B Balancing Equations

We have learned that in an equation, the expressions on both sides of the equal sign must be equivalent.

For example,

$3 + 4 = 14 \div 2$	because $7 = 7$
$4 \times 6 - 3 = 3 \times 7$	because $21 = 21$
$3x + 3 + 5 = x + 8 + 2x$	because $3x + 8 = 3x + 8$

If the expressions on both sides are not equivalent, it is not an equation.

For example,

$3 \times 4 \neq 5 + 8$	because $12 \neq 13$
$8 \times 2 + 3 \neq 3^2 + 1$	because $19 \neq 10$
$2y + 3 - y \neq y + 10 - 6$	because $y + 3 \neq y + 4$

If we add the same number to, or subtract the same number from, both sides of an equation, the expressions on both sides remain equivalent.

Similarly, if we multiply or divide both sides of an equation by the same number, the expressions on both sides remain equivalent. For example,

If $4 + 2 = 2 \times 3$,

then
$(4 + 2) + 4 = (2 \times 3) + 4$
$(4 + 2) - 1 = (2 \times 3) - 1$
$(4 + 2) \times 2 = (2 \times 3) \times 2$
$(4 + 2) \div 3 = (2 \times 3) \div 3$

If we add, subtract, multiply, or divide each side by different numbers, the expressions on each side will not be equivalent. For example,

If $4 + 2 = 2 \times 3$,

then
$(4 + 2) + 4 \neq (2 \times 3) + 3$
$(4 + 2) - 2 \neq (2 \times 3) - 1$
$(4 + 2) \times 2 \neq (2 \times 3) \times 3$
$(4 + 2) \div 3 \neq (2 \times 3) \div 5$

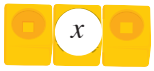
CLASS ACTIVITY 1

Objective: To explore the properties of equations.

Materials: Balance scale labeled with an equal sign in the middle.
Two different colored connecting cubes such as yellow and green to represent the variables and constants.
Small stickers with (x) to label the yellow (variable) cubes and (=) to label the middle of the scale.

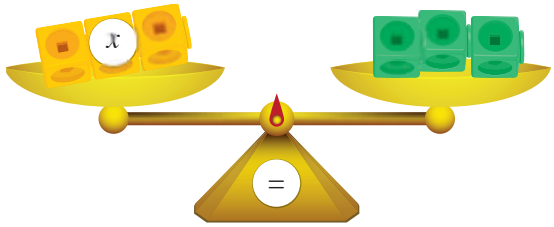
Questions

1. Put 3 yellow cubes together and label them with an x, as shown.

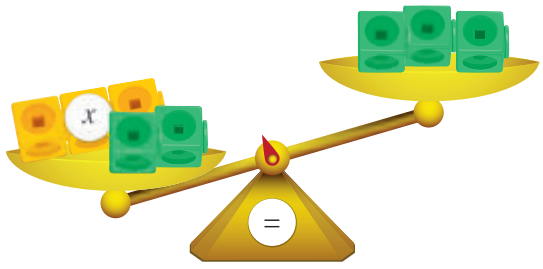


This shows that $x = 3$.

Place 3 yellow cubes together on the left side and 3 green cubes on the right side of the balance scale, as shown.



2. Add 2 green cubes to the left side of the balance scale, so we have $x + 2$ on the left side of the scale.



3. Next, add 2 green cubes to the right side of the scale. Fill in the □ with the appropriate number.

$$x + 2 = 3 + \square.$$

4. Now, the balance scale shows the equation $x + 2 = 5$.
Take 2 green cubes off the left pan.

(a) What happens to the scale?

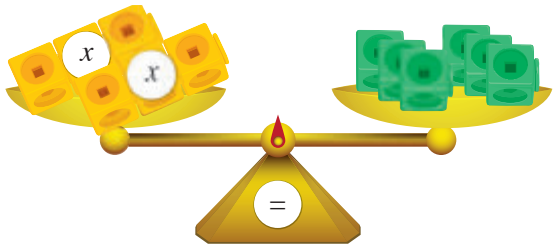
(b) Take 2 green cubes off the right pan now. What happens to the scale?

