

Apply measurement in solving problems

Units of measurement

When measuring quantities such as length, mass, etc., appropriate measurement units must be used. In the **metric system**, each quantity measured has a **base unit**.

Quantity	Base unit
Length (distance)	metre (m)
Mass (weight)	gram (g)
Capacity (liquid volume)	litre (L)

Prefixes are used to derive larger or smaller units. Some commonly used prefixes are:

- **kilo** (k) which means 1 000
- **centi** (c) which means $\frac{1}{100}$
- **milli** (m) which means $\frac{1}{1\,000}$

Note: 1 tonne = 1 000 kg (an exception to the use of prefixes). The abbreviation for tonne is t.

Examples

1. The length of a book would be measured in centimetres (cm).
2. The mass of a sack of potatoes would be measured in kilograms (kg).
3. The volume of medicine to be taken daily would be measured in millilitres (mL).
4. The distance between Auckland and Wellington would be measured in kilometres (km).
5. The mass of a tablet for pain relief would be measured in milligrams (mg).
6. The volume of water in a swimming pool would be measured in kilolitres (kL).



Exercise A: Units of measurement

Answers
p. 311

Give an appropriate unit for each of the following measurements.

1. The length of a netball court.

2. The mass of a schoolbag.

3. The volume of fruit juice in a can.

4. The distance from New Zealand to Australia.

5. The time taken to sprint 100 m.

6. The weight of a teaspoon of sugar.

7. The mass of a truckload of apples.

8. The depth of a goldfish bowl.

9. The width of a coin.

10. The amount of water in a swimming pool.

Using metric units

To **convert** from one unit of measurement to another, multiply or divide by the appropriate power of 10 (i.e. by 10, 100, 1 000, etc.).

- When converting from a larger unit to a smaller unit, *multiply* (to get more of the smaller unit).
- When converting from a smaller unit to a larger unit, *divide* (to get fewer of the larger unit).

Always think about how sensible the answer seems!

Examples

1. $2.55 \text{ kg} = 2.55 \times 1\,000 \text{ g}$
larger unit to smaller, so multiply
($1 \text{ kg} = 1\,000 \text{ g}$)
 $= 2\,550 \text{ g}$
2. $275 \text{ cm} = 275 \div 100 \text{ m}$
smaller unit to larger, so divide
($100 \text{ cm} = 1 \text{ m}$)
 $= 2.75 \text{ m}$
3. $7.6 \text{ cm} = 7.6 \times 10 \text{ mm}$
larger unit to smaller, so multiply
($1 \text{ cm} = 10 \text{ mm}$)
 $= 76 \text{ mm}$

You also need to be familiar with quantities such as:

- **temperature** – measured in degrees Celsius ($^{\circ}\text{C}$)
- **money** – measured in dollars (\$) and cents (c)
- **time** – non-metric units include seconds (s) minutes (min), hours (h), days, years, etc.

Problems involving time include the use of **24-hour times**:

- the first two digits give the hours after midnight
- the final two digits give the minutes.

Examples

1. A 12-hour time of 6:45 a.m. converts to 0645 in 24-hour time. **4 digits needed**
2. A 24-hour time of 2015 converts to 8:15 p.m. in 12-hour time.
20 hours after midnight is $20 - 12 = 8$ p.m.
3. The time that elapses between 2:47 a.m. and 11:04 p.m. is:
13 minutes plus 8 hours and 4 minutes plus 12 hours
This gives a total of 20 hours and 17 minutes
 $2:47 \text{ a.m.} + 13 \text{ min} = 3:00 \text{ a.m.}$
 $3:00 \text{ a.m.} + 8 \text{ h } 4 \text{ min} = 11:04 \text{ a.m.}$
 $11:04 \text{ a.m.} + 12 \text{ hours} = 11:04 \text{ p.m.}$

Exercise B: Using metric units

Answers
p. 311

1. Convert each of the following measurements to the unit given in brackets.

a. $3\,575 \text{ cm (m)}$

b. 6.5 cm (mm)

c. 8.5 kg (g)

d. $57\,000 \text{ mg (g)}$

e. $3\frac{3}{4} \text{ hours (min)}$

f. 38.75 t (kg)

g. 445 mm (cm)

h. $98\,000 \text{ c ($)}$

i. 68.9 m (cm)

j. $13 \text{ decades (years)}$

k. $99\,650 \text{ kL (L)}$

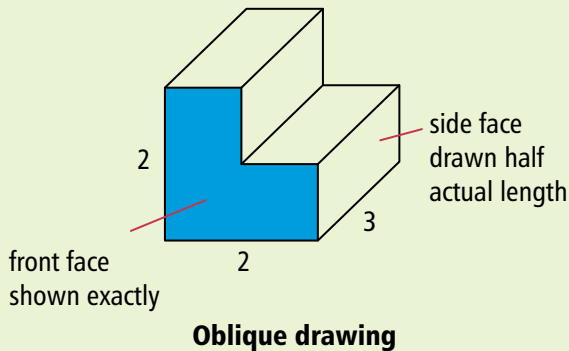
l. $87\,500 \text{ kg (t)}$

m. 0.45 kg (g)

