Example

Q. A family decides to have three children. Explain how this situation could be modelled using a coin. (Assume half of all babies born are female and half are male.)

A. A fair coin has two equally likely sides, so is useful for modelling situations involving a probability of $\frac{1}{2}$.

Each time the coin is flipped, the outcome corresponds to the sex of the baby. For example:

- Heads: 'baby is female' — p(head) = p(female baby) = $\frac{1}{2}$
- Tails: 'baby is male' — p(tail) = p(male baby) = $\frac{1}{2}$

In each trial, toss the coin three times and record the results. For example, using H for Heads, and T for Tails:

HHT corresponds to (Female, Female, Male), i.e. two females and one male.

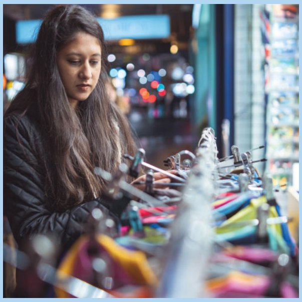
TTT corresponds to (Male, Male, Male), i.e. three males.

A spinner is a useful tool for simulations as it can have its face divided up into any number of sectors, of any size (depending on the angle at the centre of the spinner).

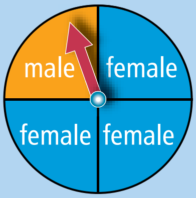
Example

Q. How can the following situation be simulated?

25% of customers in a shop are male. At a checkout, how many customers will need to be served on average in order to get two male customers in the group?



A. A spinner could be used: divide the face into quarters, with one quarter representing male customers, and the other three quarters representing female customers.



In each trial the arrow on the spinner is spun until two 'males' result.

For each trial, the total number of spins is counted.

For example, a trial resulting in two males might be FMFFM, which has 5 spins.

The average number of spins (customers) can then be calculated.

 ISBN 978-1-988586-71-7

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1. a. A fair die is rolled. What is the probability of getting:

i. a five?

ii. a number that is a 2, 3 or 4?

iii. a number greater than 1?

iv. an odd number?

b. In a simulation Zac uses a die to model a situation in which a student is chosen from a class. He lets the numbers 1 and 2 represent choosing a boy; and the numbers 3, 4, 5, 6 represent choosing a girl. In Zac's experiment what is the probability of

i. choosing a boy? ______

ii. choosing a girl? ______

2. A spinner has its face divided into ten equal parts labelled as shown. The arrow on the spinner is spun.

a. What is the probability that the arrow points to:

i. the number 2?

ii. an odd number?

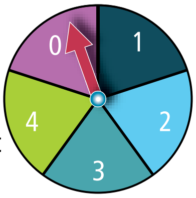
iii. a number less than 4?

iv. a number greater than or equal to 2?

b. The spinner is used in a simulation involving ways of travel to a school. If 30% of the students travel to school by car, what number could be used on the spinner to represent these students?

c. In a simulation, the numbers 1 and 4 are used to represent male students in the school. What percentage of the school is male?

3. A spinner has its face divided as shown.



A class is made up of 40% boys and 60% girls. A student is chosen at random from the group. Explain how you could use the spinner to choose the student.

4. A die has twelve identical faces labelled 1–12.

In a factory, three quarters of the workers are in area A and the rest are in area B. A team of four workers was randomly selected from the factory and no workers from B were picked. How likely is this? Explain how you could use a die like this in a simulation to estimate this probability.

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Expected numbers and simulations

Simulations may involve **expected numbers**. If there are n trials and in each trial an event has a probability p of happening, then the expected number of occurrences of the event is:

$$\text{Expected number} = n \times p$$

For example, if the probability of a male customer coming into a shop is $\frac{1}{3}$, then out of the next 12 customers coming into the shop you would expect $12 \times \frac{1}{3} = 4$ of these customers to be male (on average).

Example

Q. A school tuck shop knows from past sales that one customer in four buys a pie (at $3 each), and one customer in two buys a fruit drink (at $2 each). The tuck shop is considering a specially priced lunch pack offer which includes both a pie and a fruit drink (at $4 each). If the tuck shop serves around 300 customers per day, how many of these lunch packs should it produce? Design a simulation to model this situation.

A. The probability a student buys a pie is $\frac{1}{4}$ and the probability a student buys a fruit drink is $\frac{1}{2}$.
Using a random number function on a calculator, produce pairs of numbers from 1 to 4.