(c) Fill in the □ with the appropriate number.

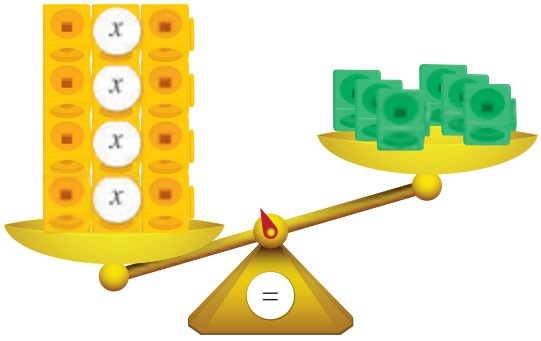
$$x + 2 - 2 = 3 + 2 - \square.$$

5. Add the same number of green cubes to each side and observe what happens.
6. Add a different number of green cubes to each side and observe what happens.

7. Make a new equation $2x = 6$, as shown below.



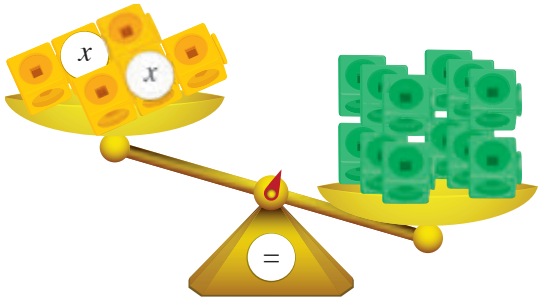
8. Add yellow cubes on the left side of the scale, as shown below.



(a) What do you need to do to the right side to balance the scale?
(b) Fill in the □ with the appropriate number.

$2x \times 2 = 6 \times \square$.

9. Remove half of the x's from the left side of the scale, so that you have $2x$ on the left side and 12 cubes on the right side.



(a) What do you need to do to the right side to balance the scale?
(b) Fill in the □ with the appropriate number.

$$\frac{4x}{2} = \frac{12}{\square}.$$

From the results of **Class Activity 1**, we see that in order to maintain equality, we need to increase or decrease both sides of the equation by the same value.

- If we add the same number to, or subtract the same number from, both sides of an equation, it maintains its equality.
- If we multiply or divide both sides of the equation by the same number, it maintains its equality.

Example 4

Solve the equation $x + 17 = 36$.

Solution

Method 1

Algebraically,

$$x + 17 = 36$$

What number plus 17 equals 36?

$$x + 17 - 17 = 36 - 17$$

Subtract 17 from both sides of the equation so that the variable is by itself.

$$x = 19$$

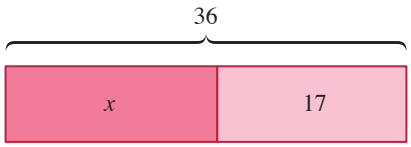
Check by substitution.

Check: $19 + 17 = 36$

$36 = 36$ ✓

Line up the equal signs in your equations and check your answer after you solve for x.

Method 2



From the model,

$x + 17 = 36$

$x = 36 - 17$

Subtract the part we know, 17, from the total, 36, to find x.

$= 19$

REMARKS

Total – Part = Part

Try It! 4

Solve each equation and check you answer.

(a) $p + 7 = 23$

(b) $y + 4 = 11.5$

(c) $x + \frac{1}{2} = \frac{3}{4}$

(d) $s + 48.3 = 66$

Example 5 Solve the equation $x - 8 = 14$.

Solution

Method 1

Algebraically,

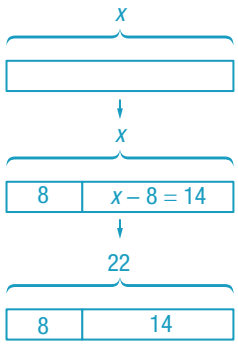
$x - 8 = 14$

$x - 8 + 8 = 14 + 8$ What number minus 8 equals 14?

