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NUMBER CONCEPTS

$\sqrt{7}$ π ***-15.2***

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1.4 Laws of exponents

$\frac{3}{7}$

9.23251... ***-122*** $\sqrt{5}$ 2^3 ***0.221*** ***0.05***

$\left(\frac{2}{3}\right)^4$ $11\frac{2}{3}$ ***10.01***

1.1 The Real Number System

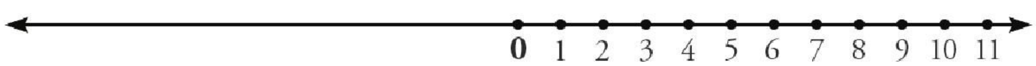
The system of **real numbers** that we use in our everyday lives consists of a collection of smaller sets of numbers that has evolved over several centuries. It began with numbers used to count objects, used for trading and other commercial purposes. It was then extended and refined as a need for numbers to represent parts of objects and locations on the number line became important. A discussion of the sets of numbers that comprise the real number system is next. Each of these sets of numbers builds on those contained in the preceding set.

Natural Numbers

- **Natural numbers** can be thought of as counting numbers. They can be used to identify how many objects are contained in a collection.
- Since a collection of objects has at least one item in it, the natural numbers begin with the number 1 and then proceed to represent additional objects in the set.
- Counting numbers can be listed as follows: 1, 2, 3, 4, 5, 6, 7, 8, …

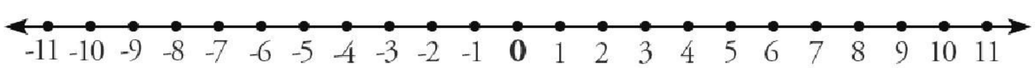
Whole Numbers

- **Whole Numbers** consist of the Natural Numbers in addition to the number 0.
- Although 0 does not represent an object in a set, it is an important addition to the number system
- Whole numbers can be listed as follows: 0, 1, 2, 3, 4, 5, 6, 7, 8, …
- They correspond to locations on the number line as follows:



Integers

- The set of **integers** builds on the set of Whole numbers by adding the negative values of each.
- As a result it includes numbers such as -1, -2, -3, -4, -5, … (Note: There is no negative value for 0.)
- Negative values of whole numbers are used in many situations such as to represent a minus temperature (-22^0 C), distance below sea level (-8 m below the sea), or a golf score that is under par (-4 strokes under par).
- Integers correspond to locations on the number line as follows.



Rational Numbers

- The next extension to our number system is the set of **rational numbers**. By including these numbers, we can begin to look at parts of the counting objects discussed earlier (e.g. one-half of an item or quantity, one quarter of a degree in temperature).
- A **rational number** is any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
- This includes the natural numbers (e.g. 1, 2, 3, …), the whole numbers (e.g. 0, 1, 2, 3, …), and the integers (e.g. ...-3, –2, -1, 0, 1, 2, 3 …).
- Natural, whole, and integer numbers are rational since each can be written in the form $\frac{a}{b}$ (e.g. $1 = \frac{2}{2}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{4}$).
- The set of rational numbers begin to fill in many locations on the number line. However, there still remain locations that do not have corresponding numbers associated with them.

<u>Examples of Rational Numbers</u>:

- All fractions and mixed numbers, both positive and negative

  e.g. $\frac{2}{3}$, $\frac{-3}{4}$, $\frac{5}{2}$, $-3\frac{1}{4}$ (note: $\frac{0}{7} = 0$ is rational but $\frac{7}{0}$ is **not rational** since it is not defined when the denominator equals 0)
- All integers e.g. -11, -3, 0, 1, 5, 68
- All terminating and repeating decimals, both positive and negative

  e.g. 0.8, -0.32, $0.333\bar{3}$, $7.12121\overline{2}$

Irrational Numbers

- To complete our system of real numbers it is necessary to add an additional set. These additional numbers are called **irrational numbers**.
- **Irrational numbers** are any numbers that cannot be written as the quotient ($\frac{a}{b}$) of two integers where $b \neq 0$.

<u>Examples of Irrational Numbers</u>:

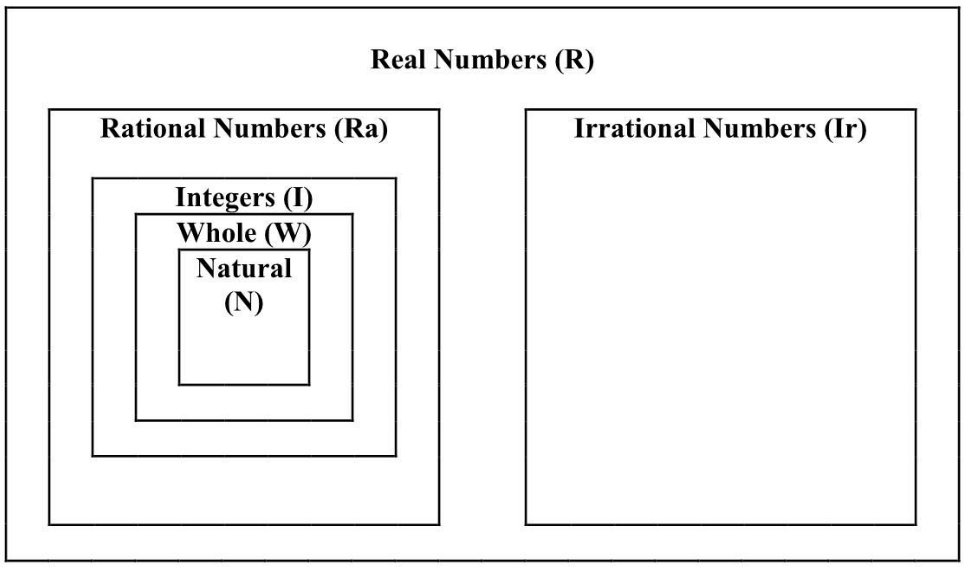
- Numbers that are roots of whole numbers that <u>cannot</u> be simplified to obtain a rational number