- The first number represents whether a student buys a pie: 1 represents buying a pie; 2, 3, 4 represent not buying a pie.
- The second number represents whether a student buys a fruit drink: 1, 2 represent buying a fruit drink; 3, 4 represent not buying a fruit drink.

30 trials will be carried out, with the results recorded in a table. A trial will be considered a 'success' if a student buys both a pie and a fruit drink.

A few lines of a typical table used for recording outcomes are shown.

Trial	1st number	2nd number	Pie	Drink	Both
1	3	2	no	yes	no
2	1	1	yes	yes	yes
3	1	4	yes	no	no

The proportion of students buying both a pie and a drink is calculated by dividing the number of 'yes' responses in the 'Both' column by 30 (the total number of trials).

For example, if there were 4 trials in which 'yes' was recorded in the 'Both' column, then the probability of a student buying both a pie and a fruit drink is estimated as $\frac{4}{30}$.

The recommended number of lunch packs which should be produced for 300 customers is

$300 \times \frac{4}{30}$

$= 40$ lunch packs

It is important to remember that results of simulations **vary** from simulation to simulation. So results from simulations must be regarded as estimates of the actual situation.

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1. A standard deck of cards has 52 cards, of which 12 are face cards (labelled Jack, Queen or King).
Hatty is playing a game in which you need to deal out cards, one at a time, from a well-shuffled deck until you get a face card. In one game Hatty had to turn over 10 cards before she got a face card, which seemed like a lot!
Design a simulation that would investigate the average number of cards that would need to be turned over before getting a face card.

Trial	Outcome of trial
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
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30	

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More complex simulations

The variables in some situations can be quite complex – your simulation needs to be carefully described, so that someone else could carry it out.

Example

The company *Gamesup* is demonstrating its products at a computer games trade fair which runs from 10 a.m. to 8 p.m. From 6 p.m. to 8 p.m. *Gamesup* has 8 computers available for customers to use for 20-minute 'demo' game sessions. From past experience at trade fairs, the staff at *Gamesup* know that customers arrive to use the computers every 2 to 6 minutes, with probabilities as shown in the table.

Minutes between arrivals	2	3	4	5	6
Probability	0.1	0.2	0.3	0.3	0.1

If a computer is not available, the customer will leave.

Gamesup is interested in finding out whether 8 is the best number of computers to have available (so that the chance of a customer being refused a game is minimised), and whether its 20-minute demonstration game should perhaps be replaced with a game of a different length.

Design and run simulations to investigate this situation.

Solution

Assumptions

- The times between arrivals and their probabilities are assumed to be current and applicable to this trade fair.
- It is assumed that customers arrive independently and that no two customers arrive together.
- The last customer must begin play by 7:40 p.m. (so play finishes by close of show at 8 p.m.).

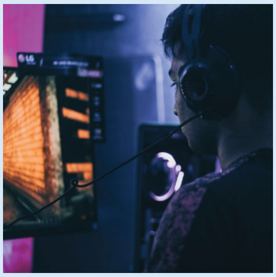
Simulation description

The first simulation is designed using 8 computers running 20-minute demo games.

The probabilities of arrival times are fractions over 10, so generate ten random numbers from 0 to 9 (using a random number generator on a calculator).

Assign one number out of the ten possible numbers to represent a time between arrivals of 2 minutes (since the probability of a customer arriving after 2 minutes is $\frac{1}{10}$).

Assign two numbers out of the ten possible numbers to represent a time between arrivals of 3 minutes (since the probability of a customer arriving after 3 minutes is $\frac{2}{10}$), and so on.



If the number is:

- 0 – next customer arrives after 2 minutes
- 1 or 2 – next customer arrives after 3 minutes
- 3, 4, or 5 – next customer arrives after 4 minutes
- 6, 7, or 8 – next customer arrives after 5 minutes
- 9 – next customer arrives after 6 minutes.

Each trial simulates the time of arrival of a customer, and the number of computers available at the time of the customer's arrival (if the number available is at least 1, then a computer is allocated). An allocated computer becomes available again after 20 minutes, when it re-enters the pool of computers available for the next customer.

Results of the simulation will be recorded in a table, as shown below.

