Mathematics and Statistics 1.5

Internally assessed
3 credits

AS 91030

Apply measurement in solving problems

Units of measurement

When measuring quantities such as length, mass, etc., appropriate measurement units must be used. In the **metric system,** each quantity measured has a **base unit**.

Quantity		Base unit	
Length	(distance)	metre	(m)
Mass	(weight)	gram	(g)
Capacity	(liquid volume)	litre	(L)

Prefixes are used to derive larger or smaller units. Some commonly used prefixes are:

- **kilo** (k) which means 1 000
- **centi** (c) which means $\frac{1}{100}$
- **milli** (m) which means $\frac{1}{1\,000}$

Note: 1 tonne = 1 000 kg (an exception to the use of prefixes). The abbreviation for tonne is t.

Example

1. The length of a book would be measured in centimetres (cm).
2. The mass of a sack of potatoes would be measured in kilograms (kg).
3. The volume of medicine to be taken daily would be measured in millilitres (mL).
4. The distance between Auckland and Wellington would be measured in kilometres (km).
5. The mass of a tablet for pain relief would be measure in milligrams (mg).
6. The volume of water in a swimming pool would be measured in kilolitres (kL).

Exercise A: Units of measurement

Ans. p. 45

Give an appropriate unit for each of the following measurements.

1. The length of a netball court.
2. The mass of a schoolbag.
3. The volume of fruit juice in a can.
4. The distance from New Zealand to Australia.
5. The time taken to sprint 100 m.
6. The weight of a teaspoon of sugar.
7. The mass of a truckload of apples.
8. The length of a season.
9. The width of a coin.
10. The quantity of ice-cream in a scoop.
11. The mass of a pill.
12. The depth of a goldfish bowl.
13. The amount of water in a swimming pool.
14. The time taken for a kettle to boil.

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Ans. p. 45

Using metric units

To **convert** from one unit of measurement to another, multiply or divide by the appropriate power of 10 (i.e. by 10, 100, 1 000, etc.).

- When converting from a larger unit to a smaller unit, *multiply* (to get more of the smaller unit).
- When converting from a smaller unit to a larger unit, *divide* (to get fewer of the larger unit).

Always think about how sensible the answer seems!

Example

1. 2.55 kg = 2.55 × 1 000 = 2 550 g [larger unit to smaller, so multiply (1 kg = 1 000 g)]

2. 275 cm = 275 ÷ 100 = 2.75 m [smaller unit to larger, so divide (100 cm = 1 m)]

3. 7.6 cm = 7.6 × 10 = 76 mm [larger unit to smaller, so multiply (1 cm = 10 mm)]

You also need to be familiar with quantities such as:

- **temperature** – measured in degrees Celsius (°C)
- **money** – measured in dollars ($) and cents (c)
- **time** – non-metric units include seconds (s) minutes (min), hours (h), days, years, etc.

Problems involving time include the use of 24-hour times

- the first two digits give the hours after midnight
- the final two digits give the minutes.

Example

1. A 12-hour time of 6:45 a.m. converts to 0645 in 24-hour time. [4 digits needed]

2. A 24-hour time of 2015 converts to 8:15 p.m. in 12-hour time. [20 hours after midnight is 20 – 12 = 8 p.m.]

3. The time that elapses between 2:47 a.m. and 11:04 p.m. is:

13 minutes plus 8 hours and 4 minutes plus 12 hours

This gives a total of 20 hours and 17 minutes

[2:47 a.m. + 13 min = 3:00 a.m.
3:00 a.m. + 8 h 4 min = 11:04 a.m.
11:04 a.m. + 12 hours = 11:04 p.m.]

