6. Mrs Paris buys the girls some beads to make necklaces. She gives $\frac{2}{5}$ of the beads to Georgina and $\frac{1}{2}$ of them to Daphne.

a. What fraction of the beads has Mrs Paris given away?

b. Which girl got more beads?

c. If there were 500 beads to start with, how many beads are left after Mrs Paris gave the beads to the girls?

7. Mrs Paris is planning to buy new gloves for the whole family. At the glove counter all gloves usually cost $20 per pair.

But there is a special offer available:

a. How much would Mrs Paris usually pay for four pairs of gloves?

Mrs Paris buys four pairs of the gloves on special.

b. How much does she pay?

c. What percentage discount does Mrs Paris receive?

Mathematics Unit Standard 26626

Internally assessed
3 credits

Interpret statistical information for a purpose

Chapter 3: Measures of centre and spread

Calculators, computers or other appropriate technology are permitted for assessment in this unit standard. The examples in the text explain fully how to solve problems, but the student is encouraged to use appropriate approved technology at all times when solving problems.

Introduction

When we want to find out information about a **population**, it usually takes too long and costs too much to carry out a **census** (involving every member of the population). Instead, we investigate a **sample** (part of the population) in order to find out more about the population.

Statistics is the study of **data** (information) that is collected, analysed and interpreted. Everywhere you look, you will find statistical information and graphs – in newspapers, on television and online.

You need to be able to understand what the data is telling you, so that you can form sensible conclusions for yourself.

In this unit standard you will use the language of statistics to examine **investigations** and discuss their purpose. In each case the **data set** will be provided, and you will be asked to make comments on:

- **measures of centre**, such as the **mean**, **median** or **mode**
- **measures of spread**, such as the **range** and **interquartile range**

When working with these measures, you may need to check whether or not the values are reasonable or sensible; this process is called **estimation**.

Data values are often organised into tables or displayed in **graphs** and plots. You will need to know how to **analyse** (make sense of) these displays.

This may include:

- picking out any **trends** and patterns
- describing any interesting features of the data, e.g. **outliers** (extreme values) and discussing their effects on measures of centre or spread

A well-drawn graph can summarise information and communicate trends and features 'at a glance'.

Polls

One example of statistics in everyday life occurs when **polls** are carried out before a general election.

In a poll, the opinions of a sample of voters are used to try to predict who will win the election on voting day.

Statisticians believe that the result from a sample of voters will be true for the whole population of voters, providing the sample is **representative** (a typical group from the whole population).

Example

The *One News Colmar Brunton Poll* is taken just before a general election. Many people (say 1 000) of voting age are surveyed to find out which party they will vote for. The answers to this question are recorded and, based on the results, a prediction is worked out.

Perhaps the hardest part is randomly selecting 1 000 people. Taking a poll is an expensive process, so careful consideration is given as to which sample of people will be interviewed.

The people in the sample may be **randomly selected** from the phone book. (This means that each person in the phone book is just as likely to be selected as any other person in the phone book.) The 1 000 people chosen in this way is called a **random sample**.

Then the members of the sample have to be contacted, perhaps by telephone (this is often done automatically by computer).

Some people might refuse to answer so replacements are needed to make up numbers. Some respondents may answer 'don't know' (the 'don't knows' are included in the responses).

When the responses come in they have to be analysed to see which party is predicted to win the General Election.

In this unit of work you will be expected to see if you can draw any **conclusions** about the values in a data set, or describe the ways in which data values differ from each other.

You will need to write down results you find, in order to communicate them to others.

Measures of centre

Measures of centre are 'average' values which are used to represent a data set. These include the mean, the median and the mode.

The mean

The **mean** of a set of data values is found by adding up the values, and dividing the answer by the number of values.

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

The mean may be used to compare two groups.

Example

Suppose there are two classes of boys, Years 11A and 11B, at a West Coast highschool. Their teachers decide to find out which class, on average, has heavier male students.

The 15 boys in 11A and the 12 boys in 11B were weighed in kilograms, to the nearest whole kilogram. The weights, in kilograms, are listed below:

11A: 72 79 67 69 82 78 83 104 70
65 76 81 67 74 75

Total weight for 11A = 1 142 kg

11B: 76 82 65 54 65 79 75 68 74
65 86 78

Total weight for 11B = 867 kg

Obviously you would expect the total weight of the boys in 11A to be more than the total weight of the boys in 11B, because there are 3 extra boys in 11A.