  e.g. $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$ (Note: $\sqrt{9}$ is <u>rational</u> since it is equal to 3.)
- Numbers whose decimal representation does not repeat in a pattern

  e.g. 0.1357421… (Note: $0.33\bar{3}$ is <u>rational</u> since it repeats a pattern and is equal to $\frac{1}{3}$.)
- Special numbers such as π (which is equal to 3.1415927…)

Real Numbers

- The set of **real numbers** consists of all rational and all irrational numbers.
- All locations on a number line correspond to a real number and we can think of the set of real numbers as filling all locations on the number line.
- The diagram below shows the relationship among the sets of numbers discussed so far.



<u>Note</u>:

- As shown in the above diagram, the set of rational numbers includes each of the following.

  Natural Numbers: $N = \{1,2,3,4,...\}$
  Whole Numbers : $W = \{0,1,2,3,4,...\}$
  Integers: $I = \{... -3,-2,-1,0,1,2,3,4,...\}$
  Rational: Ra = All of the above <u>plus</u> any other number that can be written in the form $\frac{a}{b}$, $b \neq 0$ (e.g. 0.5, 1.24, $\frac{2}{5}$, $-1\frac{1}{4}$, 7)
- All rational and all irrational numbers make up the set of real numbers.

Identification of Rational and Irrational Numbers

Recall that:

- Rational numbers can be shown in several different formats, as long as they can be rewritten in the form $\frac{a}{b}$, $b \neq 0$.
  1. Natural numbers, whole numbers, and integers
     Examples: 7, -43, 0, 2761, - 403
  2. Proper fractions, mixed numbers, or improper fractions
     Examples: $\frac{3}{11}, -\frac{2}{9}, 3\frac{1}{4}, \frac{7}{5}, -8\frac{1}{10}, -\frac{7}{3}$
  3. Decimals (terminating or repeating)
     Examples: 0.8, -0.25, $0.2233\overline{3}$, $2.61\overline{61}$
- Irrational numbers cannot be shown as common fractions.
  1. Decimals that do not terminate or repeat in a pattern (e.g. 0.12323569…)
  2. Roots of numbers that are not rational (e.g. $\sqrt{2}, \sqrt{11}, -3\sqrt{5}$,..)
  3. Special numbers like π

Examples with Solutions

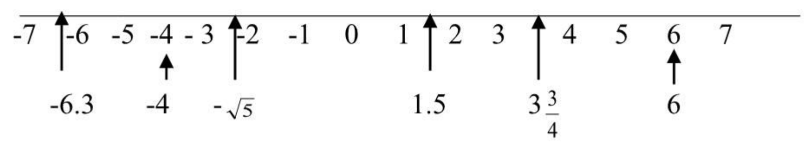
Identify whether each of the following are rational or irrational numbers. Give a reason for your answer.

Number	Rational or Irrational?	Reason
1. -1.25	*Rational*	*It can be written as* $-\frac{125}{100}$ *or* $-\frac{5}{4}$.
2. 0.6010347…	*Irrational*	*The decimal doesn't terminate or repeat the same pattern.*
3. $-\sqrt{25}$	*Rational*	*It can be written as -5.*
4. 0.1111 …	*Rational*	*It repeats the same pattern and can be written as* $\frac{1}{9}$.
5. $\sqrt{13}$	*Irrational*	*The decimal version doesn't repeat the same pattern e.g.* $\sqrt{13} = 3.6055513...$
6. 4.01	*Rational*	*It has a terminating decimal (e.g. it could be written as* $4\frac{1}{400}$ *or* $\frac{401}{400}$*).*

7. $-7\frac{1}{2}$	*Rational*	*It could be written as* $\frac{-15}{2}$.
8. $2125\frac{1}{4}$	*Rational*	*It could be written as* $\frac{8501}{4}$.
9. 2.333…	*Rational*	*The decimal repeats the same pattern and is equal to* $2\frac{1}{3}$ *or* $\frac{7}{3}$.
10. 5.0100382…	*Irrational*	*The decimal doesn't terminate or repeat the same pattern.*

Comparing and Ordering Rational Numbers

- Each rational number corresponds to a point on the number line. Several examples are shown next.



- It should be noted that numbers **increase** in magnitude as you go from left to right on the line.
  e.g. $1 < 3$; $2.1 < 4$; $-7 < -6$; and $2 > 1.8$; $-3 > -5$; $-1 > -10.5$

Examples with solutions:

To compare the magnitudes of rational numbers where one is written in decimal and the other in common fraction form, write both in the same form and then compare.