Exercise B: Using metric units

1. Convert each of the following measurements to the unit given in brackets.

   a. 3 575 cm (m)

   b. 6.5 cm (mm)

   c. 8.5 kg (g)

   d. 57 000 mg (g)

   e. $3\frac{3}{4}$ hours (min)

   f. 38.75 t (kg)

   g. 445 mm (cm)

   h. 98 000 c ($)

   i. 68.9 m (cm)

   j. 13 decades (years)

   k. 99 650 kL (L)

   l. 87 500 kg (t)

   m. 0.45 kg (g)

   n. 35 000 mL (L)

2. a. What time is it 35 minutes after 23:45?

   b. Half-way through a 130-minute movie the time is 3 p.m. At what time did the movie start?

c. A nurse has a sleep before starting her night shift, which begins at 10 p.m. Once she wakes from her sleep it takes her $1\frac{1}{2}$ hours to get ready and drive to the hospital. The nurse has a 24-hour time alarm clock. For what time should she set the alarm so that she arrives at the hospital 15 minutes before her shift begins?

d. A lighthouse flashes 7 times every 20 seconds. How long does it take to flash one million times? Give your answer in days, hours, minutes and seconds.

e. A passenger boards a plane in New Zealand at 2215 on Friday and is due to arrive in London 26 hours later. If times in England are 11 hours behind times in New Zealand, what is the day and time in London when he arrives there?

3. a. A tap drips 8 times per minute. Each drip is 0.25 mL. How many litres of water are lost in a week?

b. 1 litre of water weighs 1 kg. How many tonnes of water are in a pool whose capacity is 47.5 kL?

c. Flour is sold in 1.25 kg bags. A cake recipe requires 225 g of flour. How many cakes can be made with a single bag of flour?

d. The pages of a book are numbered 1–350. If the pages are a total of 2.1 cm thick, what is the thickness of a single page?

e. 450 g of coffee beans fill a 1.25 L container. What weight of coffee beans fit into a 750 mL container?

4. Bella started playing a game on the computer at 3.51 p.m. and she played for an hour and a quarter. She then had a break before playing for another 65 minutes until dinner time at 7 p.m. How long was Bella's break between the two playing sessions?

5. A company charged a consumer $152 for 3.8 hours of cell phone calls. Find the cost of a call lasting 3 minutes.

6. An internet 'world time zones' website reports that when it is Tuesday 7:08 a.m. in Adelaide it is Monday 3:39 p.m. in Lima.

a. What time is it in Adelaide when it is Saturday 7:45 a.m. in Lima?

b. What time is it in Lima when it is Thursday 3:05 p.m. in Adelaide?

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Derived units

When two quantities with different units are compared using division, a **derived unit** is formed.

For example, when distance (in kilometres) is divided by time (in hours) to get speed, the derived unit will be kilometres per hour (km/h or km h^{-1}).

Example

1. A runner covers 240 metres in 30 seconds.

Average speed $= \frac{240}{30}$ metres per second [dividing distance by time]

= 8 metres per second (or 8 m/s or 8 m s^{-1})

2. 10 cubic centimetres of lead has a mass of 113 grams.

Density of lead $= \frac{113}{10}$ grams per cubic centimetre [dividing mass by volume]

= 11.3 grams per cubic centimetre, or 11.3 g/cm^3

3. 50 cubic metres of gas costs $500.

Cost of gas $= \frac{\$500}{50}$ per cubic metre [dividing cost by volume]

= $10 per cubic metre (or $10/$m^3$)

When comparing quantities, the same derived unit is used for each quantity.

Example

Q. A 1 L container of orange juice costs $2.45 while a 375 mL can of the same juice is on special for 90 cents. Which is better value?

A. Expressing both rates in mL per cent:

the 1 L container gives

$\frac{1\,000}{245}$ = 4.08 mL per cent [1 L = 1 000 mL]

the 375 mL can gives

$\frac{375}{90}$ = 4.17 mL per cent

Comparing the two, the can is better value [more mL for each cent spent]

To convert one derived unit to another, work with the original units separately.

For example, 25 cents per hour = (25 × 24) cents per day [24 hours in 1 day].

This simplifies to 600 cents per day or $6 per day.

Example

A tap drips at the rate of 5 mL per minute. To express this rate in L per day, first convert to mL per day.

5 mL per minute = (5 × 60 × 24) mL per day [60 min = 1 hour, and 24 h = 1 day]

= 7 200 mL per day

= 7.2 L per day [1 000 mL = 1 L]

Ans. p. 45

Exercise C: Derived units

1. Sione earns $120 for working $7\frac{1}{2}$ hours at his job in a butcher's shop. What is his hourly pay rate?

2. Jan goes on a tramp and completes a 52 km walk over 3 days. What is her average daily tramping rate?

3. Zac is putting advertising leaflets in envelopes to raise money for his athletics club. After 3 hours he has done 360 envelopes.
   a. How many envelopes per hour can Zac do?
   b. How many minutes per envelope does Zac take?
   c. Zac still has another 140 envelopes to finish. If he continues to work at this rate, how much longer will it take him to finish?