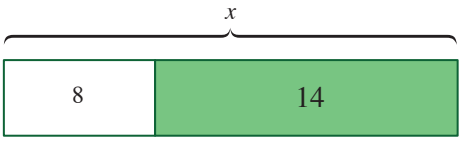
$x = 22$

Check: $22 - 8 = 14$

$14 = 14$ ✓



Method 2



From the model,

$x - 8 = 14$

$x = 14 + 8$ Add the two parts to find the total.

$= 22$

REMARKS

Part + Part = Total

Try It! 5 Solve each equation and check your answer.

(a) $n - 5 = 12$

(b) $y - 87 = 56$

(c) $x - \frac{1}{3} = \frac{1}{6}$

(d) $r - 0.75 = 1.25$

Example 6 Solve the equation $48 = x + 19$.

Solution

Method 1

Algebraically,

$48 = x + 19$

$48 - 19 = x + 19 - 19$

$29 = x$

$x = 29$

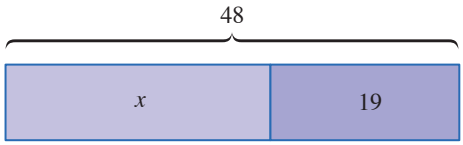
Check: $48 = 29 + 19$

$48 = 48$ ✓

REMARKS

When presenting the final solution of an equation, we generally write the variable on the left-hand side like $x = 29$, instead of $29 = x$.

Method 2



From the model,

$48 = x + 19$

$x + 19 = 48$

$x = 48 - 19$

$x = 29$

Try It! 6 Solve each equation and check your answer.

(a) $43 = m + 18$ **(b)** $5 = x + \frac{4}{3}$

(c) $35 = y - 17$ **(d)** $\frac{3}{4} = x - 2$

REMARKS

We could also rewrite the equation so the variable is on the left side before solving it.

Example 7 Solve the equation $4x = 52$.

Solution

Method 1

$4x = 52$

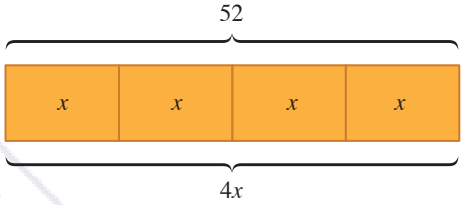
$\frac{4x}{4} = \frac{52}{4}$ Divide both sides of the equation by 4.

$x = 13$

Check: $4 \times 13 = 52$

$52 = 52$ ✓

Method 2



From the model,

$4x = 52$

$x = 52 \div 4$ 4 units → 52

1 unit → $52 \div 4 = 13$

$x = 13$

Try It! 7 Solve each equation and check your answer.

(a) $6x = 84$ **(b)** $5n = 27.5$

(c) $2.5p = 7.5$ **(d)** $0.5y = 13$

Example 8 Solve $\frac{x}{3} = 5$.

Solution

Method 1

$$\frac{x}{3} = 5$$

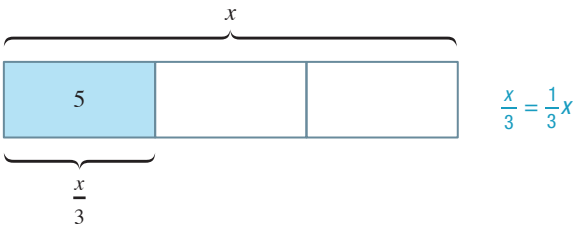
$$\frac{x}{3} \times 3 = 5 \times 3$$ Multiply both sides of the equation by 3.

$$x = 15$$

Check: $\frac{15}{3} = 5$

$5 = 5$ ✓

Method 2



$\frac{x}{3} = \frac{1}{3}x$

$$\frac{x}{3} = 5$$

$$x = 3 \times 5$$ 1 unit $\rightarrow 5$

$= 15$ 3 units $\rightarrow 3 \times 5 = 15$

Try It! 8 Solve each equation and check your answers.

