Achievement Standard 91523

Demonstrate understanding of wave systems

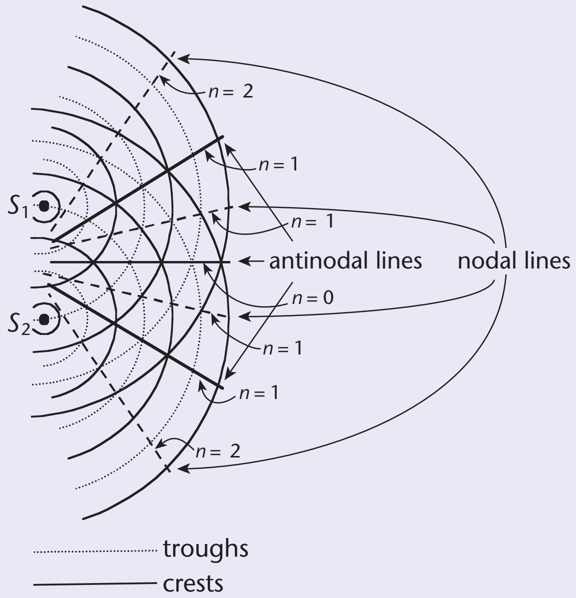
PHYSICS

3.3

Externally assessed 4 credits

Interference

Interference happens when two waves having the same frequency, each from individual point sources, pass through each other. Because of the way the troughs and crests from each of the waves add together when they meet, a pattern is formed which consists of lines along which the wave amplitude is increased in between lines along which the wave amplitude has been reduced to nothing.



Along the **antinodal lines**, the two waves arrive *in phase*. This means crests from one wave source meet crests from the other and so the combined wave along this line will have high amplitude. This type of interference is called **constructive** interference.

Along the **nodal lines**, the two waves arrive with *opposite phase* (180° out of phase). This means crests from one wave source meet troughs from the other forming a line of low-amplitude wave. This type of interference is called **destructive interference**.

If the waves from each source have the *same* amplitude, constructive interference will give a wave that has amplitude that is *double* the amplitude of the individual waves. Destructive interference will produce a wave that has *zero* amplitude and so the wave will be completely destroyed.

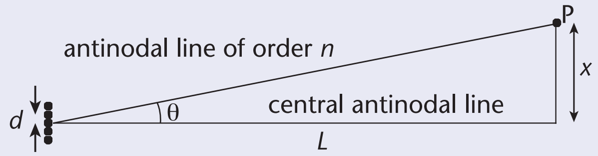
Nodal and antinodal lines are numbered, independently, from the middle outward. The number is called the **order number**, *n*.

The way nodal and antinodal lines are experienced depends on what sort of waves produced them. For light waves, at any position on a nodal line there will be darkness, at any position on an antinodal line there will be brightness; for sound waves, nodal lines are experienced as silence, antinodal lines as loudness.

The pattern can be sharpened and spread by using multiple sources instead of just two.

If the waves are light waves, an additional condition for the interference pattern to be produced is that the waves must be **coherent**. Splitting a single light wave into two or more sources most readily achieves coherence. This can be done by shining light through a series of narrow slits, allowing diffraction to create a point source of the wave at each slit.

The angle that any outer antinodal line ($n > 0$) makes with the central antinodal line ($n = 0$) depends on the wavelength of the wave.



ESA PUBLICATIONS ISBN 978-1-988548-81-4

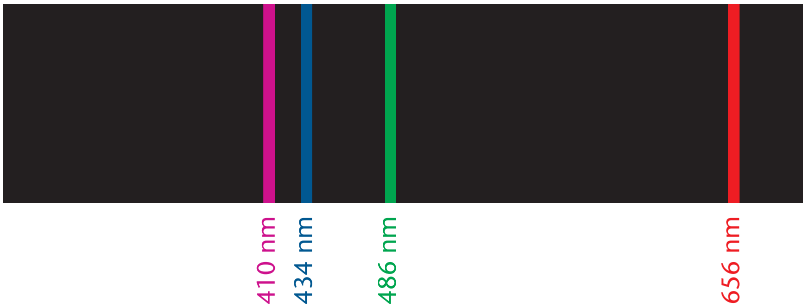
Questions: Interference

Question One: Diffracting spectral lines

Year 2018
Ans. p. 107

All elements emit a number of distinct fixed wavelengths of light known as spectral lines that are unique to each element. Hydrogen emits four visible light lines, as shown below.

