MATHEMATICS AND STATISTICS 1.13

Internally assessed
3 credits

Investigate a situation involving elements of chance

AS 91038

Introduction: chance

Situations involving **chance** can arise in everyday life, e.g. flipping a coin in order to decide which team gets the ball at the start of a game, playing a game of cards, or buying a raffle ticket.

In this achievement standard you will explore a situation involving chance by:

- posing a question
- planning an experiment to explore the situation
- gathering data by performing the experiment
- selecting and using appropriate displays including experimental probability distributions
- identifying and communicating patterns in the data
- summarising your findings in a conclusion.

In some situations you may be able to compare your experimental distribution with a theoretical distribution.

Probability

Some events are more **likely** to happen than others, e.g. when two dice are rolled together, it is **unlikely** that you will get two sixes, but it is very likely that the two numbers will add up to a number bigger than 2.

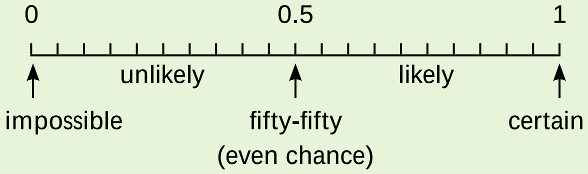
Numbers from 0 to 1 are used to describe how likely an **event** is to occur. These numbers are called the **probability** of the event.

- An **impossible** event has probability = 0
  Unlikely events have probabilities close to 0
- A **certain** event has probability = 1
  Likely events have probabilities close to 1
- An event which is just as likely to occur as it is not to occur has probability = $\frac{1}{2}$ or 0.5

Probability scale

A **probability scale** is a number line between 0 and 1, which is used to show the values that a probability can have.

Probability scale



The probability of an event occurring is written P(event).

Example

P(an ant weighs more than 1 kg) = 0

P(getting a six when a fair die is rolled) = $\frac{1}{6}$ or 0.167 (3 d.p.)

P(a fair coin comes up heads when flipped) = 0.5

P(getting less than six when a fair die is rolled) = $\frac{5}{6}$ or 0.833 (3 d.p.)

P(a kindergarten child is less than 6 years old) = 1

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Ans. p. 37

Experimental probability

An event is any happening of interest, e.g. scoring a try in a rugby game; getting a letter in the post; or getting a head when a coin is tossed.

In most practical situations, the actual probability of an event is unknown, so a **probability experiment** is run. This involves a series of trials – a trial is a 'success' if the event occurs.

An **experimental probability** is then worked out for the event.

$$\text{Experimental probability} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Example

I want to know how likely it is that when I throw a ball of paper at a bin 2 metres away that the ball of paper lands in the bin. So I throw the ball of paper at the bin 20 times.

Each throw is a trial.

If the ball of paper goes in the bin then that is a successful trial.

If I get the ball of paper into the bin 11 times out of 20 attempts, then my experimental probability of getting the ball of paper in the bin is $\frac{11}{20}$.

The experimental probability is an **estimate** of the actual probability.

Example

Melanie plays Goal Shoot for her netball team. Out of her last 100 attempts at shooting a goal, Melanie got 73 goals.

Melanie success rate in shooting goals is $\frac{73}{100}$ or 0.73 or 73%

This is the experimental probability of success for Melanie.

Melanie uses the experimental probability to estimate that her overall success rate at shooting goals is 73%. This means that she thinks there is a 73% chance that her next shot at goal will be successful.

Exercise A: Experimental probability

1. The face of a spinner is divided into parts labelled 0, 1, 2 and 3. In an experiment the spinner is spun 30 times. The table shows the results.

Score	Frequency
0	6
1	7
2	13
3	4
Total	30

For this experiment, what is the experimental probability of getting:

a. a 2?

b. a 0?

c. an odd number?

d. a number less than 2?

e. a number greater than 3?

f. a number less than 4?

2. A bag contains 25 marbles, some blue and some red. A marble is selected from the bag, its colour noted then returned to the bag. This is repeated 100 times. Altogether 69 selections were blue marbles and the rest were red marbles.

a. What is the experimental probability of getting a blue marble when a selection is made?

b. What is the experimental probability of getting a red marble when a selection is made?

3. In a town a record is kept of the gender of the babies that are born. Of the last 300 births in the town there were 157 boys and the rest were girls. For this town, what is the experimental probability of

   a. a boy baby?

   b. a girl baby?

4. A 'hand' of four cards is dealt from a standard pack of 52 cards, and the number of face cards (Jack, Queen, King or Ace) is counted for each hand. The cards are then returned to the pack, shuffled and another 'hand' of four cards is dealt. This process is repeated for a total of 24 hands. The results are shown in the table.