4. When Iskra babysits she gets paid at a rate of $12.50 per hour. How much does she get paid for an evening of work starting at 7.15 p.m. and finishing at 11.45 p.m.?

5. Zhao is cleaning used paving stones. He cleans 75 paving stones in $2\frac{1}{2}$ hours.
   a. How many minutes per stone is that?

   b. How long would it take him to clean 110 stones if he continued to work at this rate?

6. Jane buys a 2.5 kg bag of 'Doggo' pet food for her dog Jasper's daily dinner. A bag lasts 2 weeks. At what daily rate is Jasper eating?

7. a. Which is better value, 3.5 kg for \$8.70 or 5 kg for \$12.90?

   b. Which is the better pay rate: \$145 for 8 hours' work or \$180 for 10 hours' work?

8. A car uses 20 L of petrol to travel 285 km. How many litres per 100 km is that?

9. Convert a speed of 12 m s^{-1} (metres per second) to a speed in km h^{-1} (km per hour).

10. Gas costs \$11.50 per m^3.
    a. In one month, a household used 9.8 m^3 of gas. What is the bill to pay?

    b. The next month the bill was \$145. How much gas was used?

11. On his holiday, Hohepa converts \$NZ500 to \$531.25 in Singapore dollars. At this exchange rate:
    a. how much is NZ\$1 equivalent to in Singapore dollars?

    b. how much is \$1 Singapore equivalent to in New Zealand dollars, to the nearest cent?

    c. Hohepa's dad exchanged a certain number of New Zealand dollars into Singapore dollars, and received a hundred more Singapore dollars than the number of New Zealand dollars he exchanged. How many New Zealand dollars did Hohepa's Dad exchange?

12. The density of gold is 19.3 g/cm^3.
    a. Find the mass of a 45 cm^3 piece of gold.

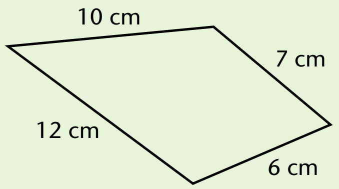
    b. Find the volume in cm^3 of a quantity of gold that has a mass of 173.7 mg.

Ans. p. 45

Perimeter

The **perimeter** of a 2-dimensional figure is the total distance around the boundary of the figure. Units for perimeter are the same as those for length.

Example

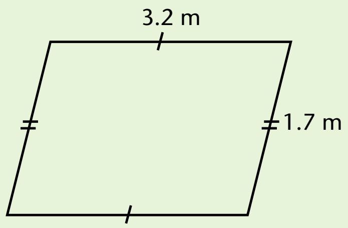


Perimeter = 10 + 7 + 6 + 12
= 35 cm

Use any symmetries or other properties of figures to find any unknown lengths of sides.

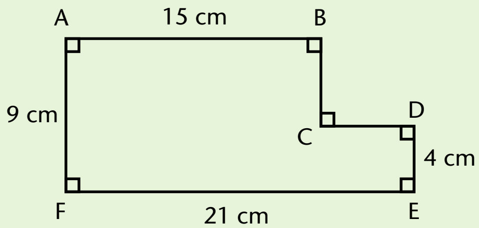
Example

1. The figure shown is a parallelogram, so opposite sides are equal in length (as shown by matching marks on opposite sides).



Perimeter = 3.2 + 1.7 + 3.2 + 1.7 [adding all side lengths]
= 9.8 m

2.



Looking at vertical heights:

BC = 9 – 4 [since AF = BC + DE]
= 5 cm

Looking at horizontal widths:

CD = 21 – 15 [since AB + CD = FE]
= 6 cm

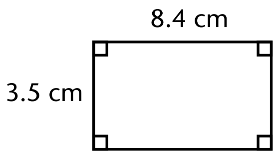
Adding the lengths of all edges:

Perimeter = 15 + 5 + 6 + 4 + 21 + 9
= 60 cm

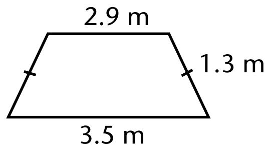
Exercise D: Perimeter

1. Find the perimeter of the following figures (not drawn to scale).

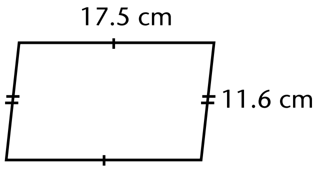
a.