The visible spectrum of hydrogen [1 nm = 1 × 10^{-9} m]



Light from a hydrogen source can be passed through a diffraction grating to form an interference pattern. The wavelength of each spectral line can then be determined by measuring the angle to its first order maximum.

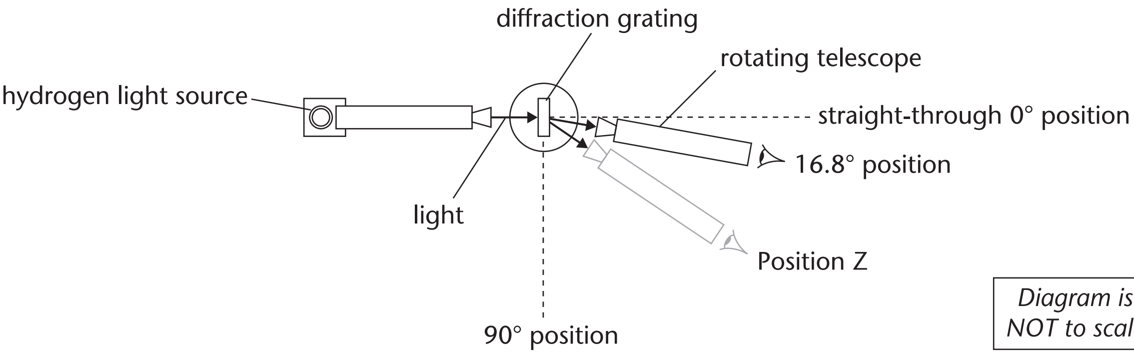


Diagram is NOT to scale

a. The lines on a diffraction grating are spaced 1.68 × 10^{-6} m apart.

Show that the wavelength of the spectral line with a first order maximum at 16.8° is 486 nm.

 ISBN 978-1-988548-81-4

3.3

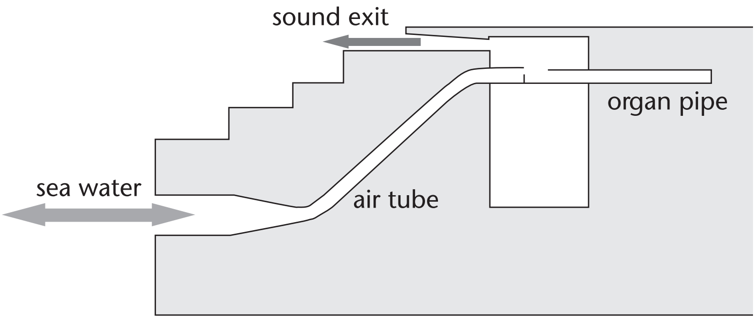
Year 2014
Ans. p. 109

Question Five: The Sea Organ

The Sea Organ in Zadar, Croatia, is a musical instrument that creates its musical notes through the action of sea waves on a set of pipes that are located underneath the steps shown in the picture. The sound from the pipes comes out through the regular slits in the vertical part of the top step.

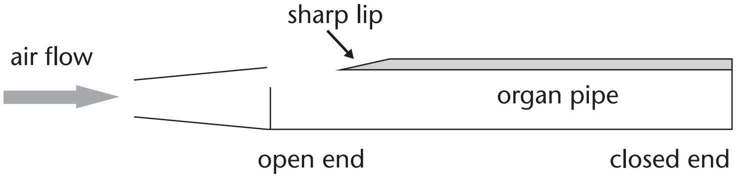
To produce a sound, the organ pipes must have air blown into them, so each organ pipe is connected to the top end of a tube, as shown in the diagram on the right.

The action of the waves pushes water in and out of a tube, creating a flow of air at the upper end of the tube.



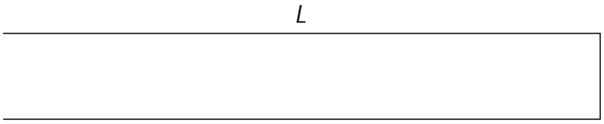
The diagram on the right shows the inside of an organ pipe.

These organ pipes have one closed end.



a. Calculate the length, L, of an organ pipe, with one closed end, that produces a fundamental standing wave of wavelength 2.60 m.

You may find the diagram on the right useful.



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Achievement Standard 91524

Demonstrate understanding of mechanical systems

PHYSICS

3.4

Externally assessed 6 credits

Translational motion

Centre of mass

The term *system* is used to describe two or more objects when they need to be considered as a single object.