(a) $\frac{n}{4} = 28$ **(b)** $\frac{x}{7} = 2.5$

(c) $\frac{y}{8} = 17$ **(d)** $\frac{m}{5} = 9.5$

Example 9 Solve $\frac{2}{3}x = 16$.

Solution

Method 1

$$\frac{2}{3}x = 16$$

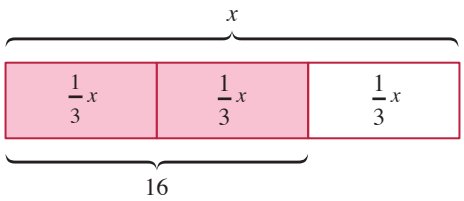
$$3 \times \frac{2}{3}x = 3 \times 16$$ Multiply both sides of the equation by 3 to get a whole-number coefficient.

$$2x = 48$$

$$\frac{2x}{2} = \frac{48}{2}$$ Divide both sides of the equation by 2.

$$x = 24$$

Method 2



$\frac{2}{3}x = 16$ — 2 units → 16

$\frac{1}{3}x = 16 \div 2 = 8$ — 1 unit → 16 ÷ 2 = 8

$\frac{3}{3}x = 3 \times 8 = 24$ — 3 units → 3 × 8 = 24

Method 3

$\frac{2}{3}x = 16$ — $\frac{2}{3}$ of what number is 16?

$\frac{3}{2} \times \frac{2}{3}x = \frac{3}{2} \times 16$ — Multiply both sides of the equation by the reciprocal of $\frac{2}{3}$. $\frac{3}{2} \times \frac{2}{3} = 1$

$x = \frac{3 \times 16}{2}$

$x = 24$ — **Check:** $\frac{2}{3} \times 24 = 16$, $16 = 16$ ✓

Thus, $x = 24$

Try It! 9

Solve each equation and check your answers.

(a) $\frac{3}{4}x = 12$ **(b)** $\frac{4}{5}y = 28$

(c) $\frac{6}{7}n = 1.8$ **(d)** $\frac{3}{8}p = \frac{1}{2}$

Example 10

Brianna had some money in her savings account. She deposited \$87 more and now she has \$324. How much money did she have at first? Write an equation and solve the problem.

Solution

Let \$$m$ represent the amount of money Brianna had in her savings account at first.

Algebraically,

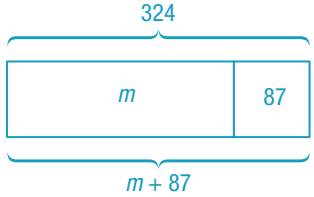
$m + 87 = 324$

$m + 87 - 87 = 324 - 87$

$m = 237$

She had \$237 at first.

Using a model,



From the model,

$m = 324 - 87$

$= 237$

REMARKS

The variable represents what we are trying to find.

Try It! 10

On Monday, a sixth grade class collected some cans for a recycling project. On Tuesday they collected 352 cans. They collected 641 cans in total on these two days. How many cans did they collect on Monday? Write an equation and solve the problem.

Example 11

Danny had some ribbon. He used 96 cm of it to wrap a present. He has 208 cm of ribbon left. How many centimeters of ribbon did he have at first? Write an equation and solve the problem.

Solution

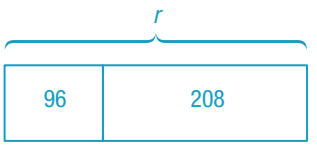
Let r cm represent the length of ribbon Danny had at first.

Algebraically,

$$\begin{aligned} r - 96 &= 208 \\ r - 96 + 96 &= 208 + 96 \\ r &= 304 \end{aligned}$$

He had 304 cm of ribbon at first.

Using a model,



From the model,
$r = 96 + 208$
$= 304$

REMARKS

If you can write the equation without drawing the model, you do not need to draw the model.

Try It! 11

A bakery made some rolls in the morning. By noon, 156 rolls were sold and 87 rolls were left. Write an equation from the problem and solve it to find how many rolls the bakery made in the morning.

Example 12

Abel has one-third as much money as Marvin. If Abel has \$64, how much money does Marvin have? Write an equation and solve the problem.

