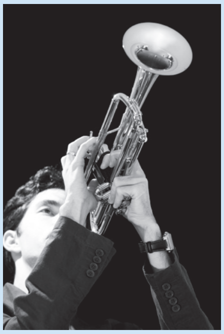
Musical instruments

In *Standing waves in strings and pipes*, you saw how it is possible to set up a standing wave in a string or a pipe of fixed length. However, in order to hear a musical note, a sound wave must travel from the instrument to our ears. When we listen to music we are able to distinguish between the different types of instrument. Middle C played on a guitar sounds completely different from middle C played on a piano.

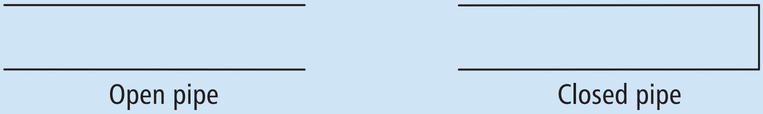
Musical notes

A musical note is produced when a standing wave it set up in the taut string of a stringed instrument or in the pipe of a wind instrument. To set up a standing wave, a travelling wave must first be generated.

In a stringed instrument, the travelling wave is generated by plucking or stroking the string. The string is fixed at both ends, so the standing wave must have a node at each end. The vibration of the string between these nodes creates a sound wave that travels outwards, away from the instrument, to the ears of the listener.



In a wind instrument, the travelling wave is generated by 'blowing' the instrument. This sends a longitudinal sound wave down the pipe. The pipe *must* be open at *one* end. The other end may be open or closed.



The pressure variations that occur at the open end of a pipe create a sound wave that travels outwards, away from the instrument, to the ears of the listener.

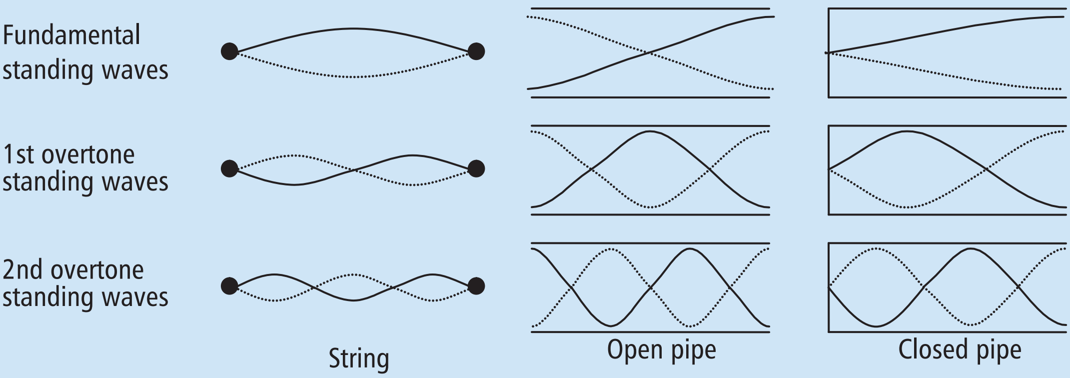
Fundamental and overtones

When a travelling wave has been generated in a string or a pipe, a standing wave will be set up *only* if the frequency of the travelling wave is a resonant frequency.

The action of plucking or stroking a string, or blowing a pipe, generates, over the period of time it takes to carry out the pluck or blow, a wide range of different frequency waves. Most of the different frequencies quickly die away because they do not resonate. The waves that *do* have a resonant frequency *all* set up *their own* standing wave, simultaneously, in the string or pipe.

It is important to recognise that *many* standing waves will be set up at the same time, and all these will *add* together to give a **resultant** standing wave. It is the frequency of the resultant standing wave that determines the **pitch** of the note that is heard.

The standing wave with the longest wavelength (lowest frequency) is called the **fundamental**. The standing wave with the next longest wavelength (next lowest frequency) is called the 1st **overtone**, and so on.