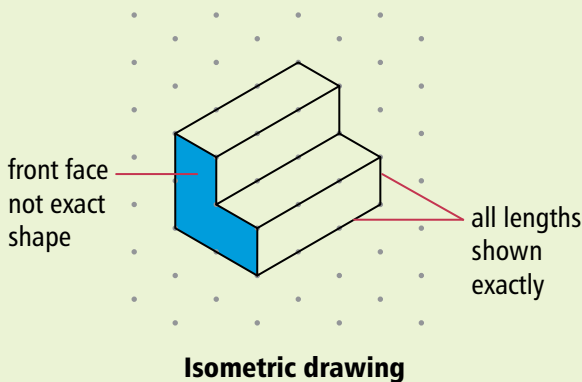
n. $35\,000 \text{ mL (L)}$

Views and isometric drawings

2-dimensional drawings of solids can be done in various ways, e.g. **oblique drawings** are commonly used. Side faces are drawn on an angle.



Another useful technique is the **isometric drawing**, which is done on an isometric grid of equilateral triangles.

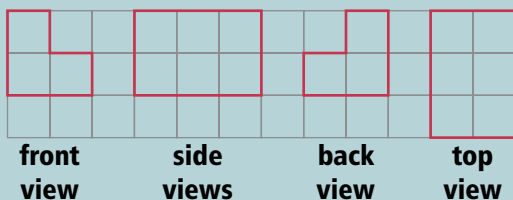


Views

Solids can also be **viewed** from different directions, e.g. top view, front and back views or side views.

Example

The figure above has the following views.



Note: The top view is drawn facing front of figure.

For solids made from cubes a **plan view** shows a top view with block heights marked.

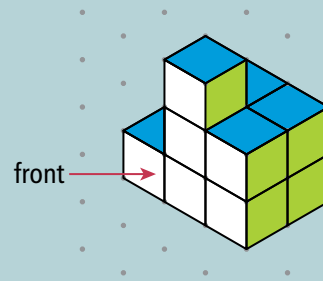
Example

The plan view of a solid made from 11 cubes is shown. Do an isometric drawing of the solid.

1	2	2
1	3	2

front

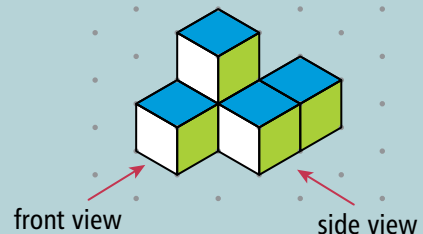
Solution:



You may be required to draw a figure from a different view point – constructing a plan view first can help with this.

Example

An isometric drawing of a solid has front and side views as shown. Redraw the solid with the old side view as the front view.



Solution:

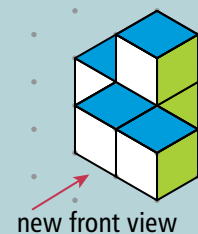
A plan view of the solid is drawn from the old side view.

The isometric drawing below shows the solid drawn with the old side view now as front view (using the plan view to help).

plan view from old side view

1	2	0
0	1	1

now the front view

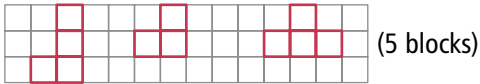


Exercise B: Views and isometric drawings

Answers
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1. Top, side and front views of solids made from cubes are shown. Make an isometric drawing of each solid. (The total number of blocks in each solid is given in brackets alongside.)

a. top front right side



b. top front right side



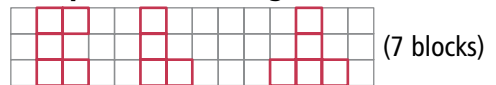
c. top front right side



d. top front right side



e. top front right side



2. Make isometric drawings of the solids whose plan views are as shown.

a.