One solution might be to take three boys out of 11A and then the total weights of the 12 boys in each class. But which three boys should be taken out?

The best way to make the comparison fair is to find the average weight by dividing each total by the number of students in each class.

11A: Average weight is 1 142 ÷ 15 = 76 kg, to the nearest whole number.

11B: Average weight is 867 ÷ 12 = 72 kg, to the nearest whole number.

Conclusion: the boys in 11A are heavier than the boys in 11B, on average.

The mean value of a data set can be thought of as a representative value for the data set. In other words, the mean will 'stand in place of' the whole set of data.

In the example above:

- the mean value of the weights of the boys in 11A is 76 kg. The mean of 76 kg is used here to represent (stand in place of) the weights of all the boys in 11A.
- the mean value of the weights of the boys in 11B is 72 kg. The mean of 72 kg is used here to represent (stand in place of) the weights of all the boys in 11B.

Note: The mean value is usually somewhere near the middle of the data set. This is helpful if you are estimating the mean.

Outliers

A data set may contain an **extreme value** (an unusually large or unusually small value).

These unusually large or small values are easily picked out because they lie outside the usual range of values – it is for this reason that they are often called **outliers**.

Outliers can have a big effect on the value of the mean, by making it larger or smaller than it would be otherwise.

For example, in the class 11A in the example above, there is a boy weighing 104 kg, which is an unusually heavy weight. If this value was removed then the total weight of the remaining 14 boys would be 1 142 – 104 = 1 038 kg, giving a mean weight of 1 038 ÷ 14 = 74 kg (to the nearest kilogram). This is 2 kg lower than the overall mean of 76 kg (when the outlier is included).

Note: The people who gather data should always check the values of outliers, because they may not be correct values but may be due to errors in measurement or recording.

Number crunching

In a statistical investigation, the larger the data set, the more reliable the sample is for making predictions or drawing conclusions. So statistical investigations usually involve a large number of data values.

For example, in the *Colmar Brunton* poll, taken just before an election, there were 1 000 people contacted over the phone. That is a large quantity of data to process!

In the past, this may have meant a lot of 'number crunching' needed to be done. But these days, calculators and computers make short work of data analysis.

Estimation

It is always sensible to check calculations (even when they have been done on a calculator) to make sure your answers are reasonable and make sense.

For example, when the mean of a data set is calculated, a simple check is to see if its value lies roughly near the middle of the data set (if it doesn't, is it because of outliers?).

In the following exercise, questions are in everyday contexts, but the data sets are reasonably small to avoid having to do a lot of 'number crunching'.

Ans. p. 141

Exercise A: Mean of a data set

You may use a calculator or a computer, but explain what you did.

1. Suppose we want to compare the goal-scoring abilities of two football teams, *United* and *Rovers*. The goals each team has scored in their entire last 17 games are given in the table.

Goal scoring	
United	*Rovers*
2	3
3	3
9	4
2	1
2	4
9	3
3	0
1	5
4	2
4	2
3	4
0	3
1	4
1	4
3	2
10	2
0	4
Total = 57	Total = 50

a. Calculate the mean number of goals scored by each team. Show your working, and give answers to 1 decimal place.

i. *United* mean =

ii. *Rovers* mean =

b. Write down at least two sentences comparing the goal scoring patterns of the two teams. To help you, the data is ranked in order of size in the table below.

Goal scoring	
United	*Rovers*
0	0
0	1
1	2
1	2
1	2
2	2
2	3
2	3
3	3
3	3
3	4
3	4
4	4
4	4
9	4
9	4
10	5

c. The two teams are going to play in the same competition. When they play each other, who do you think would win? Give a reason for your answer.

MATHEMATICS UNIT STANDARD 26627

Internally assessed
3 credits

Use measurement to solve problems

Chapter 5: Use standard units of measurement

US 26627

Calculators, computers or other appropriate technology are permitted for assessment in this unit standard. The examples in the text explain fully how to solve problems, but the student is encouraged to use appropriate approved technology at all times when solving problems.