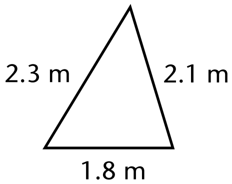
b.



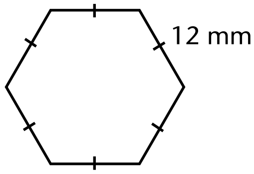
c.



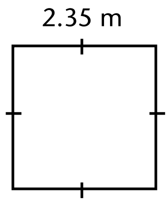
d.



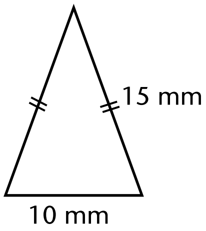
e.



f.

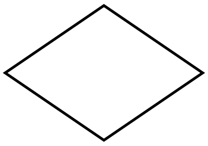


g.

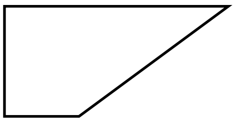


2. Find the perimeter of the following figures. You will need to take any necessary measurements.

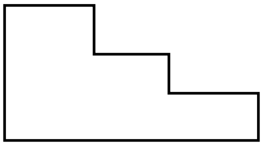
a.



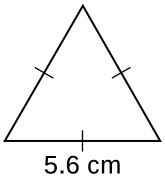
b.



c.



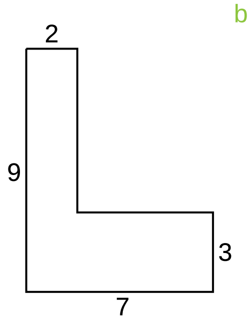
3. A piece of wire is bent to form an equilateral triangle of side length 5.6 cm.



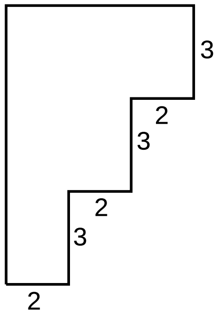
If the wire had been bent into a square instead, what would be the side length of the square?

4. Find the perimeters of these composite shapes, which are not drawn to scale (do not take measurements off the figures). All dimensions are in centimetres and all angles are right angles.

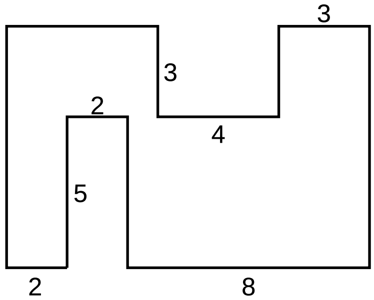
a.



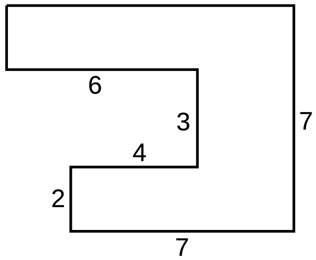
b.



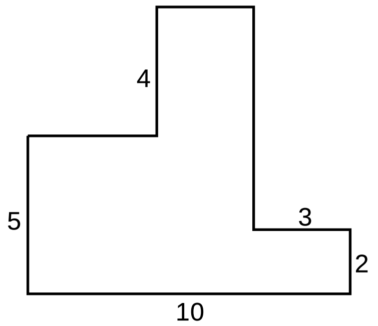
c.



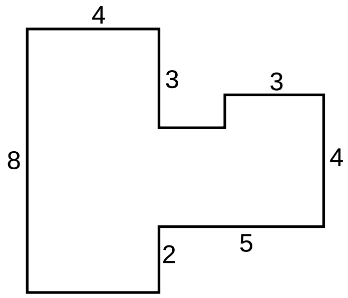
d.



