Tips For Solving Problems in the 3 Major Scoring Categories

The tips given below for the topics that are tested in the math section of the ACT is an inclusive but not an exhaustive list.

PREPARING FOR HIGHER MATH (57–60%)

Higher math is divided into 5 subcategories.

NUMBER & QUANTITY

- Absolute value is used to measure distance, which can never be negative. $|-5| = 5$ since –5 is 5 units to the left of 0 on the number line. $|3-7| = 4$ since the distance between 3 and 7 is 4 units on the number line.
- When combining (adding) numbers, remember;
  - Same sign, same sum: For example, –15 + –4 = –19. The sign of the answer is the same as the signs of the numbers being summed.
  - Different signs: Subtract the absolute value of the smaller number from the larger and take the sign of the larger for the answer. For example, $-15+4 = |-15| - |4| = 15 - 4 = 11$ so, the final answer is –11 since $|-15| > |4|$. Also, note that 4 + –15 = $|-15| - |4| = 15 - 4 = -11$ since $|-15| > |4|$.
- When combining (subtracting) numbers remember:
  - You can change subtraction to addition and change the sign of the number being subtracted, then use the rules for adding above. For example, –15 – 4 = –15 + –4 = –19, and –15 – –4 = –15 + 4 = –11.
- Products/Quotients. The sign of the product of an even number of negative numbers is negative, and an odd number of negatives is negative. For example, $-3 \times -4 \times -5 = -60$ and $-2 \times -3 \times -4 \times -5 = 120$. If the sign of the quotient of 2 numbers is positive, the signs of the numbers are the same and negative if the signs are different.
- Apply the order of operations (PEMDAS) especially when performing more than 2 operations. PEMDAS stands for parenthesis, exponents, multiplication, division, addition, subtraction. The order is:
  1. Parenthesis - simplify what is inside of them using PEMDAS first.
  2. Exponents
  3. Multiplication & Division - performed from left to right
  4. Addition & Subtraction - performed from left to right

For example, $24 \div 6 \times 3 = 4 \times 3 = 12$, not $24 \div 18 = \frac{4}{3}$. Also, $6 + 4 \times 3 = 6 + 12 = 18$, not $10 \times 3 = 30$.

- Be careful when finding the square of a negative and the negative of a square. For example, if $x = -3$, then $x^2 = (-3)^2 = 9$ not, $-3^2 = -9$.
  However, $-x^2 = -(-3)^2 = -9$.
- The set of Natural numbers are also known as the counting numbers: 1, 2, 3, … The set of Integers includes negatives and zero: …, –3, –2, –1, 0,1, 2, 3, … Natural numbers are used to index a term in a sequence. For example, the sequence $\{5, 8, 11, \ldots, a_n = 3n + 2, \ldots\}$ has its 1st term $a_1 = 5$, 2nd term $a_2 = 8$, and 3rd term $a_3 = 11$.
- The square root of a negative number is an indication that you are working with a Complex number, $a + bi$, where a and b are Real numbers, bi is imaginary and $i = \sqrt{-1}$. So, $4 + \sqrt{-9}$ should be simplified to $4 + 3i$ before performing any operations to it and another value. Also, $i^2 = -1$ so,
  $\sqrt{-4} \times \sqrt{-9} = 2i \times 3i = 6i^2 = 6(-1) = -6$ not, $\sqrt{-4} \times \sqrt{-9} = \sqrt{36} = 6$.
- The radical form for the exponential expression $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ where m and n are Integers. To write and simplify an expression in exponential form, put it in radical form first. For example, $64^{\frac{2}{3}} = \sqrt[3]{64^2} = (\sqrt[3]{64})^2 = 4^2 = 16$. Also, $x^{-m} = \frac{1}{x^m}$. So, $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$.
- Conjugate. Sometimes you will need to simplify and rational(fraction) expression that has a radical in the denominator. To take the radical out of the denominator, multiply the given expression by a form of 1 that contains the conjugate(opposite sign) of the denominator. For example, $\frac{2}{3+\sqrt{5}}$ can be simplified by multiplying it by 1 as $\frac{3-\sqrt{5}}{3-\sqrt{5}}$. Now, $\frac{2}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{2(3-\sqrt{5})}{9-5} = \frac{2(3-\sqrt{5})}{4} = \frac{3-\sqrt{5}}{2}$. Note that $\sqrt{5}$ is no longer in the denominator due to $3\sqrt{5}$ and $-3\sqrt{5}$ reducing to 0.