1. Compare 0.1 with $\frac{3}{20}$ (convert both to fractions first).

   - Change 0.1 to $\frac{1}{10}$.
   - The common denominator is 20, $\therefore \frac{1}{10} = \frac{2}{20}$.
   - $\frac{2}{20} < \frac{3}{20}$ or $0.1 < \frac{3}{20}$

2. Compare 3.15 with $3\frac{1}{11}$ (convert both to decimals first).

   - Change $3\frac{1}{11}$ to a decimal $\rightarrow 3.09\overline{09}$.
   - $\therefore\ 3.15 > 3.09\overline{09}$ or $3.15 > 3\frac{1}{11}$

Exercises 1.1

Identify which of the following are rational and which are irrational numbers. Give a reason for your answer.

Number	Rational or Irrational	Reason
1. 0.013		
2. $5\frac{1}{2}$		
3. 7.0900134...		
3. 0.666…		
4. -10.001		
5. $\sqrt{49}$		
6. 0.122357…		
7. 0.212121…		
8. 210.013		
9. $\sqrt{8}$		
10. -5.999…		
11. 3.009		
12. $-345\frac{1}{3}$		

Use a check mark to indicate which set(s) each number belongs to.

		Set of Numbers				
Question	**Number**	**N**	**W**	**I**	**Ra**	**Ir**
13.	0.7					
14.	-45					
15.	$\frac{15}{7}$					
16.	0.13243…					

Question	Number	N	W	I	Ra	Ir
17.	$0.\bar{7}$					
18.	2					
19.	$1\frac{5}{8}$					
20.	-160					
21.	0					
22.	$\sqrt{81}$					
23.	0.93					
24.	$\sqrt{15}$					

Note: N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, Ir = Irrational Numbers

25. Locate the following numbers on the number line.

$3.1,\ 2\frac{5}{8},\ -\frac{13}{12},\ -\sqrt{6},\ -\sqrt{16}$

-4 -3 -2 -1 0 1 2 3 4

26. Arrange the following numbers from smallest to largest.

a. -0.57, -0.507, -5.07, -5.70

b. $3.4,\ -\frac{11}{3},\ -3.4,\ -3.5$

c. $-\frac{3}{8},\ -\frac{2}{3},\ -0.6,\ -0.4$

27. Put the correct symbol (>, =, <) between each pair of numbers.

a. 0.15 ☐ $\frac{7}{40}$

b. -1.8 ☐ $-\frac{9}{5}$

c. -2.8 ☐ $-\frac{13}{5}$

28. Express each term in common fraction form (as a quotient of two integers)

Student Work

a. 0.17

b. $-0.\overline{5}$

c. $-1\frac{2}{3}$

d. 3.07

29. Which rational number is greater?

Student Work

a. $-0.\overline{6}$ or -0.6?

b. -0.25 or $-\frac{1}{3}$?

c. $-\frac{2}{3}$ or $-\frac{4}{5}$?

Extra for Experts

30. List the set of all integers greater than -4 and less than $\frac{1}{2}$.

31. Is $\sqrt{\frac{4}{9}}$ rational or irrational? Give a reason for your answer.

32. Is the sum of the following numbers rational or irrational? Give a reason for your answer.

$$0.1 + 0.01 + 0.001$$

33. Is the sum of the following numbers rational or irrational? Give a reason for your answer.

$$0.333\ldots + 0.666\ldots + 0.999\ldots$$

34. If the sum of 3.82 + 12.<u>ab</u> is an integer, what digits must go in place of <u>ab</u>?

35. If the sum of $-12 + -8\frac{1}{2} + n$ is a natural number, what is the smallest number that can replace ***n***?

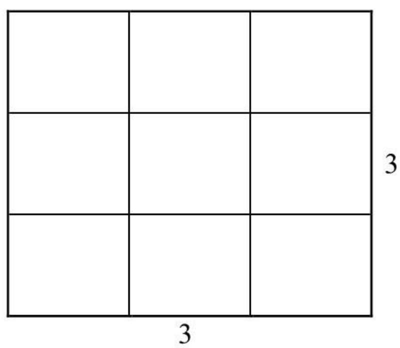
1.2 Square Root of a Number

A list of perfect squares that are useful to remember is shown below. Most of these should look familiar.

Number	**Square of No.**	**Number**	**Square of No.**
1	1	9	81
2	4	10	100
3	9	11	121
4	16	12	144
5	25	13	169
6	36	14	196
7	49	15	225
8	64	25	625

To inspect the square root of a number pictorially, we can examine a perfect square using a grid.