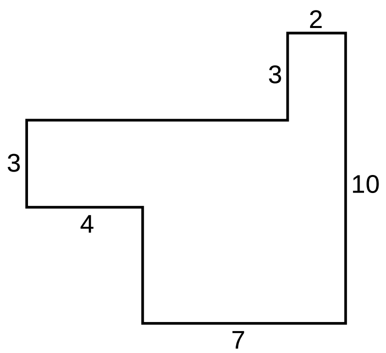
e.



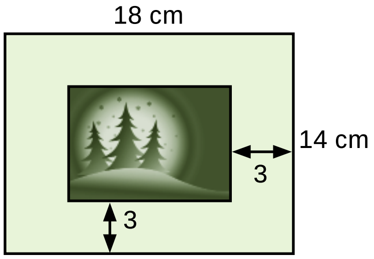
f.



g.



5. The figure shows a picture in a frame. The border of the picture is 3 cm wide (all the way round).



Find the perimeter of the picture.

6. a. Prove that the perimeter, p of a rectangle of length l and width w is:

$p = 2(l + w)$

b. Prove that the width, w, of a rectangle of length l and perimeter p is

$w = \frac{p - 2l}{2}$

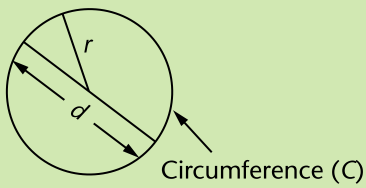
Circumference

The perimeter of a **circle** is called the **circumference** and is given by the formulae:

$C = \pi d$ where d is the **diameter** of the circle

or

$C = 2\pi r$ where r is the **radius** of the circle



Answers

Exercise A: Units of measurement (page 1)

1. m 2. kg 3. mL
4. km 5. s 6. g
7. t 8. months 9. mm
10. mL 11. mg 12. cm
13. kL 14. min

Exercise B: Using metric units (page 2)

1. a. 35.75 m b. 65 mm c. 8 500 g
   d. 57 g e. 225 min f. 38 750 kg
   g. 44.5 cm h. $980 i. 6 890 cm
   j. 130 years k. 99 650 000 L
   i. 87.5 t m. 450 g n. 35 L
2. a. 0020 b. 1:55 p.m. c. 20:15
   d. 33 days, 1 hour, 39 min, 2.86 seconds
   e. Saturday 1:15 p.m.
3. a. 20.16 L b. 47.5 t c. 5
   d. 0.12 mm e. 270 g
4. 49 min 5. $2
6. a. Saturday 11:14 p.m. in Adelaide
   b. Wednesday 11:36 p.m. in Lima

Exercise C: Derived units (page 4)

1. $16 per hour 2. $17\frac{1}{3}$ km per day
3. a. 120 b. 0.5 c. 1 hour 10 min
4. $56.25
5. a. 2 min/stone b. 3 hours 40 min
6. 180 g per day
7. a. 3.5 kg for $ 8.70
   b. $145 for 8 hours' work
8. 7.0 litres/100 km (1 d.p.) 9. 43.2 km h^{-1}
10. a. $112.70 b. 12.6 m^3
11. a. $1.0625 Singapore dollars
   b. $0.94 c. $1 600
12. a. 868.5 g b. 9 cm^3

Exercise D: Perimeter (page 6)

1. a. 23.8 cm b. 9.0 m c. 58.2 cm
   d. 6.2 m e. 72 mm f. 9.4 m
   g. 40 mm
2. a. 6 cm b. 8 cm
   c. 10.2 cm
3. 4.2 cm
4. a. 32 cm b. 30 cm c. 56 cm
   d. 40 cm e. 38 cm f. 36 cm
   g. 42 cm
5. Picture is 12 cm by 8 cm so perimeter is 40 cm
6. a. $p = w + w + l + l$
   $p = 2w + 2l$
   $p = 2(w + l)$
   b. $p = 2w + 2l$ from part a.
   $2w = p - 2l$ rearranging
   $w = \frac{p - 2l}{2}$ dividing by 2

Exercise E: Circumference (page 9)

1. a. 33.9 cm b. 37.7 m
   c. 46.2 mm d. 39.3 cm
   e. 82.9 mm
2. a. 22.3 mm (1 d.p.) b. 12.1 m (1 d.p.)
   c. 108.0 cm (1 d.p.) d. 68.0 cm (1 d.p.)
3. a. 6.9 cm (1 d.p.) b. 6.2 cm (1 d.p.)
4. 12.7 cm (1 d.p.) 5. 400 m