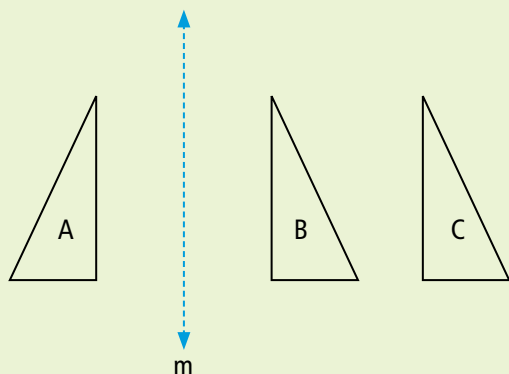
2	3	2
1	2	1



Composite (combined) transformations

Sometimes one transformation follows another to make a **composite transformation**.

For example, in the diagram below, triangle A is reflected in the mirror line m to its image B. Triangle B is then translated 2 cm right horizontally to its image C.

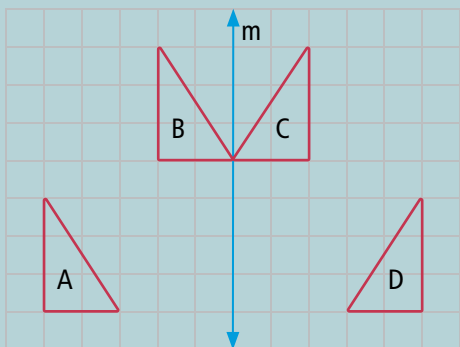


The transformations that may be combined with each other are reflection, rotation, translation, and enlargement.

A composite transformation is sometimes equivalent to a single transformation.

Example

A sequence of transformations is applied to triangle A.



A is translated $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ to B

B is reflected in m to C

C is translated $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ to D

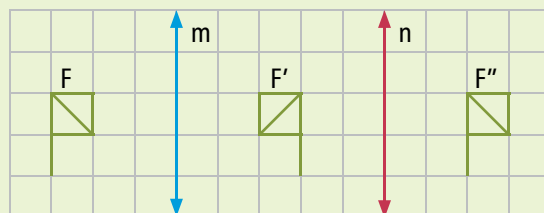
The transformation that takes A directly to D is reflection in m .

A reflection in one mirror may be followed by a reflection in a second mirror.

Reflections in two parallel mirrors

In the figure below the object F is reflected in the mirror line m to its image F' .

The image F' is now the object for a second reflection, this time in the mirror line n , to its image F'' .



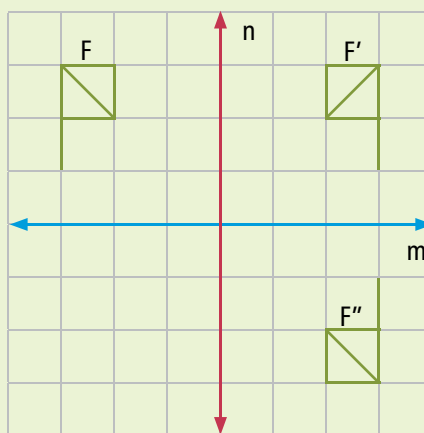
It can be seen that the composite transformation (of F to F'') is equivalent to a single transformation which is a translation of 10 units right (a translation by the vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$).

Note: If the order of these two reflections is reversed, so that the first reflection is in the mirror line n and the second reflection is in mirror line m , the results would be quite different.

Reflections in two perpendicular mirrors

In the figure below the object F is reflected in the mirror line n to its image F' .

The image F' is now the object for a second reflection, this time in the mirror line m , which is perpendicular to mirror line n . This gives a final image F'' .



It can be seen that the composite transformation (of F to F'') is equivalent to a single transformation which is a rotation of 180° about the point of intersection of the mirrors.

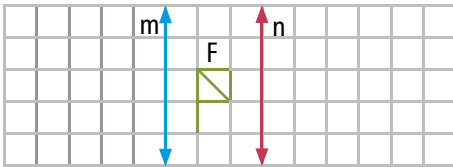
Exercise G: Composite (combined) transformations

Answers p. 324

For each of the following, draw the results of composite transformations and identify an equivalent single transformation.

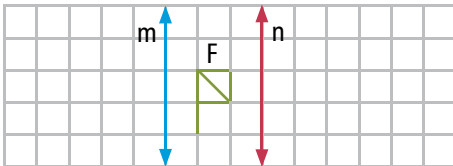
1. Flag F is shown below.

- a. i. Reflect F in the mirror line m to its image F' . Then reflect F' in the mirror line n to its image F'' .



