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Cavendish Gravitational Balance KSCICGB

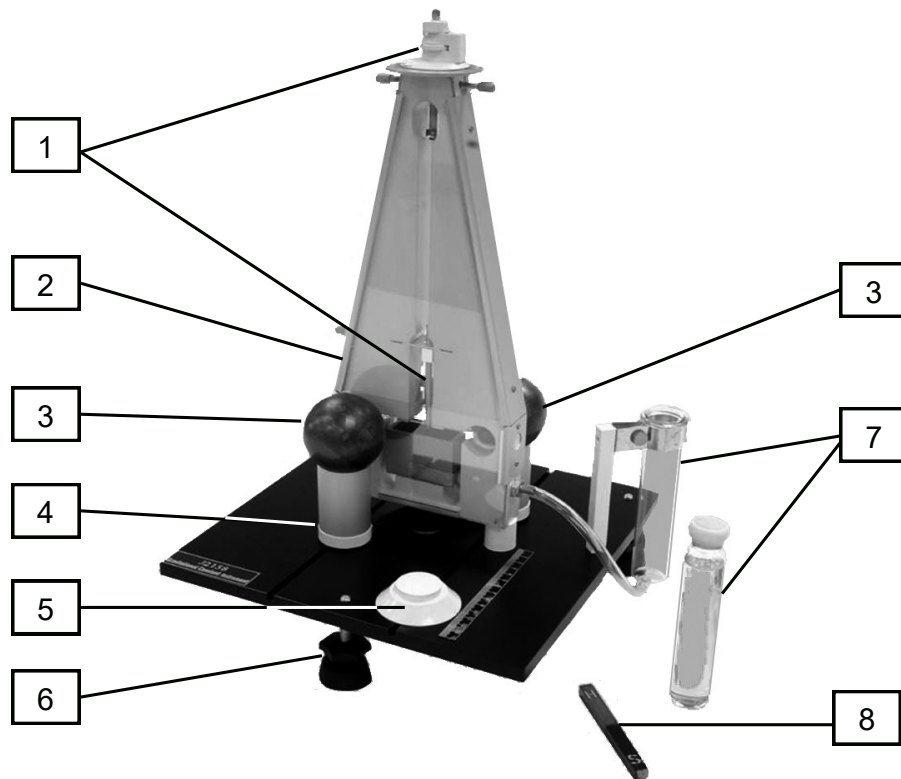


Figure 1

DESCRIPTION OF THE INSTRUMENT

The CGB001 Cavendish Gravitational Balance is a miniature version of the apparatus used by Henry Cavendish in 1797-8 to measure the density of the Earth. The experiment allows the value of the gravitational constant, G , to be measured, although Cavendish did not use his version for that purpose. The experiment is remarkable for the ability to measure an extremely tiny force using simple mechanical means.

The apparatus contains a pendulum system (1, *Figure 1*) consisting of an adjustable suspended central rod carrying a small mirror for the optical lever detection system, a light aluminum cross-piece with two 20g lead balls 10 cm apart, and a light damping vane. The pendulum is mounted in a massive aluminum housing (2). Two large 1.5 kg plastic-coated lead balls (4) rest atop light aluminum cylinders on a swivel (4) that enables the balls to be swung from one side to the other of the apparatus. They can also be placed onto two circular sliding mounts (6) that allow the distance between the pendulum and the attracting masses to be varied. The base rests on three leveling feet (6). An oil reservoir and damping oil (7) as well as a damping magnet (8) are provided.

