SINGLE SLIT EXPERIMENT - KSCISSE

1. Key Concepts

- Wave nature of EM radiation
- Particle Nature of light
- Heisenberg's uncertainity

2. Introduction

The discovery of diffraction in 1665 by Francesco Grimaldi. Diffraction occurs when an opaque object placed in front of a bright point source, the shadow cast on the screen is seemed to composed of consecutive bright and dark patterns. he referred it to as “diffraction” which means “to break into different directions”. This pattern in the shadow of the aperture baffled many natural philosophers and physicists. In order to explain this phenomenon, many theories were put forth.

In, 1678 Christian Huygens proposed the wave theory of light which was able to partially explain the phenomenon of diffraction. Sir Issac newton, in 1704 proposed the corpuscular theory of light, this failed to explain the diffraction phenomenon. In 1804, Thomas Young performed the famous double slit experiment that gave further evidence to the wave nature of light. Around the same time, Augustin-Jean Fresnel modified the the Huygens theory and proposed a mathematical method to calculate the diffraction patterns. This came be known as ‘Huygens-Fresnel theory of diffraction’. Actually, even to this day an exact solutions to the patterns obtained in the diffraction are not available. Huygens-Fresnel model will suffice for simpler situations.

There is no major physical distinction between interference and diffraction. The term interference is used when superposition of only a few waves occur and the term diffraction is used when a large number of waves are involved.

3. Objective

4. To determine the slit using known wavelength

5. To determine the wavelength of the laser light from measurements of the intensity distribution of the diffraction pattern of the light due to single slit.

6. To demonstrate the Heisenberg’s uncertainty principle.

4. Theory

When a beam of light strikes an obstacle whose dimensions is comparable to its wavelength, then some of the light is diffracted, some reflected, some of it refracted, absorbed and re-radiated.

Diffraction is a phenomenon of bending of waves when it encounters obstacles or narrow opening (slits). Diffraction is a universal phenomena in waves

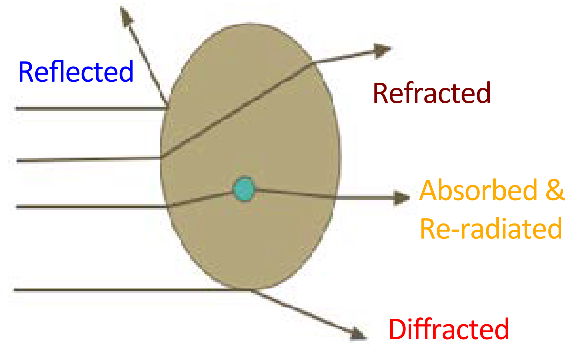


Image 1 Interaction of waves with matter

4.1 Classical theory of phenomenon of diffraction.

Christian Huygen's proposed a theory to explain the phenomena of diffraction. It is called the Huygen's principle, in which he used wavefront model to explain diffraction. To make it easier to visualize wavefront, when a stone is dropped in water, the stone disturbs the water and causes a number of circular ripples that travel outward. As time progresses the radius of the ripples increase. They start out small and become big.

Consider a ripple of radius r form the point source, here along this circle of radius r, the points are in some phase of oscillation. This is referred to as wavefront. In a wavefront all the points have the same phase of oscillation.

The radius of the wavefront increase as it travels outwards from the coherent point source. The circular wavefronts can be approximated as nearly straight wavefronts known as plane wavefronts. According to Huygen's principle, any wavefront can be thought to be composed of a collection of coherent point sources emanation in all directions (Image 2). Each such point source has emits circular waves called as wavelets

Consider a beam of light propagating to the right, composed of planar wavefronts (image-3) encounters an obstacle of width a.

