Achievement Standard 91267 (Mathematics and Statistics 2.12)

Apply probability methods in solving problems

Practice assessment

Solutions

1. a. $P(45 < X < 47.3) = P(-1.429 < Z < 1.857)$
$= P(0 < Z < 1.429) + P(0 < Z < 1.857)$
$= 0.892$ (3 dp) [tables]

b. $P(X > 46.5) = P(Z > 0.714)$
$= 0.5 - P(0 < Z < 0.714)$
$= 0.2376$
$= 23.8\%$

c. $P(X < 45) = P(Z < -1.429)$
$= 0.5 - P(0 < Z < 1.429)$
$= 0.0765$

Expected number $= 24\,000 \times 0.0765$
$= 1\,836$

Number of bars is 1 800 (2 sf)

d. i. 95% are below k grams, so require $P(X < k) = 0.95$

From tables $P(Z < 1.645) = 0.95$

95% of values are less than 1.645 standard deviations above the mean

Required weight $= 46 + 1.645 \times 0.7 = 47.2$ g (1 dp)

ii. The weights of 5% of bars are above the middle 90% and the weights of 5% of bars are below the middle 90%

So 95% of bars weigh less than the upper limit (of the middle 90% of weights), so using the working in (i) the upper limit of the weights of the middle 90% of bars is 47.2 g

By symmetry lower limit is $46 - 1.645 \times 0.7 = 44.8$ g

90% of the bars lie between 44.8 g and 47.2 g

e. A normal distribution is symmetrical about the mean, unimodal and with values in the tails less likely.

The experimental distribution is unimodal with values in the range 46.5 g to 47 g most likely (these values are a little higher than the mean of 46 g).

The experimental distribution is not symmetrical, but is skewed to the left, with a gap in the 44.5–45 gram weight range, and fewer than might be expected in the 47.5–48 gram weight range.

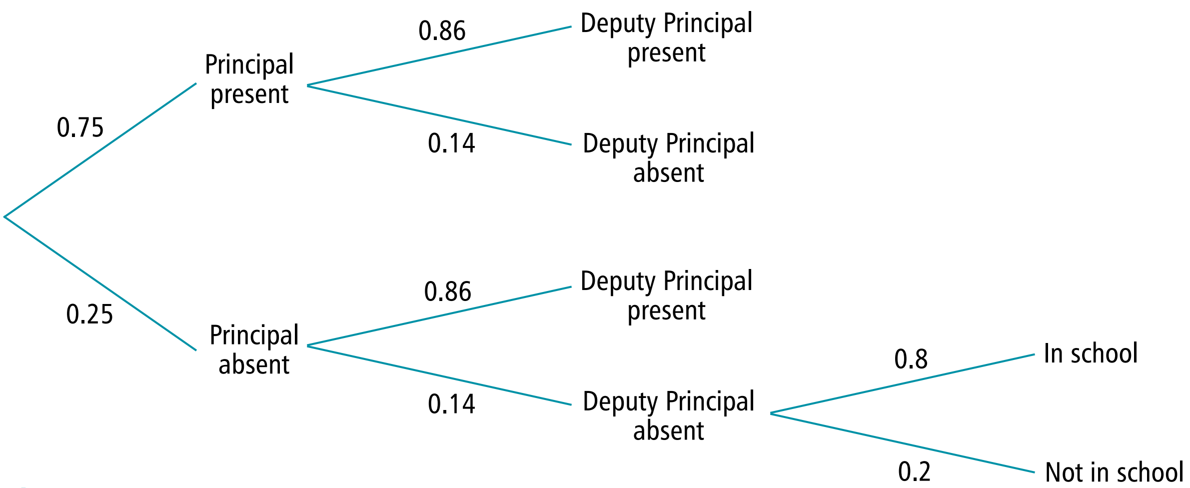
Values in the tails of the experimental distribution are generally less likely, although the 48–48.5 gram weight range is more likely than would be expected.

These differences would suggest that the machine may not be working properly.

Although less than 1% of a production run has been sampled, a random sample size of 200 bars is reasonably large for reliable results.

For this reason, the differences between the experimental distribution of Chocco bar weights and the theoretical normal distribution of these weights cannot simply be explained by sampling variability.

2. a.

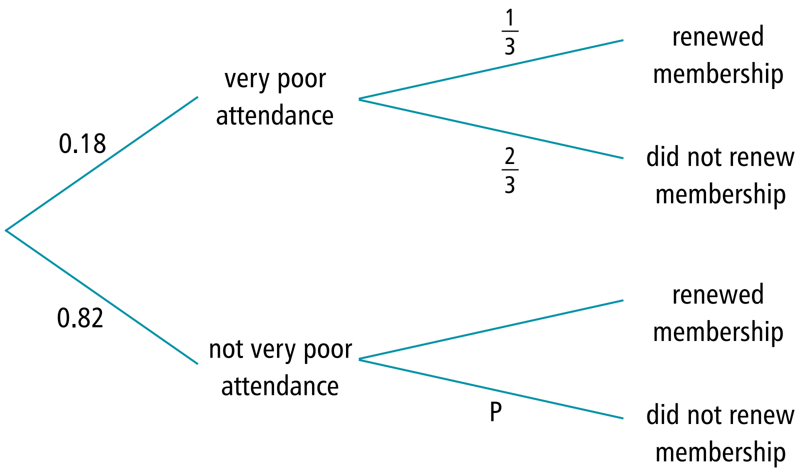


i. $0.75 \times 0.86 = 0.645$

ii. $1 - 0.25 \times 0.14 = 0.965$

iii. $0.25 \times 0.14 \times 0.2 = 0.007$

b.



i. $0.18 \times \frac{2}{3} = 0.12$ or 12%

ii. P(very poor attendance but renewed) $= 0.18 \times \frac{1}{3} = 0.06$

Expected number from 4 580 members $= 0.06 \times 4\,580 = 275$ (to nearest whole number)

iii. Let p = probability that a member who did not have very poor attendance did not renew.

Solve $0.18 \times \frac{2}{3} + 0.82 \times p = 0.325$

$$0.12 + 0.82p = 0.325$$
$$0.82p = 0.205$$
$$p = 0.25$$

25% (or $\frac{1}{4}$) of customers who did not have very poor attendance did not renew their membership.

3. a. $\frac{45}{678}$ or 0.066 (3 dp)

b. $\frac{268}{281}$ or 0.954 (3 dp)

c. Assuming that the rate of returns in 2011 carries on into 2012:

expect $\frac{14}{153} \times 184 = 16.8$ to be returned

i.e. 16 or 17 *Turboclean* Model A vacuum cleaners.

d. $\frac{9}{18}$ or 0.5

e. i. $\frac{27}{434}$ or 0.062

ii. P(VacMax Model B is returned) $= \frac{9}{109}$

Expected number $= \frac{9}{109} \times 100 = 8.257$

Expect 8 or 9 to be returned

iii. $\frac{18}{244}$ or 0.074 (3 dp)

f. i. $\frac{27}{434} \div \frac{45}{678} = 0.94$ (2 dp)

ii. *Turboclean* vacuum cleaners are approximately 6% less likely to be returned than the average rate of returns of vacuum cleaners.

g. The claim does not seem to be correct. For 2011 sales for *VacMax*, the relative risk of Model B being returned compared to Model A is $\frac{9}{109} \div \frac{9}{135} = 1.24$, so Model B is 24% more likely to be returned than Model A.