ALGEBRA

- Be careful about how you use the distributive property when simplifying expressions. Check to see if you are distributing a negative along with the value of a number or variable. For example, $4x - 2(5x - 4) = 4x - 2 \times 5x - 2 \times -4 = 4x - 10x + 8 = -6x + 8$. A common error when simplifying an expression like this is to only distribute –2 to $5x$ but not 2 to –4.
- Only like terms can be combined (added or subtracted), although any 2 terms can be multiplied. Like terms must have the same variables and have the same degree for each variable. So, $4x^2y^3$

and $5x^2y^3$ can be added to get $9x^2y^3$, but $4x^2y^3$ and $5xy^3$ cannot be added since the variable x is not of the same degree for each term. However, $4x^2y^3$ and $5xy^3$ can be multiplied to get $20x^3y^6$.

- When factoring, look for a GCF (greatest common factor) first before using various techniques such as difference of squares or a sum of 2 cubes. For example, $x^3 - 64x$ has a GCF of x to factor to $x(x^2 - 64)$ then factors again using the difference of squares formula, shown below, to get $x(x-8)(x+8)$. A common error is to see $x^3 - 64x$ as a difference of 2 cubes, shown below, where $a = x$ and $b = 4x$.
- Special factoring includes;
  - Perfect square trinomial (3 terms): $a^2 \pm 2ab + b^2 = (a \pm b)^2$
  - Difference of 2 squares: $a^2 - b^2 = (a-b)(a+b)$. There is no sum of 2 squares formula since $a^2 + b^2$ does not factor.
  - Sum or difference of 2 cubes: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.
- A factor can be a number, variable, or expression. The quadratic $x^2 - 16 = (x-4)(x+4)$ has 2 factors: $(x - 4)$ and $(x + 4)$. Factors can be identified since they are always multiplied to other factors. The 3 factors to $2(x - 5)(x + 3)$ are 2, $(x - 5)$, and $(x + 3)$.
- When simplifying rational expressions, only common factors can be canceled in the numerator and denominator. For example, the common factor in the numerator and denominator for the following expression is $(x + 6)$. $\frac{x^2-36}{(x-3)(x+6)} = \frac{(x-6)(x+6)}{(x-3)(x+6)} = \frac{(x-6)}{(x-3)}$. A common error is to simplify further, $\frac{(x-6)}{(x-3)} \neq \frac{-6}{-3}$ with the x's treated as a common factor. However, x is only part of the factors $x - 6$ and $x - 3$, so it cannot be canceled.
- To determine when a rational function is not defined, set the factors in the denominator equal to zero and solve before doing any simplifying. The rational expression in the previous example is undefined when $x = -6$ or 3 since $x - 3 \neq 0$ or $x + 6 \neq 0$. If you used the simplified version $\frac{x-6}{x-3}$ then, the expression would be undefined at only $x = 3$ since $x - 3 \neq 0$.

When solving an equation that factors,

i. Set one side equal to zero and completely factor the non-zero side.
ii. Set each factor equal to zero since at least one of the factors must be 0 in order for the product of the factors to equal 0.
iii. Solve each equation for the unknown variable or variables that are set equal to 0.
iv. When solving a quadratic equation in the form $ax^2 + bx + c = 0$ and it does not factor, the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ can be used to find its roots or x-intercepts. If the

discriminant $b^2 - 4ac$ is positive, the roots will be 2 real numbers, and if it is negative the roots will be 2 complex conjugate numbers.

v. If time permits, check that the answer makes sense by plugging it back into the original equation.