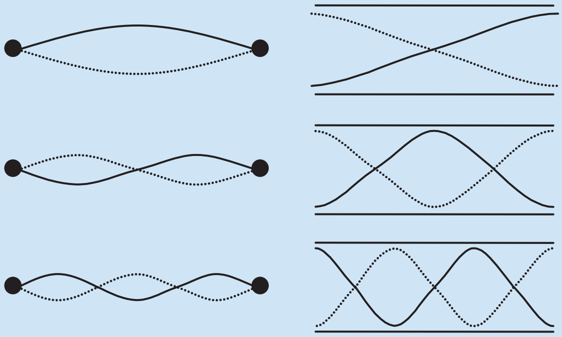


The fundamental standing wave is also called the 1st **harmonic**. Each overtone has a **harmonic number**. The overtone that has twice the frequency of the 1st harmonic is called the 2nd harmonic, and so on. This means that:

$$f(n\text{th harmonic}) = n \times f(\text{1st harmonic})$$

Because $v = f\lambda$, $f \propto \frac{1}{\lambda}$ and so $\lambda(n\text{th harmonic}) = \frac{1}{n} \times \lambda(\text{1st harmonic})$

If the pipe or string has the *same* restriction at both ends (either open/open for a pipe, or fixed/fixed for a string), the wavelength of the 1st and 2nd overtones relates to the wavelength of the fundamental – as shown in the following diagram (the length of the string/pipe is L).



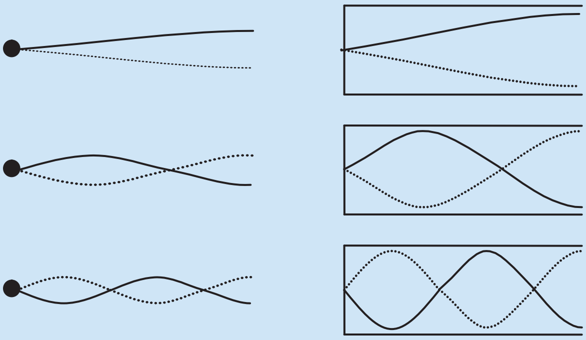
$\lambda = 2L$

$\lambda = L = \frac{\lambda(\text{1st harmonic})}{2}$

$\lambda = \frac{2L}{3} = \frac{\lambda(\text{1st harmonic})}{3}$

This shows that the 1st overtone is the 2nd harmonic, the 2nd overtone is the 3rd harmonic, etc.

If the pipe or string has a *different* restriction at both ends – either open/closed for a pipe, or fixed/free for a string – the wavelength of the 1st and 2nd overtones relate to the wavelength of the fundamental, as shown in the following diagram.



$\lambda = 4L$

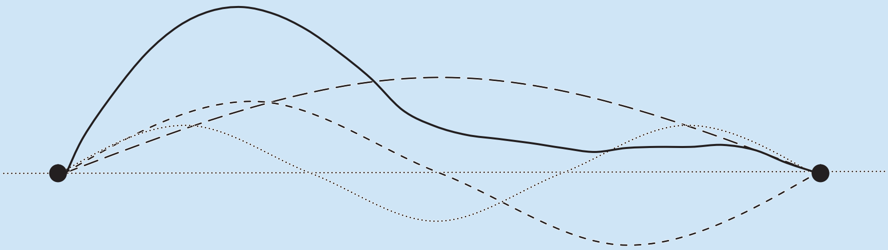
$\lambda = \frac{4L}{3} = \frac{\lambda(\text{1st harmonic})}{3}$

$\lambda = \frac{4L}{5} = \frac{\lambda(\text{1st harmonic})}{5}$

This shows that the 1st overtone is the 3rd harmonic, the 2nd overtone is the 5th harmonic, etc. The *even*-numbered harmonics do not exist.

Pitch and timbre

In the diagram, the dotted lines are the fundamental and first two overtones on a string. The solid line is the **resultant** wave shape.



Notice that the wavelength of the *resultant* wave is the same as the wavelength of the *fundamental*. This means that the *pitch* of the note that is heard is the same as the pitch of the *fundamental*.

The *shape* of the wave determines what the note sounds like – its **timbre**. The resultant shape of the wave will depend on how many overtones are present and their relative strength (amplitude). This will vary greatly between different instruments, because each instrument has its own way of generating the travelling waves that set up the standing waves. Therefore, each different instrument has its own characteristic timbre.