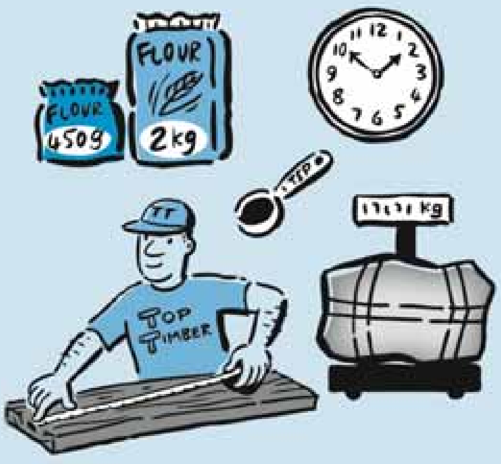
Introduction

Students credited with this unit standard are able to use measurement to solve problems. Students will be able to:

- carry out a variety of measurements, such as measuring length, capacity or time, using appropriate units
- interpret location and direction, such as in orienteering
- convert between units in the metric system, e.g. 4 cm = 40 mm; or $\frac{1}{2}$ hour = 30 minutes.

Having made appropriate measurements, the student will be able to:

- use the measurements in calculations, such as finding area or volume, using appropriate units
- estimate an answer in order to check whether or not a solution they have calculated is sensible.



Measuring length

The **metric units** for length are millimetre (mm), centimetre (cm), metre (m), and kilometre (km).

A **ruler** is used for measuring lengths up to 30 cm. When measuring with a ruler, record lengths in centimetres, to one decimal place (1 dp). Your measurement will then be correct to the nearest millimetre.

Example

Each centimetre on the ruler is divided into ten millimetres, so each division is 0.1 cm or 1 mm.

Point A:
The arrow is pointing to the 2nd division past 2 cm.

A is pointing to 2.2 cm or 22 mm.

Point B:
The arrow is pointing to the 7th division past 0 cm.

B is pointing to 0.7 cm or 7 mm.

Point C:
The arrow is pointing between the 3rd and 4th divisions past 5 cm, so the closer mark is used, which is 4 mm.

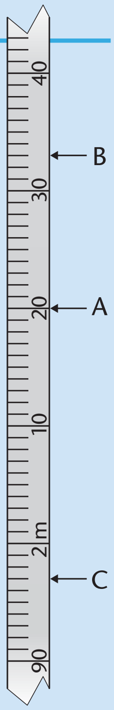
C is pointing to 5.4 cm or 54 mm.

A **tape measure** is used for measuring longer distances. A tape measure is marked in metres and centimetres. You should record your measurements, in metres, to 2 decimal places (2 dp). This means that your answer will then be accurate to the nearest centimetre, which is usually accurate enough for longer distances.

When measuring with a tape, someone will have to hold firmly the other end of the tape, so that you can pull the tape reasonably tightly to get rid of kinks, curves or bumps.

US 26627

Example



The figure represents part of a typical measuring tape which has been reduced in size.

The numbers 10, 20, 30, … show the number of centimetres since the last **metre** marking (only the 2 m mark, just above point C, is showing on this part of the tape).

Each small division on the tape shows 1 cm which is 0.01 m.

Point A: Look below point A until you find the number of whole metres, which is 2 m. The number of extra centimetres past 2 m is 20 cm.

The reading for point A is 2.20 m (2 dp).

Point B: The number of whole metres is 2 m. The number of extra centimetres is 33 cm.

The reading for point B is 2.33 m (2 dp).

Point C: Below point C you would find the number of whole metres is 1 m. The number of centimetres is 97 cm.

The reading for point C is 1.97 m (2 dp).

When measuring the distance between two markings on a sports pitch, measure between the centres of each marking.

Ans. p. 149

Exercise A: Measuring lengths around you

Use your measuring skills, demonstrated in the examples above, to measure the lengths given below. Measure using a ruler or a tape measure, whichever is more suitable for the task.

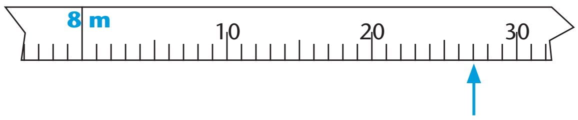
If you are using a ruler, measure in centimetres to 1 dp, which is to the nearest millimetre. If you are using a tape measure, measure in metres to 2 dp, which is to the nearest centimetre.

1. Record each of the following measurements to the nearest mark on the scales.

a. The figure shows Tamati and a height scale.

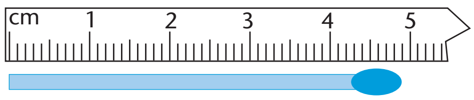
How tall is Tamati in metres?

b. The arrow shows the mark for the length of a class room.



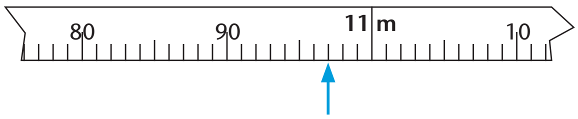
What is the length of the room in metres?

c. A ruler is used to measure the length of a matchstick.



