Mathematics and Statistics 1.12

Externally assessed
4 credits

Demonstrate understanding of chance and data

Probability trees

Probability trees

Probability trees are useful for displaying sequences of events, and calculating their probabilities. The various possible outcomes are shown at the end of each branch pathway. Multiply 'along the branches' to find the probability of a particular outcome.

Example

A school has to nominate a boy and a girl to represent the school at a function. The three boys being considered are Adam, Hemi and James. The two girls being considered are Louise and Sandy. The school decides to select the students randomly so that each boy is equally likely to be chosen (each has probability $\frac{1}{3}$) and each girl is equally likely to be chosen (each has probability $\frac{1}{2}$).

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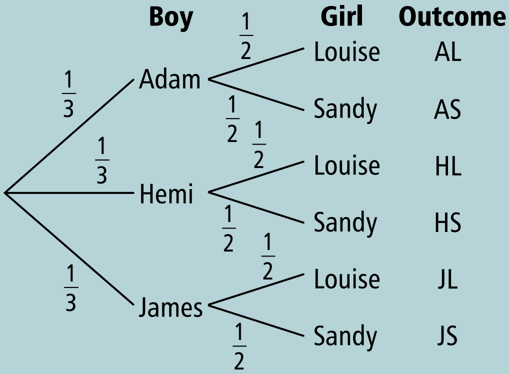
The tree diagram below illustrates the situation. Each branch is labelled with its probability. At the end of each branch pathway the six possible outcomes are shown (AL means Adam and Louise are chosen, etc.).



To find the probability of Adam and Louise being chosen, multiply along the branches. Thus

$P(AL) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Note: Alternatively in this case, since each of the six outcomes is equally likely, each outcome has probability $= \frac{1}{6}$

To find the probability of an event made up of several outcomes, add 'between the ends of the branches'.

Example

In the example above, Louise wants to go to the function with either Hemi or James.

P(Louise is chosen with Hemi or James) $=$ P(HL or JL)

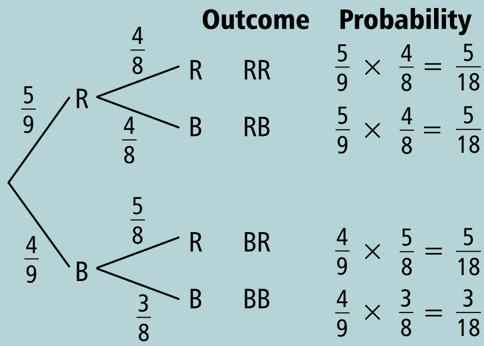
$= \frac{1}{6} + \frac{1}{6}$ adding together the probabilities of the HL and the JL branch

$= \frac{1}{3}$

Example

Q. A bag contains 5 red and 4 blue marbles. Two marbles are selected at random from the bag. Find the probability that the marbles are the same colour.

A. The tree diagram shows the situation.



$P(\text{same colour}) = P(RR \text{ or } BB)$

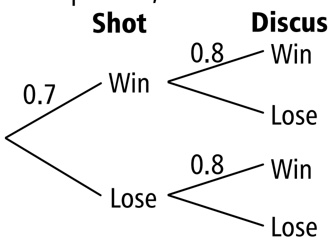
$= P(RR) + P(BB)$

$= \frac{5}{18} + \frac{3}{18}$

$= \frac{8}{18}$ or $\frac{4}{9}$

Exercise A: Probability trees

1. Bella enters in the shot-put and discus events in an athletics competition. Bella has a 0.7 chance of winning the shot-put competition and a 0.8 chance of winning the discus competition, as shown in the tree diagram.



   a. Complete the tree diagram.

   b. What is the probability Bella wins the shot-put but loses the discus competition?

   c. What is the probability Bella wins at least one of the events?

2. In a healthfood store, $\frac{2}{3}$ of the customers are female. 75% of the customers make a purchase.

   a. Put this information on a tree diagram.

   b. What is the probability that a customer is a male who makes a purchase?

   c. What is the probability that a customer is a female who doesn't make a purchase?

   d. What is the probability that 2 customers in a row both make a purchase?