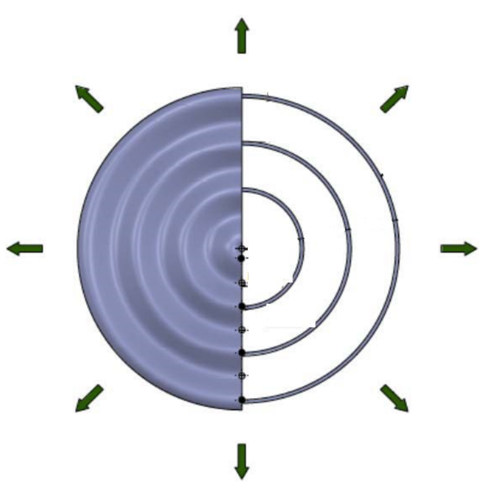


Image 2.Periodic wave surface (flat wave)

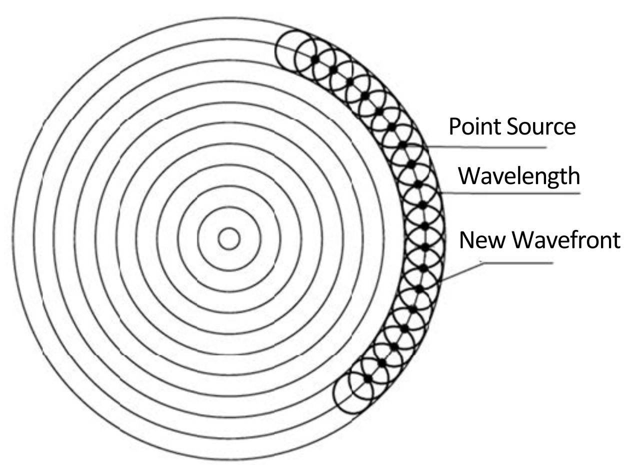
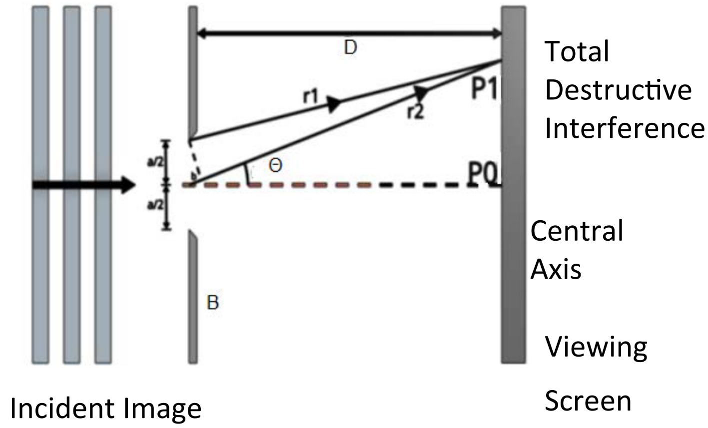


Image 3. Wavefront points become close coherent point sources of secondary waves



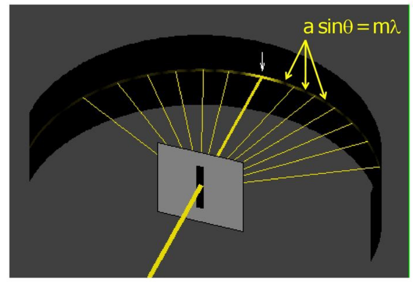


Image5. Plane wave propagating to the right

$$d \times \sin\theta = m\lambda \quad (1)$$

A basic set up to observe diffraction is shown in figure 4. It consists a slit B, screen at places at a distance of D from the slit. The wavefronts are partially obstructed by the slit B. The wavelets originating from the unobstructed part of the wave interfere on the screen resulting in diffraction pattern on the screen. The intensity distribution of the diffraction pattern is shown in figure 5. We can see that the intensity distribution is symmetric along about the central axis. The primary peak is called the central maxima. The corresponding peaks are called secondary, tertiary maxima.
Mathematically, the relation between the slit width (d), and the wave wavelength (λ) of the diffracted wave is given as follows.
where m: order of diffraction and θ is the angle between the central axis and a point on the screen.
The intensity distributions are governed by the relative wavelength difference between the two interfering waves.