Solution

Let \$$x$ represent the amount of money Marvin has.

Then, $\$\left(\frac{1}{3}x\right)$ represents the money Abel has.

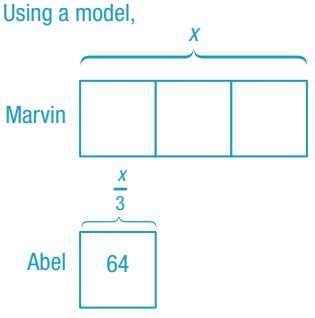
$$\frac{1}{3}x = 64$$

$$3 \times \frac{1}{3}x = 3 \times 64$$

$$x = 192$$

Therefore, Marvin has \$192.

Using a model,



1 unit $\rightarrow$ 64
3 units $\rightarrow 3 \times 64 = 192$

Try It! 12

Jesse has 4 times as many foreign stamps as local stamps. If he has 267 local stamps, how many foreign stamps does he have? Write an equation and solve the problem.

Example 13

Rebecca read 5 times as many pages in her book this week as last week. If she read 135 pages this week, how many pages did she read last week? Write an equation and solve the problem.

Solution

Let x pages represent the number of pages Rebecca read last week.
Then, the number of pages she read this week is $5x$.

Algebraically,

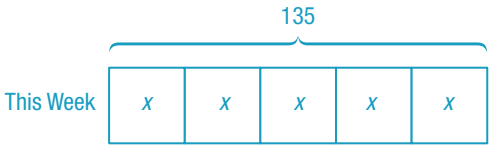
$$5x = 135$$

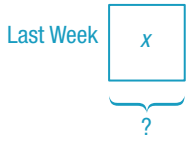
$$\frac{5x}{5} = \frac{135}{5}$$

$$x = 27$$

She read 27 pages last week.

Using a model,





From the model,
5 units $\rightarrow$ 135
1 unit $\rightarrow$ 135 ÷ 5 = 27

Try It! 13

Steve spent 3 times as much on a coat as on a pair of shoes. If he spent \$90 on the coat, how much did he spend on the pair of shoes? Write an equation and solve the problem.

Example 14

Lily bought a sweater and a coat. The sweater cost $\frac{3}{5}$ as much as the coat. If the sweater cost \$36, how much did the coat cost?

Solution

Method 1

Let \$$x$ be the cost of the coat.

Then, $\$\left(\frac{3}{5}x\right)$ represents the cost of the sweater.

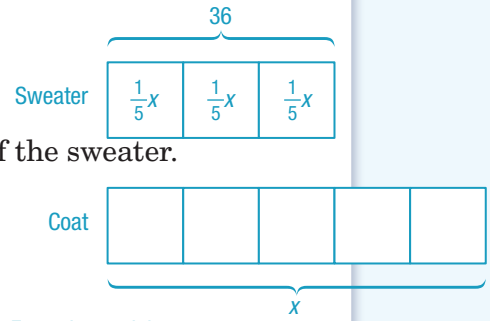
$$\frac{3}{5}x = 36$$

$$\frac{1}{5}x = 36 \div 3 = 12$$

$$\frac{5}{5}x = 12 \times 5 = 60$$

Thus, $x = 60$.

Using a model,



From the model,
3 units $\rightarrow$ \$36
1 unit $\rightarrow$ \$36 ÷ 3 = \$12
5 units $\rightarrow$ 5 × \$12 = \$60

Method 2

$$\frac{3}{5}x = 36$$

$$\frac{5}{3} \times \frac{3}{5}x = \frac{5}{3} \times 36$$

Multiply both sides of the equation by the reciprocal of $\frac{3}{5}$.

$$x = 60$$

The coat cost \$60.

Try It! 14

Ping took $\frac{5}{8}$ as much time as Daniel to solve a puzzle. She took 45 minutes. Write an equation for the problem and solve it to find how much time Daniel took to solve the puzzle.