3. In an in-store promotion, customers buying phone cards can select a scratch card from a large pile. The scratch cards have the letters of the word PHONE on them, and each letter is equally likely to turn up. When a customer has enough letters to make up the word PHONE, he or she wins a free phone card.

   a. Beth has two cards. What is the probability both cards have the letter P?

   b. Vinder has two cards. What is the probability at least one is a vowel?

   c. Kim has two cards. What is the probability the cards are different?

   d. Zan has three cards. What is the probability the cards are all different?

4. Jenna enters three events at the school sports day. From past performance she gives herself a 0.75 chance of winning the 100 m sprint, a chance of 0.8 of winning the 200 m sprint and a chance of 0.6 of winning the high jump. If these probabilities are correct, what is the probability Jenna wins only one event?

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Conditional probability

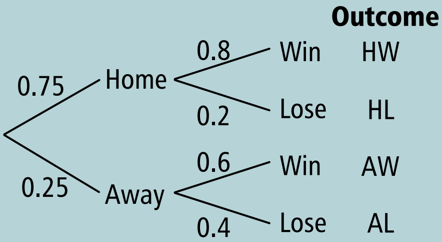
In more complex situations, the probability of a second (or subsequent) event occurring depends on the occurrence (or not) of the first event. **Conditional probabilities** such as these must be labelled carefully on the branches.

Example

Over a tennis season, 75% of Kristin's games are held at her school and 25% are played at another venue. When playing tennis at 'home', Kristin has an 80% chance of winning her match. When playing tennis 'away', Kristin wins only 60% of her matches.

Getty Images – Steve Debenport

The probability tree shows this information.



The following probabilities can be worked out using the tree:

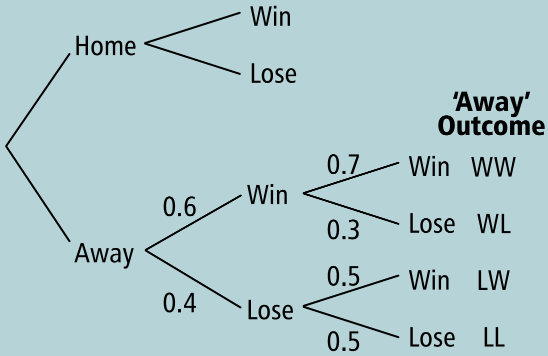
1. P(Kristin plays at 'home' and wins)
   $= 0.75 \times 0.8$ multiplying along the branches
   $= 0.6$
2. P(Kristin wins her match) = P(Kristin plays at 'home' and wins OR Kristin plays 'away' and wins)
   = P(HW) + P(AW)
   $= 0.75 \times 0.8 + 0.25 \times 0.6$
   multiply along the branches and add between the branches
   $= 0.75$

For some conditional probability problems only part of a tree diagram is used.

Example

In the example above, suppose Kristin is playing an 'away' game and she plays a singles match then a doubles match. When Kristin plays an 'away' singles match and wins, then the probability she wins her doubles match is 0.7. However, if she is playing 'away' and loses her singles match, then the probability she wins her doubles match is 0.5.

The probability tree has a third set of branches added.



When Kristin plays 'away':

P(Kristin wins exactly one of her singles and doubles games) = P(WL) + P(LW)

$= 0.6 \times 0.3 + 0.4 \times 0.5$

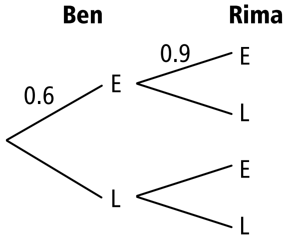
$= 0.38$

Note: It is stated that Kristin is playing an away game so there is no probability attached to the event 'Away'. It has definitely occurred, so multiplication of probabilities are restricted to the lower part of the probability tree, beyond this event.

To solve inverse problems, these processes will need to be reversed.