Musical instruments

Answers p. 68

1. Preliminary question

a. For a string of length, *L*, fixed at both ends:

i. draw a diagram and derive an expression for the frequency of the fundamental wave in terms of the speed of the wave, *v*, and the length, *L*

ii. draw diagrams and derive expressions for the frequency of the 1st and 2nd overtone waves in terms of the speed of the wave, *v*, and the length, *L* and use these derived expressions and the expression found in **i.** to state the relationship between the frequency of the overtones and the frequency of the fundamental.

b. Repeat **i.** and **ii.** for an open pipe of length, *L*.

c. Repeat **i.** and **ii.** for a closed pipe of length, *L*.

d. Explain why even-numbered harmonics do not exist in a closed pipe.

Check that you understand the answers to this **Preliminary question** before carrying on.

2. Speed of sound in air = 340 m s^{-1}

Speed of light = 3.00×10^8 m s^{-1}

Calculate the wavelength of the following waves. (You may find it useful to first draw the wave shape in the diagram provided.)

a. The 4th harmonic in an open pipe 4.0 m long.

b. The 5th harmonic in a string, fixed at both ends, 1.2 m long.

c. The 3rd harmonic in a closed pipe, 1.5 m long.

3. A tuning fork has a frequency of 160 Hz.

a. Calculate the wavelength of the sound wave it creates.

b. Calculate the shortest length of pipe with which it will resonate if:

i. the pipe is open

ii. the pipe is closed.

4. An organ pipe is closed at one end. The organist causes the air inside the pipe to vibrate in its fundamental (1st harmonic) mode. The organ pipe is 1.5 m long.

1.5 m

a. On the diagram alongside, draw the 1st overtone (3rd harmonic) wave and calculate its wavelength and frequency.

b. Explain why there is no 2nd harmonic in a closed pipe.

c. The organ was transported in an aircraft. The organist found that the pitch of the note changed when the organ was played aboard the aircraft when the aircraft was at high altitude. Explain why this happened.

5. A guitar string is 0.65 m long. The frequency of the fundamental is 85 Hz.

a. Calculate the wavelength of the standing wave.

b. Calculate the speed of the wave in the string.

c. On the diagram, draw the shape of the standing wave that is the 2nd harmonic. Label the nodes and antinodes, and calculate its frequency.

d. A cello is plucked and plays the same note as the guitar, producing the same frequency. Explain why the note from the cello *sounds* different from the note from the guitar.

e. Explain what the guitarist must do to change the pitch of the note produced by the guitar string.

6. The Indian bamboo flute, a wind instrument, is effectively a simple open pipe. The length of the pipe can be changed by covering or exposing the holes along the top.

a. When all the holes are covered, the effective length of the pipe is the complete length. On diagram **a.**, draw the shape of the fundamental wave.

b. The effective length of the pipe can be changed by uncovering holes. The last uncovered hole becomes the 'end' of the pipe. On diagram **b.**, draw the shape of the new fundamental wave.

c. Explain what happens to the pitch of the note when the flute is blown first with all holes closed then with the last two holes uncovered.

a.

b.

Musical instruments – definitions, formulae and essential learning

travelling wave when a wave travels through a medium, energy is carried from one place to another.

standing wave in a standing wave, the energy oscillates between kinetic and potential but does not move through the medium; a standing wave is created when two waves having the same frequency travel through each other in opposite directions.

nodes and antinodes a standing wave consists of a series of nodes and antinodes – at a node, the medium has zero amplitude vibration; at an antinode, the medium has maximum amplitude vibration.

phase particles vibrating in the medium are in phase if they both move in the same direction and turn at the same time. Particles are out of phase by 180° (have opposite phase) if they turn at the same time but move in opposite directions.

phase in standing waves between any two adjacent nodes in a standing wave, all the particles of the medium vibrate in phase with one another and with opposite phase to the particles on the other side of the nodes.

resonance if the medium in which a standing wave is set up has particular positions that have to be nodes or antinodes (such as anchored strings and fixed-length pipes), only certain frequencies of the wave will have wavelengths that 'fit' in with the restrictions imposed – these frequencies are called resonant frequencies.