Example 1: The square root of 9



(what)(what) = 9

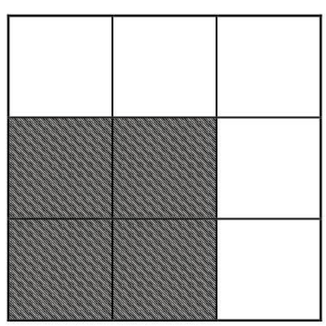
(?) (?) = 9

$\sqrt{(?)(?)} = ?$

Side of square = 3

$\therefore \sqrt{9} = 3$

Example 2: The square root of four ninths ($\sqrt{\frac{4}{9}}$)



Side of shaded square = 2

Side of larger square = 3

$\therefore \sqrt{\frac{4}{9}} = \frac{2}{3}$

Square Roots by Factoring

- When we multiply a number by itself the product is called the **square of the number**. (e.g. 9 is the square of 3 since 3 x 3 = 9, 49 is the square of 7 since 7 x 7 = 49.)
- When we find one of two equal factors of a number the equal factor is called the **square root of the number**. (e.g. 5 is the square root of 25.)
- We usually use a radical sign to indicate a square root. (e.g. $\sqrt{36} = \sqrt{6x6} = 6$ or $\sqrt{49} = \sqrt{7x7} = 7$

  Further examples:
  - 2500 is the **square** of 50 since 50 x 50 = 2500
  - 50 is the **square root** of 2500 since $\sqrt{2500} = \sqrt{50x50} = 50$
  - 9 is the **square** of 3 since 3 x 3 = 9 and 3 is the **square root** of 9 since $\sqrt{9} = \sqrt{3x3} = 3$

- It is important to note that the square root of a number can be either positive or negative. (e.g. $\sqrt{100} = \sqrt{(10)(10)} = 10$ OR $\sqrt{100} = \sqrt{(-10)(-10)} = -10$)
- When finding the square root of a number we usually take only the positive square root of a number which is called the **principal square root**.
- When solving an equation, then both the positive and the negative values of the square root are taken. e.g. If $x^2 = 144$ then $x = \pm\sqrt{144} = \pm 12$

Remember

When we find the square root of a number, we usually take only the positive value (the **principal square root**).

e.g.

1. $\sqrt{81} = \sqrt{(9)(9)} = 9$
2. $\sqrt{\frac{9}{25}} = \sqrt{\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)} = \frac{3}{5}$

When we solve an equation that involves a square root, we usually take both the positive and negative values.

e.g.

1. $x^2 = 36$; $x = \pm\sqrt{36} = \pm 6$; since (6)(6) = 36 and (-6)(-6) = 36
2. $a^2 = \frac{1}{4}$; $a = \pm\sqrt{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \pm\frac{1}{2}$; since $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$ and $(-\frac{1}{2})(-\frac{1}{2}) = \frac{1}{4}$

Examples with Solutions

Question	Answer

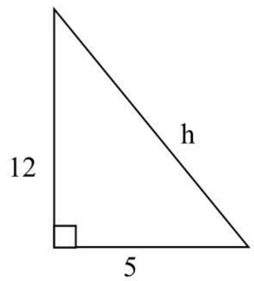
Use factoring to find the square root of each of the following numbers.

Question	Answer
1. $\sqrt{1}$	$\sqrt{1} = \sqrt{1x1} = 1$
2. $\sqrt{144}$	$\sqrt{144} = \sqrt{12x12} = 12$
3. $\sqrt{400}$	$\sqrt{400} = \sqrt{20x20} = 20$
4. $\sqrt{0.04}$	$\sqrt{0.04} = \sqrt{0.2x0.2} = 0.2$
5. $\sqrt{\frac{9}{100}}$	$\sqrt{\frac{9}{100}} = \sqrt{\left(\frac{3}{10}\right)\left(\frac{3}{10}\right)} = \frac{3}{10}$
6. $\sqrt{\frac{1}{900}}$	$\sqrt{\frac{1}{900}} = \sqrt{\left(\frac{1}{30}\right)\left(\frac{1}{30}\right)} = \frac{1}{30}$
7. $\sqrt{1\frac{7}{9}}$	$\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \sqrt{\frac{4}{3}x\frac{4}{3}} = \frac{4}{3}$
8. $\sqrt{6\frac{1}{4}}$	$\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \sqrt{\frac{5}{2}x\frac{5}{2}} = \frac{5}{2}$

Use factoring to solve each of the following equations.

Question	Answer
9. $x^2 = 121$	$x = \pm\sqrt{121} = \pm\sqrt{11x11} = \pm 11$
10. $x^2 - 5 = 620$	$x^2 = 620 + 5 = 625$ $x = \pm\sqrt{625} = \pm\sqrt{25x25} = \pm 25$
11. $3n^2 + 1 = 109$	$3n^2 = 108$ $n^2 = 36$ $n = \pm\sqrt{36} = \pm\sqrt{6x6} = \pm 6$

12. Recall that the square of the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. Use factoring to find the value of the hypotenuse of the right triangle shown below.