The motion of a system can be treated in the same way as the motion of a single object provided it is the **centre of mass** of the system that is considered.

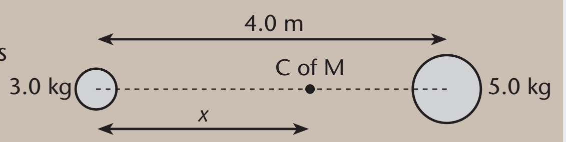
The centre of mass of a system is the position at which (if it were possible to do so) objects would balance if pivoted. It is therefore also the position about which the **torques** of the individual objects will be balanced.

About the centre of mass: $\tau_{\text{anticlockwise}} = \tau_{\text{clockwise}}$

where: $\tau = Fr$

Example

Q. A system consists of two objects 4.0 m apart. Calculate the distance, x, of the centre of mass of the system from the 3.0 kg object.



A.

$\tau_{\text{anticlockwise}} = \tau_{\text{clockwise}}$	balanced torques
$\Rightarrow 3.0 \times g \times x = 5.0 \times g\,(4.0 - x)$	substituting $\tau = Fr$ and $F = mg$
$\Rightarrow 3.0 \times x = 20 - 5.0 \times x$	cancelling g and expanding the bracket
$\Rightarrow 8.0 \times x = 20$	
$\Rightarrow x = \dfrac{20}{8.0} = 2.5\text{ m}$	

Momentum

The momentum of a system can be found from its total mass and the velocity of the centre of mass.

$$p_{\text{system of objects A and B}} = (m_A + m_B)\, v_{\text{centre of mass}}$$

The momentum of a system can also be found from the sum of the momentums of the individual objects.

$$(m_A + m_B)\, v_{\text{centre of mass}} = m_A v_A + m_B v_B$$

If there are no external forces acting on the objects in a system it is called an **isolated** system. Momentum is conserved in an isolated system, so the sum of the momentums of the individual objects stays constant. This means the *velocity* of the centre of mass will stay *constant* no matter what happens to the individual objects.

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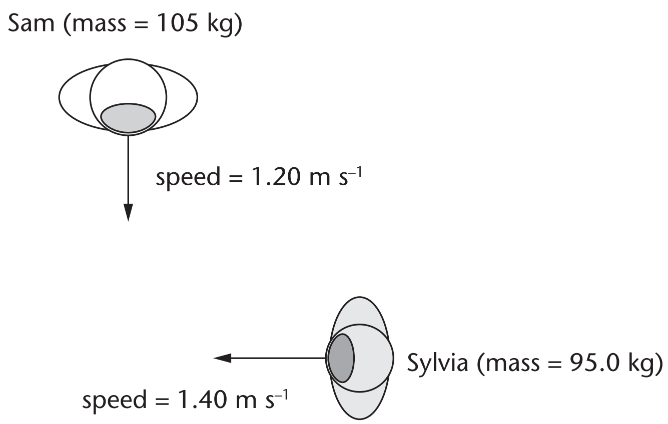
Questions: Translational motion

Question One: Space walk

Year 2017
Ans. p. 111

Two astronauts, Sylvia and Sam, are on a mission to another planet. During their journey they are doing a 'space walk' outside their spaceship.

At one time they are moving freely as shown in the diagram below. They collide and stick together.



a. Calculate the distance between Sam and the centre of mass of the system when he and Sylvia are 4.80 m apart.

b. Describe what happens to the centre of mass of the system as the astronauts move closer together and then collide.

c. Calculate the astronauts' combined speed after they collide.

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Year 2015
Ans. p. 113

Question Three: Satellites

Mass of Earth = 5.97×10^{24} kg, universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}

Digital television in New Zealand can be accessed by using a satellite dish pointed at a satellite in space. The satellite used to transmit the signals appears to stay still above the equator. The satellite, with a mass of 300 kg, is actually travelling around the Earth in a geostationary orbit at a radius of 4.22×10^7 m from the centre of the Earth.