SPECIFICATIONS

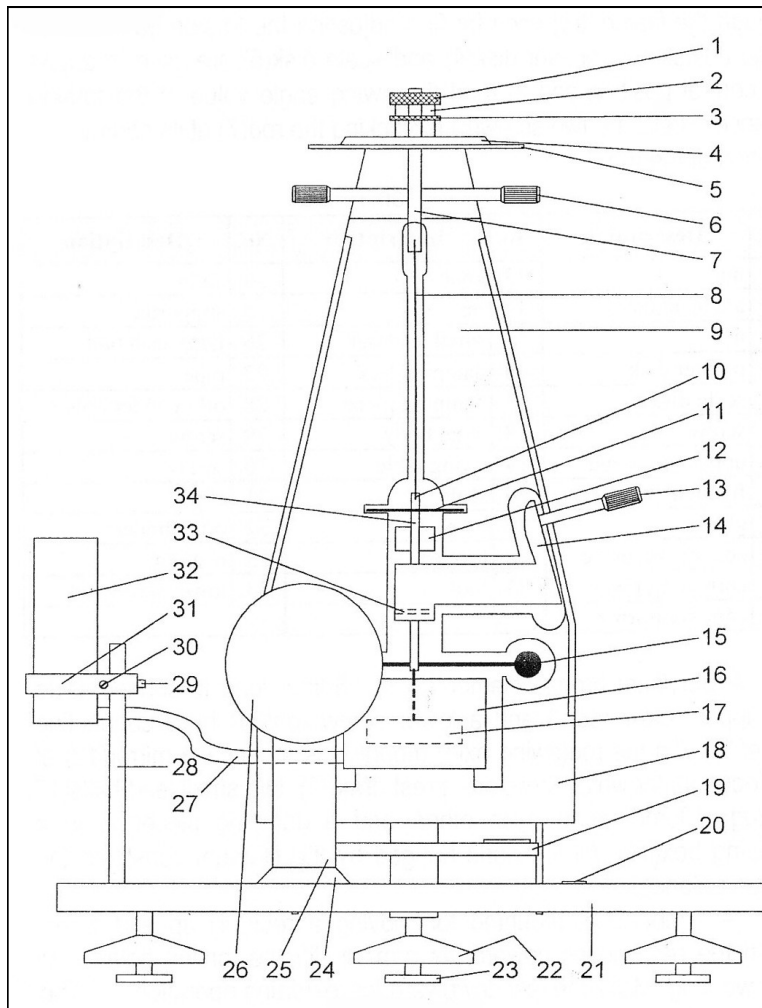


Figure 1

IDENTIFICATION OF THE COMPONENTS

- | | | | |
|-----|---------------------------------------|-----|---------------------------------------|
| 1. | Upper suspension rod locknut | 18. | Aluminum body |
| 2. | Pendulum angle adjustment block | 19. | Swinging support for large lead balls |
| 3. | Suspension rod height adjusting nut | 20. | Scale (on base—1 of 2) |
| 4. | Angle indicator disk | 21. | Base plate |
| 5. | Angle scale disk | 22. | Leveling nut (1 of 3) |
| 6. | Suspension rod locking screw (1 of 2) | 23. | Foot (1 of 3) |
| 7. | Upper suspension rod | 24. | Sliding ball support (1 of 2) |
| 8. | Torsion filament | 25. | Ball support cylinder (1 of 2) |
| 9. | Glass cover plate (1 of 2) | 26. | Large lead attracting ball (1 of 2) |
| 10. | Torsion filament connector plate | 27. | Damping oil supply tube |
| 11. | Balance centering plate (2 halves) | 28. | Damping oil reservoir support |
| 12. | Concave mirror for light pointer | 29. | Reservoir support set screw |
| 13. | Balance lock actuating screw | 30. | Reservoir clip tightening screw |
| 14. | Balance lock mechanism | 31. | Reservoir support clip |
| 15. | Small lead ball (1 of 2) | 32. | Damping oil reservoir |
| 16. | Damping oil trough | 33. | Balance locking rod |
| 17. | Balance damping vane | 34. | Lower suspension rod |

The torsion balance is housed in a solid aluminum main body (18) with glass windows (9) on both sides to eliminate drafts.

The adjusting nut (3) on the top can be used to raise or lower the upper suspension rod (7) in order to adjust the vertical position of the balance. The angle adjustment block (2), angle indicator disk (4) and the circular scale (5) are used to read the angle the balance is rotated from its equilibrium position. The locking screws (6) on the upper side of the body can fix the upper suspension rod in its angular equilibrium position. The filament (8), 150mm long and made of beryllium bronze, has connector plates (10) at each end and connects the upper and lower suspension rods (7,34). A concave mirror (12) with 2m focal length is mounted on the lower suspension rod. Under the mirror are the locking rod, a pair of lead balls of 10.0mm diameter (15) and the damping vane (17), which projects into a damping trough (16).

The locking screw (13) on the side can raise or lower the locking mechanism (14). With the locking mechanism lowered, the balance is suspended on the filament and can rotate freely. When the mechanism is raised, it lifts the balance by the locking rod (33) and presses the arm of the balance against the body, removing the load from the filament and immobilizing the balance.

The filament is extremely delicate and the load of the balance stresses it highly. The balance should only be released and allowed to swing freely when an experiment is in progress. At all other times, the balance should be locked to preserve the filament.

The main body is mounted on an aluminum base (21). The three leveling screws (22) under the base are used to level the device. The two grooves on the base serve as guides for the sliding ball supports (24). The ball support cylinders (25) for the large lead balls (26) rest on the sliding blocks. The distance between the large balls can be measured by the scales (20) along the grooves.

The oil damping system is mounted on one side of the base and connected to the damping chamber (16) through a tube (27).

Mass of large lead balls:	Approximately 1.5kg
	Difference between the two balls < 0.002kg
Mass of small lead balls:	Approximately 0.02kg
	Difference between the two balls < 0.0005kg
Arm length of the torsion balance:	$5.0 \times 10^{-2} \text{m}$
Torsion Filament:	Length: approximately 150mm
	Cross sectional area: $0.145 \pm 0.08 \text{mm}^2$
	Material: Be-Sn-Cu alloy
Period of the torsion balance	$590 \pm 10 \text{sec.}$
Scale:	140mm with 1mm divisions
	Full-scale error < 0.1mm.
Damping method:	Silicone oil
Relative error of Gravitational Constant:	< 15%
Operating temperature:	10 – 40°C
Relative humidity:	< 40%
Operating location:	Should be free from vibration, sunlight, radiant heat, magnetic and electric fields
Dimensions:	300x300x420mm
Net weight:	12kg

THEORY

The most common methods used to measure the value of the gravitational constant are the *displacement method* and the *acceleration method*.

