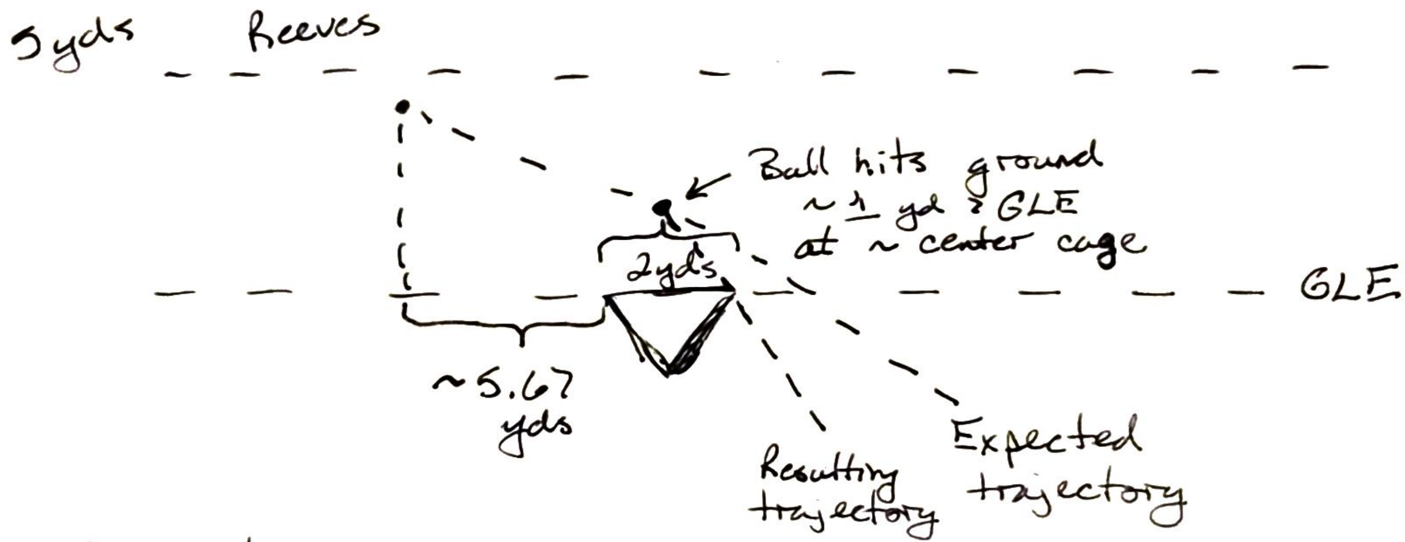
Reeves Curve Shot

Scale: ~3cm → 6ft or 2 yds



Assumptions:

- Reeves shot with foot right inside 5 yd above GLE, assume ball release at 5 yds
- Reeves is likely a little closer to near pipe than "measured" due to camera angle.
- Ball hits ground ~1 yd above GLE at equal distance between pipes
- Origin of coordinates will be center of goal posts along GLE.

Coordinates:

Ball Release = (-6.67, 5) yds

Ball hit ground = (0, 1)

Ball score = (1, 0)

How far outside the post *along GLE* was the shot supposed to be if it continued straight after it hit the ground?

Line of shot: $y = mx + \underline{1}$ ← $\underline{1}$ comes from where ball hits the ground.

↑ want slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-6.67} = -0.6$$

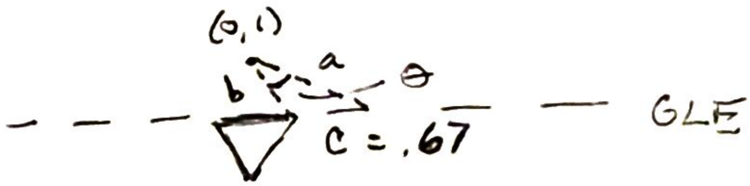
Where does ball cross GLE? $y = 0$

$$0 = -.6x + \underline{1}$$

$$x = \frac{1}{.6} = 1.67 \text{ yds}$$

Ball should have been $2/3$ yds outside of cage!

Ball should have crossed GLE = (1.67, 0)



Calculate a, b, θ

Get a + b with Pythagorean Theorem:

$$a = \sqrt{(1.67)^2 + 1^2}$$

$$\boxed{a = 1.95 \text{ yds}}$$

$$b = \sqrt{1^2 + 1^2}$$

$$\boxed{b = \sqrt{2} = 1.41 \text{ yds}}$$

Law of cosines for θ

$$c^2 = a^2 + b^2 - 2ab \cos\theta$$

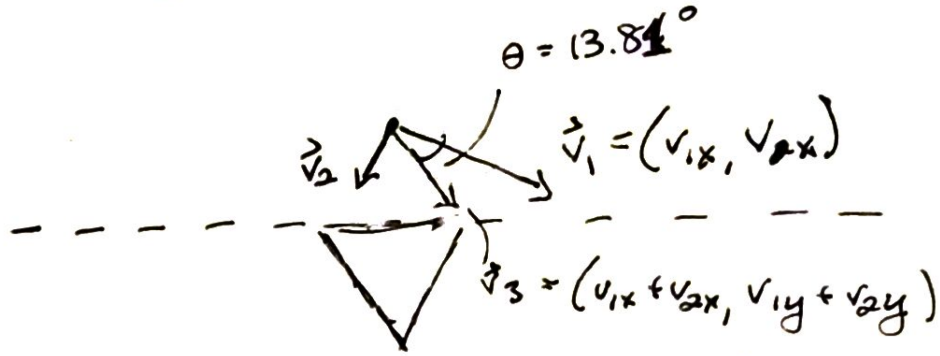
$$\cos^{-1}\left(\frac{-c^2 + a^2 + b^2}{2ab}\right) = \theta = 13.81^\circ$$

Dr. Reeves changed his shot angle by 13.81°. But how? This has all been geometry, now here is the physics.