 ISBN 978-1-988586-71-7

3. A hire company has some water blasters which it hires out for one-, two- or three-day periods. Past experience has shown that the numbers of customers who hire a water blaster for one, two or three days are in the ratio 4:2:3. Demand records show that
   - on 10% of the days there will be no customers wanting to hire a water blaster
   - on 40% of the days there will be one customer wanting to hire a water blaster
   - on 50% of the days there will be two customers wanting to hire a water blaster.

   The company hires out the water blasters seven days a week.

   Design a simulation to work out a suitable number of machines that the company should have, so that it doesn't have to turn customers away, or have too many machines sitting idle. Begin your simulation with six water blasters.

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Reporting on a simulation

All of the aspects discussed so far should be included when reporting the results of a simulation.

Your **report** must include a description of each component of the simulation process (including any calculations and spreadsheet formulae). You need to:

- identify the tool you will use to generate random numbers
- explain how the numbers will represent outcomes of the simulation
- define a trial and state and explain the number of trials you have decided to carry out
- record the results of trials clearly so that someone else could use them to verify your conclusions
- draw graphs and calculate relevant measures
- explain clearly the conclusions you have come to and why
- discuss assumptions and possible improvements.

Another person should be able to carry out the simulation independently, based on your description.

Example

Q. *Diamond* petrol stations run a competition for 10 weeks. Each time a customer spends at least $30 on petrol, they receive a scratch card with one of four symbols: Hearts, Diamonds, Clubs, or Spades. There are more of some symbols than others. The probabilities of the symbols are shown in the table.

Symbol	Diamonds ♦	Hearts ♥	Clubs ♣	Spades ♠
Probability	0.1	0.4	0.2	0.3

If a customer gets one of each of the symbols he/she receives a prize of a 'diamond' key ring.

Peter decides to fill his car with at least $30 worth of petrol weekly at a *Diamond* petrol station until the competition is over, or until he wins a key ring.

Design and run a simulation to investigate the number of weeks that Peter visits the *Diamond* petrol station in order to win a key ring. Then write a report about the conclusions you have reached from your simulation.

A. *Answers will vary, a suggested outline solution is given.*

Random numbers from 0 to 9 are generated using a random number function on a calculator (press [1] [0] [Ran#] then [=] repeatedly, taking whole-number part).

Each of these ten random numbers has a 0.1 chance of occurring, so allocate one random number for each 0.1 of probability.

For example, Hearts has probability 0.4, so four random numbers are allocated ($0.4 = 4 \times 0.1$).

The symbols are represented by the random numbers as shown alongside.

		Probability	**Random number**
Trial	♦ **(D)**	0.1	0
	♥ **(H)**	0.4	1,2,3,4
	♣ **(C)**	0.2	5,6
	♠ **(S)**	0.3	7,8,9

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Practice assessment task

The *Fast and Furious* soft-drink company is running a competition to promote its new energy drink, *ZOOM*.

The inside of the cap of each bottle is printed with one of the letters Z, O, or M. Large numbers of the letters Z, O and M are produced in the ratio 1 : 2 : 1.

To be eligible to enter the competition, a person needs to include four soft drink lids with their entry, printed with the letters Z, O, O and M.

Katie wants to enter the competition, but decides to limit herself to buying a maximum of six bottles of *ZOOM*.

Design, run and report on a simulation to investigate the average number of bottles of *ZOOM* that Katie needs to purchase in her attempt to enter the competition, and the likelihood that she will be eligible using her strategy.

Your report should:

- describe your simulation method fully (so that someone else could carry out the simulation independently), identifying tools to be used and defining trials
- include a clear record of your outcomes
- estimate the mean number of bottles of *ZOOM* Katie will buy and her chances of being eligible to enter the competition
- discuss any assumptions you have made, and how they have affected the design of your simulation
- justify conclusions in your report using evidence from your simulation.

