2. Design a base for a fountain made from at least three different solids (such as a cylinder, pyramid or hemisphere). The structure will be placed on a platform (which is also a solid) and surrounded by a fence.

a. Draw a diagram of your complete design, showing all relevant measurements.

b. Work out the perimeter of the fence.

c. Find the area of land inside the fence.

d. Find the volume of each solid in your design, and work out the total volume of all the solids.

e. Each solid in the design is made separately. Design a box that could be used to pack and transport all the solids in your design.

3. Design a simple three-dimensional cartoon vehicle, such as an aeroplane, made using at least three different solids (such as a cuboid, cylinder or cone).

a. Draw a diagram of your complete design, showing all relevant measurements.

b. Work out the surface area of each solid in your design.

c. Find the total surface area of all solids in your design.

d. Find the volume of each solid in your design, and work out the total volume of all the solids.

e. The aeroplane is displayed on a circular platform. Find the smallest area the circle can be so that no part of the aeroplane overhangs the circle.

▶ **ESA Online**

91030 practice assessment with answers available.

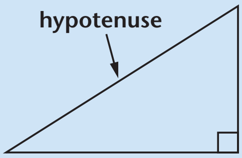
MATHEMATICS AND STATISTICS 1.7

Internally assessed
3 credits

Apply right-angled triangles in solving measurement problems

Right-angled triangles

In a **right-angled triangle**, one of the angles is 90° (a right angle). The longest side of a right-angled triangle is opposite the right angle and is called the **hypotenuse**.

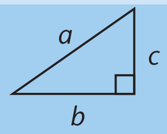


right-angled triangle

The theorem of Pythagoras

Pythagoras' theorem states an important relationship between side lengths in a right-angled triangle:

$a^2 = b^2 + c^2$

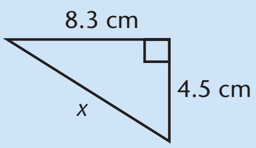


where a is the length of the hypotenuse and b and c are the lengths of the other two sides.

Using this result, the length of the hypotenuse of a right-angled triangle can be found when the other two side lengths are known.

Example

Q. Find the side length x in the triangle shown,



A. By Pythagoras' theorem, $a^2 = b^2 + c^2$

Substituting $a = x$ (the hypotenuse is labelled x), $b = 4.5$ and $c = 8.3$ gives:

$x^2 = 4.5^2 + 8.3^2$

$x^2 = 89.14$

$x = \sqrt{89.14}$ [taking square root]

$x = 9.4$ cm (1 dp) [rounding to same accuracy as original measurements]

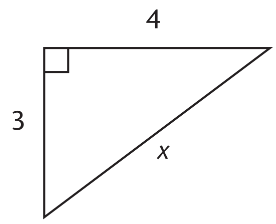
Note: If the side lengths of b and c were interchanged, the result would still be the same.

Exercise A: Finding the hypotenuse using Pythagoras' theorem

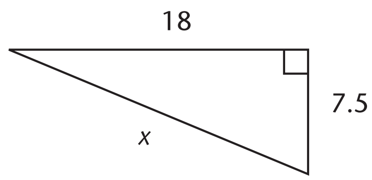
Ans. p. 271

1. Find the length of the hypotenuse marked x in the following right-angled triangles. (All measurements are in centimetres. Round answers to 1 dp where necessary.)

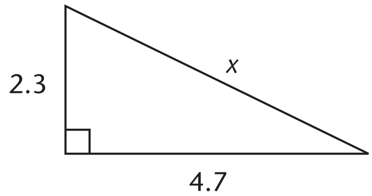
a.



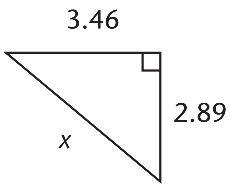
b.



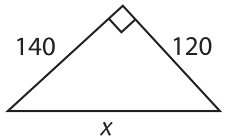
c.



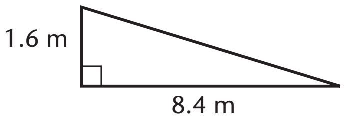
d.