EXERCISE 9.1

BASIC PRACTICE

1. Determine whether $x = 3$ is a solution of each equation.

(a) $x + 12 = 15$ **(b)** $21 - x = 17$

(c) $7 = 11 - x$ **(d)** $48 = x + 45$

2. Determine whether $y = 12$ is a solution of each equation.

(a) $7y = 82$ **(b)** $72 = 6y$

(c) $\frac{y}{3} = 4$ **(d)** $8 = \frac{y}{2}$

3. Determine whether $x = 8$ is a solution of each equation.

(a) $\frac{35}{4} = x + \frac{3}{4}$ **(b)** $\frac{3}{2}x = 14$

(c) $2.3x = 18.2$ **(d)** $6 = \frac{3}{4}x$

4. Solve each equation and check your answer.

(a) $x + 19 = 52$ **(b)** $y - 12 = 14$

(c) $61 = n + 37$ **(d)** $26 = p - 38$

5. Solve each equation and check your answer.

(a) $8x = 104$ **(b)** $\frac{n}{7} = 23$

(c) $112 = \frac{x}{4}$ **(d)** $105 = 15x$

FURTHER PRACTICE

6. Determine whether $y = 0.6$ is a solution of each equation.

(a) $5y = 2$ **(b)** $4.2 = 7y$

(c) $\frac{2}{3}y = 0.4$ **(d)** $17.4 - y = 18$

7. Solve each equation and check your answer.

(a) $x + 8 = 11.4$ **(b)** $n + \frac{3}{10} = \frac{4}{5}$

(c) $p + 0.7 = 18.5$ **(d)** $12 = x + \frac{1}{3}$

(e) $y - \frac{3}{7} = \frac{9}{14}$ **(f)** $\frac{5}{8} = y - 6$

(g) $2.5 = q - 1.7$

8. Solve each equation and check your answer.

(a) $4n = 11.2$ **(b)** $\frac{4}{5}x = 8$

(c) $3.5w = 7$ **(d)** $\frac{y}{5} = \frac{4}{3}$

(e) $3.5 = 0.7n$ **(f)** $\frac{3}{4}y = 1.5$

(g) $8.5 = \frac{x}{2}$

MATH@WORK

Write an equation for each problem and solve the problem.

9. Carlos had some money. After earning \$150 from a part-time job, he now has \$325. How much money did he have initially?

10. A blue ribbon is twice as long as a red ribbon. If the blue ribbon is 128 cm, how long is the red ribbon?

11. Janice walked to raise money for charity. Peter walked $\frac{2}{3}$ of the distance that Janice walked. If Peter walked 10 miles, how far did Janice walk?

12. A big tank holds 3 times as much water as a small tank. If the big tank holds 74 gallons more than the small tank, how much water does the small tank hold?

13. Mei goes to her gym which is 4.5 miles from her home. She cycles some distance of the journey and then runs the rest to get there. If Mei runs $\frac{1}{4}$ of the distance that she cycles, how far does she cycle?

14. Mr. Wong sold cakes at his shop. The number of cakes he sold in May was 18 more than that sold in June. If the total number of cakes sold is 150, how many cakes did he sell in June?

15. In a box, there are five more quarters than dimes. If the total amount of money in the box is \$8.25, how many dimes are there?

BRAINWORKS

16. The sum of three consecutive odd numbers is 129. Find the product of the three numbers.

17. A textbook contains the following two problems on algebraic simplification. How would you correct the printing errors to make them correct?

(a) $8x + 3 = 11x$

(b) $11y - 4 = 7y$

18. A student solved the equation $4x - 20 = 16$ as follows:

$$\begin{aligned} 4x - 20 &= 16 \\ x - 20 &= 16 \div 4 \\ x - 20 &= 4 \\ x &= 20 + 4 \\ x &= 24 \end{aligned}$$

Where did he go wrong? Show one correct way to solve this equation.

19. Three boys want to buy a present for their friend. The present costs \$120. Peter and Andrea have \$70 altogether. Andrea and Russell have \$80 altogether. Peter and Russell have \$90 altogether. Do they have enough money to buy the present?

WRITE IN YOUR JOURNAL

Uses of Inequalities

Create a situation that is represented by each of the following inequalities.

