Mathematics and Statistics 1.13

Internally assessed
3 credits

Achievement Standard 91038
Investigate a situation involving elements of chance

Introduction

This activity requires you to undertake a statistical investigation into a game of chance.

You will be assessed on the quality of your discussion and reasoning and how well you link this to the context.

Task

Part 1

Pose an investigative question to explore selecting balls from a bag containing six balls: 2 red, 2 blue and 2 green balls. A suitable investigative question reflects the probability situation, has a clear variable for investigation, requires statistical analysis, and can be meaningfully answered with data gathered.

Make a prediction and write down what you think your expected results might be.

Plan an experiment to answer your question.

In your plan:

- discuss and define the set of possible outcomes
- identify the number of trials
- list the steps needed to perform your experiment
- explain how you will record your results
- provide sufficient information to answer your investigative question.

Part 2

Carry out your experiment and record your data in a suitable format.

Part 3

Independently analyse the data and produce a report by:

- drawing at least two appropriate displays, including the experimental probability distribution, that show different features of the data in relation to the investigative question
- developing appropriate statistics from the data
- discussing your prediction in relation to the experimental probability distribution
- identifying patterns in the data
- writing a conclusion about your findings that answers your investigative question and provides supporting evidence for this answer.

Relate your comments about the distribution, and the information presented in your displays, to the context. Include the plan for your experiment from Part 1 in your report.

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Solution

Answers will vary; the following is an example of a possible investigation. Note that the investigation has more than is required for achievement of the standard (see description of assessment criteria at end of solution).

Problem

I am going to investigate the likelihood of getting two balls of the same colour, when two balls are selected together from the bag containing 2 red (R), 2 blue (B) and 2 green (G) balls.

Prediction: The first ball can be any colour, but for the second ball there is only 1 ball out of the remaining 5 balls that is the same colour, so I predict that the probability of getting two balls of the same colour is $\frac{1}{5}$ or 0.2.

Plan

I will place six identically-sized balls in a bag that is not transparent and label two balls R (for red) so that I can identify two as being red; similarly label two blue and two green.

I will conduct 50 trials. In each trial, without looking in the bag, I will put my hand into the bag and mix the balls thoroughly before removing two balls.

I will record the colours of the two balls in a table, then return the balls to the bag for the next trial.

The table will have columns for outcome, tally, frequency and experimental probability.

- The six possible outcomes are listed: RR (two reds); RB (a red and a blue); RG (a red and a green); BB (two blues); BG (a blue and a green): and GG (two greens).
- The tally column will be used to record outcomes as they occur.
- The total number of tallies will be recorded in the frequency column.
- I will then work out experimental probabilities by dividing each frequency by the total frequency.

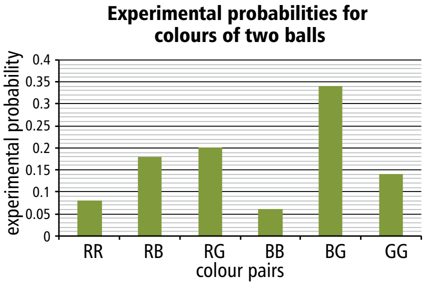
Data

The results of my trials are shown in the table below.

Outcome	Tally	Frequency	Experimental probability
RR	////	4	0.08
RB	~~////~~ ////	9	0.18
RG	~~////~~ ~~////~~	10	0.20
BB	///	3	0.06
BG	~~////~~ ~~////~~ ~~////~~ //	17	0.34
GG	~~////~~ //	7	0.14
Total		50	

Analysis and conclusion

A bar graph of the results is shown below.



There were 4 + 3 + 7 = 14 outcomes out of 50 trials with 2 balls of the same colour. So the experimental probability of a trial resulting in 2 balls of the same colour is $\frac{14}{50}$ = 0.28

From my investigation, it seems that the probability of drawing 2 balls of the same colour is around 0.28. This is higher than my prediction of 0.2. The difference could be due to experimental variability as each experiment is likely to give a different experimental probability.

The following is an example of discussion at merit and excellence level.

Model prediction

It is interesting to compare the experimental results with the theoretical probabilities.

In the following table: R1 and R2 are the two red balls; B1 and B2 are the two blue balls, and G1 and G2 are the two green balls. It is assumed that each of these six balls is equally likely to be chosen.

The possible outcomes when two balls are drawn from the bag are shown in the table below.