fundamental and overtones at the fundamental frequency of a standing wave in a fixed-length medium, the wavelength has its longest possible value – the 1st overtone frequency wave has the second-longest possible wavelength, and so on – the fundamental frequency is the *lowest* of all the possible frequencies that will resonate.

harmonics the first harmonic standing wave is the wave that has the fundamental frequency; all overtones have a frequency that is a whole-number multiple of the fundamental frequency; the harmonic number of an overtone is the whole-number multiple.

resultant wave many different standing waves can be set up in a medium at the same time – the only condition is that each standing wave must have a frequency that resonates; the resultant standing wave is the sum of all the contributing standing waves, and will have a frequency that is the same as the *lowest* frequency contributing wave.

timbre when a stringed or wind instrument is played, the wave generator produces a range of different frequencies, some of which will resonate and produce standing waves which add together to give a resultant wave; it is the particular shape of the resultant wave that gives an instrument its characteristic sound – its timbre.

Musical instruments – topic questions

Answers p. 69

1. The Irish harp

Speed of sound in air = 3.40×10^2 m s^{-1}

The picture shows an Irish harp – an instrument that is played by plucking the strings.

One of the strings of the harp is 43.2 cm long.

a. Calculate the wavelength of the fundamental note produced on this string when it is plucked.

b. On the line below sketch the 3rd overtone (4th harmonic) on this string.

Another harp string is 57.8 cm long and has a mass of 4.62×10^{-4} kg. The tension force in the string is 70.0 N.

The wave speed on this string can be calculated using the formula:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension force and μ is the mass per unit length of the string.

c. By finding the mass per unit length, show that the wave speed on this string is 296 m s^{-1}.

The tension force in the harp string is now increased.

d. State what happens to the size of the wavelength **AND** frequency of the wave on the string.

e. During a concert, a flute is played alongside the harp. Explain why the same note played on a flute **sounds** different to that played on the harp.

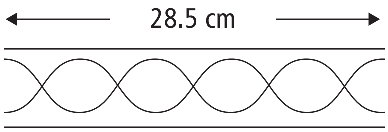
f. A particular flute can be modelled as an open pipe of length 0.61 m. Calculate the lowest possible frequency note that could be played on this flute.

2. The recorder

Answers p. 69

The speed of sound in air is 3.40×10^2 m s^{-1}.

A recorder can be modelled as an open pipe. On one occasion, the note that is played has the following standing wave pattern for one of its overtones (harmonics). The length of the pipe when it plays this note is 28.5 cm.



a. Calculate the wavelength of the standing wave shown in the diagram.

b. Which harmonic (or overtone) is shown in the diagram above?

__________ harmonic OR __________ overtone

c. By first calculating the wavelength that the fundamental standing wave would have in this length of pipe (or otherwise), calculate the frequency of the fundamental standing wave.

d. Explain how the fundamental standing wave is produced in this pipe.

e. Opening or closing holes along the length of the pipe can produce different frequency notes. At first a note is played with all holes closed. Then a note is played with the last hole open.

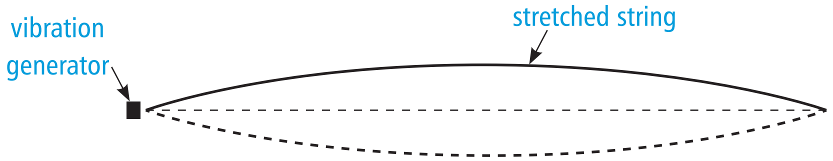
Explain how the frequency of the note produced will change.

3. Standing waves

Answers p. 69

Speed of sound in air = 3.40×10^2 m s^{-1}

When a guitar string is plucked, a standing wave is set up. Standing waves can be demonstrated in the laboratory by vibrating **one end** of a stretched elastic string with the other end fixed. The end that is vibrated can also be considered fixed, because the vibration generator oscillates with very low amplitude.



a. The vibration generator is set at a frequency of 35 Hz. When the string is stretched to a length of 1.2 m, a 1st harmonic (fundamental) standing wave is produced.
Calculate the speed of the wave in the string.