The Displacement Method

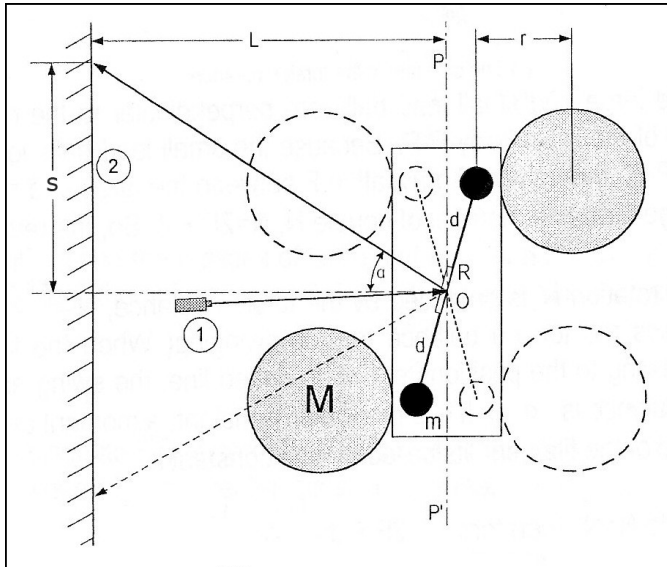


Figure 2

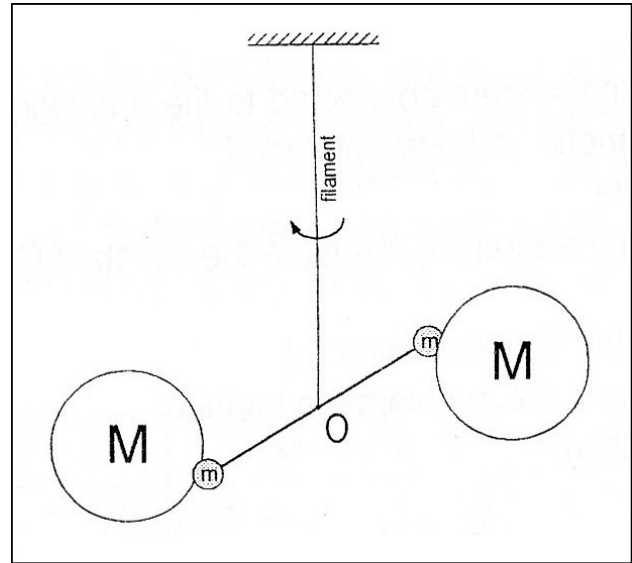


Figure 3

The displacement method is named after the final angular displacement of the torsion balance to be measured in the experiment. In *Figure 2*, the torsion balance is set up a large distance L from a wall or screen with a millimeter scale ②. The concave mirror is illuminated by a laser ①, producing a spot of light on the scale. The distance between the small lead balls, each of mass m , and the axis O is d . The concave mirror is mounted at the center of the arms connecting the two small balls. The projection system produces a large displacement of the light spot, s , for a small angular displacement of the mirror, $\alpha/2$. This allows the user to measure the angle α easily, and determine the angular displacement of the mirror from the central position, $\alpha/2$.

Two large lead balls of same mass M are placed against the glass windows of the main body, symmetric to the axis O . *Figure 3* illustrates the side view of the arrangement. The line connecting the centers of the large and small balls is perpendicular to the center line of the main body, PP' . *Figure 2* greatly exaggerates the angles of the balance for clarity; the deviation of the balance from its central position is extremely small, so the centers of the small balls are almost on the line PP' , and the torque produced by the gravitational force F between the large and small balls, N , is given by $2F \cdot d$. When the torsion balance comes to rest, the restoring torque applied by the torsion filament, N' is $k \cdot \alpha/2$, where the coefficient k represents the torque in the wire when it is twisted by 1 degree. Since $N = N'$, we have $2F \cdot d = k \cdot \alpha/2$. Therefore

$$F = k \left(\frac{\alpha}{4d} \right) \quad (1)$$

If we now move the large lead balls to their symmetrically opposite final positions, indicated by the large dotted circles in *Figure 2*, the torsion balance will be subjected to a torque in the opposite direction and will move to the new equilibrium position represented by

the dotted line and small circles in *Figure 2*. Its angle of rotation to the new equilibrium position can be determined by the displacement S ($=2s$) of the projected dot on the scale. The distance from the mirror to the screen being L , and $S \ll L$, we have

$$\alpha = S/2L \quad (2)$$

The torque constant of the wire, k , can be determined from the balance's period of oscillation T , using

$$T = 2\pi\sqrt{\frac{I}{k}}$$

derived from the harmonic motion equation

$$I \frac{d^2\theta}{dt^2} = -k\theta$$

where the moment of inertia I is given by

$$I = \sum_i (m_i r_i^2) = m_1 d_1^2 + m_2 d_2^2 = 2md^2$$

the moments of inertia of the lower suspension rod, mirror, and balance crossbar are neglected, since they are extremely small compared with those of the lead balls.

Hence the torque constant of the wire k can be expressed as

$$k = 4\pi^2 \frac{I}{T^2} = 8\pi^2 \frac{md^2}{T^2} \quad (3)$$

Substituting expressions (2) and (3) into (1), we have

$$F = \frac{k\alpha}{4d} = \left(\frac{1}{4d}\right) \cdot \left(\frac{8\pi^2 md^2}{T^2}\right) \cdot \left(\frac{S}{2L}\right) = \frac{\pi^2 mdS}{T^2 L} \quad (4)$$

The gravitational force between the large ball M and the small ball m at a separation of r is

$$F = G \frac{Mm}{r^2} \quad (5)$$

Combining expressions (4) and (5) to eliminate F , we have the formula for the gravitational constant G :

$$G = \frac{\pi^2 r^2 dS}{MT^2 L} \quad (6)$$

Where r is the distance between the centers of the large and small balls, d the length of the arm of the torsion balance, S the maximum displacement of the light dot, M the mass of the large lead balls, T the period of oscillation of the balance, and L the distance from the mirror to the scale.

Strictly speaking, distance r between the balls in *Figure 2* should be

$$r'' = r - d \cdot \left(\frac{\alpha}{2}\right) = r - \frac{dS}{4L}$$

However, the error caused by the approximation will not exceed 2% so that it can be neglected.

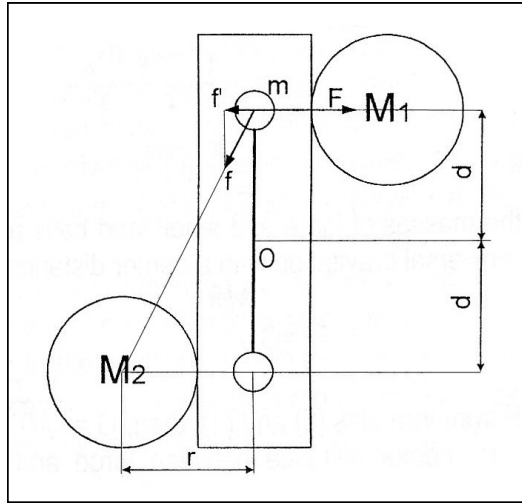


Figure 4

There is a second, more important correction, illustrated by *Figure 4*. The small ball m is subjected not only to the force F applied by the large ball M_1 , but also to a force f applied by the large ball M_2 . Therefore, the value for G obtained from expression (6) needs to be corrected.

