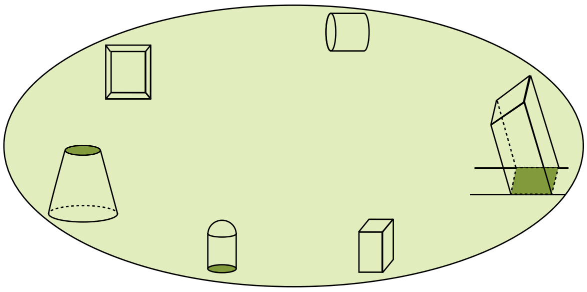
Mathematics and Statistics 1.5

Internally assessed
3 credits

Achievement Standard 91030
Apply measurement in solving problems

Some archaeologists have recently uncovered a number of stone monuments that were built about the time of Stonehenge. These monuments are all mathematical solids, and are clustered together in an Enclosure Circle, as shown in the figure below.



Some of these monuments are used in the following assessment.

1. A surveyor used a trundle wheel (as shown in the figure) to estimate the circumference of the Enclosure Circle.

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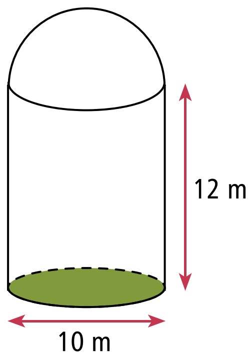
This trundle wheel measures out a distance of one metre when it rolls through one full turn.

a. Calculate the diameter of the trundle wheel in centimetres to 1 d.p.

b. One of the archaeologists estimated the diameter of the Enclosure Circle to be 50 metres. Estimate the area of the Enclosure Circle in hectares to 2 d.p.

2. One of the monuments is an upright cylinder surmounted by a hemisphere, with dimensions as shown on the figure below.



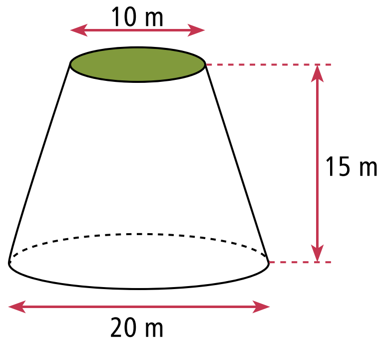
a. Find the volume of the monument in cubic metres to 1 d.p.

ISBN 978-0-947504-97-7

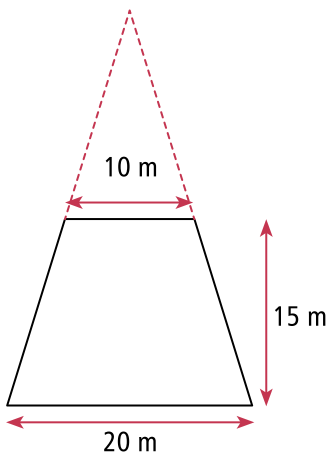
b. Find the density of the stone in kg/m^3, if the weight of the monument is 2 520 tonnes.

c. Find the exposed surface area of the monument (base is not exposed).

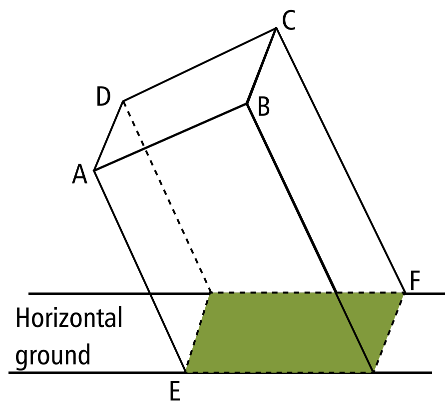
3. The monument drawn here is a truncated cone. Find the volume of the monument. Give your answer in cubic metres to 2 s.f.



A diagram of the cross-section of the original cone is shown below.



4. A monument in the shape of a cuboid has partly sunk into the soft ground at one end. ABCD is a rectangle 3 m by 6 m. AE is 8 m long and CF is 12 m plus 4 cm long. Find the volume of the monument that is above the ground. Give your answer in cubic metres to 2 d.p.