$h^2 = 12^2 + 5^2$

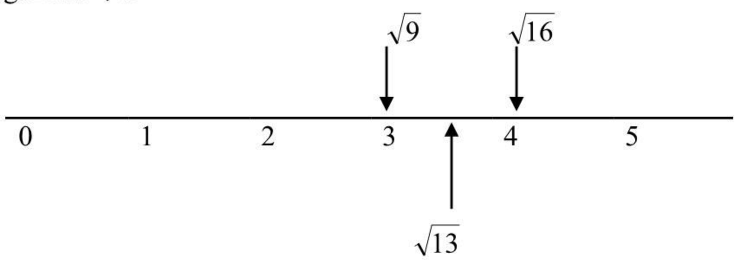
$h^2 = 169$

$h = \pm\sqrt{169} = \pm\sqrt{13x13} = \pm 13$

$h =$ *13 (Reject the negative value for 13 in this case since length is positive.)*

- If a number is not a perfect square, we can estimate its approximate value by locating it between two close perfect squares on the number line.

e.g. Find $\sqrt{13}$



$\sqrt{13}$ is closer to 4 than to 3 and $\sqrt{13} \cong 3.6$

- The most common mistake that a student makes is thinking that $\sqrt{6.4} = 3.2$, i.e., *that square roots are somehow half of a number* ***(incorrect assumption)****.*
- One can always check by reversing the process and squaring, i.e., $(3.2)^2 = 10.24$. $\therefore \sqrt{6.4} \neq 3.2$.

- Remember that taking the square root of a number and squaring the number are opposite processes.

Square Roots Using a Calculator

Finding Approximate Values of Square Roots

- Most square roots are not from perfect-square numbers and therefore do not have two equal factors. As a result we need to find an approximate decimal value for them.
- For example, $\sqrt{7}$ is not the square root of a perfect square. It is somewhere between the square roots of 4 and 9, which are both perfect squares. Hence the decimal value of $\sqrt{7}$ is between $\sqrt{4}$ and $\sqrt{9}$, and therefore it is between 2 and 3.
- We can use a calculator to find the approximate decimal value of a square root using a series of keystrokes. Each calculator is somewhat different so the next series of steps shows how to find the square root using a particular calculator.

Example 1:

Use a calculator to find $\sqrt{7}$, rounded to 2 decimal places.

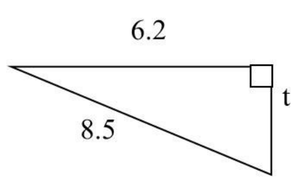
1. Push the "$\sqrt{\ }$" key.
2. Input the number 7.
3. Push the "=" key.
4. The display should read 2.6457513… (This is an irrational number.)
5. The answer, rounded to two decimal places, is 2.65.

Example 2:

$\sqrt{43} \approx 6.5574$ (This is only an approximation since (6.5574)(6.5574) = 42.99949476.) Notice that the approximation will end in non-zero digit unless it is a perfect square.

- Note that $\sqrt{43} \approx 6.5574...$ is an irrational number (the decimals continue on without repeating a pattern) whereas the square root of a perfect square such as $\sqrt{64} = 8.0$ is rational.
- An example of another irrational number is π. For example, $\pi \approx 3.14159...$ using an approximation will give somewhat different results, depending upon how many decimal points are used for the answer.

Examples with Solutions

Question	Answer or Solution
Find the square root of each number to two decimal places.	
1. $\sqrt{29}$	*5.39*
2. $\sqrt{234}$	*15.30*
3. $\sqrt{0.85}$	*0.92*
4. $\sqrt{61.04}$	*7.81*
5. $\sqrt{21.7}$	*4.66*
6. $\sqrt{0.018}$	*0.13*
7. $\sqrt{1000.04}$	*31.62*
Solve each equation below and show the answer to two decimal places.	
8. $y^2 = 53$	$y = \pm\sqrt{53} = \pm 7.28$
9. $t^2 - 4 = 122$	$t^2 = 126$ $t = \pm\sqrt{126} = \pm 11.22$
10. $\frac{1}{3}x^2 = 5.42$	$x^2 = 16.26$ $x = \pm\sqrt{16.26} = \pm 4.03$
11. $1 - \frac{1}{2}y^2 = 0.6$	$-\frac{1}{2}y^2 = 0.6 - 1 = -0.4$ $y^2 = -2(-0.4) = 0.8$ $y = \pm\sqrt{0.8} = \pm 0.89$
12. $1 + n^2 = 8.33$	$n^2 = 7.33$ $n = \pm\sqrt{7.33} = \pm 2.71$
13. $5x^2 + 1 = 2.15$	$x^2 = 1.15 \div 5 = 0.23$ $x = \pm\sqrt{0.23} = \pm 0.48$
14. Find the value, to two decimal places, of the unknown side of the right triangle shown below.	
	$(6.2)^2 + t^2 = (8.5)^2$ $t^2 = 72.25 - 38.44$ $= 33.81$ $t = \pm\sqrt{33.81} = \pm 5.81$ *Answer = +5.81 (Reject the negative value since the length of a side is positive.)*