- ii. What single transformation would map F to F'' ?
- _____

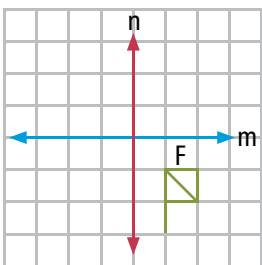
- b. i. Reflect F in the mirror line n to its image F' , then reflect F' in the mirror line m to its image F'' .



- ii. What single transformation would map F to F'' ?
- _____

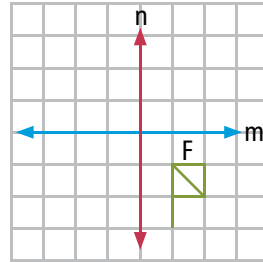
2. Flag F is shown below.

- a. i. Reflect F in the mirror line m to its image F' , then reflect F' in the mirror line n to its image F'' .



- ii. What single transformation would map F to F'' ?
- _____

- b. i. Reflect F in the mirror line n to its image F' , then reflect F' in the mirror line m to its image F'' .



- ii. What single transformation would map F to F'' ?
- _____

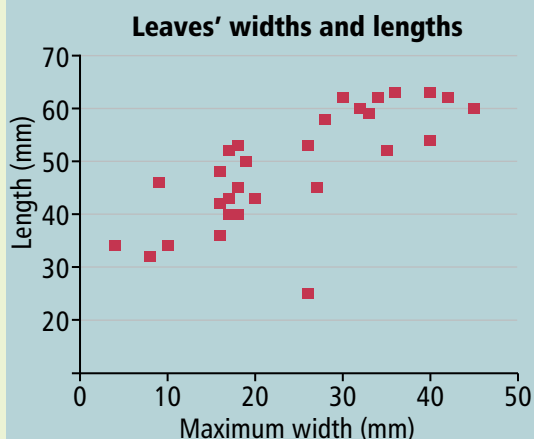
3. a. A flag F is shown. Rotate F about the point O, clockwise through 90° , then rotate the image about O through 180° anticlockwise. Draw F'' , the image of F, after this composite transformation.



- b. Describe fully the single transformation which is equivalent to the composite transformation in part a.
- _____
- _____

- c. If the two rotations were applied in reverse order, would the image F'' be the same or different?
- _____
- _____

Jacinta drew a scatter graph to display the group's bivariate data.

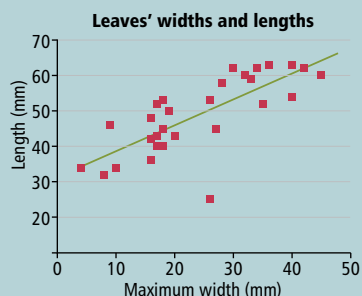


Looking at the points on the scatter graph, Jacinta noticed that there was a positive relationship between the maximum width of a leaf and its length, so that as the maximum width of the leaf increased, the length of the leaf also tended to increase.

Jacinta also noticed some other interesting features of the scatter graph.

- There was a cluster of points in the region $16 \text{ mm} \leq x \leq 20 \text{ mm}$ (i.e. between maximum leaf widths in the range 16–20 mm). Jacinta thought that these could be a group of leaves that were growing in a similar position on the bush; or leaves of similar stage of maturity due to receiving similar amounts of rain/sun, etc.
- There was an outlier at the point (26,25), so there could be an unusual leaf which is wider than it is long (width 26 mm and length 25 mm). Another possible explanation for this unusual point is that it is the result of a measurement or recording error (e.g. two width measurements may have been recorded for this leaf, instead of a pair of width and length measurements). This would need to be checked if possible.

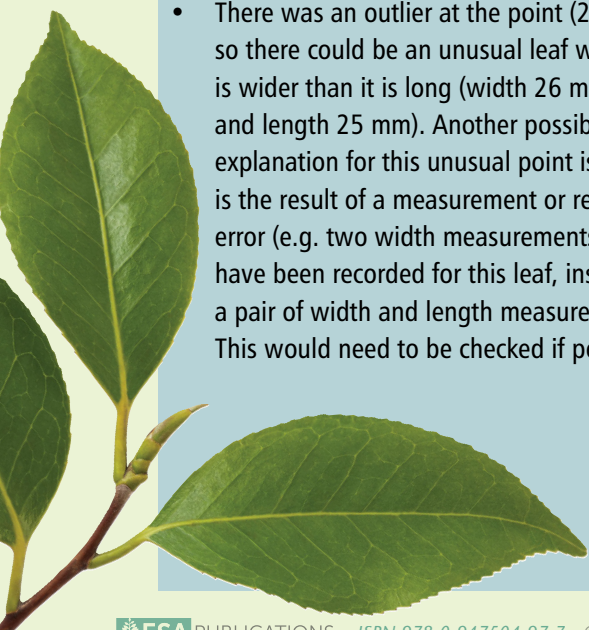
Jacinta drew a trend line on her scatter graph.