The string is fixed at this length and the frequency of the generator is increased until the 3rd harmonic (2nd overtone) standing wave is produced.

b. Calculate the new frequency of the generator.

c. How does this increase in frequency change the wavelength of the wave on the string, and by what factor?

With the generator still set at the **higher** frequency (producing the 3rd harmonic), the string is **tightened**, keeping the length the same, and the standing wave disappears.

d. Explain why a standing wave does not occur when the string is tightened.

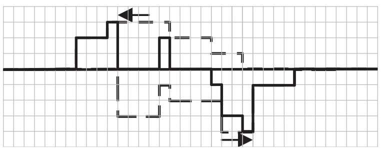
e. The string is now stretched, and when its length reaches 1.8 m, a 2nd harmonic (1st overtone) standing wave is produced by the vibration generator, which is still at the higher setting.
Calculate the new speed of the wave.

f. In the laboratory experiment described at the start of this question, when resonance occurs, only one harmonic is produced in the string. When a guitar string is plucked, several harmonics are produced.
Explain why, in the laboratory experiment, only **one** harmonic is produced, whereas **many** harmonics are produced at **the same time** on a plucked guitar string.

Answers

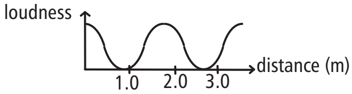
Waves – NCEA Level 2 Physics revision (page 1)

1. a. Amplitude.
   b. By the pitch.
   c. Sound needs a medium to travel through. As there is a vacuum between Earth and the Sun, there is no medium through which the sound can travel.
   d. $v = f\lambda$, which means frequency and wavelength are inversely proportional; rising pitch means increasing frequency and so decreasing wavelength.
   e. Sound is a longitudinal wave, so the particles of the medium vibrate parallel to the direction of movement of the wave about a fixed point.
   f. Sound travels faster in a solid than in a gas, so the sound through the railway line will reach the workman first.
   g. A sound wave is generated by a vibrating object creating a series of compressions and rarefactions of the medium, which is a sound wave.
2. a. $v = f\lambda \Rightarrow \lambda = \dfrac{3.00 \times 10^8}{9.4 \times 10^6} = 31.915 = \mathbf{32\ m}$
   b. $f = \dfrac{1}{T} = \dfrac{1}{2.6 \times 10^{-10}} = 3.846 \times 10^9 = \mathbf{3.8 \times 10^9\ s^{-1}}$
   c. $v = f\lambda \Rightarrow f = \dfrac{3.00 \times 10^8}{5.4 \times 10^{-7}} = 5.556 \times 10^{14} = \mathbf{5.6 \times 10^{14}\ s^{-1}}$
   d. $v = f\lambda,\ f = \dfrac{1}{T} \Rightarrow v = \dfrac{\lambda}{T} = \dfrac{1.3}{4.0 \times 10^{-3}} = 325 = \mathbf{330\ m\ s^{-1}}$
3. a. Each turn of the spring will vibrate in a direction that is at right angles to the length of the spring.
   b. The vibrations of all the turns will have the same amplitude but each turn will reach a maximum displacement slightly later than the turn in front of it, and slightly before the turn before it.
4. a. The displacements of the two pulses must be added at all positions in the medium.
   b.



Standing waves (page 5)