- A function is linear if it is to the 1st degree, or its graph has the same slope between any 2 points. $y = \frac{3}{2}x + 1$ is linear since x is raised to the 1st degree. In slope-intercept from, $y = mx + b$, has a slope of m and a y-intercept of b.
- Slope is also referred to the rate of change in 2 quantities. Given 2 points for a line, the slope is $m = \frac{\Delta y}{\Delta x}$. When you see word problems where the rate of change is constant, the problem can typically be solved with a linear function. For example, if the rate at which the population of a state grows each year is 2%, then 0.02 is the slope that is used to write a linear function for the population of the state in a given year.
- If an x-intercept is given for the graph of an equation, then (x. 0) is a point on its graph. If a y-intercept is given, then (0, y) is a point on its graph. If the equation is a function, it can only have only one y-intercept. Any polynomial function can have at most n x-intercepts where n is the degree of the polynomial.
- To find the distance, d, between 2 given points,(x_1, y_1) and (x_2, y_2), use $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- To find the midpoint between two points, (x_1, y_1) and (x_2, y_2), $(x_m, y_m) = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ which is just the average of the two x and y values for the points.
- To write a system of 2 equations using matrices use $AX = B$ where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. For example, the system,

  $3x + 4y = 12$

  $5x - y = 10$ can be written as,

  $$\begin{bmatrix} 3 & 4 \\ 5 & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

  A × X = B

- The determinant of a 2x2 matrix is $a \times d - b \times c$, with a, b, c, and d as shown below: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

FUNCTIONS

- A function is a set of ordered pairs, (x, y) where no two ordered pairs have the same x-coordinate. The set {(0, 1), (3, 2), (4, 5)} is a function but {(0, 1), (3, 2), (3, 5)} is not a function since the x-coordinate of 3 is in two ordered pairs. It can be tricky to determine if a given equation is a function. If you can graph the equation and no one vertical line drawn through its graph intersects it more than one point (also referred to the vertical line test), the equation is a function. If the vertical line intersects the graph more than once, the equation is not a function. A circle is not a function since it is possible to draw a vertical line (many, in this case) through its graph intersecting it twice.
- Function f, in function notation, can be written as $y = f(x)$ and is read y equals f at x.It is not f times x. To determine $f(2)$ for $f(x) = 3x^2 + x$, substitute $x = 2$ into f to get, $f(2) = 3(2)^2 + 2$=14. So, the value of f at 2 is, $f(x) = 14$ and (2, 14) is an ordered pair of f for a point on the graph of the function f.
- Be careful when performing a composition of 2 or more functions. The composition $f(g(x))$ or $f \circ g(x)$means to evaluate f using the function value for g, rather than the x value for g. To perform such a composition, substitute the y value of $g(x)$ as the x value of f. For example, if $f(x) = 3x^2 + x$ and $g(x) = 2x - 1$, then $f(g(3)) = f(2(3) - 1) = f(5) = 3(5)^2 + 5 = 80.$ Notice the y-value of 5 for g becomes the x-value for f.
- The graph of a function in the from $y = af(x - h) + k$ transforms the graph of a parent function $y = f(x)$ depending on the constants a, h, and k. The value for a stretches f vertically if $|a| > 0$ and compresses it if $0 < a < 1$. If $a < 0$, the graph of f is reflected across the x-axis. The value for h shifts the graph of f horizontally right h units if $h > 0$ and left if $h < 0$. Be careful to use the value for h in the function and not $-h$ when determining this shift. The value for k shifts the graph of f vertically upwards if $k > 0$ and downwards if $k < 0$. So $y = 2(x + 3)^2 + 4$ vertically stretches the graph of $y = x^2$, the parent function, by a factor of 2, then shifts it 3 units to the left since $h = -3$, and finally 4 units upwards.
- A quadratic function in the form $y = a(x - h))^2 + k$ is the graph of a parabola that has its vertex at (h, k). If $a > 0$, the vertex is the maximum point on the graph and a minimum point of $a < 0$. So, the vertex of (3, 2) for $y = -4(x - 3)^2 + 2$ is a maximum point since $a = -4$ since the graph of the parabola is reflected across the x-axis so it opens downward.
- A function has an inverse only if its inverse is also a function.
- A logarithmic function is just an inverse to an exponential function. For example, $x = b^y$can be written as $y = log_b x$ read as log base b of x. Notice that the base for an exponential function and

its log are both b. It is usually easiest to change a log to its exponential form when evaluating it. For example, $log_2 32 = 5$ since $2^5 = 32$.

