

13. The perimeter of a badminton court is 42 yd 2 ft. The width of the badminton court is 20 ft. What is the ratio of the width to the length of the badminton court?



BRAINWORKS

14. Tricia used the following method to find out whether the ratios 6 : 10 and 9 : 15 are equivalent.

Since $6 \div 10 = 0.6$
 and $9 \div 15 = 0.6$
 Thus, $6 : 10 = 9 : 15$.

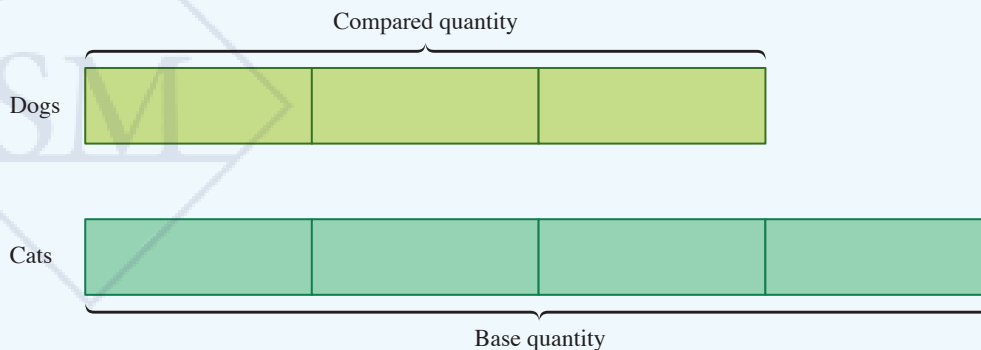
Can Tricia's method be used to find out whether 10 : 16 and 15 : 40 are equivalent? Show your working.

15. A bag contains some red balls and blue balls. The ratio of the number of red balls to the number of blue balls is 4 : 7. If the total number of balls in the bag is not more than 40, what are the possible numbers of blue balls in the bag? Explain how you find the answers using equivalent ratios.

5.2 Ratios and Fractions

In the previous section, we saw that finding equivalent ratios is no different from finding equivalent fractions. Let's look at some real-world problems that exemplify how ratios and fractions are related.

The ratio of the number of dogs to the number of cats in a pet shop is 3 : 4.

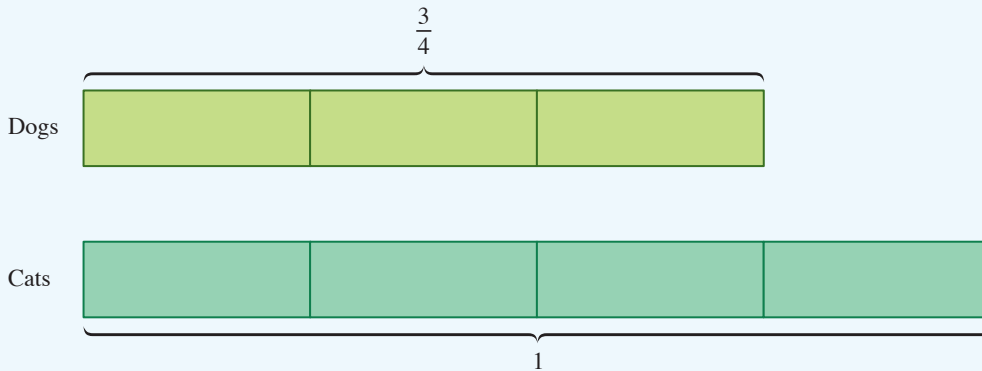


REMARK

The **base quantity** is often taken as **1 unit** because it is easier to make comparisons.