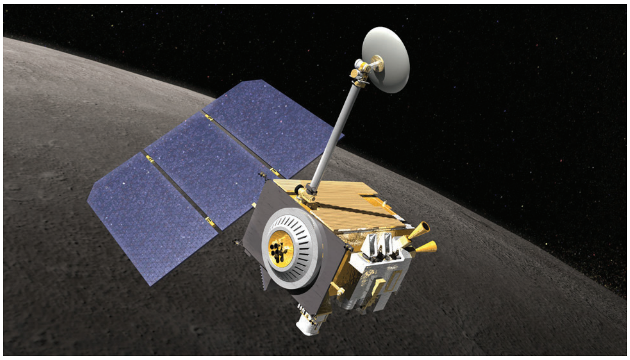
a. Name the force that is keeping the satellite in this circular orbit, and state the direction in which this force is acting.

b. Calculate the force acting on the satellite.

c. Show that the speed of the satellite is 3.07×10^3 m s^{-1}.

d. Kepler's law states that, for any orbiting object, $T^2 \propto r^3$, where r is the radius of the orbit, and T is the time period for the orbit.

NASA uses a robotic spacecraft to map the Moon. The Lunar Reconnaissance Orbiter orbits the Moon at an average height of 50.0×10^3 m with a period of 6.78×10^3 s. The Moon has a radius of 1.74×10^6 m.



Use Kepler's law to estimate the mass of the Moon. In your answer you should:

- use the relevant formulae to derive Kepler's law
- use Kepler's law to determine the mass of the Moon.

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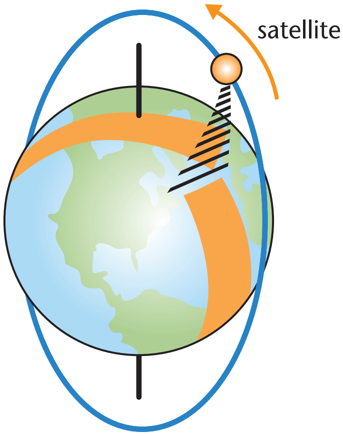
Questions: Rotating systems

Year 2018
Ans. p. 114

Question One: Rotating satellite



Weather satellites can be launched into orbits that circle around the North and South poles. This enables the satellite's camera to view the whole of the Earth's surface as the Earth spins underneath. The orbital period of a typical weather satellite is 101 minutes.

a. Show that in order to keep the camera pointed towards the Earth's surface, the satellite must spin at an angular velocity of 1.04×10^{-3} rad s^{-1}.

This angular velocity is achieved by firing two 5.00 N thrusters attached on opposite sides of the satellite's 1.60 m diameter.

b. Explain why two thrusters must be used rather than simply one on one side of the satellite with double the thrust force.

c. The thrusters are fired for 6.48×10^{-3} s to set the satellite rotating at the required angular velocity.

Show that the rotational inertia of the satellite is 50.0 kg m^2.

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Oscillating systems

Motion and force

Simple harmonic motion (**SHM**) is oscillatory (back and forth) motion in a straight line through a central (mean or equilibrium) position.

The **amplitude**, A, of a SHM is the maximum displacement from the central position.

The **period**, T, of a SHM is the time for one complete oscillation to be completed.

The **frequency** of a SHM can be expressed in two ways.

- The *cyclic* frequency, f, is the number of oscillations completed per second and is related to the period by:

$$f = \frac{1}{T}$$

- The *angular* frequency, ω, is related to the cyclic frequency by:

$$\omega = 2\pi f$$

An object moving with simple harmonic motion is accelerating all the time. As with any other motion, the acceleration is caused by an unbalanced force ($F = ma$). The direction of the force is the same as the direction of the acceleration.

Not all straight-line oscillatory motion is SHM. An object has SHM if:

- its speed, v, is zero when its displacement, y, from the mean position (at the end point) is maximum, and its speed is maximum when its displacement is zero (at the centre)
- its acceleration, a (and therefore the force acting on it), is maximum when its displacement from the mean position (at the end point) is maximum, and its acceleration is zero when its displacement is zero (at the centre)
- its acceleration (and therefore the force acting on it) is always proportional to its displacement from the mean position
- its acceleration (and therefore the force acting on it) is always in a direction *opposite* to its displacement, i.e. always towards the *centre* of the motion. (Because the force always acts towards the centre of the motion, it is called a *restoring* force.)

These rules of motion for SHM are expressed in the following formulae:

Time, t, is zero at the equilibrium position	Time, t, is zero at the end position
$y = A \sin \omega t$	$y = A \cos \omega t$
$v = A\omega \cos \omega t$	$v = -A\omega \sin \omega t$
$a = -A\omega^2 \sin \omega t$	$a = -A\omega^2 \cos \omega t$
$a = -\omega^2 y$ ($F = -m\omega^2 y$)	$a = -\omega^2 y$ ($F = -m\omega^2 y$)

The restoring force acting on a mass bouncing on a spring that has spring constant, k, is:

$$F = -ky$$

The negative in the formula is because the force, F, is always opposite to the displacement, y.