Exercises 1.2

1. Use factoring to find the square root of each of the following numbers.

a. $\sqrt{9}$ b. $\sqrt{225}$

c. $\sqrt{\frac{25}{49}}$ d. $\sqrt{\frac{81}{16}}$

e. $\sqrt{0.81}$ f. $\sqrt{0.0004}$

2. Without a calculator, find the approximate values of each of the following to one decimal.

a. $\sqrt{7}$ b. $\sqrt{28}$

c. $\sqrt{70}$ d. $\sqrt{200}$

3. Without a calculator, solve for x.

a. $x^2 = 36$ b. $x^2 = 50$

c. $x^2 = 0.81$ d. $x^2 = 0.0625$

e. $x^2 = 40\ 000$

4. Which of the following square roots are between 7 and 8? (Do not use a calculator.)

a. $\sqrt{40}$ b. $\sqrt{58}$

c. $\sqrt{71}$ d. $\sqrt{63}$

e. $\sqrt{51}$

5. Use factoring to solve each of the following equations.

a. $y^2 + 2 = 83$ b. $2a^2 - 4 = 94$ c. $t^2 + 1 = 1.09$

6. Using a calculator, find the square root of each number to 2 decimal places.

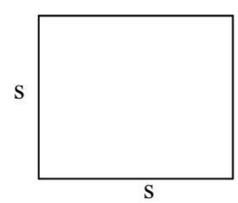
   a. $\sqrt{7.55}$ b. $\sqrt{129.45}$ c. $\sqrt{0.308}$

7. Solve each equation below using a calculator. Show the answer to 2 decimal places.

   a. $y^2 + 6 = 111$ b. $3n^2 – 2 = 112.5$

   c. $w^2 – 0.22 = 34.7$ d. $0.5t^2 – 1 = 22.8$

8. The area of a square can be found by the formula $A = s^2$, where s is the length of one side. If a square has an area of 55 cm^2, what is the length of one side?



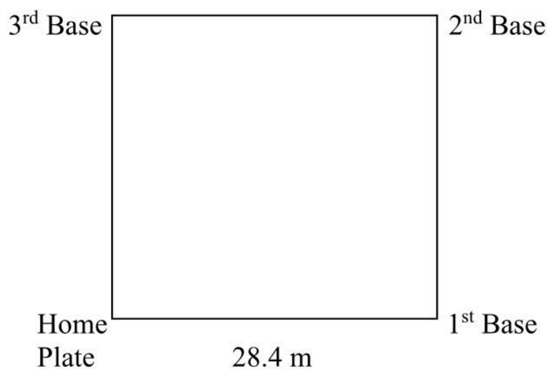
9. The area of a circle can be found by the formula $A = \pi r^2$, where π = 3.14 (to 2 decimal places) and r = the radius of the circle.

   Find the length of the radius (to 2 decimal places) of each of the following circles.

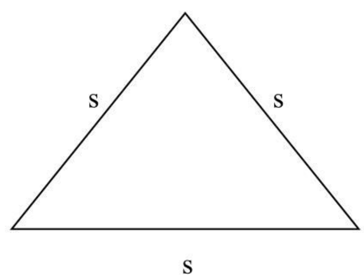
   a. A circle with an area of 64.5 cm^2

   b. A circle with an area of 1.005 m^2

10. A baseball diamond is in the shape of a square. The distance from home plate to first base is 28.4 m. What is the distance from home plate to second base?



11. The area of an equilateral triangle is given by the formula $A = \frac{\sqrt{3}s^2}{4}$ where s is the length of each side.

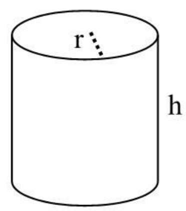


a. What is the area of an equilateral triangle with sides of length 5 cm?

b. What is the area of an equilateral triangle with sides of length 4.12 cm?

c. If an equilateral triangle has an area of 35.6 cm^2, how long is each side?

12. The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the top and h is the height.



a. What is the volume of a cylinder with a radius of 5 cm and a height of 12 cm? (Use $\pi = 3.14$.)

b. What is the height of a cylinder with a radius of 2.5 cm and a volume of 35.8 cm^3? (Use $\pi = 3.14$.)

1.3 Powers, Bases, and Coefficients

Natural Numbers Written as a Base to a Power (or exponent)

- The following numbers are factored into prime factors.
  - (i) 125 = 5 x 5 x 5
  - (ii) 81= 3 x 3 x 3 x 3
- Notice that 5 appears three times as a factor for the first number and 3 appears four times as a factor for the second number.
- When a number is multiplied by itself 2 or more times, we can write it a shorter way, by using a raised number, called an exponent or power.
  e.g. 5 x 5 x 5 can be written as 5^3 and 3 x 3 x 3 x 3 can be written as 3^4
- The raised number is called a **power or exponent** and the number it is written to is called a **base.**

Example 1:

Exponent
or Power of 3
↓
e.g. 5 x 5 x 5 = 5^3
↑
Base of 5

- The 5 is called the base. It is the number which we are going to multiply by itself.
- The 3 is called the power or exponent. It tells us how many times we are going to multiply the base by itself.

Example 2:

Exponent
or Power of 5
↓
e.g. 3 x 3 x 3 x 3 x 3 = 3^5
↑
Base of 3

- The 3 is called the base. It is the number which we are going to multiply by itself.
- The 5 is called the power or exponent. It tells us how many times we are going to multiply the base by itself.

So 5^3 and 3^5 are two totally different expressions.
$5^3 = 125$ and $3^5 = 243$

Rational Numbers as a Base to an Integer Power

- A base may include any non-zero number. Several examples with bases that are rational numbers follow.