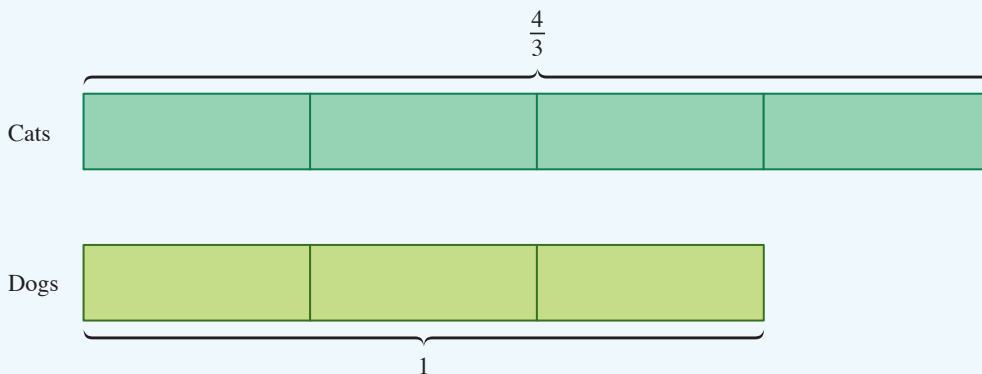


In the ratio 3 : 4, we compare the number of dogs to the number of cats by taking the number of cats as the base quantity, i.e., 1 unit.



The number of dogs is $\frac{3}{4}$ as many as the number of cats.

In the ratio 4 : 3, we compare the number of cats to the number of dogs by taking the number of dogs as the base quantity, i.e., 1 unit.



The number of cats is $1\frac{1}{3}$ or $\frac{4}{3}$ as many as the number of dogs.

In a ratio $a : b$, the quotient $\frac{a}{b}$ is called the value of the ratio and it tells us the relative size of a compared to b .

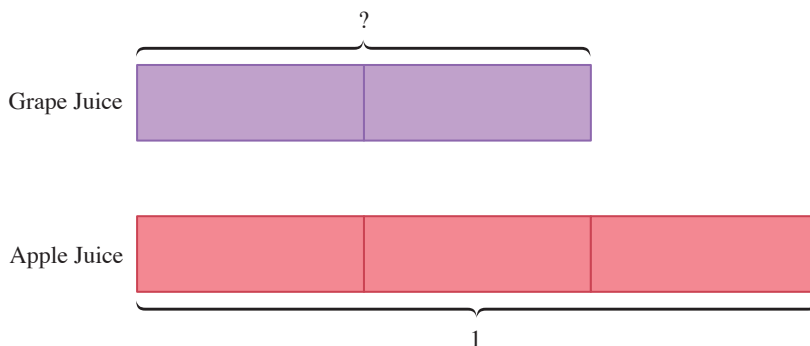
For example, in the ratio 3 : 4, the first quantity is $\frac{3}{4}$ as large as the second quantity.

Example 11

A juice recipe calls for mixing grape juice with apple juice in the ratio 2 : 3. Express the amount of grape juice as a fraction of the amount of apple juice.

Solution**Method 1**

We compare the amount of grape juice to the amount of apple juice by taking the amount of apple juice as the base quantity.



$$3 \text{ units} \rightarrow 1$$

$$1 \text{ unit} \rightarrow \frac{1}{3}$$

$$2 \text{ units} \rightarrow \frac{2}{3}$$

Method 2

Divide the number of units of grape juice by the number of units of apple juice.

$$2 \text{ units} \div 3 \text{ units} = \frac{2}{3}$$

The amount of grape juice is $\frac{2}{3}$ of the amount of apple juice.

Try It! 11

The ratio of the weight of Package A to the weight of Package B is 7 : 8. Express the weight of Package A as a fraction of the weight of Package B.

DISCUSS

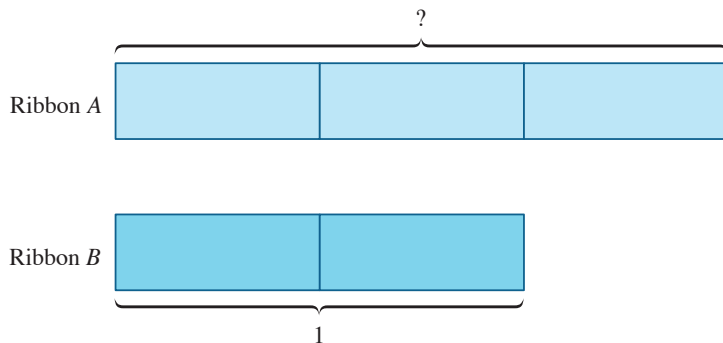
Can you see that it is $\frac{2}{3}$ just by looking at the model? Explain.