- The special exponential function $y = e^x$ is called the natural exponential function where $e \approx 2.718$. The inverse is $y = log_e x$ or $y = ln\ x$ which is called the natural logarithm. You should have a natural log button labeled LN or ln on your calculator and the e button is probably the 2nd function button of LN since it is its inverse.
- Use these 3 basic rules for logs (they also work for natural logs) when working with logs.
    I. Multiplication Rule: $log_b\,(MN) = log_b M + log_b N$
    II. Division Rule: $log_b\,(\frac{M}{N}) = log_b M - log_b N$
    III. Exponent Rule: $log_b (M)^n = n \cdot log_b M$

Be careful how you use the exponent rule in conjunction with the multiplication and division rules. For example, completely expand $log_5(\frac{x^2y}{z^3})$:

$$\log_5\left(\frac{x^2y}{z^3}\right) = \log_5\left(x^2y\right) - \log_5 z^3 = \log_5 x^2 + \log_5 y - \log_5 z^3 = 2\log_5 x + \log_5 y - 3\log_5 z$$
.

Notice the exponent rule is used after expanding the log using the division and multiplication rules. However, writing $2log_5 x + log_5 y - 3log_5 z$as a single log you have:

$$2\log_5 x + \log_5 y - 3\log_5 z = \log_5 x^2 + \log_5 y - \log_5 z^3 = \log_5\left(x^2y\right) - \log_5 z^3 = \log_5\left(\frac{x^2y}{z^3}\right)$$

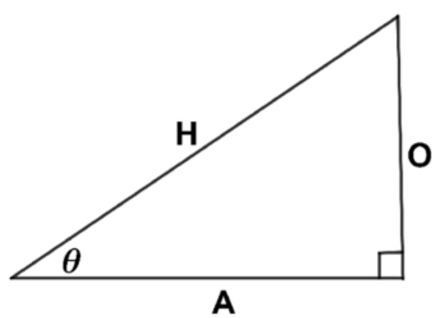
where the rules are used in reverse.

- When working with sequences look for 2 special types, arithmetic and geometric. An arithmetic sequence has a common difference between any 2 successive terms, and a geometric sequence has a common ratio between any 2 successive terms. The difference in the sequence {3, 7, 11, 15, …} is 15 – 11 = 11 – 7 = 7 – 3 =4 so, the sequence is arithmetic. The common ratio in the sequence {3, 6,12, 24, …} is $\frac{24}{12} = \frac{12}{6} = \frac{6}{3} = 2$, so the sequence is geometric. If a sequence does not follow either of these patterns, it is neither arithmetic nor geometric.

GEOMETRY

- Sometimes you will need to write the formula measuring some given dimension for a geometric object using another dimension of the object. For example, area of a circle in terms of its circumference, *C*, is $A = \frac{C^2}{4\pi}$. This is accomplished by substituting the circumference written in terms of *C* as $r = \frac{C}{2\pi}$ into the area formula, $A = \pi r^2$.
- Review the formulas for the geometric shapes and solids.
- For a triangle that has angles *A, B,* and *C,* and corresponding sides *a, b,* and *c*, the Pythagorean theorem can be used only if it is a right triangle. You will want to use the laws of sines or cosines to determine missing side lengths or angles for a non-right triangle. Use the law of sines when you are given SSA (2 sides and a non-included angle) or AAS (2 angles and a non-included side). use $\frac{a}{sin\ A} = \frac{b}{sin\ B} = \frac{c}{sin\ C}$ to determine an unknown side and $\frac{sin\ A}{a} = \frac{sin\ B}{b} = \frac{sin\ C}{c}$to find an unknown angle.
- Use the law of cosines when you are given SAS (2 sides and an included angle) or SSS (all 3 sides). To find a unknown side use $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\ A$or $A = cos^{-1}\ (-\frac{a^2-b^2-c^2}{2bc})$ to find an unknown angle.
- For the right triangle shown below with vertex angle θ, opposite side O, adjacent side A, and hypotenuse H, the 6 trigonometric ratios are;

$$\sin\theta = \frac{O}{H} \qquad \csc\theta = \frac{H}{O}$$
$$\cos\theta = \frac{A}{H} \qquad \sec\theta = \frac{H}{A}$$
$$\tan\theta = \frac{O}{A} \qquad \cot\theta = \frac{A}{O}$$



Use the acronym SOHCAHTOA to help determine the correct ratios to use for sine, cosine, and tangent.