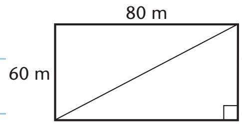
e.



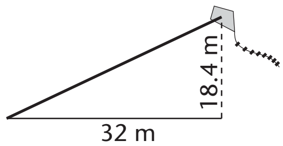
2. A ramp is 1.6 metres high and is 8.4 metres long horizontally. Find the length of the ramp.



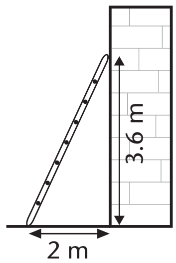
3. The length of a hockey pitch is 80 metres, and its width is 60 metres. Calculate the length of the diagonal of the pitch.



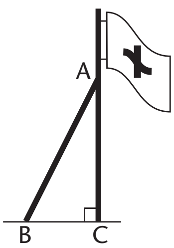
4. Boz is flying a kite. The horizontal distance of the kite from Boz is 32 metres and its height above the ground is 18.4 metres. How long is the string, assuming that the string is straight and not sagging?



5. A ladder rests against a vertical wall and reaches 3.6 metres vertically up the wall. The foot of the ladder is 2 metres from the base of the wall. Calculate the length of the ladder.



6. A flag is flying on its pole, but the flagpole is rather unstable. A stay is fastened to the flagpole at one end, A, and to the ground at its other end, B. If the length of AC is 8 metres and the length of BC is 4 metres, calculate the length of the stay AB.

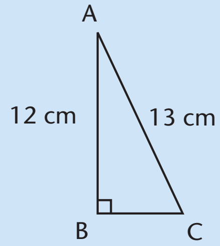


Finding other sides of a right-angled triangle using Pythagoras' theorem

By rearranging Pythagoras' formula, any side length of a right-angled triangle can be found if the other two side lengths are known.

Example

In the right-angled triangle ABC, the length of AB is 12 cm and the length of AC is 13 cm. Find the length of BC.



Solution:

Let BC be x cm long. The hypotenuse is AC.

By Pythagoras, $a^2 = b^2 + c^2$, where a is the hypotenuse:

$13^2 = x^2 + 12^2$

$x^2 = 13^2 - 12^2$ [subtracting 12^2 from both sides and swapping sides]

$x^2 = 25$

$x = 5$ [taking square root]

BC is 5 cm long.

Pythagoras' theorem can also be used in reverse to prove that a triangle is right-angled.

If the three sides of a triangle obey Pythagoras' theorem, then the triangle is right-angled.

For example, a triangle whose side lengths are 8 m, 15 m and 17 m is right-angled since

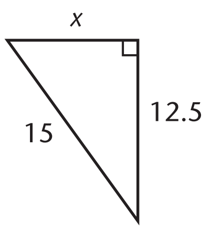
$8^2 + 15^2 = 17^2$ [since 64 + 225 = 289]

Exercise B: Finding other sides of a right-angled triangle using Pythagoras' theorem

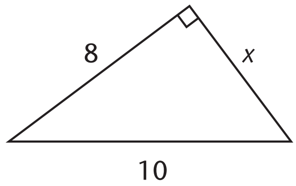
Ans. p. 271

1. Find the side lengths marked x in the following right-angled triangles. (All measurements are in centimetres. Round answers to 1 dp where necessary.)

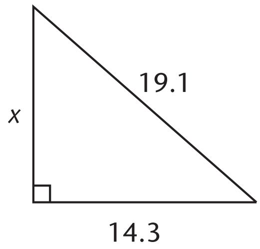
a.



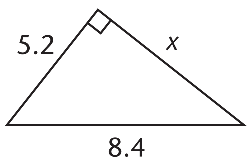
b.



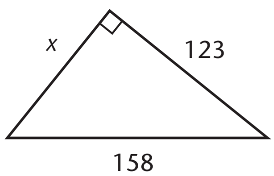
c.



d.



e.



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2. The longest side of a right-angled triangle is 12 m. The shortest side of the same triangle is 7.2 m. What is the length of the third side?