Example 12

The ratio of the length of Ribbon A to the length of Ribbon B is 3 : 2. How many times is Ribbon A as long as Ribbon B?

Solution**Method 1**

Consider the length of Ribbon B as the base quantity.



$$2 \text{ units} \rightarrow 1$$

$$1 \text{ unit} \rightarrow 1 \div 2 = \frac{1}{2}$$

$$\begin{aligned} 3 \text{ units} &\rightarrow 3 \times \frac{1}{2} = \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

Method 2

Divide the number of units in Ribbon A by the number of units in Ribbon B.

$$3 \text{ units} \div 2 \text{ units} = \frac{3}{2}$$

Ribbon A is $\frac{3}{2}$ or $1\frac{1}{2}$ times as long as Ribbon B.

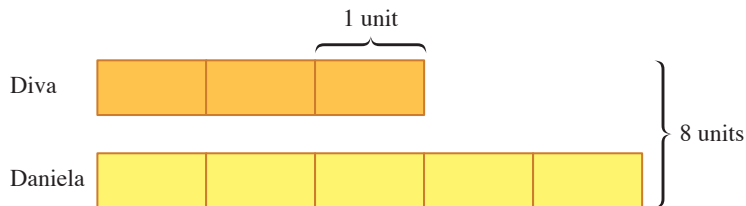
Try It! 12

The ratio of the weight of Package A to the weight of Package B is 5 : 4. How many times is Package A as heavy as Package B?

Example 13

Diva and Daniela shared some money in a ratio of 3 : 5.

- (a) What fraction of the money did Diva get?
 (b) What fraction of the money did Daniela get?

Solution

We are comparing each person's amount to the total amount of money they have.

- (a) Diva's amount of money represents 3 units out of a total of 8 units.

$$3 \div 8 = \frac{3}{8}$$

Diva got $\frac{3}{8}$ of the money.

- (b) Daniela's amount of money represents 5 units out of a total of 8.

$$5 \div 8 = \frac{5}{8}$$

Daniela got $\frac{5}{8}$ of the money.

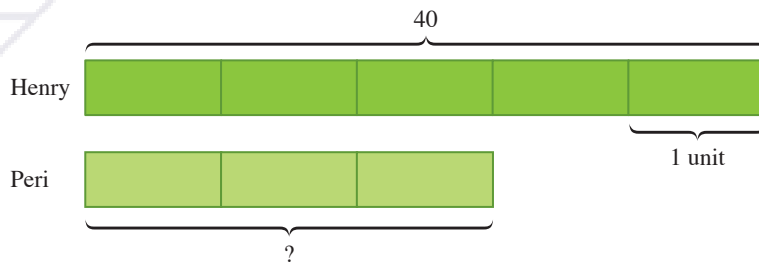
Try It! 13

An artist makes green paint by mixing yellow and blue paint in a ratio of 5 : 4.

- (a) What fraction of the mixture is yellow paint?
 (b) What fraction of the mixture is green paint?

Example 14

Henry and Peri share carnival tickets in a ratio of 5 : 3. If Henry has 40 tickets, how many tickets does Peri have?

Solution

Method 1

$$5 \text{ units} \rightarrow 40$$

$$1 \text{ unit} \rightarrow 40 \div 5 = 8$$

$$3 \text{ units} \rightarrow 3 \times 8 = 24$$

Method 2

Since Peri has $\frac{3}{5}$ as many tickets as Henry, we multiply 40 by $\frac{3}{5}$.

$$40 \times \frac{3}{5} = 24$$

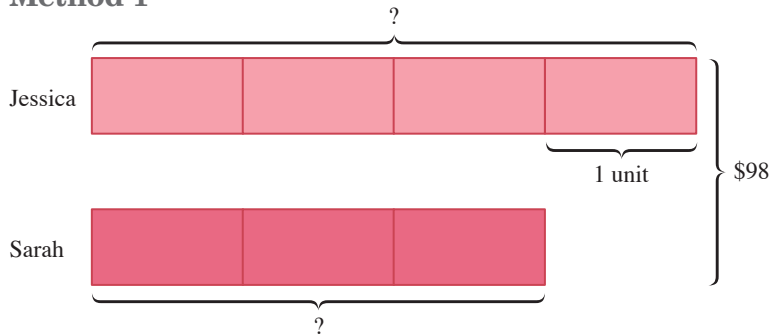
Peri has 24 tickets.