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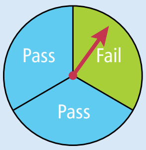
Answers

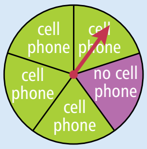
Probability and simulations (page 3)

1. a. i. $\frac{1}{6}$ ii. $\frac{1}{2}$
iii. $\frac{5}{6}$ iv. $\frac{1}{2}$
b. i. $\frac{1}{3}$ ii. $\frac{2}{3}$
2. a. i. 0.2 ii. 0.4
iii. 0.6 iv. 0.9
b. the number 3
c. 50%
3. 40% is $\frac{2}{5}$ so use two out of the five numbers to represent boys, e.g. 0, 1
60% $= \frac{3}{5}$ so use three out of the five numbers to represent girls, e.g. 2, 3, 4
4. Three quarters of 12 is 9, so use 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent a worker in area A (any group of nine numbers between 1 and 12 is a valid choice) and use the remaining numbers (here 10, 11, 12) for workers in area B. Each roll of the die gives a number corresponding to the area from which the worker comes. Roll the die four times in each trial and count the number of workers from area B.
For example, the result could be 6, 11, 2, 9 which corresponds to A, B, A, A, so this selection has 1 worker from B.
The proportion of trials resulting in no workers from B can then be calculated as an estimate of the probability.
5. *Answers may vary from suggested solutions.*
a. A die is used. 1, 2 and 3 represent a prize, 4, 5 and 6 represent 'better luck next time'. In each trial, count the number of rolls before a 1, 2 or 3 turns up. Repeat many times, then work out the average number of rolls.
b. Use a die. 1 represents a person who drinks more than three cups. 2–6 represent a person who drinks fewer than three cups. Trial: Roll the die until two 1s turn up. Repeat many times, then find the average number of rolls.
c. Use a spinner with 10 equal sectors labelled 1–10. Sector 1 is Y13 students. Trial: Spin four times and give ✓ if at least one sector is 1, otherwise ✗. Repeat many times.
Probability $= \frac{\text{number of ticks}}{\text{number of trials}}$
d. Use a die. Assign the numbers 1–4 to female teachers and the number 5 to a male teacher (ignore the number 6 whenever it occurs). In each trial the die is rolled four times to get four numbers below six (re-roll the die whenever a six is the result). For example, the results may be 3, 5, 6 (ignore), 1, 1. This corresponds to 3 females (3, 1, 1) and 1 male (5). Repeat many times then work out the proportion of these groups of four that have two males in them.
6. a.



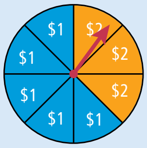
Divide spinner into three equal sectors. Two out of three sectors represents 'pass'; one out of three is 'fail'.
In each trial, the spinner is spun until twelve 'pass' results are achieved, and the number of spins is counted.

b.



80% $= \frac{4}{5}$, so divide spinner face into five equal parts, four corresponding to 'has cell phone' and the fifth corresponding to 'no cell phone'. For each trial, the spinner is spun six times and the number of results corresponding to having a cell phone counted. A successful trial is one in which four or more students have a cell phone. The probability equals the number of successful trials divided by the number of trials.

c.



The ratio 3:5 has 8 parts, so divide spinner face into eight equal parts, three corresponding to $2 coins and five corresponding to $1 coins. For each trial, the spinner is spun ten times and the total value of the results calculated. These totals are then averaged.

d.

The proportion of customers wanting a treatment that costs less than $100 is $\frac{333}{500}$, so the probability that a customer wants a treatment less than $100 is approximately $\frac{2}{3}$ (and the probability a customer wants a treatment costing at least $100 is approximately $\frac{1}{3}$).
Divide the spinner into thirds: label one part 'at least $100' (≥ $100) and label the other two parts 'under $100' (<$100).
For each trial, spin the spinner five times, noting the costs of treatment for each spin.
Repeat many times, then work out the proportion of these trials that have at least two customers wanting a treatment that costs at least $100.

Random numbers (page 8)

1. a. Generate 10 random numbers, ignoring repeats. These students are chosen for leadership camp.
b. The numbers are completely random and will differ each time random numbers are generated.
c. *Answers will vary*.
d. i. *Answers will vary.*
ii. *Student discussion could include a reflection on whether the number on both lists was surprising (would have expected more/fewer, etc.), and comments on the 'randomness' of the selection.*
2. a. i. Press [2] [5] [Ran#] [+] [1].
Press [=] six times. Take whole-number part only.
ii. *Answers will vary (all whole numbers between 1 and 25, repeats allowed).*
b. i. Press [1] [0] [Ran#].
Press [=] five times. Take whole-number part only.

 ISBN 978-1-988586-71-7

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