- Jacinta noticed that points near the right end of the line where $x \geq 30 \text{ mm}$ (representing wider leaves with a maximum width of at least 30 mm) seemed to be part of a flatter trend, with all leaves having a length around 60 mm approximately. These may be the typical dimensions of mature (bigger) leaves from this type of bush. This flattening of the trend for points representing leaves of larger maximum width may indicate that a single trend line is not useful for describing the relationship between the maximum width and length of all of these leaves. It may be better to draw two trend lines, one for leaves of maximum width lower than 30 mm and another for leaves of maximum width greater than 30 mm.
- The relationship between the maximum width of a leaf and its length is moderately strong so that predictions could be made from the graph.

For example, when the maximum width of a leaf is 22 cm the leaf would be predicted to be about 48 mm long (Jacinta drew a vertical line from $x = 22 \text{ mm}$ up to the point (22,48) on the trend line). Jacinta would not be very confident about this prediction as the relationship between the variables is not a very strong one.

The scatter of the points about the trend line means that any prediction could be up to around 10 mm more or less than the prediction from the trend line. This means that the length of a leaf of maximum width 22 cm is predicted to lie between $48 \text{ mm} \pm 10 \text{ mm}$, i.e. between 38 mm and 58 mm.



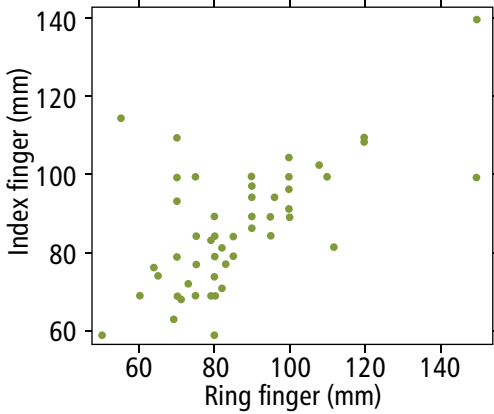
Exercise D: Trend lines and other features of scatter graphs

Answers p. 333

1. For each of the following graphs, discuss the strength of the relationship, and any other features of interest such as clusters, outliers and end points. You may wish to draw a trend line on your graph if the relationship is linear.

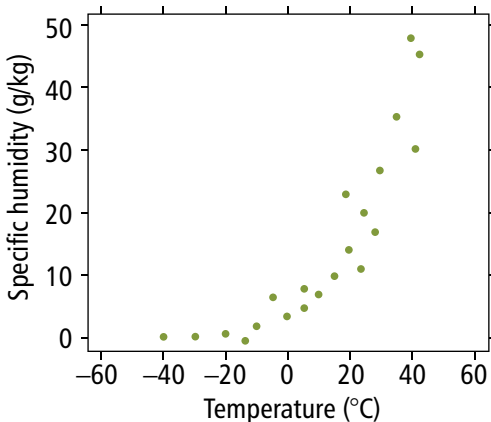
a. Length of index finger and ring finger.

Index finger versus ring finger



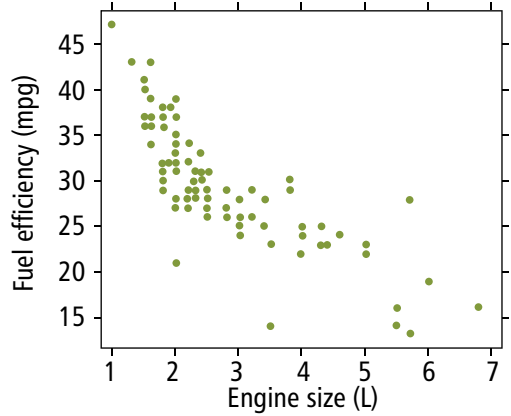
b. Atmospheric temperature and humidity

Humidity versus temperature in the atmosphere



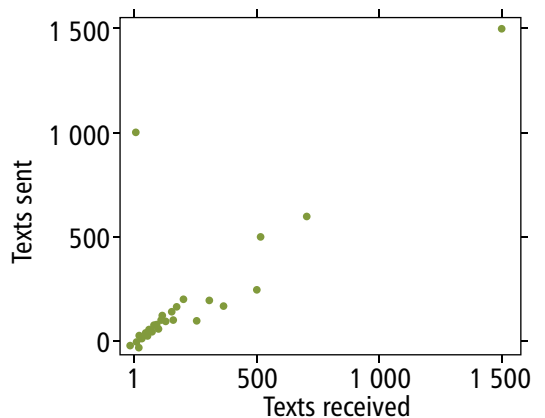
c. Engine size and fuel efficiency (miles per gallon).