1. a. $v = f\lambda \Rightarrow \lambda = \dfrac{340}{85} = \mathbf{4.0\ m}$
   b. 4.0 m Same as that of the travelling wave.
   c. i. An antinode.
      ii. At all times the individual wave displacements are such that the combination of their displacements results in zero displacement of the medium.
   d. i. The loudness of the sound changes as the amplitude of the vibration of the air molecules (the medium) changes. In a standing wave, each part of the medium vibrates with a different amplitude to the positions either side, and so, as the girl moves her position, she will experience the sound wave at different amplitudes.
      ii. An antinode.
   e. In a standing wave, the amplitude of vibration of the medium stays constant at any position. As loudness depends on amplitude, the loudness also stays constant.
   f. In a travelling wave, all positions in the medium vibrate with the same amplitude, and hence the loudness does not change.
2. a. AE or BF.
   b. Nodal: *two of* A, C and E; Antinodal: *two of* B, D and F.
   c. In phase.
   d. The turns between A and C are in phase with each other and have opposite phase with the turns between A and C.
   e. At B and D, the turns have opposite phase (out of phase by 180°). At B and F, the turns are in phase.
   f. At A there is no energy change. The medium is stationary.
      At B, the energy changes between kinetic and potential energy.
3. a. The same.
   b. The standing wave has the sum of the amplitudes of the travelling waves.
   c. In a standing wave, each position vibrates with its own constant amplitude that varies between zero and maximum. In a travelling wave, all positions in the medium vibrate with the same amplitude.
   d. In a standing wave, within adjacent nodes, all positions in the medium vibrate in phase. In a travelling wave, each position in the medium lags the position in front and leads the position behind.
   e. In a standing wave, the energy does not move. In a travelling wave, energy is carried from one place to another in the direction of travel of the wave.
4. a. The reflected wave will have the same frequency as the incident wave and will be travelling in the opposite direction to it.
      These are the conditions for a standing wave to be set up.
   b. Nodes.
   c. At the end wall, the wave has a node because the rigidity of the wall means the medium is not able to vibrate. As the miner is standing close to a node, the sound will be loud.
      A position of no vibration is a node.
   d.



   e. $v = f\lambda,\ \lambda = 3.6\ m \Rightarrow v = 95 \times 3.6 = 342 = \mathbf{340\ m\ s^{-1}}$
   f. If the frequency doubles the wavelength halves, and so the positions of maximum loudness would be half the distance apart.

Standing waves in strings and pipes (page 10)

1. a. i. $1\frac{1}{2}\lambda = L \Rightarrow \lambda = \frac{2L}{3} = \frac{2 \times 1.2}{3} = \mathbf{0.80\ m}$

ii. $2\lambda = L \Rightarrow \lambda = \frac{1}{2}L = \mathbf{0.60\ m}$

There are 2 waves fitted into the length L

b. $v = f\lambda$, $\lambda = 2L \Rightarrow v = 756 = \mathbf{760\ m\ s^{-1}}$

c. i. $f = \frac{v}{\lambda} \Rightarrow f_2 = \frac{756}{1.2} = \mathbf{630\ Hz}$

ii. $f_3 = \frac{756}{0.80} = 945 = \mathbf{950\ Hz}$

iii. $f_4 = \frac{756}{0.60} = 1\,260 = \mathbf{1\,300\ Hz}$

d. i. L is 1¼ wavelengths, and so $L = \frac{5\lambda_7}{4} \Rightarrow \lambda_7 = \frac{4L}{5}$

$= \frac{4 \times 1.2}{5} = \mathbf{0.96\ m}$

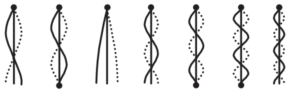
ii. L = 1¾ wavelengths, and so $L = \frac{7\lambda_8}{4} \Rightarrow \lambda_8 = \frac{4L}{7}$

$= \frac{4 \times 1.2}{7} = 0.6857 = \mathbf{0.69\ m}$

e. i. $f = \frac{v}{\lambda} \Rightarrow f_1 = \frac{756}{4 \times 1.2} = \mathbf{160\ Hz}$

ii. $f_2 = \frac{3 \times 756}{4 \times 1.2} = \mathbf{470\ Hz}$

2. a. i. ii. iii. iv. v. vi. vii.



b. i. $\frac{3}{4}\lambda = 72 \Rightarrow \lambda = \mathbf{96\ cm}$

ii. $\lambda = \mathbf{72\ cm}$

iii. $\frac{1}{4}\lambda = 72 \Rightarrow \lambda = \mathbf{2.9\ m}$

iv. $1\frac{1}{4}\lambda = 72 \Rightarrow \lambda = \mathbf{58\ cm}$

v. $1\frac{1}{2}\lambda = 72 \Rightarrow \lambda = \mathbf{48\ cm}$

vi. $2\lambda = 72 \Rightarrow \lambda = \mathbf{36\ cm}$

vii. $2\frac{1}{4}\lambda = 72 \Rightarrow \lambda = \mathbf{32\ cm}$

3. a. $\frac{1}{4}\lambda = L \Rightarrow \lambda = \mathbf{0.84\ m}$

b. i.

