Mathematics and Statistics 1.7

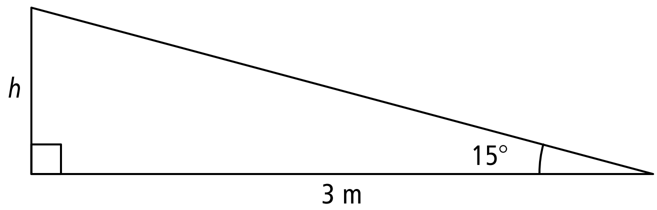
Internally assessed
3 credits

Achievement Standard 91032
Apply right-angled triangles in solving measurement problems

Diagrams are not to scale.

Question 1

A farmer has to build a ramp to load sheep onto a truck.



Getty Images – Bobtokyoharris

The ramp is to make an angle of 15° with the ground.

The horizontal distance between the beginning and the end of the ramp is 3 metres.

a. Calculate the height of the end of the ramp, h, to 2 d.p.

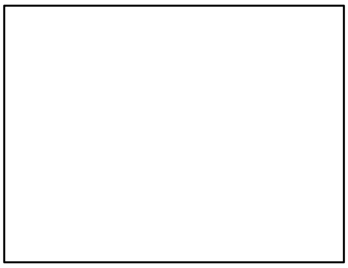
b. How long is the ramp?

c. If the angle of 15° is doubled to 30°, and the distance of 3 metres stays the same, what can you say about the value of h? Give a reason(s) to support your answer.

 ISBN 978-0-947504-97-7

Question 2

The **size** of a rectangular TV screen is given by the length of its diagonal, rounded to the nearest centimetre.



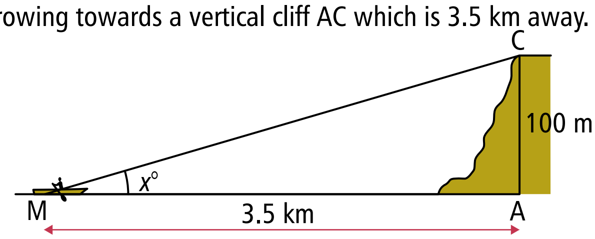
a. Use the theorem of Pythagoras to calculate the size of a TV screen with a length of 45 cm and a width of 34 cm.

b. A larger TV has a screen size (diagonal length) of 110 cm. If the length of the screen is 90 cm, calculate its width to the nearest centimetre.

ESA PUBLICATIONS ISBN 978-0-947504-97-7

Question 3

A man M is in a boat at sea and is rowing towards a vertical cliff AC which is 3.5 km away.



The height of the cliff is 100 metres.

The angle of elevation of the top of the cliff from the position of the man at M is denoted by $x°$.

Calculate the value of x.

Question 4

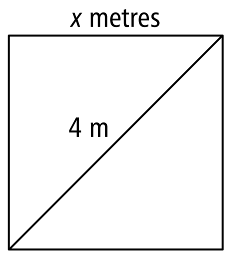
Andrew is a keen gardener and is planning to put a square bed of flowers in his lawn.

He has pegged out the diagonal of the square flower bed to be 4 metres long.

He makes a scale drawing of the diagonal and pegs out the rest of the square bed.

Calculate the length of the side of the square, x, in metres.

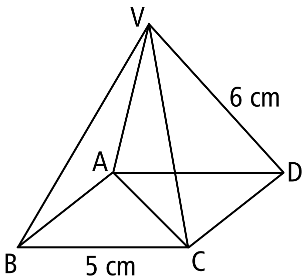
Round your answer to 2 d.p.



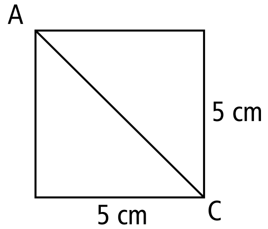
ESA PUBLICATIONS ISBN 978-0-947504-97-7

Question 5

An upright square-based pyramid is drawn below. All the sloping edges are of length 6 cm. The square base has sides of length 5 cm.