Try It! 14

The ratio of the volume of water in Bottle A to the volume of water in Bottle B is 7 : 4. If there are 28 ounces of water in Bottle B, what is the volume of water in Bottle A?

Example 15

Jessica and Sarah shared \$98 in a ratio of 4 : 3. How much money did each person get?

Solution**Method 1**

$$7 \text{ units} \rightarrow \$98$$

$$1 \text{ unit} \rightarrow \$98 \div 7 = \$14$$

$$3 \text{ units} \rightarrow 3 \times \$14 = \$42$$

Sarah's money

$$4 \text{ units} \rightarrow 4 \times \$14 = \$56$$

Jessica's money

Method 2

Jessica got $\frac{4}{7}$ of the total money.

$$98 \times \frac{4}{7} = \$56$$

Jessica's money

Sarah got $\frac{3}{7}$ of the total money.

$$98 \times \frac{3}{7} = \$42 \quad \text{or} \quad 98 - \$56 = \$42$$

Sarah's money

Jessica got \$56 and Sarah got \$42.

Try It! 15

The ratio of the number of U.S. coins to the number of foreign coins in Timothy's collection is 8 : 5. He has a total of 273 coins.

- (a) How many foreign coins does he have?
 (b) How many U.S. coins does he have?

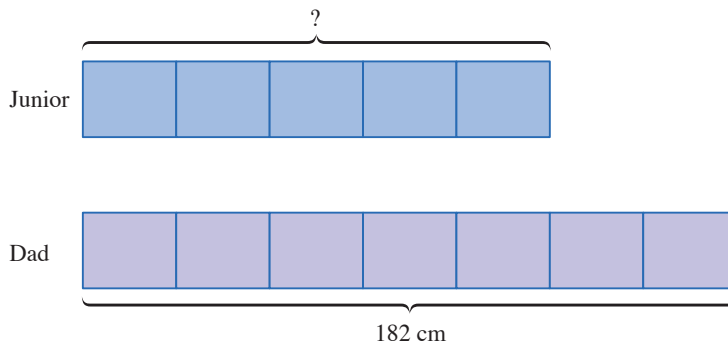
Example 16

Junior is $\frac{5}{7}$ as tall as his dad. His dad is 182 centimeters tall.

- (a) Find the ratio of Junior's height to his dad's height.
 (b) Find Junior's height.

Solution

- (a) $\frac{5}{7}$ is the value of the ratio 5 : 7.



The ratio of Junior's height to his dad's height is 5 : 7.

(b) Method 1

$$\begin{aligned} 7 \text{ units} &\rightarrow 182 \text{ cm} \\ 1 \text{ unit} &\rightarrow 182 \text{ cm} \div 7 = 26 \text{ cm} \\ 5 \text{ units} &\rightarrow 5 \times 26 \text{ cm} = 130 \text{ cm} \end{aligned}$$

Method 2

Find $\frac{5}{7}$ of 182 cm.

$$182 \text{ cm} \times \frac{5}{7} = 130 \text{ cm}$$

Junior is 130 cm tall.