In *Figure 4*, the distance between m and M_2 is

$$\sqrt{r^2 + 4d^2}$$

But $F = G \frac{Mm}{r^2}$ and $M_1 = M_2$ so,

$$f = G \cdot \left(\frac{Mm}{r^2 + 4d^2} \right) = \left(\frac{r^2}{r^2 + 4d^2} \right) \cdot F \quad (7)$$

If the component of f in the opposite direction to F is f' , then

$$f' = f \cdot \frac{r}{\sqrt{r^2 + 4d^2}} \quad (8)$$

Substituting expression (7) into expression (8), we have

$$f' = F \cdot \left(\frac{r^2}{r^2 + 4d^2} \right) \cdot \left(\frac{r}{\sqrt{r^2 + 4d^2}} \right) = F \cdot \frac{r^3}{(r^2 + 4d^2)^{3/2}} = \beta \cdot F$$

where

$$\beta = \frac{r^3}{(r^2 + 4d^2)^{3/2}} \quad (\text{approx. } 0.074)$$

Here d is the arm length of the torsion balance (0.05m), and r the distance between the centers of the large and small balls, which can be determined from

$$r = \frac{H}{2} + h + \frac{D}{2}$$

Where H is the thickness of the main body (approx. 0.025m), h the thickness of the glass window (approx. 0.002m), and D the diameter of the large ball (approx. 0.0635m).

The force actually acting on the small ball is

$$F - f' = F - \beta \cdot F = (1 - \beta) \cdot F$$

Substituting this into expression (4), we have

$$(1 - \beta) \cdot F = \frac{\pi^2 m d S}{T^2 L}$$

Since

$$F = G \cdot \frac{Mm}{r^2}$$

the corrected expression for G is

$$G = \left(\frac{1}{1 - \beta} \right) \cdot \left(\frac{\pi^2 r^2 d S}{M T^2 L} \right) \quad (9)$$

The Acceleration Method

The acceleration method determines the value of G from the acceleration produced by the gravitational force.

As in the case of final displacement method, the large balls are first moved to the positions against the glass windows and when the small balls have come to rest, the torque produced by the gravitational force will balance the torque in the wire due to the rotation.

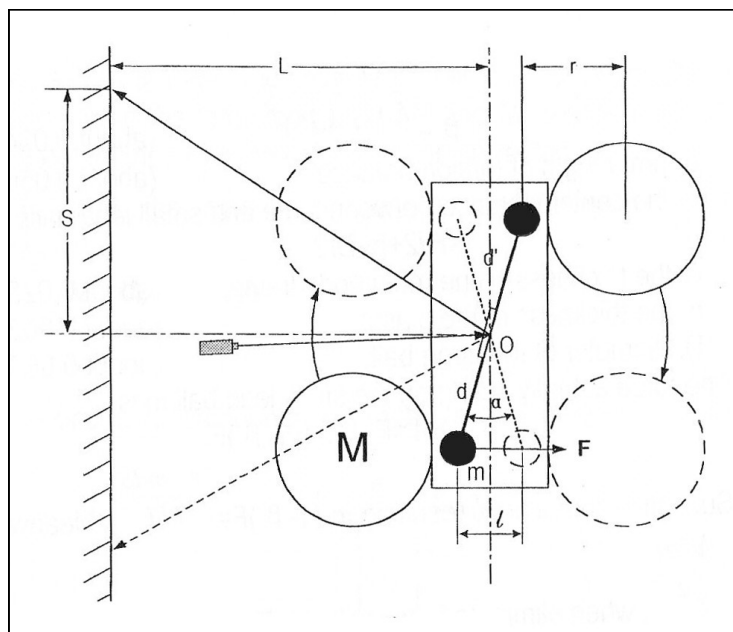


Figure 5

When the large balls are now moved to the dotted position shown in *Figure 5*, the initial force on the small ball F' will be the sum of the gravitational force F and the elastic restoring force, f . Therefore, we have

$$F' = F + f = 2F = 2G \cdot \frac{Mm}{r^2} \quad (10)$$

The small ball moves a small distance l under F' . The motion can be regarded as a uniformly accelerated over the first short period of time (within 90 seconds). Therefore

we can apply Newton's Law

$$F' = ma$$

where a is the acceleration.

Substituting into (10), we have

$$G = \frac{Fr^2}{2Mm} = \frac{ar^2}{2M} \quad (11)$$

Expression (11) can be corrected to take into account the gravitational force applied by the second large ball:

$$G = (1 + \beta) \cdot \frac{ar^2}{2M} \quad (12)$$

If the distance traveled by the small ball is l , we have

$$l = \frac{1}{2} at^2$$

In order to solve this equation for a , we need to find the distance l that the small ball has traveled in the first t seconds after the large ball was moved into the dotted position (*Figure 5*).

From *Figure 5* we have

$$\frac{S}{L} = \frac{2l}{d}$$

Therefore

$$l = \frac{dS}{2L} = \frac{1}{2} at^2$$

So the acceleration a can be determined from

$$a = \frac{Sd}{t^2 L} \quad (13)$$

Substituting (13) into (12), we have

$$G = (1 + \beta) \cdot \frac{Sdr^2}{2MLt^2}$$

The acceleration method is simpler than the displacement method. However, although it requires 1–2 hours to stabilize the instrument in its initial position, this is still only half the time required for the two stabilizations needed in the displacement method. Also, the error in the G value obtained by the acceleration method is usually larger.