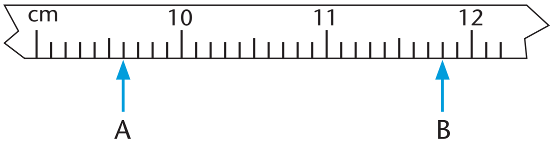
What is the length of the matchstick in millimetres?

d. The width of a tennis court at Plato High School is marked on the measuring tape by an arrow.



What is the width of the court in metres?

2. **a.** There are two measurements marked on part of the enlarged ruler shown below.



What is the distance between A and B:

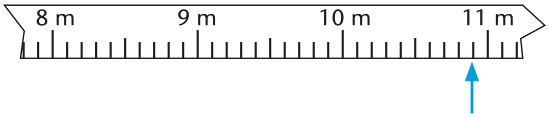
i. in cm?

____

ii. in mm?

____

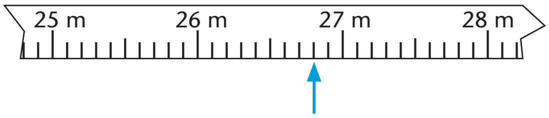
b. The arrow on the following tape shows how far Sara walked.



i. What is the value of each small division on the tape? ______________

ii. How far had Sarah walked? __________

Yvette used an arrow on the following tape to show how far she walked.



iii. What is the **total** distance walked by the two girls?

____

iv. How much further has Yvette walked than Sara?

____

The following exercises are suggested as practice in measuring the lengths of various objects. Work with a partner if possible, so you can check each other's results.

3. **a.** Measure the length of your classroom.

____

b. Estimate the width of your classroom. Check how close you were.

____

4. Measure the length of your desk.

____

5. Measure the length of your pen.

____

6. Measure the length of the side of a small square in your maths book.

____

7. Measure the width of this book.

____

8. Measure the thickness of your ruler.

____

9. **a.** Measure the length of a swimming pool.

____

b. Estimate the width of the swimming pool. Check how close you were.

____

10. Measure the height of the filing cabinet in your classroom.

____

11. **a.** Measure the length of a cricket pitch, between the wickets.

____

b. Estimate the width of the cricket pitch. Check how close you were.

____

12. Measure the diameter of a 20c coin.

____

13. Measure the length of a netball court.

____

Measuring mass

Mass is measured using the following units: **grams** (g) and **kilograms** (kg).

Grams are used to measure the mass of small objects. For example, in a café the mass of one sachet of butter sold with your muffin may be 9 grams (9 g).

Kilograms are used to measure the mass of a larger object. For example, a large bag of potatoes may have a mass of 5 kilograms (5 kg), or a car may weigh about 1 000 kg.

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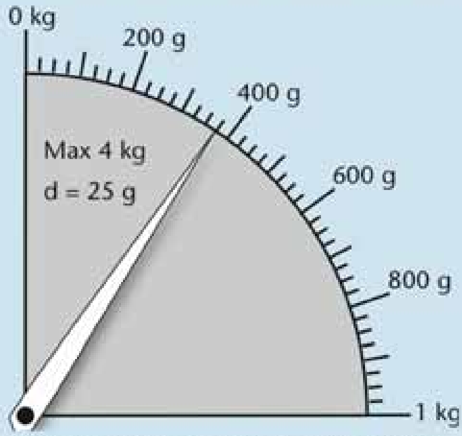
Note: For the purposes of this Unit Standard, mass and weight are considered to be the same quantity.

To find the mass of an object, you will need some weighing **scales**. There are various types but a good set of kitchen scales is fine. Make sure they are metric and weigh in kilograms and grams, not pounds (lb) and ounces (oz). A set of scales that weigh up to about 4 or 5 kg is suitable.

Part of the dial of a set of scales is drawn below. Each kilogram is subdivided into smaller parts whose unit is grams. The size of each small **graduation** is often defined on the scale (d = 25 g).



If this information is not stated, you will need to work out the value of each small graduation. This is worked out in the following way:

Each 200 g is divided up into 8 markings, so:

1 marking is 25 g [200 ÷ 8 = 25]

Always read the mass of the object to the nearest mark on the scale.

Example

What is the reading on the scale in the figure above? Give your answer in grams and kilograms.