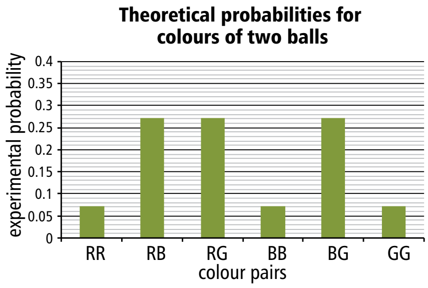
	R1	R2	B1	B2	G1	G2
R1	*	R1R2	R1B1	R1B2	R1G1	R1G2
R2	R2R1	*	R2B1	R2B2	R2G1	R2G2
B1	B1R1	B1R2	*	B1B2	B1G1	B1G2
B2	B2R1	B2R2	B2B1	*	B2G1	B2G2
G1	G1R1	G1R2	G1B1	G1B2	*	G1G2
G2	G2R1	G2R2	G2B1	G2B2	G2G1	*

There are 30 possible outcomes, of which 6 outcomes (shaded) are two balls of the same colour (* shows combinations which cannot occur).

The theoretical probability distribution of outcomes is in the table below. For example, two reds occurred 2 times out of 30 so the theoretical probability of two reds (RR) is $\frac{2}{30}$ or $\frac{1}{15}$.

Outcome	Frequency	Theoretical probability
RR	2	$\frac{1}{15}$
RB	8	$\frac{4}{15}$
RG	8	$\frac{4}{15}$
BB	2	$\frac{1}{15}$
BG	8	$\frac{4}{15}$
GG	2	$\frac{1}{15}$
Total	30	

A bar graph of the theoretical probabilities is drawn below.



The bar chart of the experimental probabilities has a shape which is approximately similar to the shape of the bar chart of the theoretical probabilities, with the differently coloured pairs (RB, RG, BG) having taller bars than the same colour pairs (RR, BB, GG).

However, in the theoretical probability the three same colour pairs (RR, BB, GG) had bars of the same height, and the three differently coloured pairs (RB, RG, BG) had bars of the same height. By comparison, in my experiment GG turned up quite a bit more than would be expected from the theoretical probabilities. Also BG had a taller bar than would be expected from the theoretical probabilities.

The theoretical probability of getting two balls the same colour is $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15}$ or 0.2.

Conclusion

I think the probability of getting two balls the same colour is 0.2.

This is what I predicted, and the result is confirmed by theoretical probability so I am confident about this, even though 0.2 is quite a bit lower than the 0.28 that I got in my experiment.

Comparing my experimental probabilities with the theoretical probabilities I note that the numbers of times 2 reds and 2 blues occurred in my experiment are almost the same as would be expected, but the number of times 2 greens occurred is double the number that would be expected. Also, the numbers of times RB and RG occurred are both lower than expected but the number of times BG occurred was higher.

There is no reason I can think of for this happening, as I thought the balls were identical except for colour and the same procedure was used each time. However, my experimental results may suggest something was different about the green balls (but this is not shown with the number of RG's that occurred).

However, the differences between the theoretical probability and my experimental probability could simply be due to chance, because each experiment of 30 trials is likely to produce a different estimate of the true probability. If I had carried out more trials, I believe that the experimental results would become closer to the theoretical results, and closer to the actual probability, because increasing the number of trials reduces variability in experimental probabilities.

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For Achieved you must do the following

- *pose an appropriate investigative question*
- *plan a suitable experiment*
- *describe the set of possible outcomes and choose a sample size*
- *gather the data as per plan*
- *create at least two appropriate data displays, one of which is an experimental probability distribution*
- *identify at least two patterns in the data in context*
- *identify at least two patterns in the data in context*
- *answer the investigative question consistent with the analysis.*

For Merit you must have enough for Achieved, plus:

- *describe the set of possible outcomes and choose a sample size and give reasons why specific plan elements were chosen*
- *compare theoretical and experimental probabilities*
- *answer the investigative question giving reasons for your answer.*

For Excellence you must have enough for Merit, plus

- *describe the set of possible outcomes and choose a sample size, with statistical reasons for why specific plan elements were chosen, e.g. indicate how the data analysis will answer the question and/or consider the effect of related variables*
- *demonstrate an understanding of applications of probability, explaining the most important features of the data and how they relate to the context*
- *reflect on the investigation, including possible effects of other factors and/or make additional conjectures about future performance.*

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