Number of face cards	**Number of hands**
0	4
1	10
2	8
3	2
4	0
Total	24

   a. What is the experimental probability of a getting no face cards?

   b. What is the experimental probability of getting 3 face cards?

   c. What is the experimental probability of getting at least 1 face card?

   d. What is the experimental probability of getting 4 face cards?

   Another hand is dealt and it has 4 face cards.

   e. What is the experimental probability now of a getting 4 face cards?

5. Lucy is collecting bottle tops so that she can enter a competition. Each bottle top is labelled with one of the five letters of the word P R I Z E.

   After collecting 10 bottle tops, Lucy has 4 P's, 2 R's, 1 Z and 3 E's.

   a. What is Lucy's experimental probability of getting a P?

   b. What is Lucy's experimental probability of getting an I?

   c. What is Lucy's experimental probability of getting an R or a Z?

   Lucy continues to collect bottle tops, and from her next 10 bottle tops she gets 5 P's, 1 R, 2 I's and 2 Z's. Using all 20 bottle tops:

   d. What is the experimental probability of getting a P now?

   e. What is the experimental probability of getting an I now?

   f. What is the experimental probability of getting an R or a Z now?

6. A town newspaper carried out an online survey about opening hours and pricing at the local public swimming pool.

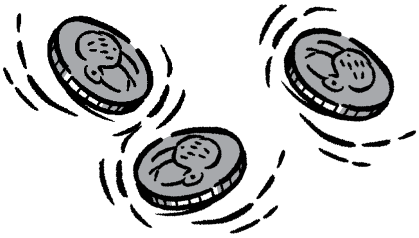
   The paper reported that out of 2 143 responses, 1 178 people were in favour of longer opening hours for the public swimming pool, and that 1 757 were against admission fees rising.

   A completed survey is selected at random. What is the probability it is from a person who is:

   a. in favour of longer opening hours?

   b. in favour of increasing admission fees?

7. Three coins are tossed together 50 times and the outcomes recorded. The number of heads in each toss of the three coins is counted. The table shows the results (H means head, T means tail).



Result	**Number of heads**	**Frequency**
HHH	3	8
HHT	2	5
HTH	2	7
HTT	1	6
THH	2	7
THT	1	8
TTH	1	6
TTT	0	3
Total		50

a. Using the information in the table, the event ‘getting 1 head when 3 coins are tossed’ happened 6 + 8 + 6 = 20 times. Use the table to complete the following table of outcomes.

Number of heads	0	1	2	3
Frequency		20		

b. What is the experimental probability of getting three heads?

c. What is the experimental probability of getting three tails?

d. What is the experimental probability of getting exactly 1 head?

e. What is the experimental probability of getting at most 1 head?

8. In a school, Year 10 students can choose from three summer sports for their recreation activity.

The table shows the numbers in some of the categories.

	Swimming	**Tennis**	**Cricket**	**Totals**
Girls	44	66		170
Boys				
Totals	66	140		400

a. Complete the missing values in the table.

b. A student is picked from the group at random. Find the probability the student is

  i. a girl

  ii. a boy who chose swimming

  iii. a tennis player

c. A girl is picked out at random. What is the probability:

  i. she chose tennis?

  ii. she did not choose cricket?

d. A swimmer is picked at random. What is the probability it is a boy?

e. A boy is picked at random. Which is more likely: he is a cricket player, or he is not a cricket player?

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9. Andy is collecting letters from a competition at his local supermarket COUNTUP.

Each time he spends $20 at the supermarket, Andy receives a small envelope containing one of the letters C, O, U, N, T, P. When a customer has collected enough letters to spell the word COUNTUP he/she wins a prize.

After several shopping trips, Andy has collected the following letters:

O U T T P T U O N U

U T P U U P T N P O

a. How much money did Andy have to spend at the supermarket in order to receive these letters?

b. Has Andy received enough letters to win a prize?

c. What is the experimental probability of the following letters?

i. C

ii. O

iii. U

iv. N

v. T

vi. P

d. Do you think that the supermarket produced equal quantities of each letter to distribute to customers? Justify your answer.

The next time he shopped, Andy received four more letters: N, C, O, C.

e. What is the experimental probability now of the following letters?

i. C

ii. O

iii. U

iv. N

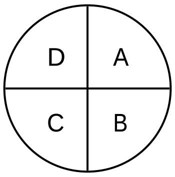
v. T

vi. P

f. What is Andy's situation now with respect to winning?

g. Has this latest set of numbers affected your opinion in part d.? Explain.