ii. $1\frac{3}{4}\lambda = L \Rightarrow \lambda = \mathbf{0.12\ m}$

iii. $v = f\lambda = 5.5 \times 0.12 = \mathbf{0.66\ m\ s^{-1}}$

c. $v = f\lambda \Rightarrow f = \frac{0.66}{0.84} = 0.7857 = \mathbf{0.79\ s^{-1}}$

d. $\frac{3}{4}\lambda = L \Rightarrow \lambda = 0.28$ m, $v = f\lambda \Rightarrow f = \frac{0.66}{0.28} = 2.357 = \mathbf{2.4\ s^{-1}}$

4. a. A node.

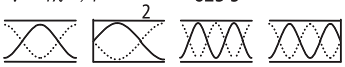
b. An antinode.

c. 

d. $\lambda = L = \mathbf{2\ m}$

e. $v = f\lambda \Rightarrow f = \frac{1\,250}{2} = \mathbf{625\ s^{-1}}$

5. a.



i. ii. iii. iv.

b. i. $1\lambda = 0.500 \Rightarrow f = \frac{340}{0.500} = \mathbf{680\ s^{-1}}$

ii. $\frac{3}{4}\lambda = 0.500 \Rightarrow f = \frac{340 \times 3}{4 \times 0.500} = \mathbf{510\ s^{-1}}$

iii. $2\lambda = 0.500 \Rightarrow f = \frac{340 \times 2}{0.500} = \mathbf{1\,360\ s^{-1}}$

iv. $1\frac{3}{4}\lambda = 0.500 \Rightarrow f = \frac{340 \times 7}{4 \times 0.500} = \mathbf{1\,190\ s^{-1}}$

Musical instruments (page 15)

1. a. i.

$\lambda_{fund} = 2L \Rightarrow f_0 = \frac{v}{2L}$

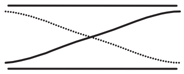
ii. λ_{1st}

$\lambda_{1st} = L \Rightarrow f_1 = \frac{v}{L} = \frac{2v}{2L} = 2 \times f_0$

λ_{2nd}

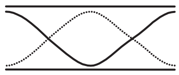
$\lambda_{2nd} = \frac{2L}{3} \Rightarrow f_2 = \frac{3v}{2L} = 3 \times f_0$

b. i.



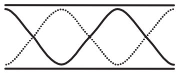
$\lambda_{fund} = 2L \Rightarrow f_0 = \frac{v}{2L}$

ii. λ_{1st}



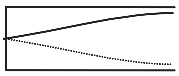
$\lambda_{1st} = L \Rightarrow f_1 = \frac{v}{L} = \frac{2v}{2L} = 2 \times f_0$

λ_{2nd}



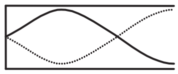
$\lambda_{2nd} = \frac{2L}{3} \Rightarrow f_2 = \frac{3v}{2L} = 3 \times f_0$

c. i.



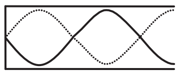
$\lambda_{fund} = 4L \Rightarrow f_{fund} = \frac{v}{4L}$

ii. λ_{1st}



$\lambda_{1st} = \frac{4L}{3} \Rightarrow f_1 = \frac{3v}{4L} = 3 \times f_0$

λ_{2nd}



$\lambda_{2nd} = \frac{4L}{5} \Rightarrow f_2 = \frac{5v}{4L} = 5 \times f_0$

d. The harmonic number of a standing wave is the number of times its frequency is greater than the frequency of the 1st harmonic. In a closed pipe there is no wave that has twice, or 4 times or 6 times, etc., the frequency of the 1st harmonic so these harmonics do not exist.

2. a. 2λ in $L \Rightarrow \lambda = \frac{1}{2} \times 4.0 = \mathbf{2.0\ m}$

b. $2\frac{1}{2}\lambda$ in $L \Rightarrow \lambda = \frac{2}{5} \times 1.2 = \mathbf{0.48\ m}$

c. ¾λ in $L \Rightarrow \lambda = \frac{4}{3} \times 1.5 = \mathbf{2.0\ m}$ 2nd harmonic does not exist.