Try It! 16

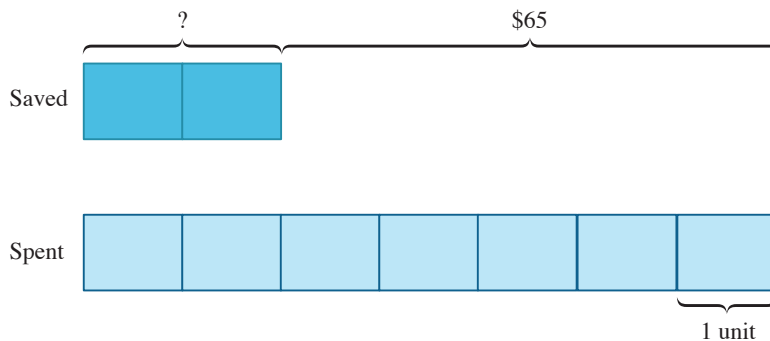
The width of a rectangular field is $\frac{2}{3}$ of its length.

The length of the field is 120 m.

- (a) What is the ratio of the width to the length of the field?
 (b) What is the width of the field?

Example 17

The ratio of the amount of money Emma saved to the amount of money she spent last month was 2 : 7. She spent \$65 more than she saved. How much money did she save?

Solution**Method 1**

$$5 \text{ units} \rightarrow \$65$$

$$1 \text{ unit} \rightarrow \$65 \div 5 = \$13$$

$$2 \text{ units} \rightarrow 2 \times \$13 = \$26$$

Method 2

From the model, we see that the amount she saved is $\frac{2}{5}$ of the difference between what she spent and what she saved.

$$\$65 \times \frac{2}{5} = \$26$$

Emma saved \$26.

DISCUSS

How much did she spend?

Try It! 17

The ratio of the number of adults to the number of children at the theater is 3 : 7. There are 152 more children than adults.

- How many adults are at the theater?
- How many children are at the theater?

EXERCISE 5.2



BASIC PRACTICE

- Express each ratio as a fraction in simplest form.
 - 2 : 5
 - 4 : 6
 - 9 : 7
 - 15 : 10
- The ratio of the volume of a small bottle of soda to the volume of a large bottle of soda is 3 : 8.
 - Express the volume of the small bottle as a fraction of the volume of the large bottle.
 - Express the volume of the large bottle as a fraction of the volume of the small bottle.
- The ratio of the distance Jeannie walked on Monday to the distance she walked on Tuesday is 7 : 5. How many times as far did she walk on Monday than on Tuesday?
- In a bag of beads sold at the store, the ratio of red beads to blue beads is 8 : 9.
 - What fraction of the beads are red beads?
 - What fraction of the beads are blue beads?
- The ratio of the amount of money Kim spent on a saddle to the amount of money she spent on a bridle is 17 : 3. If she spent \$1,500 altogether, how much did each item cost?
- Mr. Sirko is $\frac{4}{3}$ times as tall as his grandson Dylan.
 - Find the ratio of Mr. Sirko's height to Dylan's height.
 - If Mr. Sirko is 184 cm tall, how tall is Dylan?
- The length of a table is $\frac{11}{8}$ as long as its width.
 - Find the ratio of the length of the table to its width.
 - If the width of the table is 152 cm, what is the length of the table?



MATH@WORK

- Donna wants to make 200 milliliters of strawberry drink by mixing water and strawberry syrup in a ratio of 11 : 9.
 - How much water will she need to use?
 - How much syrup will she need to use?
- The ratio of Vicki's savings to Donna's savings is 4 : 7. Vicki saved \$225 less than Donna. How much money did Vicki save?
- The ratio of the number of boys to the number of girls in the school band is 13 : 3. There are 30 fewer girls than boys.
 - How many boys are there?
 - How many girls are there?



FURTHER PRACTICE

- The ratio of the number of dogs to the number of cats in a pet store is 7 : 12. If there are 48 cats, how many dogs are there?
- The ratio of the amount of chocolate syrup to the amount of milk required to make chocolate milk at a dairy is 2 : 7. How many gallons of milk are needed for 32 gallons of chocolate syrup?