10. At a school fair a spinner has its face divided into quadrants, with each quarter labelled A, B, C or D.



A player pays $1 and spins twice. If the letters match, he wins $2. Piri watches the results for 12 players and counts 3 wins.

a. What is the experimental probability that a player will lose at this game?

b. How much money would the school have made or lost from these 12 players?

c. Do you think the school will make money from this game?

Theoretical probability

In some situations, the chance of an event can be worked out theoretically, using the properties of the situation, e.g the rolling of a die, or the flipping of a coin.

Example

When a die is rolled, there are six numbers that could result: 1, 2, 3, 4, 5, 6.

Each number is equally likely to occur, so each number has a one-in-six chance of occurring.

P(getting a 4) = $\frac{1}{6}$ [1 out of 6 numbers is a 4]

P(even number) = $\frac{3}{6}$ or $\frac{1}{2}$ [3 out of 6 numbers are even]

P(getting a number greater than 1) = $\frac{5}{6}$

[5 out of 6 numbers are greater than 1]

If an event is made up of **equally likely** outcomes, then the **theoretical probability** of the event is:

$$\text{P(event)} = \frac{\text{number of outcomes in event}}{\text{total number of outcomes in event}}$$

Probabilities of combinations of events

Events can be combined in various ways to make new events. The words AND, OR and NOT have particular meanings when used to combine events.

By considering how each combination is defined, the corresponding outcomes can be listed.

Example

Consider the set of numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

If A is the event ‘number is a factor of 24’ then A = {1, 2, 3, 4, 6, 8, 12}

If B is the event ‘number is a factor of 18’ then B = {1, 2, 3, 6, 9}

The event ‘A AND B’ means ‘number is a factor of *both* 24 and 18’ which is the set of outcomes which are in both A and B.

This is the set {1, 2, 3, 6}

The event ‘A OR B’ means ‘number is a factor of 24 or a factor of 18 or a factor of both numbers’. This is the set of outcomes which lie in A or B or in both.

This is the set {1, 2, 3, 4, 6, 8, 9, 12}

The event NOT A means ‘number is *not* a factor of 24’ which gives the set of outcomes which are not in A. This is the set {5, 7, 9, 10, 11}

Once the outcomes have been listed, the probability can be worked out using previous methods for finding the probability of an event.

Example

Two fair 5-sided dice are rolled together and the sum of the numbers from the two rolls are added.

The results are shown in the table.

Scores	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

Each face on a ‘fair’ die is equally likely to turn up when the die is rolled. So each of the twenty-five outcomes in the table is equally likely, with a theoretical probability of $\frac{1}{25}$.

1. P(getting a sum of 6 AND both dice are odd) = $\frac{3}{25}$

   [(5,1), (3,3) and (1,5)]

2. P(getting a sum of 4 OR 5) = $\frac{7}{25}$

   [(3,1), (2,2), (1,3), (4,1), (3,2), (2,3), (1,4)]

3. P(NOT getting a sum of 5) = $\frac{21}{25}$

   [four sums are 5, so 25 – 4 = 21 sums are not 5]

Independent events have no effect on each other’s probabilities, so the probability of two independent events occurring together is calculated by multiplying their probabilities.
This result can be extended to more than two independent events.

Example

A bag contains 5 red marbles and 8 blue marbles.

1. A marble is selected at random, its colour noted, then the marble is returned to the bag. A second marble is then selected at random, and its colour noted. Find the probability that the first marble is red and the second is blue.

   The events ‘first is red’ and ‘second is blue’ are independent (the first marble was returned to the bag so the conditions for selecting a second marble are exactly the same as the conditions for selecting the first marble).

   P(first marble is red) = $\frac{5}{13}$

   P(second marble is blue) = $\frac{8}{13}$ so

   P(first marble is red AND second marble is blue) = $\frac{5}{13} \times \frac{8}{13}$

   = $\frac{40}{169}$ or 0.237 (3 d.p.)

2. Three marbles are selected (with replacement after each selection).

   P(all 3 marbles are red) = $\frac{5}{13} \times \frac{5}{13} \times \frac{5}{13}$

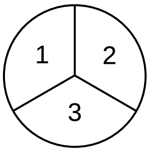
   [independent so multiply probabilities]

   = $\frac{125}{2\,197}$ or 0.057 (3 d.p.)

Exercise B: Theoretical probability

1. A bag contains 12 identically shaped coloured marbles: 5 red, 4 blue and 3 green. A marble is selected without looking at the colour. What is the probability that the marble selected is:
   - a. blue?
   - b. green?
   - c. red?