The Inverse Proportion Verification

The apparatus allows the separation of the small and large balls to be varied by supporting the large balls on two light aluminum cylinders (24 & 25 in *Figure 1*) that slide in grooves on the instrument base. This method can be used to verify the relation between the gravitational force and the distance between the objects:

$$F \propto \frac{1}{r^2}$$

From expression (4)

$$F = \frac{\pi^2 m d S}{T^2 L}$$

therefore $F \propto S$

So the relationship between F and r will be verified if we can show experimentally that

$$S \propto \frac{1}{r^2}$$

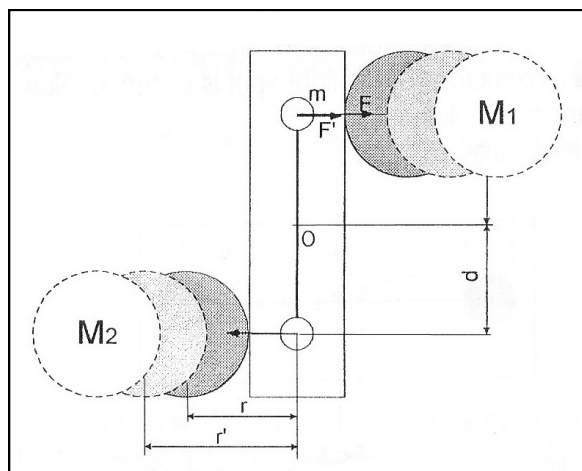


Figure 6

Figure 6 illustrates the change in the distance r between the large and small balls. Record the displacement S of the light dot and the corresponding distance r then plot $1/r^2$ vs. S on graph paper to verify that the relationship between F and $1/r^2$ is linear. Since each small ball is also subjected to the force of the second large ball, the S values obtained above will be too small and need to be corrected by replacing S with $(1+\beta)S$.

SETUP

The experiments described in this manual require the following accessories:

- Laser (He-Ne or diode) and support material
- Stopwatch (electronic or mechanical, 1/100 s)
- 3m tape measure
- Screen and support material
- Vernier caliper
- Balance (2000g capacity x at least 1g resolution)
- Calculator.

Leveling

- Choose a location as free of vibration as possible and place the instrument on a solid, level surface.
- Set up the laser and the screen as indicated in Figure 2
- Measure and record the masses of the large lead balls.
- Gently turn the locking knob (16, Figure 1) counter-clockwise to unlock the torsion balance so that it hangs free on the suspension filament.
NOTE: The suspension filament is extremely delicate and is highly stressed when bearing the load of the torsion balance. Avoid sudden or jerky movements when releasing or locking the balance.
- Use the adjusting nut (3) to center the small balls vertically in their cutouts in the main body.
- Adjust the position of the laser so that the light beam is incident at the center of the mirror and reflects onto the screen.

- Use the leveling screws (22) to adjust the angle of the apparatus so that the lower suspension rod (34) hangs in the center of the hole in the balance centering plate (11).
- Apply torsion to the filament using the angle adjustment block (2) until the small balls just touch the glass window, then finely adjust the leveling screws for the largest displacement of the light dot on the screen. Then return the angle adjustment block to its original position.

Centering

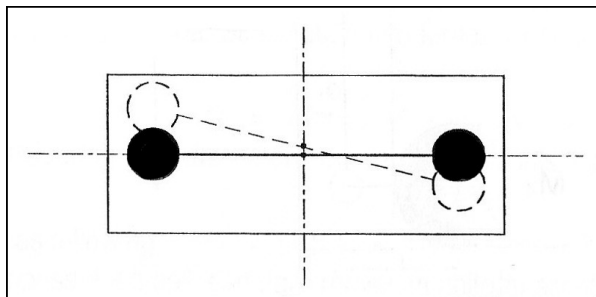


Figure 7

In equilibrium, the torsion balance should be centered as shown by the black balance in Figure 7. Otherwise the user should do the following:

- *Determine the equilibrium position:* Let the torsion balance oscillate with a large amplitude so that the small balls touch the glass windows. Mark the two end positions of the light beam on the screen and connect them with a straight line. The center of the line will be the desired equilibrium position of the balance, x .
- *Confirm the equilibrium position:* Use the damping magnet to reduce the amplitude of the swing. (The lead balls are diamagnetic and experience a weak repulsive force near the pole of a magnet. Applying this force alternately to each lead ball as it passes the center point of the swing will reduce the swing amplitude) Then record the end positions of the light dot on the screen as shown in Figure 8.

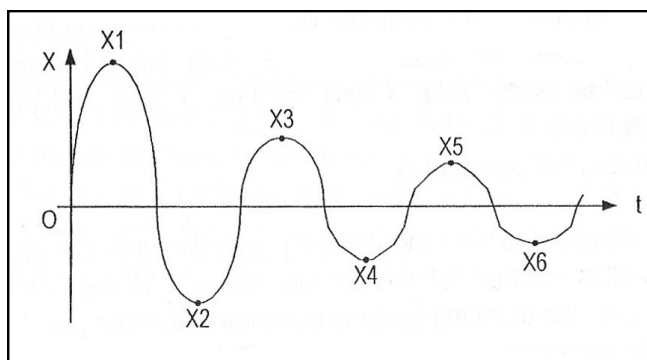


Figure 8

- The equilibrium position x can be calculated as follows. For the damped motion we have:

$$\frac{x_1 - x_2}{x_1 - x'} = \frac{x_3 - x_4}{x_3 - x_2} = k$$

or

$$x' = \frac{x_2^2 - x_1 x_3}{2x_2 - x_1 - x_3}$$

- If $x' = x$, the balance is in its equilibrium position. If not, adjust the angle adjustment block (2) and repeat the measurement until $x = x'$.

EXPERIMENTS

A The Displacement Method

1. Measurement of the period of the torsion balance

- With the balance swinging with a moderate amplitude and not touching the glass at the end points, observe the light dot on the screen and start the timer when the light dot passes the equilibrium position. Stop it at the same point in the cycle after the torsion balance has completed 2 or 3 oscillations. Average the times to find the period T . NOTE: *Very small amplitudes will yield less accurate results.*
- *Alternative method:* With the balance swinging with a moderate amplitude and not touching the glass at the end points, observe the light dot on the screen and record its position every 15 s for 15-20 minutes. Draw a graph of the position of the light dot vs. time and determine T from the graph.