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Solution

1. a. $\pi d = 1$ $C = \pi d$

$d = \frac{1}{\pi}$

$d = 0.318\ 309\ \ldots$

Diameter of trundle is 31.8 cm (1 d.p.).

b. $r = 25$ radius is half diameter

Area $= \pi \times 25^2$ area $= \pi r^2$

$= 1963.495\ \text{m}^2$

$= 0.19635$ ha   1 ha $= 10\ 000\ \text{m}^2$

$= 0.20$ ha (2 d.p.)

The area of the enclosure is estimated to be 0.20 ha.

2. a. Volume of cylinder $= \pi \times 5^2 \times 12$ $v = \pi r^2 h$

$= 942.48\ \text{m}^3$ (2 d.p.)

Volume of sphere is $\frac{4}{3}\pi r^3$, so:

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$ hemisphere is half a sphere

$= 261.80\ \text{m}^3$ (2 d.p.)

Total volume $= 942.48 + 261.80$ adding volumes of cylinder and hemisphere

$= 1204.28\ \text{m}^3$ (2 d.p.)

The volume of the monument is 1204.3 m^3 (1 d.p.)

b. Converting tonnes to kilograms, gives:

2 520 t = 2 520 000 kg   1 tonne = 1 000 kg

Density $= \frac{2\ 520\ 000}{1\ 204.3}\ \text{kg/m}^3$ dividing kg by m^3

$= 2092.5\ \text{kg/m}^3$

Density of the stone is 2092.5 kg per cubic metre

c. Curved face of cylinder has surface area:

SA $= 2\pi \times 5 \times 12$ SA $= 2\pi rh$

$= 377.0\ \text{m}^2$

Curved face of hemisphere has surface area:

SA $= \frac{1}{2} \times 4 \times \pi \times 5^2$ SA sphere $= 4\pi r^2$

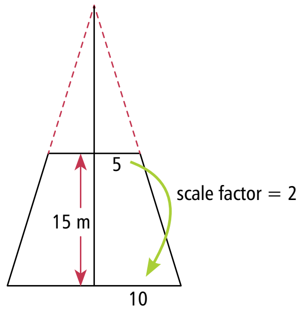
$= 157.1\ \text{m}^2$

Exposed surface area $= 377.0 + 157.1$

$= 534.1\ \text{m}^2$ (1 d.p.)

Exposed surface area of monument (excluding base) is 534.1 m^2.

3. The cross-section of the original cone (before the small cone was removed) had a height of 30 m   similar triangles, scale factor = 2



So, volume of original cone is:

Volume $= \frac{1}{3} \times \pi \times 10^2 \times 30$ $v = \frac{1}{3}\pi r^2 h$

$= 3\ 141.6\ \text{m}^3$

Volume of small (removed) top cone is:

Volume $= \frac{1}{3} \times \pi \times 5^2 \times 15$ $v = \frac{1}{3}\pi r^2 h$

$= 392.7\ \text{m}^3$

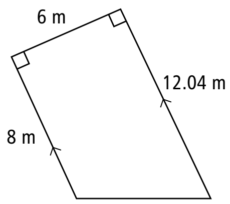
Subtracting gives the volume of the truncated cone:

Volume $= 3\ 141.6 - 392.7$

$= 2\ 748.9\ \text{m}^3$

The volume of the monument is 2 700 m^3 (2 s.f.).

4. Figure is a trapezium with cross-section as shown below:



Area of trapezium $= \frac{1}{2}(8 + 12.04) \times 6$ $A = \frac{1}{2}(a + b)h$

$= 60.12\ \text{m}^2$

Volume of prism $= 60.12 \times 3$ $V = A \times h$

$= 180.36\ \text{m}^3$ (2 d.p.)

The volume of the monument that is above ground is 180.36 m^3.

ESA PUBLICATIONS ISBN 978-0-947504-97-7