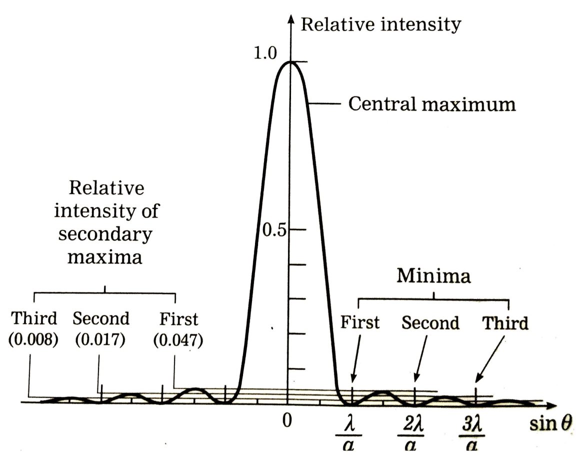


Image 5. Intensity distribution of diffracted light

$$a \times \sin\theta = m\lambda \quad (1)$$

The intensity pattern for the diffraction because of a single slit is given by a mathematical relation is given below.

$$I = I\_0 \times (\sin\beta/\beta)^{\wedge}2 \quad (2)$$

where, $\beta = \pi D \sin\theta/\lambda$ D is the slit width, λ is the wavelength and θ is the angle from the central peak.

Effect of the width of the slit/obstacle on the diffraction pattern.

Case 1: The width of the slit/obstacle equal to the wavelength of the light. $a=\lambda$
Here, we can see the width of the central maxima is very large and the intensity variation is small across the angular region. And the secondary maxima are not observable.

Case 2: The width of the slit/ obstacle is $a=5\lambda$
Here, we can see that the central maxima has a much smaller width and the intensity variation is much greater across the angular region. And the secondary maxima are observable.

Case 3: The width of the slit/ obstacle is $a=10\lambda$
Here, we can see that the central maxima has a much smaller width and the intensity variation is much greater across the angular region. And the secondary maxima, tertiary maxima are also observable.

4.2 Quantum mechanical treatment of the phenomenon of diffraction.
In quantum mechanics, electromagnetic radiation is assumed to be composed of quantized packets of energy called photons. A fundamental behavior of these particles was first hypothesized by Werner Heisenberg and is called Heisenberg uncertainty principle.
The Heisenberg uncertainty principle states that the position and velocity cannot both be measured,exactly, at the same time (actually pairs of position, energy and time). This is because the change in a velocity of a particle becomes more ill defined as the wave function is confined to a smaller region. This arises because of the measurement problem, and, the intimate connection between the wave and particle nature of quantum objects.
consider, for example, consider an ensemble of photons whose spatial probability is given by the function F_y and whose momentum by the function F_P. The uncertainty of location y and of momentum p is defined by the standard deviations as:

$$\Delta y . \Delta p \geq h/4\pi \quad (3)$$

where h = 6.6262×10-34 Js, Planck's constant.

If 'd' the width of the slit through which a photon is passing through. Then the error in measurement of the position of photon is of a maximum value equal to $\Delta y=d$.
Consider, the photon to be traveling in the x-plane. After the transmission through the slit (assuming, the slit width is comparable to the wavelength of the photon) we see that the diffraction pattern obtained is also distributed along the the other cardinal orthogonal plane. (y/z - plane depending on the orientation)