3. A triangle has sides of length 10.5 mm, 36 mm and 37.5 mm. Is the triangle right-angled?

4. A triangle ABC has side lengths AB = 33 mm, AC = 56 mm and BC = 65 mm.

a. Show that triangle ABC is right-angled.

b. Which of the vertices A, B and C is the right angle? Justify your answer.

Applications of Pythagoras' theorem

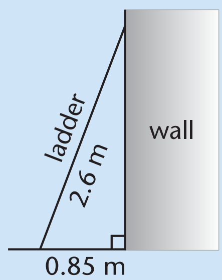
When solving practical problems involving Pythagoras' theorem:

- identify the right-angled triangle involved and make a clear drawing of it
- label the sides of the triangle with the measurements supplied (or worked out from the information given)
- label the unknown side length with a letter such as *x*.

Example

Q. A painter leans his 2.6 m ladder against a wall. The foot of the ladder is 0.85 m from the wall. How far up the wall does the ladder reach?

A. A diagram of the situation is drawn.



Let the distance up the wall that the ladder reaches be *x* metres.

Using Pythagoras' theorem:

$x^2 + 0.85^2 = 2.6^2$ [hypotenuse is 2.6 m]

$x^2 = 2.6^2 - 0.85^2$ [rearranging]

$x^2 = 6.0375$

$x = \sqrt{6.0375}$ [taking the square root]

$x = 2.4571...$

The ladder reaches 2.46 m (2 dp) up the wall.

In some diagrams, an extra line may need to be drawn in order to form a right-angled triangle (remember that **horizontal** lines form right angles with **vertical** lines).

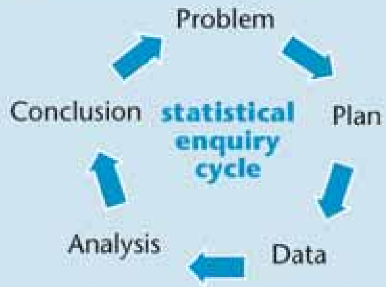
MATHEMATICS AND STATISTICS 1.12

Externally assessed
4 credits

Demonstrate understanding of chance and data

The statistical enquiry cycle (PPDAC)

The **statistical enquiry cycle** summarises the steps involved in a statistical investigation.



Statistical literacy involves understanding, evaluating and interpreting statistical investigations undertaken by others.

Data collection

Usually only part of the population, known as a **sample**, is investigated.

- The sample should be **random** (each member of the population has the same chance of being chosen) so that the sample resembles the population as much as possible.
- If the sample does not accurately reflect the characteristics of the whole population then the sample is said to be **biased**.

In a statistical investigation, data may be collected by the investigators themselves or obtained from elsewhere. It is important that this **data source** is listed.

Selecting a random sample

Suppose you are given a list of 500 students, and you have to select a random sample of 30 names from the list.

'Drawing names from a hat' is a good reliable method that has no bias.

A quicker method is to use the random numbers on your calculator, as follows:

- give each student a number from 1 to 500
- set your scientific calculator to produce random numbers from 1 to 500

  (Press: 5 0 0 Ran# + 1 = and take the whole number part, ignoring repetitions)
- obtain the first thirty random numbers
- identify the sample of 30 students by matching the numbers to the names.

Example

Select a random sample of 8 names from the following list of 15 names:

John, Will, Anne, Helen, Henry, Tom, Liz, Luke, Nathan, Jacob, Angus, Amy, Jack, Sally, Pat

Solution

Enter the names in a table and give a number to each name.

Number	Name	Number	Name
1	John	✔ 9	Nathan
2	Will	10	Jacob
✔ 3	Anne	✔ 11	Angus
✔ 4	Helen	12	Amy
5	Henry	✔ 13	Jack
6	Tom	✔ 14	Sally
✔ 7	Liz	15	Pat
✔ 8	Luke		

Using your calculator, obtain 8 numbers in the range 1–15

(Press: 1 5 Ran# + 1 = and take the whole number part only, ignoring repetitions)

A typical result might be:

13, 3, 4, 8, 13 discard (no repeats), 7, 14, 11, 9

Highlight or tick the names, as shown.