2. Measurement of G with oil damping

- Fill the oil cylinder with oil up to the mark. Lift the cylinder slowly so that the oil will flow into the damping chamber. Stop the process when the damping plate is submerged in the oil. (*Due to the buoyant force and surface tension of the liquid, the equilibrium position may deviate. In that case, use the angle adjustment to readjust the equilibrium position x .*)
- Place the large balls in the initial positions shown (with solid lines) in *Figure 2*. It will take approximately 20 minutes for the small balls to come to rest. Record the position of the light dot every minute until it stops moving.
- Carefully move the large balls to their final positions and observe the movement of the light dot. Record the position of the light dot every minute until it stops moving.
- Using the above data, draw a graph with x-axis representing time and y-axis representing the position of the light dot. Determine the maximum displacement of the light dot, S .

3. Measurement of G with air damping

- For each position of the large balls, record several end points of the swings and graph them as shown in *Figure 8*.
- Select three consecutive end points of the light dot in the graphs of both large ball positions and calculate the equilibrium positions of the light dot in each case using the mean value method:

$$Y_1 = \frac{x_2^2 - x_1x_3}{2x_2 - x_1 - x_3}$$

$$Y_2 = \frac{x_2'^2 - x_1'x_3'}{2x_2' - x_1' - x_3'}$$

- The maximum displacement is

$$S = |Y_2 - Y_1|$$

Example:

In an experiment using air damping, the experimenter measured the following quantities:

Mass of each large lead ball, M :	1.520 kg
Projection distance, L :	2.124 m
Period of the torsion balance, T :	596.3 sec.
Distance between the large and small balls, r :	4.75×10^{-2} m
Length of the balance arm, d :	5×10^{-2} m.

When the large balls were at their initial positions, three consecutive end points of the light dot were measured to be:

$$x_1 = 370 \text{ mm}$$

$$x_2 = 510 \text{ mm}$$

$$x_3 = 395 \text{ mm}$$

therefore the equilibrium position is

$$\begin{aligned} Y_1 &= \frac{x_2^2 - x_1 x_3}{2x_2 - x_1 - x_3} \\ &= \frac{(510^2 - 370 \times 395)}{(2 \times 510 - 370 - 395)} \\ &= 446.86 \text{ mm} \end{aligned}$$

When the large balls were at their final positions, three consecutive end points of the light dot were measured to be:

$$x'_1 = 475 \text{ mm}$$

$$x'_2 = 545 \text{ mm}$$

$$x'_3 = 483 \text{ mm}$$

therefore the equilibrium position is

$$\begin{aligned} Y_2 &= \frac{x'^2_2 - x'_1 x'_3}{2x'_2 - x'_1 - x'_3} \\ &= \frac{(545^2 - 475 \times 483)}{(2 \times 545 - 475 - 483)} \\ &= 512.12 \text{ mm} \end{aligned}$$

The displacement of the light dot is

$$\begin{aligned} S &= |Y_2 - Y_1| \\ &= |512.12 - 446.86| = 65.26 \text{ mm} \end{aligned}$$

Substituting S into expression (6),

$$\begin{aligned} G &= \frac{\pi^2 r^2 d S}{MT^2 L} \\ &= \frac{(3.1416^2 \times (4.75 \times 10^{-2})^2 \times 5 \times 10^{-2} \times 6.526 \times 10^{-2})}{(1.52 \times 596.3^2 \times 2.124)} \\ &= 6.329 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \end{aligned}$$

After correction,

$$\begin{aligned} G' &= (1 + \beta)G = 1.074 \times 6.329 \times 10^{-11} \\ &= 6.797 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \end{aligned}$$

The relative error, δ , is

$$\delta = \frac{|G' - G_0|}{G_0} \times 100 \% = 1.87\%$$

where the standard value G_0 is $6.672 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

B The Acceleration Method

- Do not use the damping oil for this method. Drain the oil from the main body by lowering the oil cylinder – the oil will flow back into the cylinder.
- Place the large balls in their initial positions as shown by the solid lines in *Figure 5*. Wait for 60 to 120 minutes for the light dot to come to rest.
- Carefully move the large balls into their final positions against the glass windows. Make sure they do not touch the windows.
- Start the stopwatch immediately and measure the position of the light dot every 15 seconds for 120 seconds. *Table 1* shows sample data from an experiment with $L = 2.27$ m.

t (s)	0	15	30	45	60	75	90	105	120
x (cm)	52.0	52.3	52.8	53.2	53.8	55.5	56.4	57.3	58.2

Table 1

- Plot a graph with x axis representing time squared t^2 and y axis representing the displacement S . Calculate the acceleration from the slope of the graph and substitute it into expression (11) for G' .
- The acceleration equation for the small ball

$$a = \frac{Sd}{t^2 L}$$

can be rewritten as

$$S = \left(\frac{La}{d} \right) \cdot t^2$$

Letting $Y = S$ and $X = t^2$, we have a linear equation

$$Y = \left(\frac{d}{La} \right) \cdot X$$

- The slope of the linear equation can be determined using the least squares fit procedure. In order to illustrate this, we will select the data for t between 45 and 90 sec from *Table 1* for the calculation (*Generally, greater errors occur before 30s and after 90s*). See *Table 2*.

t (s)	45	60	75	90
x (cm)	53.2	53.8	55.5	56.4
S (m, = Y)	0.0120	0.0170	0.0235	0.0330
t^2 (= X)	2025	3600	5625	8100

Table 2

From *Table 2*,

$$\begin{aligned} \Sigma X_i &= 19350 \\ \Sigma X_i Y_i &= 485.0 \\ \Sigma Y_i &= 8.55 \times 10^{-2} \\ \Sigma X_i^2 &= 1.143 \times 10^8 \end{aligned}$$

The slope, k , is given by

$$k = \frac{n \sum_i X_i Y_i - \sum_i X_i \cdot \sum_i Y_i}{n \sum_i X_i^2 - \left(\sum_i X_i \right)^2}$$

$$= (4 \times 485.0 - 19350 \times 0.0855) / (4 \times 1.143 \times 10^8 - 3.744 \times 10^8)$$

$$= 3.44 \times 10^{-6}$$

Therefore,

$$k = \frac{La}{d} = 3.44 \times 10^{-6}$$

Solving for a and substituting into expression (11),

$$G = \frac{r^2 a}{2M}$$

$$= ((4.75 \times 10^{-2})^2 \times 7.578 \times 10^{-8}) / (2 \times 1.5^2)$$

$$= 5.624 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

After correction,

$$G' = (1 + \beta)G = 6.069 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