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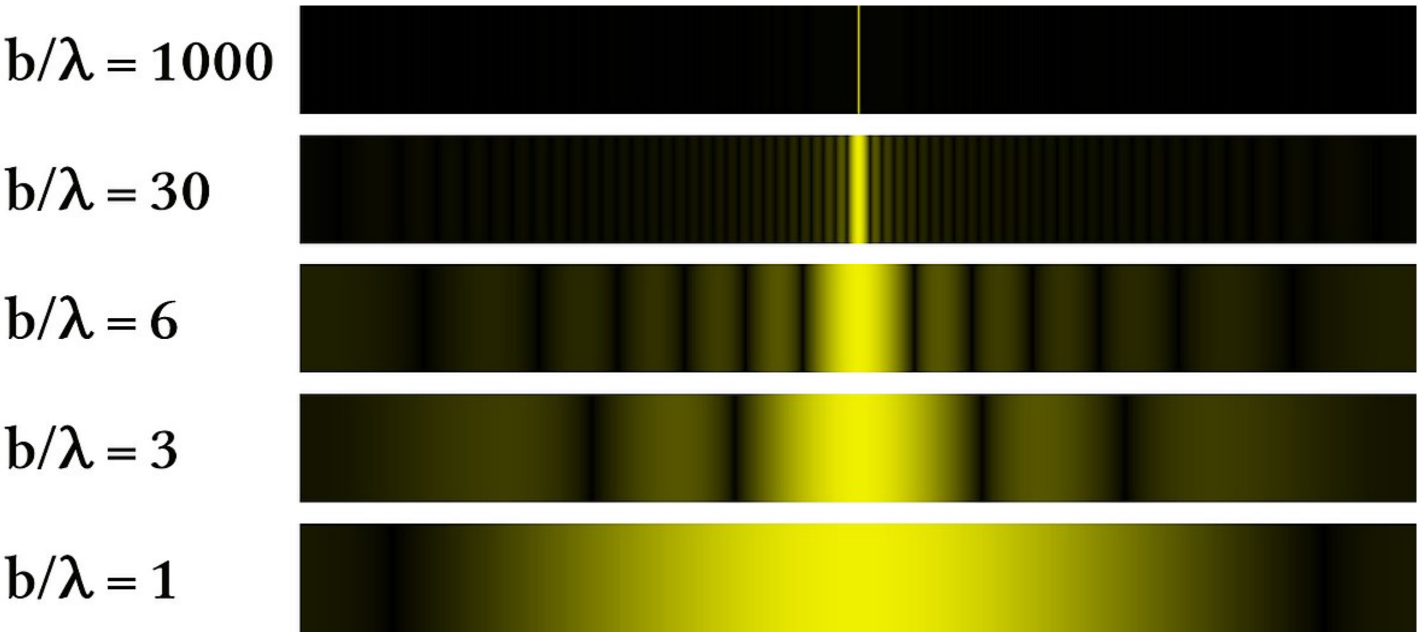

Image 6. Diffraction pattern due single slit

The probability density for the velocity component Vy is given by the intensity distribution in the diffraction pattern. We can use the first minimum to define the uncertainty of velocity

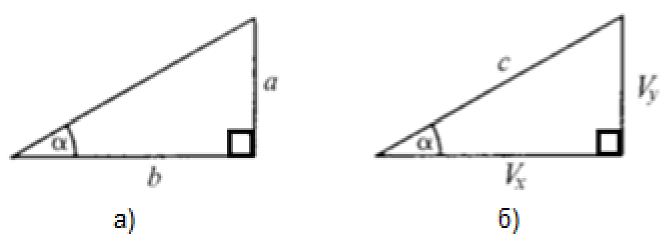

Image 7: Geometry of diffraction at a single slit a) path covered b) velocity component of a photon

)The error/ the uncertainty in the velocity measurement is given by the equation.

$$\Delta v\_y = c.\sin(\theta) \qquad (4)$$

Where θ = angle between the central line and the first minima.
We know the relation p=mv where p is the momentum, m is the mass of the partial and , v is the velocity of the particle.
The error/ the uncertainty in momentum is therefore,

$$\Delta p = mc\sin(\theta) \qquad (5)$$

We know that the De Broglie relationship links the momentum and wavelength of a particle.

$$h/\lambda = p = mc \qquad (6)$$

Thus,

$$\Delta p\_y = h/\lambda \sin(\theta) \qquad (7)$$

From the geometry of the situation we can say that sin(θ)=λ/d.

Simplifying, the above equation we get the following equation.

$$\Delta y = \Delta(p_y) = h \quad (8)$$

If the slit width y is smaller, the first minimum of the diffraction pattern occurs at larger angles of θ.
From the geometry of the situation we can say that θ is tan^(-1) a/D. Where a is the distance between the central maximum and first minima.

Substituting in equation 7 we get,

$$\Delta p_y = h/\lambda \sin(\tan^{-1} a/D) \quad (9)$$

Substituting(9) in (8) and dividing by h gives.

$$d/\lambda . \sin(\tan^{-1} (a/D)) = 1 \quad (10)$$

When $a \ll D$,$arctg\ aD \approx aD$,sin$aD \approx aD$ from where to $d\lambda \cdot aD \approx 1$.
We get an expression for the experimental determination of the width of the gap d

$$d \approx \lambda D/a \qquad (11)$$

5. Equipment

*Images may differ in appearance from the actual product.