STATISTICS AND PROBABILITY

- Know how to find the mean, median, and mode for a given set of data. Often you will be asked to find the sum or difference of 2 or more of these values.
- The mean is just the average which is found by combining the values in a set and dividing by the number of elements in the set.
- Before determining the median, order the values in the set. If there are an odd number of elements, the median is the center element in the set. If there are an even number of elements, take the average of the middle 2 elements.
- Bivariate data is just data with 2 variables. You will probably be asked to show what type of relationship the variables have with each other. For example, you may need to determine the relationship between the time of year and the amount of ice cream that is sold.
- Events are independent when the occurrence of one has no effect on the occurrence of another. If you flip a fair coin 2 times, the probability of it landing heads side up each time is $\frac{1}{2}$. The 1st flip has no bearing on the outcome of the 2nd flip. Events are dependent if the occurrence of one event has affected the occurrence of another event. For example, if 5 red marbles and 5 green marbles are placed in a bag, the probability of choosing a red marble from the bag on the 1st try is $\frac{5}{10}$ *or* $\frac{1}{2}$. The probability of choosing a red marble from the bag on the 2nd try is $\frac{4}{9}$ if the marble is not replaced from the 1st try.
- Whenever you are asked to find the probability of 1 event and a 2nd event occurring, the probabilities are multiplied. So, the probability of choosing a red marble from the bag on the 1st and 2nd try from the example above is $\frac{1}{2} \times \frac{4}{9} = \frac{4}{18}$ *or* $\frac{2}{9}$.

INTEGRATING ESSENTIAL SKILLS

- Make sure to convert a given percentage to a decimal number when using it to solve a problem. So, 75% is written as 0.75 by moving the decimal for 75 to the left 2 places, and 37.5% = 0.375. Make sure your answer makes sense. For example, 75% of 100 is $0.75 \times 100 = 75$ not, $75 \times 100 = 7500$.
- When comparing a set of numbers expressed in different ways, convert them to the same type. For example, if the values 35% and $\frac{2}{5}$ are to be compared, convert 35% to 0.35 and $\frac{2}{5}$ to 0.40.
- Know your conversion factors. Whenever converting a given unit to another unit, multiply by a ratio with the unit you are converting to in the numerator. For example, convert 11 feet to inches.

Using the conversion 12 inches = 1 foot, you get $11\ feet \times \frac{12\ inches}{1\ foot} = \frac{11\times 12}{1}\ inches =$ $132\ inches$. The trick is to use the ratio that will cancel the given units so that only the determined unit is left.

- Whenever the ratio of 2 quantities is equal to the ratio of 2 other quantities, we say the ratios are in proportion. For quantities *a, b, c,* and *d,* a proportionality statement is $\frac{a}{b} = \frac{c}{d}$. *b* and *c* are called the means, and *a* and *d* are called the extremes. The product of the means always equals the product of the means. So, *bc* = *ad* (think of cross multiplying).

MODELING

- Modeling problems include skills learned from the other topics.
- Translating a real-world situation into a mathematical problem can often be the most challenging part of any math question on the ACT. The following steps may help organize and ultimately solve a modeling problem:
  i. Identify what it is you need to find or know.
  ii. Determine what values are given. It is likely you will use those values to help determine what you need to find or know.
  iii. Think about the units being used for the given values or the values you need to find or know. Doing this can help determine a formula to use such as area or volume in order to find what it is you need to know. The formula should include the unknown value as well as the known value(s).
  iv. Set up an equation using the formula from step iii and substitute the given values.
  v. Solve the equation for the unknown value.
  vi. Check to make sure your answer makes sense. Bear in mind that there will always be answer choices that will be distractors.