Exercise B: Conditional probability

1. Ben and his friend Rima both get the bus to school. There is an early bus and a late bus and the two students catch them independently. Ben catches the early bus 60% of the time and Rima catches it 90% of the time.
   a. Complete the tree diagram for this situation.



   b. Find the probability both Ben and Rima catch the late bus.

   c. Find the probability that exactly one of the two students catches the early bus.

   The early bus always gets students to school in time for assembly, but students on the late bus miss assembly 5% of the time.

   d. What is the probability that Ben and Rima both catch the same bus and are in time for assembly?

2. Claudia works in a women's clothing boutique. When a customer comes into the shop there is a 0.15 chance she will buy a top that is on sale. If the customer buys the top, Claudia will offer her a second one at half price. 50% of customers will then buy a second top.
   a. Draw a tree diagram to represent this situation.

   b. What is the probability that a customer will buy two tops?

   If a customer buys two tops, Claudia offers to put her on the boutique's mailing list so that she can be informed of future offers. 80% of customers put their name on the list.

   c. A customer comes into the boutique and buys one top. What is the probability she will buy a second top but not put her name on the waiting list?

3. In a school $\frac{3}{5}$ are juniors and $\frac{2}{5}$ are seniors. $\frac{2}{3}$ of the juniors walk to school and $\frac{3}{8}$ of the seniors walk to school.
   a. Put this information on a tree diagram.

   b. A student is chosen at random from the school. What is the probability that:
      i. it is a junior who does not walk to school?

      ii. it is a student who walks to school?

   c. 20% of the senior students who do not walk to school drive themselves to school.
      i. What is the probability a senior student does not walk to school but does not drive themself to school either?

      ii. What proportion of the students in the school are seniors who drive themselves to school?

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4. A factory employs men and women. On any one day there is a probability of 0.05 that a male worker is absent and a probability of 0.08 that a female worker is absent. If the overall probability of a worker being absent is 0.071, what proportion of the workers are male?

5. In an exam, 20% of those who thought they had passed actually failed and 30% of those who thought they had failed actually passed. If 50% actually passed overall, what percentage thought they would pass?

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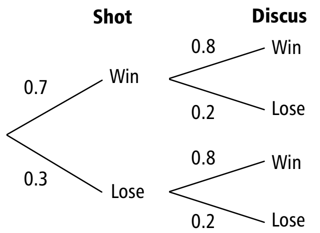
Answers

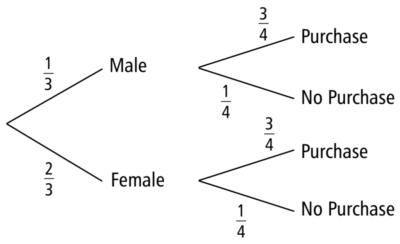
Exercise A: Probability trees

1. a.



b. 0.14 c. 0.94

2. a.



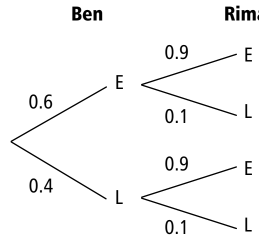
b. $\frac{1}{4}$ c. $\frac{1}{6}$ d. $\frac{9}{16}$

3. a. $\frac{1}{25}$ b. $\frac{16}{25}$ c. $\frac{4}{5}$ d. $\frac{12}{25}$

4. 0.17

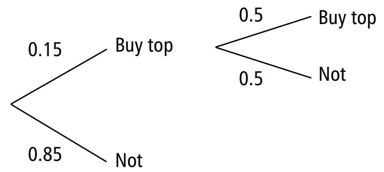
Exercise B: Conditional probability

1. a.



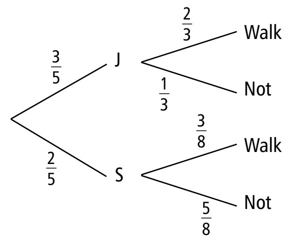
b. 0.04 c. 0.42 d. 0.578

2. a.



b. 0.075 c. 0.1

3. a.



b. i. $\frac{1}{5}$ ii. $\frac{11}{20}$ or 0.55

c. i. 0.5 ii. 5%

4. 30%

5. 40%

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