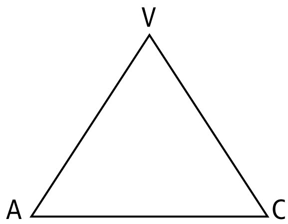


a. The base is redrawn below.



Calculate the length of AC to 2 d.p.

b. Calculate the angle the sloping edge VC makes with the square base of the pyramid. (You may find the figure here helpful.)



 ISBN 978-0-947504-97-7

Solutions

Question 1

a. The side h is opposite the angle and the side of length 3 m is adjacent to the angle, so choose tangent (tan).

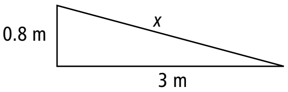
$\tan 15° = \frac{h}{3}$ — $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$h = 3\tan 15°$ — multiplying by 3

$= 0.8038...$

The height of the ramp is $h = 0.80$ m (2 d.p.)

b. Using Pythagoras



$x^2 = 0.8^2 + 3^2$

$= 9.64$

$x = 3.104\ldots$ — taking square root

the ramp is 3.10 m long (2 d.p.)

c. The relationship becomes

$\tan 30° = \frac{h}{3}$ — $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$h = 3\tan 30°$ — multiplying by 3

$= 1.732...$

So to two decimal places the value of h would be 1.73 metres, which is more than double its previous value ($2 \times 0.80 = 1.6$)

Question 2

a. $x = \sqrt{(45^2 + 34^2)}$

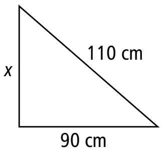
$= \sqrt{3\,181}$

$= 56.400\ldots$

The size of the screen is 56 cm (rounding to the nearest centimetre).

b. The diagonal makes a right-angled triangle with the two sides.

Let x be the width of the screen.



By the theorem of Pythagoras,

$x^2 + 90^2 = 110^2$

$x^2 = 110^2 - 90^2$ — rearranging

$x = \sqrt{(110^2 - 90^2)}$

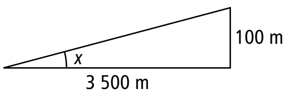
$= \sqrt{4\,000}$

$= 63.24\ldots$

The width of the screen is 63 cm (rounding to the nearest centimetre).

Question 3

a. All sides should be in the same unit, so convert 3.5 km to 3 500 m.



The side of length 100 m is opposite the angle and the side of length 3 500 m is adjacent to the angle, so choose tangent (tan).

$\tan x = \frac{100}{3\,500}$ — $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

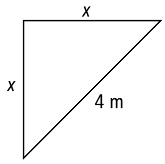
$\tan x = 0.02857$

$x = \tan^{-1}0.02857$ — taking inverse tan

$x = 1.6°$

Question 4

Using the theorem of Pythagoras in the right-angled triangle shown



square bed so missing side $= x$

$x^2 + x^2 = 4^2$

$2x^2 = 16$ — simplifying

$x^2 = 8$ — dividing by 2

$x = \sqrt{8}$ — taking square root

$x = 2.8284\ldots$

Length of the side of the square is $x = 2.83$ m (2 d.p.)

Question 5

a. Using the theorem of Pythagoras in the right-angled triangle ABC:

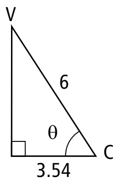
$AC^2 = 5^2 + 5^2$

$= 50$

$AC = \sqrt{50}$

$AC = 7.07$ cm (2 d.p.)

b. Draw a perpendicular line from V to the side AC to form the right-angled triangle shown below. The required angle is marked θ on the diagram.



The base of the triangle is of length half AC

Base length $= 7.07 \div 2$ — since triangle VAC is isosceles

$= 3.54$ cm (2 d.p.)

The base is adjacent and the side of length 6 cm is the hypotenuse, so choose cosine (cos).

$\cos\theta = \frac{3.54}{6}$ — $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\theta = \cos^{-1}\left(\frac{3.54}{6}\right)$

$= 53.8°$

VC makes an angle of 53.8° with the square base.

ESA PUBLICATIONS ISBN 978-0-947504-97-7