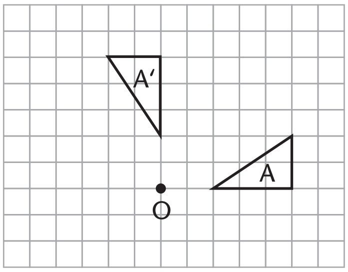
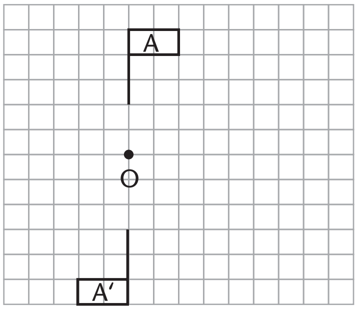


Exercise B: Rotation (page 4)

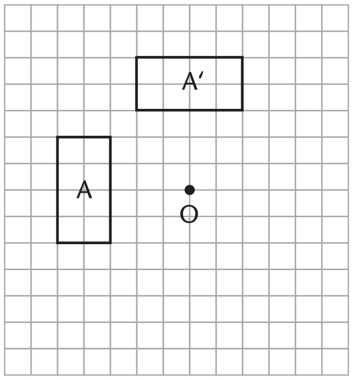
1. a.



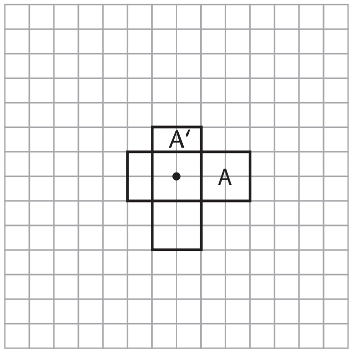
b.



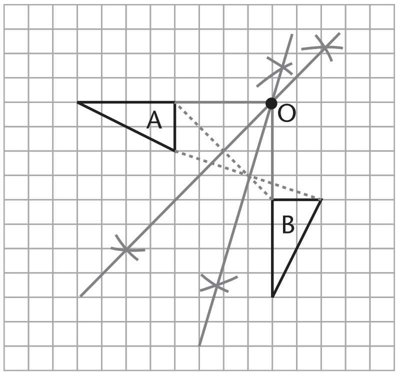
c.



d.

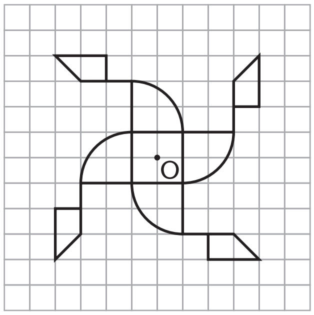


2.



A → B
Rotation centre O, angle 90°

3. Answers will vary. One possible solution is shown with centre O.



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