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Achievement Standard 91526

Demonstrate understanding of electrical systems

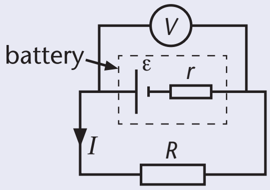
PHYSICS

3.6

Externally assessed 6 credits

Resistors in DC circuits

Internal resistance



The **internal resistance** of a battery changes some of the energy per coulomb generated inside the battery (called the **emf**, ε) into heat energy, leaving less energy per coulomb (voltage, V) available to the external circuit that is connected to the terminals of the battery.

If the *external* circuit only is considered, the terminal voltage (voltage, V, across the battery terminals) is the current × external resistance ($V = IR$). If the *internal* circuit only is considered, the terminal voltage is the emf reduced by the voltage across the internal resistance ($V = \varepsilon - Ir$).

If the *whole* circuit is considered, the emf is the current × total resistance ($\varepsilon = I(R + r)$).

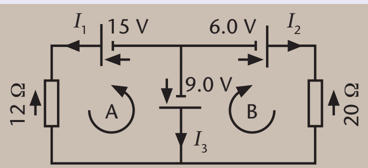
Kirchhoff's laws

Kirchhoff's laws state:

- *the sum of the currents at a junction is zero* – if the currents entering a junction are counted as positive and currents leaving the junction are counted as negative, the sum of the currents will be zero
- *for any closed loop in a circuit, the sum of the potential differences in the loop is zero* – if components around a loop in a circuit are considered one after the other, and voltages are counted as positive if potential increases through the component and negative if potential decreases, sum of all voltages will be zero.

Kirchhoff's laws are often used to solve circuit problems in which there is more than one loop and two or more of the loops have a voltage supply.

Example



Q. Calculate the currents I_1, I_2, and I_3 in the circuit.

A. To apply Kirchhoff's voltage law, the voltage across each component must be put as either positive or negative.

In the diagram, the arrows at each component all go from low potential to high potential. (Current loses energy through a resistor, gains energy through a battery.)

Consider the voltages going anticlockwise around loop A:

$+15 - (12 \times I_1) - 9 = 0$ putting $V = IR$ for the resistor

$I_1 = \frac{-6.0}{-12}$ rearranging

$I_1 = 0.50$ A

Consider the voltages going clockwise around loop B:

$+6.0 - (20 \times I_2) - 9 = 0$ putting $V = IR$ for the resistor

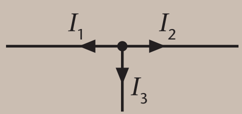
$I_2 = \frac{3.0}{-20}$ rearranging

$= -0.15$

negative sign indicates current *direction is wrong*

$I_2 = 0.15$ A

Consider Kirchhoff's current law at the junction marked:



$I_1 + I_2 + I_3 = 0$

$0.50 - 0.15 + I_3 = 0$

$I_3 = 0.15 - 0.50$

$= 0.35$ A (in the opposite direction)

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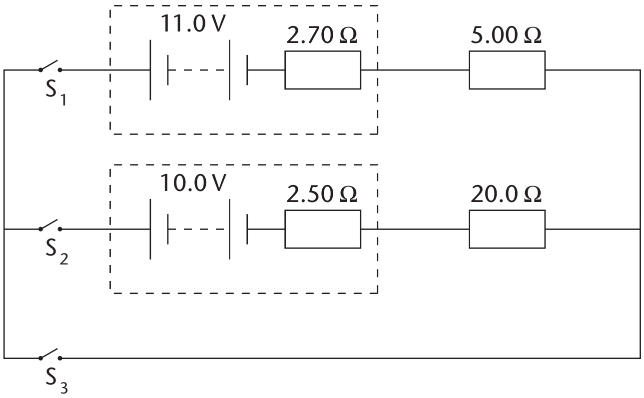
Questions: Resistors in DC circuits

Year 2014
Ans. p. 117

Question One: Batteries

The circuit diagram shows two batteries connected into a circuit. The internal resistance, r_1, of the 11.0 V battery is 2.70 Ω, and the internal resistance, r_2, of the 10.0 V battery is 2.50 Ω.