Exercise A: Selecting a random sample

Ans. p. 299

Use the random numbers on your calculator to answer the questions in Exercise A.

1. Select 8 names from the list of 12 names in the table.

Number	1	2	3	4	5	6	7	8	9	10	11	12
Name	Sarah	Millie	Phiz	Boz	Liz	John	Bill	Ann	James	Barry	Amy	Petra

List the eight numbers, and in the table highlight your choices.

Random numbers used:

2. Allocate a number to each country in the table. Select 10 countries from the list of 15. Highlight your selection and list the numbers.

Number															
Country	Brazil	Peru	Bolivia	Sudan	Libya	Korea	Angola	Zambia	USA	NZ	UK	China	Niger	Poland	Kuwait

Random numbers used:

3. At Mountain High School there is a boys' soccer team and a girls' netball team. The players in each squad are listed in the table. Select a sample of 7 players from the soccer team and 5 players from the netball team.

Soccer team															
Number															
Name	Luke	Peter	John	Ian	Carl	Andy	Sean	Karl	Mike	Kurt	Amos	Sang	Mark	Joel	Alan

Random numbers used:

Netball team												
Number												
Name	Grace	Meg	Emma	Ella	Lorna	Sarah	Prue	Emily	India	Rose	Laura	Bianca

Random numbers used:

Data collection methods

There are three main methods of gathering data from a sample.

- **Observation** – watching and accurately recording the information, e.g. counting cars (**discrete** data), measuring plant heights (**continuous** data), etc.
- Oral **interviews** – an investigator asks questions and records responses, e.g. opinions about products, or attitudes to various issues (**qualitative** data).
- Written **questionnaires** – also involve questions and responses, but in written form. Answers are recorded in various ways – in words, choosing from multichoice options, or using a scale (e.g. a rating from 0 to 10).

The initial data gathered is called **raw data**.

Data organisation

After collection, the raw data are often organised into **tables**. This allows features of the data set to be seen more clearly.

Frequency distribution tables

Raw data is often organised into a **frequency distribution table**.

Example

The times taken in seconds to complete a simple puzzle are shown below:

6, 5, 5, 7, 7, 6, 5, 9, 7, 6, 6, 6, 8, 6, 5, 9, 5, 6, 9, 5, 6, 5, 7, 8, 6

The data are organised, using **tallies**, into a frequency table.

Time (seconds)	Tally	Frequency
5	𝍸 II	7
6	𝍸 IIII	9
7	IIII	4
8	II	2
9	III	3
Total		25

Frequency tables for grouped data

For **discrete** (counted) data sets with many values, the data are grouped into **classes**, e.g. 1–5, 6–10, …

Continuous (measured) data are always grouped into intervals of values, since there are infinitely many values possible.

Example

The distance students travelled to school each day was measured (rounded to the nearest tenth of a kilometre). The results for one class are shown.

2.3	1.8	0.8	2.4	2.1	3.0	0.7	0.5	1.1	1.8
1.4	2.7	3.1	1.1	0.9	2.0	0.2	1.4	1.0	3.2
4.4	1.8	1.5	0.7	0.1	1.6	2.3	2.2	1.7	0.3

The data are organised into classes of width 0.5 km.

Distance (km)	Tally	Frequency
0.0–	III	3
0.5–	𝍸	5
1.0–	𝍸	5
1.5–	𝍸 I	6
2.0–	𝍸 I	6
2.5–	I	1
3.0–	III	3
3.5–		0
4.0–4.5	I	1
		30

The table clearly shows that one distance is much greater than the others (4.4 km).

Note: This frequency table is not the only one possible – you could use other interval widths.

The tally column is usually not included in a frequency table.

Example

The time taken to travel to school one day was recorded in a table by each student in a class.

Time taken (minutes)	Number of students
0–	9
15–	17
30–	4
45–60	1

1. In the table, the interval 0– means the time taken is at least 0 minutes but is less than 15 minutes.
2. If a student took 32 minutes to travel to school, a **tally** would be placed in the 30– class.
3. 4 + 1 = 5 students took more than 30 minutes to travel to school.
4. No students took longer than 1 hour (60 minutes) to travel to school.
5. If a student is randomly selected from the class, the time the student took to get to school would most likely be between 15 and 30 minutes, since this is the class with the highest frequency.