The relative error, δ , is

$$\delta = \frac{|G' - G_0|}{G_0} \times 100 \% = 9.08\%$$

C The Inverse Proportion Method

- Fill the damping chamber with damping oil. The equilibrium position of the balance will have shifted. Recalibrate the equilibrium position and record it.
- Place the large balls on the sliding blocks and move them until they touch the glass windows (see *Figure 6*). Record the position of the light dot every minute until it stops moving. Record the final position, x_1 .
- Set the large balls 5.3cm away from the glass (i.e. $r_2 = 5.3\text{cm}$). Record the position of the light dot every minute until it stops moving. Record the final position x_2 .
- Repeat the previous step for $r_3 = 6.5\text{cm}$ and $r_4 = 9.2\text{cm}$.
- The differences between the x_i and the equilibrium position x will be the displacements S_i of the light dot. Sample data are shown in *Table 3*.

r (m)	r^{-2} (m ⁻²)	S (m)	β	$(1 + \beta)S$ (m)
$r_1 = 4.75 \times 10^{-2}$	4.43×10^2	3.2×10^{-2}	0.079	3.5×10^{-2}
$r_2 = 5.30 \times 10^{-2}$	3.56×10^2	2.6×10^{-2}	0.103	2.9×10^{-2}
$r_3 = 6.50 \times 10^{-2}$	2.37×10^2	1.5×10^{-2}	0.162	1.7×10^{-2}
$r_4 = 9.20 \times 10^{-2}$	1.18×10^2	0.6×10^{-2}	0.310	0.8×10^{-2}

Table 3

The S_i have to be corrected. The correction coefficient β varies with r , so its values have to be calculated in each case from the formula $\beta = r^3 / (r^2 + 4d^2)^{3/2}$. The corrected

values are shown in *Table 3*.

A graph of $(1+b)S$ vs. r^2 can be plotted with the data in *Table 3*. See *Figure 9*. The graph verifies the linear relationship:

$$F \propto S \propto \frac{1}{r^2}$$

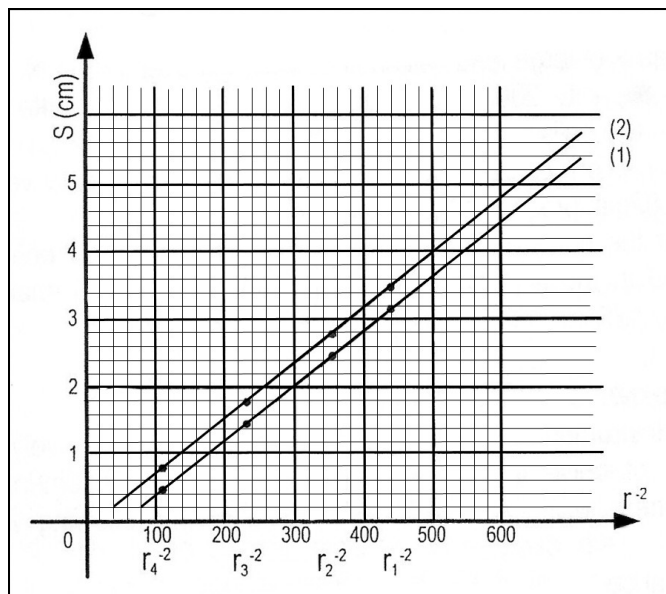


Figure 9

MAINTENANCE

- Lock up the torsion balance with the locking knob after each experiment.
- When putting away the accessories, handle the lead balls with extra caution.
- Set the oil tank to its lowest position to empty the damping chamber, then pour the oil into the bottle and replace the cap.
- Cover the instrument with the dust cover and store it in a cool, dry and clean place.
- Should the filament be accidentally broken and need replacement, follow the instructions in the appendix.

NOTES

- When filling the damping trough with oil, allow the oil to flow in only slowly to minimize the disturbance to the suspended torsion balance. Fill the trough until the vane is completely submerged by about 1 cm, but do not fill oil above the mark.
- When choosing a suitable location for the apparatus, avoid areas with uneven heating (e.g. drafts, periodic sunshine, nearby heating vents, etc.) The ideal location would have an even temperature of $20^{\circ} \pm 5^{\circ} \text{C}$. Also avoid areas near magnetic or electric fields. Uneven heating, magnetic or electric interference can lead to poor measurements.
- Place the lead balls on their supports some time before the experiment to allow their temperature to equilibrate. Do not handle the lead balls unnecessarily.
- Before the experiment, check the thickness of the apparatus body and glass covers $(H + 2h)$ to at least 0.02mm with a vernier caliper. Similarly check the diameters (D) of the large lead balls. Set $r = h + (H + D)/2$.
- If the light spot appears to be stuck or in irregular motion during an experiment,

the cause could be air convection due to temperature change, oil flowing into or out of the damping trough, a build-up of electrostatic charge, or external vibration nearby (air conditioner, compressor, refrigerator, footsteps, etc.) To obtain optimal performance, conditions should be kept as calm as possible. Avoid touching the apparatus or the bench, moving chairs around, etc., as much as possible.

APPENDIX

Changing the suspension filament

Changing a broken suspension wire on the Cavendish Gravitational Balance requires patience and a steady hand.

You will need:

A small Phillips head screwdriver (#0)
Fine pointed tweezers
Replacement wire set

IMPORTANT NOTE:

The high strength beryllium bronze wire is very brittle and easily breaks at defects such as nicks and kinks. Once kinked, the wire cannot be straightened without it breaking under load. Take great care to avoid any stresses, twists, or kinks when handling the wire during replacement, as such defects will almost certainly lead to immediate failure.