2. A spinner has its face marked in three equal sectors labelled 1, 2, 3.



   - a. The spinner is spun once. What is the probability of getting:
     - i. a 2?
     - ii. an odd number?
   - b. The spinner is spun twice and the sum of the two numbers is found (by adding them together). For example, if the first spin is a 2 and the second is a 3, then the sum of the two numbers is 5.
     - i. Complete the table of possible outcomes.

	1	2	3
1			
2			5
3			

When the spinner is spun twice and the numbers added, what is the theoretical probability of getting:

   - ii. a sum of 4?
   - iii. an odd sum?
   - iv. a sum less than 6?

3. When a coin is flipped, the result can be a Head (H) or a tail (T).
   - a. Complete the table showing all possible results when two coins are flipped. (For example Head on the first and tail on the second is recorded as HT)

	H	T
H		HT
T		

   - b. When two coins are flipped together, what is the probability of getting
     - i. two heads?
     - ii. a head and a tail in either order?
     - iii. at least one tail?
   - c. When three coins are flipped together then the possible outcomes are shown in the table. For example, if the first two flips are HT and the next flip is H, then the result is HTH; if the first two flips are TH and the next flip is T then the result is THT).

     Complete the table.

	H	T
HH		
HT	HTH	
TH		THT
TT		

   - d. When three coins are flipped together, what is the probability of getting:
     - i. three tails?
     - ii. two heads?
     - iii. no more than 1 tail?
     - iv. at least 1 head and 1 tail?

4. Soong rolls two fair 4-sided dice with faces labelled 1, 2, 3, 4. She then adds the two scores together.

Complete the table of possible outcomes.

2nd die 1st die	1	2	3	4
1	2	3	4	
2	3	4		
3				
4				

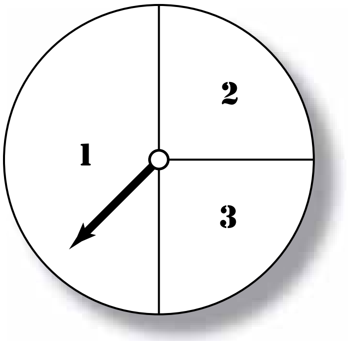
What is the probability that:

a. the sum of the two dice scores is 2?

b. the sum of the two dice is greater than 5?

c. the sum of the two dice is a prime number?

5. A spinner has its face labelled as shown.



The spinner is spun once. Find:

a. P(the result is a 1)

b. P(the result is a 2)

c. P(the result is an odd number).

d. P(the result is less than 4)

6. A coin is biased so that the probability of a Head is now $\frac{3}{5}$.

a. What is the probability of a Tail for this coin?

The coin is flipped twice. What is the probability that

b. i. both are Heads?

ii. the first is a Head and the second is a Tail?

iii. both are Tails?

c. What is the probability of one Head and one Tail in either order?

The coin is flipped three times. What is the probability that

d. i. all three are Heads?

ii. the first and third only are Heads?

iii. all three are Tails?

e. What is the probability that there is at least one Head and at least one Tail?

ANSWERS

Exercise A: Experimental probability (page 2)

1. a. $\frac{13}{30}$ b. $\frac{6}{30}$ or $\frac{1}{5}$ c. $\frac{11}{30}$
   d. $\frac{13}{30}$ e. 0 f. 1
2. a. $\frac{69}{100}$ b. $\frac{31}{100}$
3. a. $\frac{157}{300}$ b. $\frac{143}{300}$
4. a. $\frac{4}{24}$ or $\frac{1}{6}$ b. $\frac{2}{24}$ or $\frac{1}{12}$
   c. $\frac{20}{24}$ or $\frac{5}{6}$ d. 0 e. $\frac{1}{25}$
5. a. $\frac{4}{10}$ or $\frac{2}{5}$ b. 0
   c. $\frac{3}{10}$ d. $\frac{9}{20}$
   e. $\frac{2}{20}$ or $\frac{1}{10}$ f. $\frac{6}{20}$ or $\frac{3}{10}$
6. a. $\frac{1\,178}{2\,143}$ or 0.550 (3 s.f.)
   b. $\frac{386}{2\,143}$ or 0.180 (3 s.f)
7. a.

Number of heads	0	1	2	3
Frequency	3	20	19	8

   b. $\frac{8}{50}$ or $\frac{4}{25}$ c. $\frac{3}{50}$
   d. $\frac{20}{50}$ or $\frac{2}{5}$ e. $\frac{23}{50}$
8. a.

