

## Achievement Standard 91258 (Mathematics and Statistics 2.3)

## Apply sequences and series in solving problems

## Practice assessment

## Solutions

1. Time to complete project is

$$1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + 3 \quad \text{[substituting } n = 6, 7, 8, 9, 10 \text{ into } \frac{1}{2}n - 2 \text{ and adding results]}$$

$$= 10 \text{ days}$$

2. a. Arithmetic sequence with
- $a = 150$
- ,
- $d = 52$
- ,
- $n = 8$
- [5–12 March is 8 days]

$$t_8 = 150 + (8 - 1)52 \quad \text{[substituting in } t_n = a + (n - 1)d]$$

$$= 150 + 7 \times 52$$

$$= 514 \text{ words on 12 March}$$

- b. Sum of terms of arithmetic sequence, where
- $a = 150$
- ,
- $d = 52$
- ,
- $n = 8$

$$S_8 = \frac{8}{2}(2 \times 150 + 7 \times 52) \quad \text{[substituting in } S_n = \frac{n}{2}(2a + (n - 1)d)]$$

$$= 4 \times 664$$

$$= 2\,656 \text{ words}$$

- c.
- $S_{12} = \frac{12}{2}(2 \times 150 + 11 \times 52)$
- 
- = 5 232 words, so project is finished by the 12th day

$$S_{11} = \frac{11}{2}(2 \times 150 + 10 \times 52)$$

$$= 4\,510 \text{ words}$$

so project is not finished by the 11th day.

3. Geometric sequence with
- $a = 100$

$$\text{Common ratio is } r = \frac{90}{100} \quad [r = \frac{t_2}{t_1}]$$

$$= 0.9$$

$$\text{Sum to infinity} = \frac{100}{1 - 0.9} \quad \text{[substituting in } S_\infty = \frac{a}{1 - r}]$$

$$= 1\,000$$

Aunty Jo gives \$1 000 in total

4. a. Geometric sequence with
- $r = 2$

$$t_6 = 1\,024$$

$$a \times 2^5 = 1\,024 \quad [t_n = ar^{n-1}]$$

$$32a = 1\,024$$

$$a = 32 \quad \text{[dividing by 32]}$$

Amanda wrote 32 words on the first day.

- b. Require
- $n$
- such that
- $S_n = 5\,000$

$$\frac{32(1 - 2^n)}{1 - 2} = 5\,000 \quad [S_n = \frac{a(1 - r^n)}{1 - r}]$$

$$32(1 - 2^n) = -5\,000 \quad \text{[multiplying by } -1]$$

$$1 - 2^n = -156.25 \quad \text{[dividing by 32]}$$

$$2^n = 157.25 \quad \text{[rearranging and simplifying]}$$

$$\log 2 = \log 157.25 \quad \text{[taking logs of both sides]}$$

$$n = \frac{\log 157.25}{\log 2} \quad \text{[rearranging]}$$

$$n = 7.3$$

Amanda will finish writing on the 8th day.

Note: The equation  $2^n = 157.25$  could also be solved by graphics calculator [SOLVER] or by trial-and-improvement techniques.