Figure 1

Even wires which appear perfect sometimes will have minute nicks and cracks. It is not unusual for a successfully replaced wire to break as soon as the weight of the pendulum is applied to it. *Figure 1* shows a typical failure of this kind. Five replacement wires are provided, and you should expect one or two such failures in a typical set of five.

Procedure:

Removing the broken wire:

1. Remove and set aside the two long upper aluminum sheet side pieces that secure the glass front and back covers.
2. Supporting the glass panels in place with your hand, remove and set aside the two short aluminum sheet lower side pieces. Take care not to lose the small celluloid covers over the side holes (*Figure 2*).
3. Carefully remove and set aside the glass front and back panels.

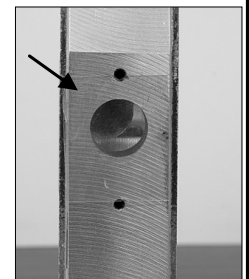


Figure 2

- Using the pendulum locking mechanism screw on the left side of the instrument, carefully raise the pendulum until it is secured against the body of the case at the top of the pendulum cutout. Be careful not to over-tighten the mechanism, as this can bend the pendulum arm.

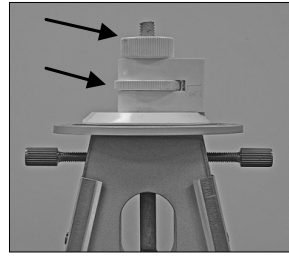


Figure 3

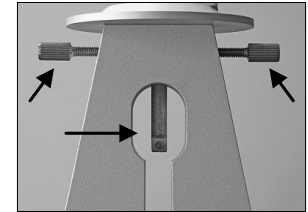


Figure 4

- Using the nut and locknut on the angle adjustment at the top of the instrument (Figure 3), raise the bottom of the upper wire suspension rod until it is in the wide cutout (Figure 4) and arrest it with the two side set screws.
- Supporting the suspension rod from behind with your finger, loosen the set screw of the upper wire lug using a small Phillips screwdriver and remove the lug with tweezers. Loosen the pendulum arresting mechanism slightly and turn the pendulum about 45°. Loosen the set screw of the lower wire lug using a small Phillips screwdriver and remove the lug from the pendulum with tweezers.

Installing the new wire:

- Carefully open the package of replacement wires. Orient the board on the packaging with the screws securing the wires facing up (Figure 5).
- Carefully remove the screws holding one of the wires with a small Phillips screwdriver. Take care not to move the lugs during this operation to avoid creating kinks in the wire.
- Close the package over the wires and board and turn the closed package upside down.
- Open the package and gently remove the board by lifting it straight up, leaving the released wire lying on the packaging.
- Carefully grasp the lug on one end of the wire with fine tweezers. Hold the lug against the packaging with the tweezers, and using your other hand gently tilt the wire and package together until the wire is hanging vertically. Gently remove the packaging, leaving the wire hanging from the tweezers.
- Being careful not to bend or kink the wire, slide the lug into place under the head of the set screw on the upper suspension rod.
- Carefully supporting the upper suspension rod from behind with your finger, gently tighten the set screw. The lug should be square on the support.
- Using the pendulum locking mechanism screw on the left side of the instrument, carefully lower the pendulum until the lug on the lower end of the wire hangs free (Figure 6).
- Allow the wire to "hang out" for 30 minutes so that any tensions in the wire can relax.
- Gently raise the pendulum until the lug hangs a little below its place on the pendulum stem. If necessary, use the point of the tweezers to gently guide the lug so that it does not "hang up"

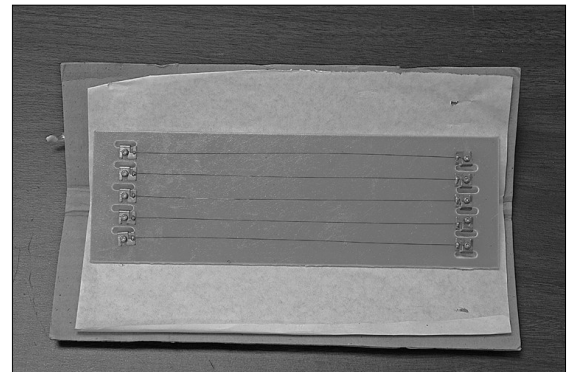


Figure 5

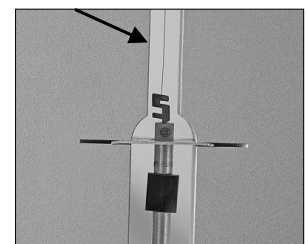


Figure 6

on the end of the pendulum stem or the head of the set screw.
DO NOT ALLOW THE WIRE TO KINK AT THE LUG SOLDER POINT!

11. If the pendulum is not already angled, loosen the pendulum arresting mechanism slightly and turn the pendulum about 45°.
12. Gently grasp the free lug with the tweezers, raise it a little so that the slot matches the set screw head position, and gently slide the lug into place under the head of the set screw.
DO NOT TWIST THE LUG ABOUT THE HORIZONTAL AXIS DURING THIS OPERATION; A KINK CAN EASILY BE FORMED AT THE ATTACHMENT POINT.
13. Carefully supporting the pendulum stem from behind with your finger, gently tighten the set screw. The lug should be square on the support, and the wire should not be taut.
14. Check that the loop of the loose wire does not project beyond the plane of the housing in the front or back. If it does, gently raise the upper suspension rod to retract it, but do not tighten the wire.
15. Re-assemble the glass plates, celluloid strips, and aluminum sheet side pieces to the instrument. The celluloid sheets attached to the glass plates go on the inside. Gently release the pendulum until it hangs free on the wire.

NOTE: If there are any kinks or hidden nicks in the wire, it may break at this point. Even if this does not happen, it may still break in the next 24 – 36 hours as the internal tensions in the wire relax under the pendulum load. Allow the pendulum to hang undisturbed for this time before adjusting its height and angle.