Fuel efficiency versus engine size



d. Daily number of texts sent and received for Year 11 students

Texts sent versus texts received



MATHEMATICS AND STATISTICS 1.13

Internally assessed
3 credits

Investigate a situation involving elements of chance

Introduction

Situations involving chance can arise in everyday life, e.g. flipping a coin in order to decide which team gets the ball at the start of a game, playing a game of cards, or buying a raffle ticket.

In this achievement standard you will explore a situation involving chance by:

- posing a question
- planning an experiment to explore the situation
- gathering data by performing the experiment
- selecting and using appropriate displays including experimental probability distributions
- identifying and communicating patterns in the data
- summarising your findings in a conclusion.

In some situations you may be able to compare your experimental distribution with a theoretical distribution.

Probability

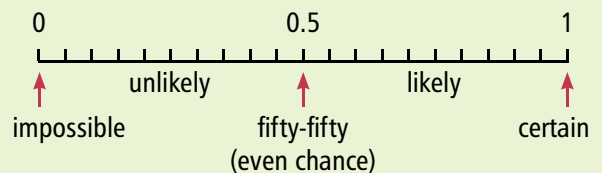
Some events are more likely to happen than others, e.g. when two dice are rolled together, it is unlikely that you will get two sixes, but it is very likely that the two numbers will add up to a number bigger than 2.

Numbers from 0 to 1 are used to describe how likely an **event** is to occur. These numbers are called the **probability** of the event.

- An **impossible** event has probability = 0
Unlikely events have probabilities close to 0
- A **certain** event has probability = 1
Likely events have probabilities close to 1
- An event which is just as likely to occur as it is not to occur has probability = $\frac{1}{2}$ or 0.5

The closer a probability is to 1, the more likely an event is to occur. The closer a probability is to 0, the less likely an event is to occur.

Probability scale



The probability of an event occurring is written P(event).

Example

$P(\text{an ant weighs more than 1 kg}) = 0$

$P(\text{a fair coin comes up heads when flipped}) = 0.5$

$P(\text{a kindergarten child is less than 6 years old}) = 1$

Experimental probability

An event is any happening of interest, e.g. scoring a try in a rugby game; getting a letter in the post; or getting a head when a coin is tossed.

In most practical situations, the **true probability** of an event is unknown, so a **probability experiment** is run. This involves a series of trials – a trial is a ‘success’ if the event occurs.

An **experimental probability** is then worked out for the event.

$$\text{Experimental probability} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Example

I want to know how likely it is that when I throw a ball of paper at a bin 2 metres away that the ball of paper lands in the bin. So I throw the ball of paper at the bin 20 times.

Each throw is a trial.

If the ball of paper goes in the bin then that is a successful trial.

If I get the ball of paper into the bin 11 times out of 20 attempts, then my experimental probability of getting the ball of paper in the bin is $\frac{11}{20}$.

The experimental probability is an estimate of the true probability.

Example

Melanie plays Goal Shoot for her netball team. Out of her last 100 attempts at shooting a goal, Melanie got 73 goals.

Melanie's success rate in shooting goals is $\frac{73}{100}$ or 0.73 or 73%.

This is the experimental probability of success for Melanie.

Melanie uses the experimental probability to estimate that her overall success rate at shooting goals is 73%. This means that she thinks there is a 73% chance that her next shot at goal will be successful.



Exercise A: Experimental probability

Answers
p. 340

1. The face of a spinner is divided into parts labelled 0, 1, 2 and 3. In an experiment the spinner is spun 30 times. The table shows the results.

Score	Frequency
0	6
1	7
2	13
3	4
Total	30

For this experiment, what is the experimental probability of getting:

- a. a 2?

- b. a 0?

- c. an odd number?

- d. a number less than 2?

- e. a number greater than 3?

2. A bag contains 25 marbles, some blue and some red. A marble is selected from the bag, its colour noted then returned to the bag. This is repeated 100 times. Altogether 69 selections were blue marbles and the rest were red marbles.
- a. What is the experimental probability of getting a blue marble when a selection is made?
- _____
- b. What is the experimental probability of getting a red marble when a selection is made?
- _____
3. In a town a record is kept of the gender of the babies that are born. Of the last 300 births in the town there were 157 boys and the rest were girls. For this town, what is the experimental probability of:
- a. a boy baby?
- _____
- b. a girl baby?
- _____
4. A 'hand' of four cards is dealt from a standard pack of 52 cards, and the number of face cards (Jack, Queen, King or Ace) is counted for each hand. The cards are then returned to the pack, shuffled and another 'hand' of four cards is dealt.