- 13.** The ratio of the number of red balls to the number of green balls in a box is 7 : 12. There are 60 green balls.
- (a) How many red balls are there?
(b) How many more green balls than red balls are there?
- 14.** Steve has $\frac{3}{4}$ as many beads as Rachel. If Steve gives $\frac{1}{3}$ of his beads to Rachel, what will be the ratio of the number of Steve's beads to the number of Rachel's?
- 15.** A painter produces purple paint by mixing red paint and blue paint in the ratio $3 : \frac{2}{3}$. What fraction of the mixture is red paint?
- 16.** The ratio of the sides of a triangle is 3 : 4 : 5. If the perimeter of the triangle is 80.4 cm, what is the length of the longest side?
- 17.** The ratio of boys to girls in a club is 5 : 6 and there are 4 more girls than boys. If another 8 boys and 4 girls join the club, what is the new ratio of boys to girls?
- 18.** A farmer looks under a fence and sees the legs of his chickens and cows. He counts 84 cow legs and 60 chicken legs.
- (a) How many cows and how many chickens are there?
(b) What is the ratio of the number of cows to the number of chickens?
- 19.** Divide 88 apples into two groups, *A* and *B*, so that the ratio of the number of apples in group *A* to the number of apples in group *B* is $\frac{2}{3} : \frac{4}{5}$. How many apples are there in group *A* and in group *B*?



WRITE IN YOUR JOURNAL

Ken is 4 years old and his mother is 24 years old. How old will Ken be when his mother is 42 years old?

A student's solution is as follows:

Let x be Ken's age when his mother is 42 years old.

Since the age ratio does not change,

$$\frac{4}{24} = \frac{x}{42}$$

$$\frac{1}{6} = \frac{x}{42}$$

$$6x = 42$$

$$x = 7$$

So, Ken will be 7 years old.

Is the solution logical? Explain your answer.



EXTEND YOUR LEARNING CURVE

The Golden Ratio, denoted by the Greek letter ϕ (phi), is approximately 1 to 1.618. The front of the Parthenon in Greece has these properties, and they are commonly found in art, architecture and nature. In mathematics, the Golden Ratio is intimately related to the Fibonacci numbers (1, 1, 2, 3, 5, 8, ...), named after the Italian amateur-mathematician, Leonardo of Pisa. Read more about the beauty, ubiquity, and utility of phi and the Fibonacci numbers.

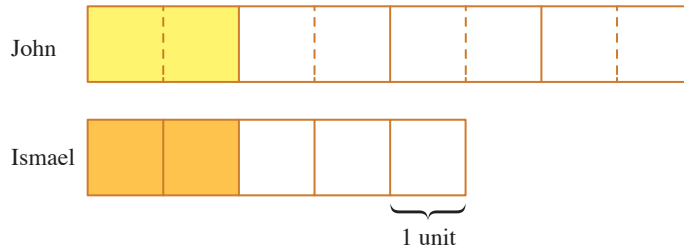
SM

Problem Solving Corner

Example 1

One quarter of John's weight is equal to two-fifths of Ismael's weight. What is the ratio of John's weight to Ismael's weight?

Solution



From the model,

$$\begin{aligned} \text{John's weight} : \text{Ismael's weight} &= 8 \text{ units} : 5 \text{ units} \\ &= 8 : 5 \end{aligned}$$

The ratio of John's weight to Ismael's weight is 8 : 5.

Try It! 1

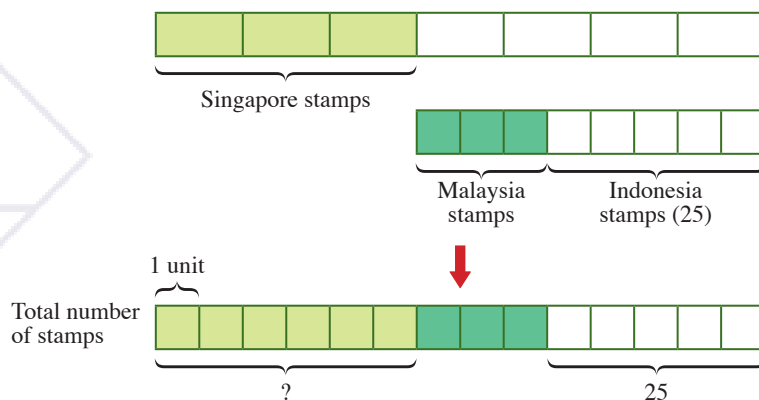
Three-fifths of Anne's age is twice of Kumar's age. Find the ratio of Kumar's age to Anne's age.