a. Switches S_1 and S_2 are closed and switch S_3 is left open. Show that the current in the circuit is 0.0331 A.

b. In which direction will the current be flowing through switch S_1? Explain your answer.

c. Switch S_3 is now closed so all three switches are closed. Show, using Kirchhoff's laws, that the current through switch S_3 is 1.87 A.

d. Switch S_1 is now opened, leaving switches S_2 and S_3 closed. After this circuit has been operating for some time, the 10.0 V battery starts to go flat. A student suspects that this is caused by an increase in the internal resistance. Explain what effect a changing internal resistance has on the power delivered to the 20.0 Ω resistor.

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Answers and explanations

Achievement Standard 91523 (Physics 3.3): Demonstrate understanding of wave systems

3.3 Interference

Question One: Diffracting spectral lines p.3

a. 486 nm is 486×10^{-9} m

$d\sin\theta = n\lambda$, $n = 1$

$\Rightarrow \lambda = 1.68 \times 10^{-6} \times \sin 16.8$

$= 4.8557 \times 10^{-7}$

$= 486 \times 10^{-9}$ m (A)

b. $d\sin\theta = n\lambda$; if d and n are constant,

$\lambda \propto \sin\theta$

$\Rightarrow \lambda \propto \theta$

When the telescope rotates, θ increases so λ must also increase. Therefore, the wavelength of the next spectral line is 656 nm. (M)

c. $\sin\theta = \frac{n\lambda}{d} \Rightarrow \frac{n\lambda}{d} < 1$ The sine of any angle must be < 1

$\frac{n\lambda}{d} < 1 \Rightarrow n < \frac{d}{\lambda} \Rightarrow n < \frac{1.68 \times 10^{-6}}{656 \times 10^{-9}} \Rightarrow n < 2.56$

As n is an integer, the maximum number of orders that can be seen is 2. (M)

d. Because d for the double slit is the same as d for the grating, the spectral lines will be seen at the same angles. The multiple interference that takes place with a diffraction grating means the lines are much sharper, so the width of the lines from the double slit will be wider. A diffraction grating has multiple slits to let the light through. As the double slit has only two slits, far less light gets through so the lines are far less bright. (E)

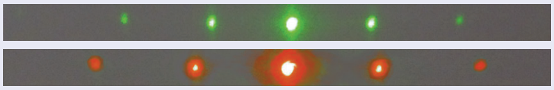
Question Two: Diffraction patterns p.5

a. $d\sin\theta = n\lambda$, $n = 1 \Rightarrow \lambda = 2.00 \times 10^{-6} \times \sin 15.4 = 5.3111 \times 10^{-7}$

$= 5.31 \times 10^{-7}$ m (A)

b. The laser light is diffracted at all slits and so travels outwards from all slits. This means that all points on the wall are receiving light from all the slits. At the position of the first antinode, on either side, the difference in the distances travelled by the light from each of the slits varies in such a way that, when they combine at the point, their phases are such that maximum combined amplitude is obtained. (M)

c.



If red light has a lower frequency than green light, as $v = f\lambda$ and v is constant, red light has a longer wavelength than green light. $d\sin\theta = n\lambda$; so, if λ increases, θ increases and so, apart from the central antinodal spot, all the bright spots are further away from the central bright spot. (M)

d. If three complete spectra fit each side of the central line, the red line of the 3rd spectrum, which will be the furthest line out, must have a diffraction angle less than 90°.

$d\sin\theta = n\lambda$ and $v = f\lambda \Rightarrow \sin\theta = \frac{nv}{df}$

$\sin 90° = 1 \Rightarrow \sin\theta < 1 \Rightarrow \frac{nv}{df} < 1 \Rightarrow d > \frac{nv}{f}$

$\frac{nv}{f} = \frac{3 \times 3.00 \times 10^{8}}{4.3 \times 10^{14}} = 2.0930 \times 10^{-6} \Rightarrow d > 2.1 \times 10^{-6}$ m (E)

Question Three: Diffraction gratings p.7

a. $n\lambda = \frac{dx}{L} \Rightarrow \lambda = \frac{1.28 \times 10^{-4} \times 0.0100}{2.10} = 6.0952 \times 10^{-7}$

$= 6.10 \times 10^{-7}$ m (A)

b. $d\sin\theta = n\lambda$

500 lines per mm = 500×10^{3} line per m $\Rightarrow d = \frac{1}{500 \times 10^{3}}$ m