ANSWERS

Achievement Standard 91026 Mathematics and Statistics 1.1

Exercise A: Factors multiples and primes (page 1)

- 1, 2, 3, 5, 6, 10, 15, 30
 - 1, 2, 3, 4, 6, 8, 12, 24
 - 1, 17
 - 1, 5, 25
 - 6
 - 1
- 9, 18, 27, 36
 - 12, 24, 36, 48
 - 42, 84, 126, 168
 - 125, 250, 375, 500
 - 36
- 17, 19, 23, 29
 - 31, 37
- 45, 54
 - 40, 48, 60
 - 41, 43, 47, 53, 59
 - 46, 51, 55, 57, 58
- 60 seconds (or 1 minute)
- 36 months (or 3 years)
- 15, 30, 45, 60
 - 20, 40, 60
 - 60
 - multiples of bigger number is quicker as step size is larger
 - 180
 - 96
- $2 \times 3 \times 5$
 - $2 \times 5 \times 7$
 - $3 \times 5 \times 11$
 - 2 mm \times 3 mm \times 5 mm
 - 2 cm \times 5 cm \times 7 cm
 - 3 m \times 5 m \times 11 m
- | | |
|----|-----|
| 1 | 140 |
| 2 | 70 |
| 4 | 35 |
| 5 | 28 |
| 7 | 20 |
| 10 | 14 |
 - | | |
|----|-----|
| 1 | 168 |
| 2 | 84 |
| 3 | 56 |
| 4 | 42 |
| 6 | 28 |
| 7 | 24 |
| 8 | 21 |
| 12 | 14 |
 - 28
 - \$5
 - \$6
- 3×17
 - 2×17
 - 17 is a common factor, and 2 and 3 have no factors in common so 17 is the HCF.
 - 19
 - 13
 - 3

Exercise B: The integers (page 4)

- 8
 - 64
 - 0
 - 36
 - 14
 - 20
 - 11
 - 39
 - 10
 - 12
 - 60
 - 12
 - 3
 - 36
 - 16
 - 27
- 3 floors below
 - 13

- 17 °C colder
- 27 °C
 - 18 °C
- 7
- 20

Exercise C: Order of operations (page 6)

- 15
 - 5
 - 24
 - 24
 - 1
 - 16
 - 27
 - 16
 - 19
 - 0
- Loss of \$250 000 per year
- \$199
- 63
- 35 g
 - 76 g

Exercise D: Powers of numbers (page 7)

- 2 025
 - 676
 - 361
 - 243
 - 64
 - 2 401
 - 1
 - 10 000
- 25
 - 64
 - 1 296
 - 23
 - 98
 - 0
- 144 cm²
 - 4 times
- 81
 - 125
- 40 joules
 - 6 750 cm³

Exercise E: Roots of numbers (page 8)

- 17
 - 32
 - 81
 - 11
 - 7
 - 12
- 7
 - 4
 - 3
 - 5
 - 2
 - 3
- 36, 38, 49
 - 144, 150, 169
 - 9, 12, 16
 - 100, 107, 121
 - $6, \sqrt{38}, 7$
 - $12, \sqrt{150}, 13$
 - $3, \sqrt{12}, 4$
 - $10, \sqrt{107}, 11$
 - 6.16
 - 12.25
 - 3.46
 - 10.34

Exercise F: Simplifying and comparing fractions (page 9)