Solution

The reading is 200 g + 7 divisions (to the nearest mark)

Reading is 200 + 7 × 25 [each division is 25 g]

The reading in grams is 375 g.

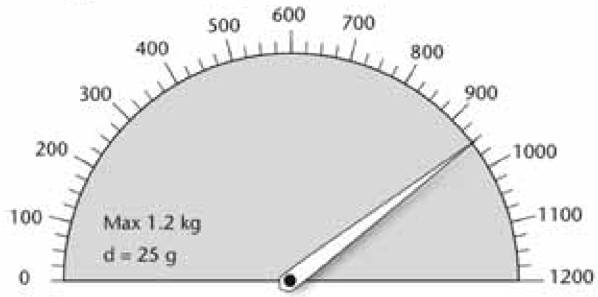
The reading in kilograms is found by dividing the grams reading by 1 000.

The reading is 0.375 kg. [375 ÷ 1 000 = 0.375]

Exercise B: Measuring mass around you

Ans. p. 149

1. The dial for a set of scales is shown. The arrow points to the weight of the object being weighed.

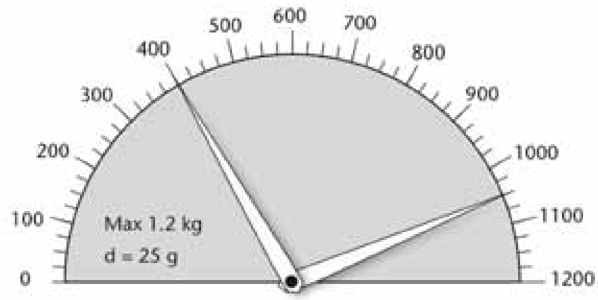


Give the weight to the nearest mark on the scale.

a. in kilograms.

b. in grams.

2. The weights of two different objects are shown on the dial below.



Find:

a. The sum (total) of the two weights, in kilograms.

b. The difference between the two weights, in grams.

c. What weight would lie exactly half way between these two weights?

d. Mark this weight on the dial using an arrow.

ANSWERS

Chapter 1: Whole numbers and integers

Exercise A: Counting numbers and basic arithmetic (page 3)

1. a. 3 287 – 402 = 2 885
   b. 469 × 87 = 40 803
   c. 6 090 ÷ 15 = 406
   d. 1 000 000 + 2 000 000 = 3 000 000
   e. 1 000 + 1 000 000 = 1 001 000
2. a. three hundred and sixty-five minus two hundred and eight equals one hundred and fifty-seven
   b. ninety thousand, three hundred and sixty plus eleven thousand equals one hundred and one thousand, three hundred and sixty
3. a. 97 = 9 × 10 + 7 × 1
   b. 208 = 2 × 100 + 0 × 10 + 1 × 1
   c. 173 425 = 1 × 100 000 + 7 × 10 000 + 3 × 1 000 + 4 × 100 + 2 × 10 + 5 × 1
   (alternatively use 10^5 for 100 000, 10^4 for 10 000, 10^3 for 1 000, 10^2 for 100)
4. b. 434 c. 5 619
   d. 3 084 e. 9 040
5. a. 2 693 b. 87 450
   c. 9 030
6. a. tens digit (0) increases by 6, number is now 9 563
   b. hundreds digit (5) increases by 4, number is now 9 930
   c. thousands digit (9) decreases by 7, number is now 2 503
   d. hundreds digit increased by 1 and tens digit increased by 2 and ones digit increased by 5, so 125 was added on.

Exercise B: Larger counting numbers and basic arithmetic (page 5)

1. a. 9^3 = 729 (check 10^3 = 1 000 which is reasonably close)
   b. 11^5 = 161 051 (check 10^5 = 100 000 which is reasonably close)
   c. 99^4 = 96 059 601 (100^4 = 100 000 000 which is reasonably close)
2. a. 42 000 b. 60 000
   c. 4 500 000 d. 3
   e. 500 f. 450
3. a. 1 646 616 = one million, six hundred and forty-six thousand, six hundred and sixteen
   b. 1976
   c. 1966
   d. 1996
4. a. 58 000 000 km (or 58 million kilometres)
   b. Neptune
   c. 550 000 000 km (or 550 million kilometres)
   d. 4
   e. In increasing distance from the sun (the lower down the table, the further the planet is from the sun).
   f. Neptune is 4 496 000 000 km from the sun, which is nearly 5 000 000 000 km (5 billion kilometres), so Milly is approximately correct (to the nearest billion).

Level 1 Numeracy Index