$\Rightarrow \sin\theta = \frac{n\lambda}{d} = \frac{1 \times 6.0952 \times 10^{-7}}{d}$

$= 1 \times 6.0952 \times 10^{-7} \times 500 \times 10^{3} = 0.30476$

$\Rightarrow \theta = 17.744 = 17.7°$ (M)

c. $d\sin\theta = n\lambda$. As d is constant, the distance between the bright spots depends on the angle θ, through which they have been bent. The sin of an angle changes in the same way as the angle itself – if one increases, so does the other. Therefore, if the wavelength is shorter, the angle through which the light has been bent is less, and so the distance between the bright spots decreases. (M)

d. The wavelengths of the colours that make up white light are all different from each other, and so each colour will be bent through a different angle and therefore constructive interference will occur at different angles for different colours. At the centre of the pattern, the light has gone straight through the grating without being bent, so the colours have not been spread and so the colour that is seen is white. Each side of the central white band, blue light, which has the shortest wavelength, is bent the least; red light, which has the longest wavelength, is bent the most. Between these two colours the other colours have differing wavelengths, and therefore differing amounts of bending, which result in the colours being spread out, with the blue light being closest to the white central band. When the colours are spread, a spectrum is seen. Between the white central band and the start of the first spectrum there is no light because all the light that has arrived in this region has undergone destructive interference. (E)

Question Four: Investigating laser light p.8

a. $n\lambda = d\sin\theta \Rightarrow d = \frac{n \times \lambda}{\sin\theta} = \frac{1 \times 532 \times 10^{-9}}{\sin 26.0}$

$= 1.2136 \times 10^{-6} = 1.21 \times 10^{-6}$ m (A)

b. $n\lambda = d\sin\theta$ which means the slit spacing d is inversely related to the diffraction angle θ

The diffraction angles will be smaller when the light is shone through the edges than when it is shone through the middle and so there will be more beams of light produced when the light is shone through the edges and the beams will have a smaller angle between them. (A)

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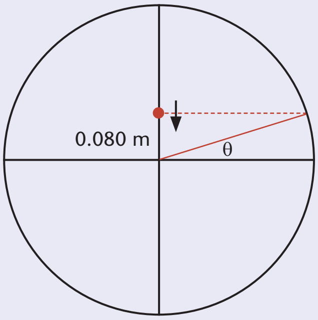
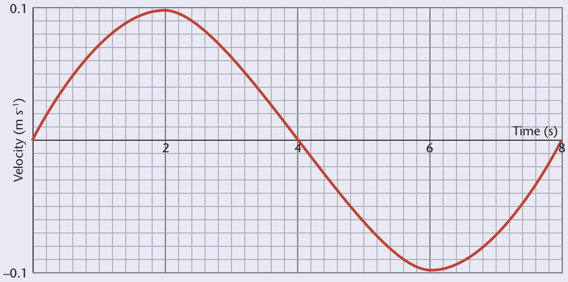
$\theta = \omega t$, $\sin\theta = \dfrac{0.080}{0.120} \Rightarrow \theta = 0.729728$ rad

$\Rightarrow t = \dfrac{0.729728}{0.785398} = 0.929118 = 0.93$ s



(M)

c. $v_{max} = A\omega = 0.120 \times 0.785398 = 0.09425$ m s^{-1}



(M)

d. Maximum restoring force = 4.40 N = $m \times a_{max}$

$a_{max} = A\omega^2 \Rightarrow F_{max} = m \times 0.120 \times 0.785398^2$

$\Rightarrow m = \dfrac{4.40}{0.120 \times 0.785398^2} = 59.441 = 59.4$ kg

The mass calculated includes the mass of the seat, the mass of the spring and the mass of the gear Sylvia is wearing. The seat is stated to be lightweight so its mass can be assumed to be negligible, but the mass of the spring and the mass of Sylvia's gear must also be assumed to be negligible. (M)

Question Three: Bouncing bumble bee

p. 64

a. An oscillatory motion is simple harmonic motion if the force causing the motion is proportional to the displacement from the central position and if the force is always directed towards the central position (in the opposite direction to the displacement). (A)

b. The timing starts when the bumble bee is at maximum displacement and so the form of the equation that has to be used is $a = -A\omega^2 \cos \omega t$.