Examples

$7^3 = (7)(7)(7) = 343$; $4^4 = (4)(4)(4)(4) = 256$

$(-3)^4 = (-3)(-3)(-3)(-3) = 81$; $(-5)^3 = (-5)(-5)(-5) = -125$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$; $\left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \left(\frac{16}{625}\right)$

$(0.3)^2 = (0.3)(0.3) = 0.09$; $(1.2)^3 = (1.2)(1.2)(1.2) = 1.728$

- Notice the role that parentheses play in determining the value of a given set of powers.

Examples: Evaluate each of the following

	Solution:
a. $(-2)^4$	*a.* $(-2)^4$ *means* $(-2)(-2)(-2)(-2) = +16$
b. (-2^4)	*b.* (-2^4) *means* $-(2 \times 2 \times 2 \times 2) = -16$
c. -2^4	*c.* -2^4 *means* $-(2 \times 2 \times 2 \times 2) = -16$
d. $-(2^4)$	*d.* $-(2^4)$ *means* $-(2 \times 2 \times 2 \times 2) = -16$

Examples with Solutions

Example	Solution
1. Write 11 x 11 x 11 x 11 as a base written to an exponent.	▪ 11 is multiplied by itself 4 times ▪ 11 is the base and 4 is the exponent ▪ $11 \times 11 \times 11 \times 11 = 11^4$
2. Write (1.2)(1.2)(1.2)(1.2)(1.2) as a base written to an exponent.	▪ *1.2 is multiplied by itself 5 times* ▪ *1.2 is the base and 5 is the exponent* ▪ $(1.2)(1.2)(1.2)(1.2)(1.2) = (1.2)^5$ ▪
3. Write $\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)$ as a base written to an exponent.	▪ $\frac{3}{5}$ *is multiplied by itself 4 times.* ▪ $\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \left(\frac{3}{5}\right)^4$

4. Write 4 x 4 x 4 x 7 x 7 x 7 x 7 using bases and exponents.	*4 is multiplied by itself 3 times and 7 is multiplied by itself 4 times.* *4 x 4 x 4 x 7 x 7 x 7 x 7 =* $(4^3)(7^4)$
5. What is the value of the following number as a single numeral? 4^3	▪ *4 is the base and 3 is the exponent.* ▪ *4 is multiplied by itself three times.* ▪ $4^3 = 4 \times 4 \times 4 = 64$
6. What is the value of the following as a single numeral? $\left(\frac{1}{5}\right)^3\left(10^4\right)$	$\frac{1}{5}$ *is multiplied by itself 3 times and 10 by itself 4 times.* $\frac{1x10x10x10x10}{5x5x5} = 2 \times 2 \times 2 \times 10 = 80$
7. Write the following number as a product of a number times a factor with a base and exponent. **75**	▪ 75 = 3 x 25 ▪ = 3 x 5 x 5 ▪ = 3 x 5^2
8. Evaluate each of the following. a. 2.1 x 10^4 b. 12 x $(1.4)^3$ c. $(1.1)^3$ x $(0.5)^2$	*a. 2.1 x (10 x 10 x 10 x 10)* *= 2.1 x 10 000 = 21 000* *b. 12 x (1.4 x 1.4 x 1.4) = 12 x 2.744* *= 32.928* *c. (1.1 x 1.1 x 1.1) x (0.5 x 0.5)* *= 1.331 x 0.25 = 0.33275*

Negative Integers as Exponents

- What happens when the power is in the denominator? Look at the following examples.

  1. $3 \times 3 \times 3 \times 3 = 3^4$ and 2. $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
  3. $\frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{3^4}$ and 4. $\frac{1}{2^6} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

- When the power is in the denominator, we can show it using a negative integer.

  e.g. $\frac{1}{5^4} = 5^{-4}$; $8^{-2} = \frac{1}{8^2}$

Variables as a Base to an Integer Power

- We can use variables, as well as rational numbers, as a base.

e.g. $n^3 = (n)(n)(n)$; $y^7 = (y)(y)(y)(y)(y)(y)(y)$

$\frac{2^4}{t^4} = \frac{2x2x2x2}{txtxtxt}$; $(ab)^3 = (ab)(ab(ab)$

$r^{-3} = \frac{1}{r^3} = \left(\frac{1}{r}\right)\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)$; $\frac{a^{-2}}{b^{-3}} = \frac{b^3}{a^2} = \frac{(b)(b)(b)}{(a)(a)}$

Examples with Solutions

Question	***Solution***
Rewrite each expression without negative exponents and then evaluate.	
1. 2^{-4}	$\frac{1}{2^4} = \frac{1}{16}$
2. $3^3 \times 5^{-2}$	$3^3 \; x \; \frac{1}{5^2} = 27 \; x \; \frac{1}{25} = \frac{27}{25}$
3. $\frac{1}{7^{-3}}$	$7^3 = 7 \; x \; 7 \; x \; 7 = 343$
4. $\left(\frac{1}{3}\right)^{-2}$	$\frac{1}{\left(\frac{1}{3}\right)^2} = 1 \div \left(\frac{1}{3}\right)^2 = 1x\left(\frac{3}{1}\right)^2 = 1x9 = 9$
Expand each of the following expressions so no exponents are showing.	
5. $2n^3$	*2(n)(n)(n)*
6. $(2n)^3$	*(2n)(2n)(2n)*
7. $\left(\frac{2}{3}y\right)^3$	$\left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right)\left(\frac{2}{3}y\right) = \frac{8}{27}(y)(y)(y)$
Rewrite each of the following expressions using bases and exponents.	
8. (5)(5)(5)(a)(a)	5^3a^2
9. (1.7)(1.7)(x)(x)(y)(y)(y)(y)	$(1.7)^2x^2y^4$