More assumptions:

- Rotational Energy is entirely converted to translational energy
- Only components in x, y plane change due to rotation of ball.
- Elastic collision with ground
- Ignore height + deal with that at end

Look at velocity vectors



$\vec{v}_1$ is velocity vector on ground impact. $\vec{v}_2$ is velocity introduced by rotational of ball & assumed to be $\perp \vec{v}_1$ because I can't measure axis of rotation through a camera. $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$. Try to relate $|\vec{v}_1|$ to $|\vec{v}_3|$.

$$\vec{v}_1 \cdot \vec{v}_3 = |\vec{v}_1||\vec{v}_3| \cos \theta$$

$$v_{1x}(v_{1x} + v_{2x}) + v_{1y}(v_{1y} + v_{2y}) = |\vec{v}_1||\vec{v}_3| \cos \theta$$

$$v_{1x}^2 + v_{1x}v_{2x} + v_{1y}^2 + v_{1y}v_{2y} = |\vec{v}_1||\vec{v}_3| \cos \theta$$

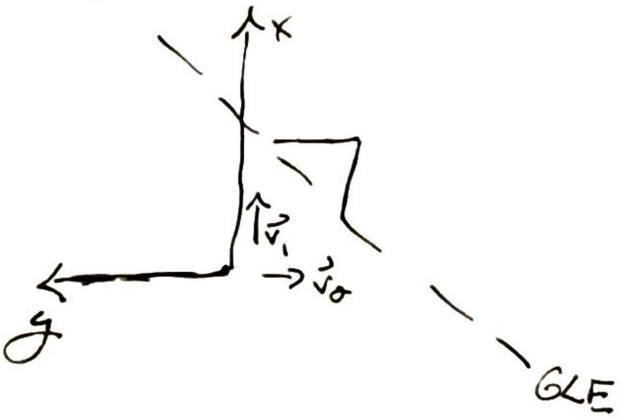
$$|\vec{v}_1|^2 + \underbrace{v_{1x}v_{2x} + v_{1y}v_{2y}}_{\vec{v}_1 \cdot \vec{v}_2 = 0 \text{ bc } \vec{v}_1 \perp \vec{v}_2} = |\vec{v}_1||\vec{v}_3| \cos \theta$$

$$|\vec{v}_1|^2 = |\vec{v}_1||\vec{v}_3| \cos \theta$$

$$|\vec{v}_3| = \frac{|\vec{v}_1|}{\cos \underset{13.81}{\theta}} = 1.03\,|\vec{v}_1|$$

So the shot is slightly faster after ground contact.

Cheat and transform coordinate system so x-axis is along original ball trajectory.



So we can solve for $|\vec{v}_2| = v_{2y}$.

$$|\vec{v}_3| = \sqrt{(v_{1x} + \underset{0}{\underset{\uparrow}{v_{2x}}})^2 + (\underset{0}{\underset{\uparrow}{v_{1y}}} + v_{2y})^2} = 1.03\,|\vec{v}_1| = 1.03\,v_{1x}$$

$$|\vec{v}_3|^2 = v_{1x}^2 + v_{2y}^2 = (1.03\,v_{1x})^2$$

$$v_{2y}^2 = (1.03\,v_{1x})^2 - v_{1x}^2$$

$$= (1.061 - 1)\,v_{1x}^2$$

$$v_{2y}^2 = .061\,v_{1x}^2$$

$$v_{2y} = 0.25\,v_{1x}$$

So the ball spinning added $0.25 v_{1x}$ to the velocity of the ball perpendicular to original trajectory.

If rotational energy before contact with ground was efficiently converted to translational energy.

$$\frac{2}{5}\frac{1}{2}\cancel{m}(R\omega)^2 = \frac{1}{2}m v_{2y}^2$$
$$= \frac{1}{2}\cancel{m}(0.25\,v_{1x})^2$$
$$\omega = \sqrt{\frac{5}{2}\left(\frac{0.25\,v_{1x}}{R}\right)^2}$$

radius of lacrosse ball

$R = 0.032\,m$
$0.034\,yd.$

$$\omega = 4.418\,v_{1x} \quad 40\,m/s \rightarrow 2984.4\ rpm!$$

How much torque is then applied? Need to assume uniform angular acceleration.

$$\bar{\alpha} = \frac{\omega - 0}{t_{release}} = \frac{\omega}{t_r} = \frac{1}{t_r}\,4.418\,v_{1x}$$

time from wind up to release

Torque: $\tau = I\bar{\alpha}$
$$= \frac{2}{5}MR^2\bar{\alpha}$$

mass of ball

$$= \frac{2}{5}(.1435)(0.032)^2\,\frac{1}{t_r}\,4.418\,v_{1x} \quad \left(\frac{kg\,m^2}{s}\,\frac{m}{s}\right)$$

$(N \cdot m)$ ✓

$$\tau = \frac{0.00026}{t_r}\,v_{1x} \quad \text{if } v_{1x} = 40\,m/s$$

$\rightarrow$ 0.5 s i guess

$\tau = 0.015$ lb-ft