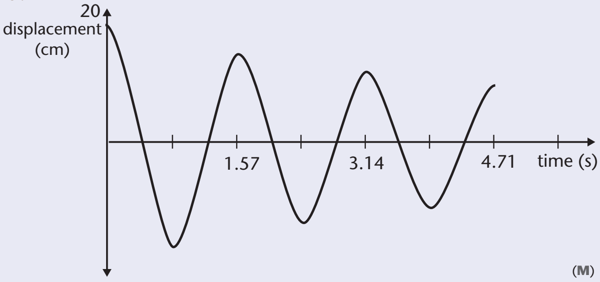
$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{1.57} = 4.0020$

$\Rightarrow a = -A\omega^2 \cos \omega t = -0.100 \times 4.0020^2 \times \cos(4.0020 \times 0.25)$

$= 0.864677 = 0.86$ m s^{-2} (E)

c. The phenomenon is resonance. For resonance to occur, Tom's pushes must have the same frequency as the natural frequency of the SHM. Also, they must be in the direction in which the bumble bee is moving – because a force in the direction of motion causes the speed and hence the kinetic energy of the system to increase. A regular input of energy into the system causes the amplitude to increase, and so Tom's repeated pushes will cause the bumble bee's motion to develop a larger amplitude. (M)

d.



(M)

Question Four: Gravity elevators

p. 65

a. Both the scales and the passenger are accelerating downwards at the same rate – the acceleration of gravity – and so the passenger will not be exerting any force on the scales. (A)

b. i. $a_{max} = -1.54 \times 10^{-6}\, y_{max}$

y_{max} is the radius of Earth.

$= -1.54 \times 10^{-6} \times 6.38 \times 10^6$

$= 9.8252 = 9.83$ m s^{-2} (M)

ii $v_{max} = A\omega$, $\omega^2 = 1.54 \times 10^{-6}$

$a_{max} = -A\omega^2$
$= -A \times 1.54 \times 10^{-6}$

$\Rightarrow v_{max} = 6.38 \times 10^6 \times \sqrt{1.54 \times 10^{-6}}$

$= 7\,917.37 = 7\,920$ m s^{-1} (M)

c. In simple harmonic motion the restoring force, and hence the acceleration, always acts towards the centre of the motion and so will always act in the opposite direction to the displacement. Also, the size of the restoring force, and hence the acceleration, is proportional to the displacement from the equilibrium position. In the equation $a = -1.54 \times 10^{-6}\, y$, the negative sign indicates acceleration is in the opposite direction to displacement, and the constant value indicates acceleration is proportional to displacement. (M)

d. N to S is half a full oscillation, so it will take half a period to travel this distance.

$f = \dfrac{1}{T}$ and $\omega = 2\pi f \Rightarrow T = \dfrac{2\pi}{\omega} \Rightarrow \dfrac{T}{2} = \dfrac{\pi}{\sqrt{1.54 \times 10^{-6}}}$

$= 2\,531.57 = 2\,530$ s (E)

Question Five: The pendulum

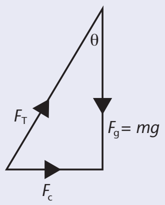
p. 66

a. The pendulum bob takes half a period to travel from one side to the other.

$T = 2\pi\sqrt{\dfrac{l}{g}} \Rightarrow \dfrac{T}{2} = \pi\sqrt{\dfrac{l}{g}} = \pi \times \sqrt{\dfrac{1.55}{9.81}} = 1.2488 = 1.25$ s (A)

b. The speed of the bob is zero when it is released. While the bob is travelling from the point of release to the centre, the forces acting on the bob are gravity and tension in the cord. The direction of travel of the bob has to be at right angles to the cord (tangential to the circular motion), and so the tension force does not affect its motion. There will be a component of gravity acting tangentially that will cause the bob to accelerate towards the centre. When the central position is reached, both gravity and tension are at right angles to the bob's motion so neither force has any effect on its motion, so the bob is instantaneously travelling at a constant speed. (E)

c. i. $\sin\theta = \dfrac{0.290}{1.55} \Rightarrow \theta = 10.783°$



The bob is moving in a horizontal circle, and so the centripetal force is horizontal. The centripetal force is the resultant force acting on the bob, so is the addition of the gravity force and the tension force.

$\cos\theta = \dfrac{mg}{F_T} \Rightarrow F_T = \dfrac{1.80 \times 9.81}{\cos 10.783} = 17.975 = 18.0$ N (M)

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