(a) $x + 4 \geqslant 10$

(b) $3x < 15$

EXTEND YOUR LEARNING CURVE

Evaluating Expressions and Solving Equations

Go online to learn how you can use a spreadsheet like Excel to evaluate algebraic expressions and to solve equations.

For example, what is the value of $\frac{1}{x} \times \frac{1}{y}$ when

(a) $x = 2$ and $y = 3$, **(b)** $x = 4$ and $y = 5$?

	A	B	C
1	x	y	$\frac{1}{x} \times \frac{1}{y}$
2	1	2	0.5
3	2	3	0.1667
4	3	4	0.0833
5	4	5	0.05
...			

Step 1: Enter in cell C2 the formula = 1/A2*1/B2

Step 2: Drag the fill handle from C2 to C5.

How would you use the spreadsheet software to solve these equations?

(a) $x + 12 = 23$ **(b)** $4x - 5 = 13$

	A	B	C	D
1	x	$x + 12$	x	$4x - 5$
2	9	21	4	11
3	10			
4	11			

Step 1: Enter = A2 + 12 in B2 and fill B3 and B4.

Step 2: Enter = 4*C2 – 5 in D2 and fill D3 and D4.

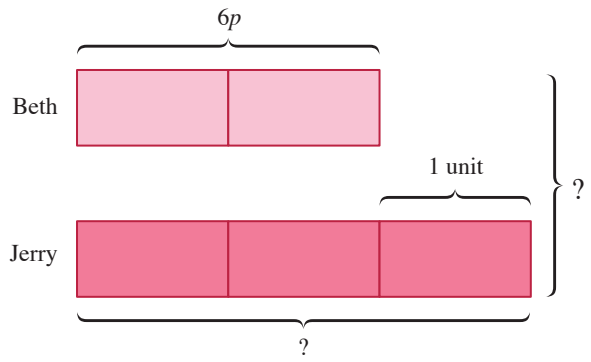
PROBLEM SOLVING CORNER

Example 1

Beth downloaded $6p$ e-books. Beth's e-book collection is $\frac{2}{3}$ of Jerry's e-book collection. How many e-books did they download altogether?

Solution

Method 1



From the model,

2 units = $6p$

1 unit = $6p \div 2$

= $3p$

3 units = $3 \times 3p$

= $9p$

$9p + 6p = 15p$

Method 2

2 units = $6p$

1 unit = $6p \div 2$

= $3p$

5 units = $5 \times 3p$

= $15p$

They downloaded $15p$ e-books.

Try It! 1

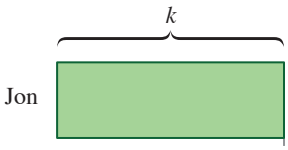
June has $6y$ friends on social media. The number of June's friends is $\frac{3}{5}$ of the number of Chapa's friends. How many friends on social media do they have in all?

Example 2

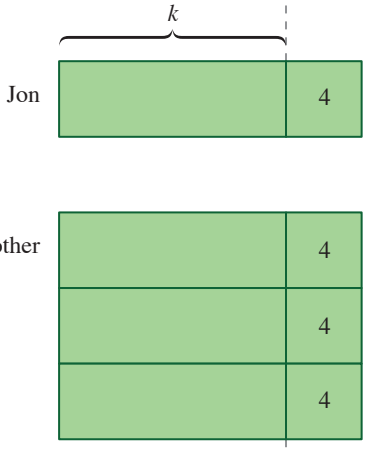
Jon was k years old 4 years ago. His mother is 3 times as old as Jon now. How old will Jon's mother be in 5 years?

Solution

4 years ago



Now



5 years later

$k + k + k + 4 + 4 + 4 + 5 = 3k + 17$

or

$3(k + 4) + 5 = 3k + 12 + 5$

$= 3k + 17$

In 5 years, Jon's mother will be $(3k + 17)$ years old.

Try It! 2

Five years ago, Alice was p years old. Alice's grandma is 4 times as old as her now. How old will Alice's grandma be in 5 years?