ANSWERS

6. 201.4 cm (1 d.p.)
7. 33 cm (1 d.p.)
8. 1.23 m (2 d.p.)
9. Arc length of 90° sector is $\frac{90}{360} \times 2\pi r = \frac{\pi r}{2}$

   Arc length of 45° sector is $\frac{45}{360} \times 2\pi r = \frac{\pi r}{4}$

   Perimeter is $2 \times 1.8r + \frac{\pi r}{2} + \frac{\pi r}{4} = 3.6r + \frac{3\pi r}{4}$

   which factorises to $\frac{r}{4}(3\pi + 14.4)$
10. 40.6 cm (1 d.p.)
11. 3.49 m (2 d.p.)

Exercise F: Area of 2-D figures (page 13)

1. a. 59.63 cm^2 b. 256 mm^2
   c. 18.75 m^2 d. 28.12 m^2 (2 d.p.)
   e. 690 mm^2 f. 3.14 m^2 (2 d.p.)
   g. 1.92 cm^2 h. 2.24 m^2
2. a. 1 250 mm^2 b. 30 000 cm^2
   c. 30 ha d. 1.9575 m^2
   e. 54 000 m^2 f. 635 ha
   g. 50 cm^2
3. a. 132.7 mm^2 (1 d.p.) b. 42.6 m^2 (1 d.p.)
   c. 131.5 cm^2 (1 d.p.) d. 27.27 cm^2
   e. 414.7 m^2 (1 d.p.)
4. 272.25 m^2
5. 4.8 cm
6. 75 m

Exercise G: Area of composite shapes (page 15)

1. a. 200 cm^2 b. 117.6 cm^2 (1 d.p.)
   c. 47.01 cm^2 d. 405.2 cm^2
2. a. 3.8 cm^2 (1 d.p.) b. 6 cm^2 (1 d.p.)
   c. 5.5 cm^2 d. 3.3 cm^2
3. 1.16 m^2
4. a. 5.5 m^2 b. 10.5 m^2 c. 16 m^2
   d. 6.5 m^2 e. 14 m^2 f. 12 m^2
5. a. 50 m^2
   b. The area has 1 significant figure, so if the depths used were accurate, the actual cross-sectional area is between 45 m^2 and 55 m^2.

      There are many reasons that the measurements may not be accurate, including the movement of the boat, the straightness of the plumb line, the unevenness of the lake bottom.

      The model (using trapeziums to approximate the actual area of the cross-section) may not be appropriate to use; also the widths of 4 m are possibly too wide for a good estimate of the area to be expected – measuring points should be closer together for a better estimate of the area.

Exercise H: Surface area of prisms (page 18)

1. a. 1 032 cm^2 b. 233.28 m^2
   c. 487.2 cm^2
2. 43.74 cm^2
3. 8 cm^2
4. a. 3.152 m^2 b. 0.50 L (2 d.p.)
5. 315 m^2
6. a. 40.24 m^2 (2 d.p.) b. 2.1 m (1 d.p.)

Exercise I: Surface area of solids with curved surfaces (page 20)

1. a. 1 168.7 mm^2 (1 d.p.)
   b. 1 963.5 cm^2 (1 d.p.)
   c. 530.1 cm^2 (1 d.p.)
   d. 75.4 cm^2 (1 d.p.)
   e. 106.9 m^2 (1 d.p.)
2. 0.6 m^2 (1 d.p.)
3. 259.2 cm^2 (1 d.p.)
4. 507 000 000 km^2 (3 s.f.)
5. 461.8 cm^2 (1 d.p.)
6. 150.8 cm^2
7. 2 545 cm^2
8. 25.0 cm (1 d.p.)

Exercise J: Converting cubic units (page 22)

1. a. 0.134 m^3 b. 2.693 cm^3
   c. 500 000 mm^3 d. 0.0259 km^3 (3 s.f.)
   e. 1 400 000 000 m^3 f. 0.000 175 m^3
2. 40 cm^3 is bigger (other lump is 3 cm^3)
3. 500 times greater

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