S.No	Equipment	Item Code	Qty
1	Optical Bench Set 0.8m	KSCIOB1	1
2	Laser Source Holder	KSCIHA003	1
3	Travelling light Sensor Holder	KSCIHA512	1
4	Adjustable Collimating Slit Holder	KSCIHA012	1
5	Data Processor	KSCIDP1	1
6	Power Supply for Light Source	KSCIPS61022D/2	1

6. Safety instructions

- Components like horizontal bench and power supply are heavy. Take adequate safety measures while handling them.
- Don't look with naked eyes into laser while working.

7. Experimental Setup

1. Place the optical bench on a stable horizontal surface such as a sturdy table top and make sure the bench is parallel to the horizontal surface using the adjustable mounts.
2. Mount the laser source securely on the upright, place the upright on the optical bench and lock the slide screw on the slider.
3. In order to measure the wavelength of given laser source and calculate slit width. Securely mount the collimating slit holder & adjust the slit width.
4. Pass the laser from collimating slit holder so that a clear diffraction pattern is achieved on the transverse saddle and then lock the positions.
5. During the experiment do not vary the distance between the sample mount and the light sensor. Use the horizontal translation screw to move the light sensor and measure the intensities of the maxima and minima.

8.Experiment

8.1 Determining the Width of a Slit

• Turn on a laser light source with a known wavelength and adjust the beam so that it falls on the hole of the adjustable slot and gives a clear diffraction pattern.

• Maintain a constant distance between the adjustable slit and the light sensor throughout the experiment.

• As soon as a clear diffraction pattern is obtained, measure and record the distance between the central maximum and the 1st order maxima on both sides of the pattern.

• Calculate the width of the slit using the formula (11) presented in the theoretical part of the work.

where, d: slit width, λ: is the wavelength of the laser, D: is the distance between the central maxima and the first minima under consideration, and w: is the distance between the slit and the light sensor.

8.2. Determination of laser wavelength

• Turn on the laser and adjust it so that the beam hits the adjustable slit.

• The width of the slit should remain unchanged from the previous experiment.

• After obtaining a clear diffraction pattern, measure and record the distance between the central maximum and the first and upper order maxima on both sides of the diffraction pattern.

• The wavelength of laser radiation can be obtained using the following formula:

$$d\sin\theta = m\lambda$$

where d is the width of the gap, θ is the angle between the central maximum and the maxima in question, is the order of the maxima in question, and is the laser wavelength.

Considering that $tg\ \theta = aD, a \ll D, arctg\ aD \approx aD, \sin aD \approx aD$, we get $damD \approx m\lambda$

where D is the distance from the slot to the illumination sensor, am is the distance from the axis of symmetry to the minimum of the m order on the diffraction pattern.

8.3. Demonstration of Heisenberg uncertainty principle and calculation of normalized value of constant $\boldsymbol{h}$

• Turn on a laser light source with a known wavelength and adjust the beam so that it falls on the hole of the adjustable slot and gives a clear diffraction pattern.

• Adjust the slit to obtain a diffraction pattern.

• Record the distance between the slit and the light sensor.

• Measure the distance between the central maximum and the 1st order maxima.

• Using formula (10) from the theoretical section, find the normalized $\boldsymbol{h}$ value.

9. Measurement & Results

9.1.Determination of slit width (d)

1. Known Wavelength of the laser λ ____________________
2. Distance b/w central maximum & first minima (a) _____________________
3. Distance b/w slit & sensor (D) ____________________

9.2. To find the wavelength of laser

1. Order of the minima (m): _________________
2. Distance b/w central maximum & first minima (a): _____________________
3. Distance b/w slit & sensor (a) ____________________
4. Slit width (d): _____________________
5. Wavelength of the laser λ ___________

9.3. Normalized Value h

Slit width d	Distance from center to 1st minimum		$\frac{d}{\lambda}\sin\left(\tan^{-1}\frac{a}{D}\right)$
	Right	Left	

10. References

a. Eugene Hetch, Optics (5th edition)
b. Diffraction lecture notes and history of diffraction https://statweb.stanford.edu/~candes/teaching/math262/Lectures/Lecture15.pdf