10. $\left(\frac{1}{3}xy\right)\left(\frac{1}{3}xy\right)\left(\frac{1}{3}x\right)\left(\frac{1}{3}x\right)$	$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)(xy)(xy)(x)(x) = \left(\frac{1}{3}\right)^4 x^4y^2$
11. $\frac{(2a)(2a)}{(3b)(3b)(3b)}$	$\frac{2^2a^2}{3^3b^3} = \frac{4a^2}{27b^3}$

Exercises 1.3

1. Evaluate each of the following expressions.

a. 2^3 b. 3^2

c. 4^3 d. 3^4

e. 5^2 f. 2^5

g. 6^3 h. 3^6

i. 5^3 j. 3^5

2. Evaluate each of the following expressions.

a. $3^2 + 4^2$ b. $(3 + 4)^2$

c. $2^3 + 2^3$ d. $(2 + 2)^3$

e. -3^2 f. $(-3)^2$

g. -3^3 h. $(-3)^3$

i. $4^2 + 2^2$ j. $(4 + 2)^2$

3. Find the value of each of the following.

a. 5^4 b. $\left(\frac{2}{3}\right)^3$

c. $(0.03)^2$ d. $10x(2.1)^3$

e. $(2.01)^2\ (1.3)^2$ f. 3^{-4}

g. $(3.1)^2\,(0.3)^3$

h. $\dfrac{1}{0.2^{-3}}$

i. $\dfrac{2^{-3}}{0.5^{-2}}$

j. Simplify: $y^{-2}y^6$

k. Simplify: $3(2n)^3 3n^4$

4. Rewrite using only positive exponents.

$(2^{-2})(3y)^2(x^{-1}y^{-1})$

<u>Extra for Experts</u>

5. Evaluate: $2^{-3}\left(\dfrac{2}{3}\right)^{-2}$

6. Simplify: $\left(\dfrac{1}{x}\right)^{-2} x^5$

7. Simplify: $\left(\dfrac{2}{3x}\right)^{-2}(2x)^2$

8. Evaluate: $(1.1)^5(1.1)^{-2}$

9. Evaluate: $\left(\dfrac{2}{3}\right)^{-2}\left(\dfrac{1}{3}\right)^{3}(2)^4$

APPENDIX

ANSWERS TO EXERCISES AND UNIT TESTS

UNIT 1 – ANSWERS

Page 7: Exercises 1.1

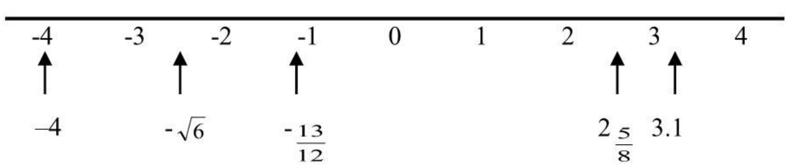
Number	Rational / Irrational	Reason
1. 0.013	*Rational*	*Can be written as 13/1000*
2. 5 1/2	*Rational*	*Can be written as 11/2*
3. 7.0900134...	*Irrational*	*Decimal doesn't terminate or repeat*
3. 0.666…	*Rational*	*Repeating decimal equal to 2/3*
4. -10.001	*Rational*	*Can be written as -10 001/10 000*
5. $\sqrt{49}$	*Rational*	*Equal to 7*
6. 0.12231353…	*Irrational*	*Decimal doesn't terminate or repeat*
7. 0.212121…	*Rational*	*Repeating decimal equal to 21/99*
8. 210.013	*Rational*	*Terminating decimal*
9. $\sqrt{8}$	*Irrational*	*Decimal value doesn't terminate or repeat 2.828427…*
10. -5.333…	*Rational*	*Decimal repeats and can be written as -16/3*
11. 3.009	*Rational*	*Decimal terminates and can be written as 3009/1000*
12. -345 1/3	*Rational*	*Can be written as -1036/3*

Place an (√) if the number belongs to the set of numbers.

		Set of Numbers				
Question	**Number**	**N**	**W**	**I**	**Ra**	**Ir**
13.	0.7				√	
14.	-45			√	√	
15.	$\frac{15}{7}$				√	
16.	0.13243…					√
17.	$0.\overline{7}$				√	
18.	2	√	√	√	√	
19.	$1\frac{5}{8}$				√	
20.	-160			√	√	
21.	0		√	√	√	
22.	$\sqrt{81}$	√	√	√	√	
23.	0.93				√	
24.	$\sqrt{15}$					√

Note: N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, Ir = Irrational Numbers

25. First, change $-\sqrt{6}$ to – 2.449 and $-\sqrt{16}$ to -4
Next, plot locations on the number line as shown below.



26a. -5.70, -5.07, -0.57, -0.507 b. $-\frac{11}{3}$, -3.5, -3.4, 3.4 c. $-\frac{2}{3}$, - 0.6, -0.4, $-\frac{3}{8}$