	Swimming	**Tennis**	**Cricket**	**Totals**
Girls	44	66	60	170
Boys	22	74	134	230
Totals	66	140	194	400

   b. i. $\frac{170}{400}$ or 0.425
   ii. $\frac{22}{400}$ or 0.055
   iii. $\frac{140}{400}$ or 0.35
   c. i. $\frac{66}{170}$ or 0.388 (3 s.f.)
   ii. $\frac{110}{170}$ or 0.647 (3 s.f.)
   d. $\frac{22}{66}$ or 0.333 (3 s.f.)
   e. P(boy plays cricket) = $\frac{134}{230}$, P(boy does not play cricket) = $\frac{96}{230}$, so it is more likely that he is a cricket player.
9. a. At least $400
   b. No (no C)
   c. i. 0 ii. $\frac{3}{20}$ or 0.15
   iii. $\frac{6}{20}$ or 0.3 iv. $\frac{2}{20}$ or 0.1
   v. $\frac{5}{20}$ or 0.25 vi. $\frac{4}{20}$ or 0.2
   d. It seems unlikely as there were almost twice as many U's and T's as there were O's, N's and P's. The number of C's may be proportionally lower as none were collected after 20 letters.

   However, after only 20 letters the experimental probabilities can vary quite a lot and be poor estimates of the actual probabilities. So it could be true that there were equal numbers of each letter produced.
   e. i. $\frac{2}{24}$ ii. $\frac{4}{24}$ iii. $\frac{6}{24}$
   iv. $\frac{3}{24}$ v. $\frac{5}{24}$ vi. $\frac{4}{24}$
   f. Andy now can spell COUNTUP twice, so he qualifies for two prizes.
   g. C's are still least common, and U and T are still more likely than O, N and P. But it seems less sure now that the probabilities of U, T, O, N and P are different, as the

numbers of times these letters turned up are all quite similar. With only 24 letters collected, the experimental probabilities are likely to be unreliable as estimators of the actual probabilities, so there could be equal numbers of each letter (except perhaps for C).

10. a. $\frac{9}{12}$ or $\frac{3}{4}$ b. School made $6

c. Out of every 12 games the school gets $12 and pays out $6 on average. So the school should make money if people play.

Exercise B: Theoretical probability

(page 7)

1. a. $\frac{4}{12}$ or $\frac{1}{3}$ b. $\frac{3}{12}$ or $\frac{1}{4}$ c. $\frac{5}{12}$

2. a. i. $\frac{1}{3}$ ii. $\frac{2}{3}$

b. i.

	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

ii. $\frac{3}{9}$ or $\frac{1}{3}$ iii. $\frac{4}{9}$ iv. $\frac{8}{9}$

3. a.

	H	T
H	HH	HT
T	TH	TT

b. i. $\frac{1}{4}$ ii. $\frac{2}{4}$ or $\frac{1}{2}$ iii. $\frac{3}{4}$

c.

	H	T
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

d. i. $\frac{1}{8}$ ii. $\frac{3}{8}$ iii. $\frac{4}{8}$ or $\frac{1}{2}$

iv. $\frac{6}{8}$ or $\frac{3}{4}$

4.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

a. $\frac{1}{16}$ b. $\frac{6}{16}$ or $\frac{3}{8}$ c. $\frac{9}{16}$

5. a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. 1

6. a. $\frac{2}{5}$

b. i. $\frac{9}{25}$ ii. $\frac{6}{25}$ iii. $\frac{4}{25}$

c. $\frac{12}{25}$

d. i. $\frac{27}{125}$ ii. $\frac{18}{125}$ iii. $\frac{8}{125}$

e. $\frac{90}{125}$

7. a.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. i. $\frac{3}{36}$ or $\frac{1}{12}$ ii. $\frac{6}{36}$ or $\frac{1}{6}$

iii. $\frac{11}{36}$ iv. $\frac{27}{36}$ or $\frac{3}{4}$

v. $\frac{33}{36}$ or $\frac{11}{12}$

c. i. $\frac{1}{36}$ or 0.028 (3 s.f.)

ii. $\frac{25}{144}$ or 0.174 (3 s.f.)

iii. $\frac{961}{1\,296}$ or 0.742 (3 s.f.)

8. a. $\frac{8}{36}$ or 0.222 (3 d.p.)

b. $\frac{22}{36}$ or 0.611 (3 d.p.)

9. a.

Green

White	1	2	2	3
1	2	3	3	4
1	2	3	3	4
2	3	4	4	5
3	4	5	5	6

b. i. $\frac{1}{2}$ ii. $\frac{1}{4}$

Exercise C: Comparing theoretical and experimental probabilities

(page 11)

1. a. i. $\frac{4}{36}$ or $\frac{1}{9}$ ii. $\frac{9}{36}$ or $\frac{1}{4}$

iii. $\frac{18}{36}$ or $\frac{1}{2}$

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