Example 2

In an album, $\frac{3}{7}$ of the stamps are Singapore stamps.

The rest are Malaysia stamps and Indonesia stamps. The ratio of the number of Malaysia stamps to the number of Indonesia stamps is 3 : 5. If there are 25 Indonesia stamps, how many Singapore stamps are there?

Solution



From the model,

$$5 \text{ units} \rightarrow 25$$

$$1 \text{ unit} \rightarrow 25 \div 5 = 5$$

$$6 \text{ units} \rightarrow 6 \times 5 = 30$$

There are 30 Singapore stamps.

Try It! 2

On social media, $\frac{2}{5}$ of Bill's fans are from the United States. The rest are from Singapore and Canada. The ratio of the number of Singapore fans to the number of Canada fans is 4 : 5. If there are 40 Canada fans, how many fans does Bill have altogether?

Example 3

The ratio of Lisa's money to Joan's money was 4 : 7 initially. After Lisa received \$120 from her grandmother and Joan gave away \$210 to a charity, they both had the same amount of money. How much money did Lisa have initially?

Solution

Before



After



From the model,

$$3 \text{ units} \rightarrow \$120 + \$210 = \$330$$

$$1 \text{ unit} \rightarrow \$330 \div 3 = \$110$$

$$4 \text{ units} \rightarrow 4 \times \$110 = \$440$$

Lisa had \$440 initially.

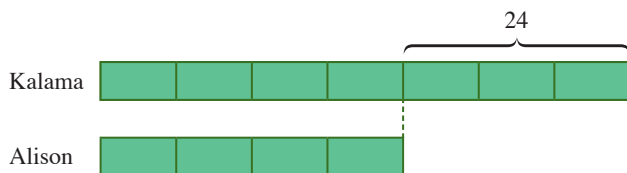
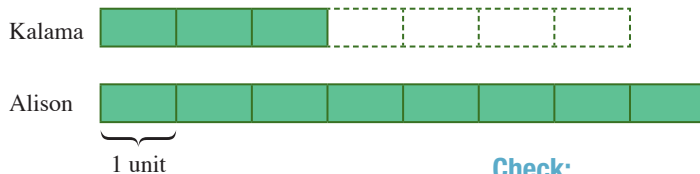


Try It! 3

The ratio of the number of Leon's apps to the number of Harry's apps was 3 : 5 originally. After Leon downloaded 12 new apps and Harry deleted 20 apps, they each had an equal number of apps. How many apps did Leon download originally?

Example 4

The ratio of the number of Kalama's postcards to the number of Alison's postcards is 7 : 4. Alison has 24 fewer postcards than Kalama. How many postcards must Kalama give to Alison so that the ratio of the number of Kalama's postcards to the number of Alison's postcards becomes 3 : 8?

Solution**Before****After**

From the model,

$$3 \text{ units} \rightarrow 24$$

$$1 \text{ unit} \rightarrow 24 \div 3 = 8$$

$$4 \text{ units} \rightarrow 4 \times 8 = 32$$

Check:**Before**

$$\begin{array}{lcl} \text{Kalama} & : & \text{Alison} \\ 56 & : & 32 \end{array}$$

After

$$\begin{array}{lcl} \text{Kalama} & : & \text{Alison} \\ 56 - 32 = 24 & : & 32 + 32 = 64 \\ 3 & : & 8 \end{array}$$

Kalama must give 32 postcards to Alison.

REMARK

The total number of postcards does not change. There are $7 + 4 = 11$ units before, and $3 + 8 = 11$ units after.

So, the size of the units stay the same.

Try It! 4

The ratio of the number of Ruth's beads to the number of Mario's beads is 2 : 5. Mario has 42 more beads than Ruth. How many beads must Mario give to Ruth so that the ratio of the number of Ruth's beads to the number of Mario's beads becomes 9 : 5?