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Exercise B: Frequency distribution tables

Ans. p. 299

1. As part of a survey, students recorded the number of pieces of fruit they ate the previous weekend. The results are shown for one class:

3	1	4	5	3	4	2	0	1	1	4	3	6	4	5
2	3	4	0	1	1	4	3	5	4	5	3	5	2	1

a. Present these data in a frequency table.

b. Comment on any interesting features of the data that can be seen.

2. At the beginning of the season, the junior cyclists at a school are timed over an 8-km course.

Their times (in minutes rounded to 2 dp) are as follows:

19.21	17.65	17.23	19.28	17.01	23.45
22.11	23.08	21.01	19.97	17.65	18.34
16.56	18.57	23.62	14.76	19.36	18.89
15.90	21.04	19.34	18.52	16.77	19.43

The data are presented in a grouped frequency table (using interval widths of 1 minute).

Time (min)	Tally	Frequency
14–	I	1
15–	I	1
16–	II	2
17–	IIII	4
18–	IIII	4
19–	~~IIII~~ I	6
20–		0
21–	II	2
22–	I	1
23–24	III	3
	Total	24

Comment on any interesting features of the data that can be seen.

3. Millie weighed a random sample of school bags from 50 Year 11 students. Her results are in the table alongside.

Weight (kg)	Number of students
0–	9
2–	18
4–	12
6–	8
8–10	3
	50

a. Explain what the interval 4– means.

b. Which frequency would change if another bag of weight 5.2 kg is entered into the table?

c. Millie says, 'Over half of the bags in the sample weigh more than 5 kg'. Is this claim supported by the data in the table?

d. A bag was left behind after the survey was done. What would be the most likely range of weights for the bag?

e. Millie says, 'Most bags of Year 11 students weigh less than 6 kg'. Comment on her statement, discussing what factors would make her claim likely to be true or false.

ANSWERS

Achievement Standard 91026 Mathematics and Statistics 1.1

Exercise A: Factors multiples and primes (page 1)

1. **a.** **i.** 1, 2, 3, 5, 6, 10, 15, 30
ii. 1, 2, 3, 4, 6, 8, 12, 24
iii. 1, 17
iv 1, 5, 25
b. 6

2. **a.** **i.** 9, 18, 27, 36
ii. 12, 24, 36, 48
iii. 42, 84, 126, 168
iv. 125, 250, 375, 500
b. 36

3. **a.** 17, 19, 23, 29
b. 31, 37

4. **a.** 45, 54 **b.** 40, 48, 60
c. 41, 43, 47, 53, 59 **d.** 46, 51, 57

5. 60 seconds (or 1 minute)

6. 36 months (or 3 years)

7. **a.** 15, 30, 45, 60 **b.** 20, 40, 60
c. 60
d. multiples of bigger number is quicker as step size is larger
e. **i.** 180 **ii.** 96

8. **a.** **i.** $2 \times 3 \times 5$ **ii.** $2 \times 5 \times 7$
iii. $3 \times 5 \times 11$
b. **i.** 2 mm × 3 mm × 5 mm
ii. 2 cm × 5 cm × 7 cm
iii. 3 m × 5 m × 11 m

9. **a.**

1	140
2	70
4	35
5	28
7	20
10	14

b.

1	168
2	84
3	56
4	42
6	28
7	24
8	21
12	14

c. **i.** 28 **ii.** \$5 **iii.** \$6

Exercise B: The integers (page 3)

1. **a.** –8 **b.** 64
c. 0 **d.** –36
e. 14 **f.** 20
g. 11 **h.** –39
i. 10 **j.** 12
k. –60 **l.** –12
m 3 **n.** 36
o. –16 **p.** –27

2. **a.** 3 floors below **b.** 13

3. 17 °C colder

4. **a.** –27 °C **b.** –18 °C

5. –7

INDEX