- $\frac{4}{5}$
 - $\frac{4}{7}$
 - $\frac{5}{8}$
 - $\frac{4}{1}$ or 4
 - $\frac{3}{4}$
 - $\frac{3}{8}$
 - $\frac{5}{7}$
 - $\frac{3}{5}$
- $\frac{3}{13}$
 - $\frac{9}{5}$
 - $\frac{8}{15}$
 - $2\frac{2}{5}$
 - $\frac{2}{3}$
 - $1\frac{1}{4}$
 - $\frac{16}{25}$
- $\frac{4}{5} < \frac{9}{10}$
 - $\frac{7}{9} < \frac{7}{8}$
 - $1\frac{3}{5} > 1\frac{1}{2}$
 - $\frac{3}{4} > \frac{5}{8}$
 - $\frac{5}{9} > \frac{3}{7}$
 - $\frac{6}{7} < \frac{9}{8}$
 - $\frac{2}{3} < \frac{7}{9}$
 - $\frac{3}{5} > \frac{7}{12}$
 - $\frac{4}{3} < \frac{16}{10}$
 - $\frac{4}{10} > \frac{3}{11}$
- $\frac{5}{9} = \frac{40}{72} = \frac{20}{36}$
 - $\frac{3}{7} = \frac{15}{35} = \frac{12}{28}$
 - $\frac{4}{3} = \frac{100}{75} = \frac{120}{90}$
 - $\frac{12}{5} = \frac{144}{60} = \frac{60}{25}$
 - $\frac{7}{14} = \frac{1}{2} = \frac{15}{30}$
- $\frac{1}{4}$

Trial	1st number	2nd number	Difference	A points (difference is more than 2)	B points (difference is less than 2)
1	4	5	1	0	1
2	4	4	0	0	1
3	6	1	5	1	0
4	3	5	2	0	0

Add up A's points and divide by 30: this is the experimental probability of A winning.

Add up B's points and divide by 30: this is the experimental probability of B winning.

Add up the number of times a difference of 2 resulted and divide by 30. This is the experimental probability of no points being scored in a round.

Draw a dot plot or bar graph of the experimental probabilities.

In the conclusion, the question is answered, based on the analysis of your results.

(Answers will vary from investigation to investigation.)

You may wish to investigate the theoretical situation. When two dice are rolled together and the difference is worked out, the results are as shown.

Difference	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

These results can be summarised as follows.

Difference	Frequency	Theoretical probability
Less than 2	16	$\frac{16}{36}$
More than 2	12	$\frac{12}{36}$
2	8	$\frac{8}{36}$
Total	36	1

Draw a bar graph of the theoretical probabilities.

Compare the experimental probability distribution with the theoretical probability distribution and comment on differences (which are due to sampling variability).

3. Question: What is the most likely number of different suits in each deal of 4 cards?

From my experience dealing cards I predict that there will be 2 or 3 suits. *(Question and predictions will vary.)*

Variable is the number of different suits in a deal of 4 cards (could be 1, 2, 3, 4 different suits per deal).

30 trials so results are reliable for estimating the actual probability.

Reduce variability: shuffle cards thoroughly between each deal.

For each trial: deal 4 cards. Record the suit of each card in a table, then count up how many different suits are in the hand.

Set out results in a table as shown.

(Table design and experimental outcomes will vary, this is just an example.)

Trial	Card 1 suit	Card 2 suit	Card 3 suit	Card 4 suit	Number of different suits
1	Heart	Club	Club	Spade	3
2	Diamond	Heart	Heart	Diamond	2
3	Club	Club	Club	Club	1

Find the average number of different suits per deal by adding up the numbers in the final column and dividing by 30.

Draw a bar graph of the experimental distribution of the possible numbers of suits when 4 cards are dealt (1, 2, 3 or 4) and comment on its shape, or any features of interest.

In the conclusion, the question is answered, based on the analysis of your results.

(Answers will vary from investigation to investigation.)

4. Question: Is the game fair – does each player have the same chance of winning?

The game is probably not fair as there are different numbers of A and B squares.

(Question and predictions will vary.)

Variable is 'who wins a round of the game after 3 coins are flipped' (A or B).

30 trials so results are reliable for estimating the actual probability.

Reduce variability: flip coins so that they spin several times so results are random.

For each trial: flip 3 coins. Record the result (Head or Tail) for each coin, then work out the moves on the board (Head: move right, Tail: move left) to find the player's final position after the turn.

If the final position is labelled A, then A wins a point; if the final position is labelled B, then B wins a point.

Set out results in a table as shown (H: Head, T: Tail, R: move right, L: move left)

For example, if the result is TTH, then the player moves left, then left, then right to finish on a square labelled B, so B gets the point.

(Table design and experimental outcomes will vary, this is just an example.)

Trial	Results	Moves	Final position
1	HTT	RLL	B
2	THT	LRL	B
3	TTT	LLL	A

Count up the number of A's and divide by 30: this is the experimental probability of A winning a point.

Count up the number of B's and divide by 30: this is the experimental probability of B winning a point.

Draw a bar graph of the experimental distribution of the points and comment on its shape, or any features of interest.

In the conclusion, the question is answered, based on the analysis of your results.

(Answers will vary from investigation to investigation.)

You may wish to investigate the situation theoretically. There are 8 equally likely outcomes when 3 coins are flipped