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NUMBER CONCEPTS

$\sqrt{23}$ **5 : 7** $\sqrt{\frac{3}{8}}$

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$\frac{3}{4} = \frac{x}{12}$ $0.3 < \frac{1}{3}$ $\sqrt{43} < 6.5574...$

1.1 Rational and Irrational Numbers

Rational Numbers

- A **rational number** is any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
- This includes the natural numbers (the counting numbers such as 1, 2, 3, …), and the whole numbers (add 0 to the counting numbers to get 0, 1, 2, 3, …) and integers (add the negatives of counting numbers to the set to get ... –2, -1, 0, 1, 2, …).
- Natural, whole, and integer numbers are rational since each can be written in the form $\frac{a}{b}$ (e.g. $1 = \frac{1}{1} = \frac{2}{2}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{4}$).

Examples of Rational Numbers:

- All fractions and mixed numbers, both positive and negative
  e.g. $\frac{2}{3}$, $\frac{-3}{4}$, $\frac{5}{2}$, $-3\frac{1}{4}$ (Note: $\frac{0}{7} = 0$ is rational, but $\frac{7}{0}$ is **not rational** since it is not defined when the denominator equals 0.)
- All integers
  e.g. –11, -3, 0, 1, 5, 68
- All terminating and repeating decimals, both positive and negative
  e.g. 0.8, -0.32, $0.333\bar{3}$, $7.1212\overline{12}$

Irrational Numbers

- A number that cannot be written as the quotient ($\frac{a}{b}$) of two integers is called an **irrational number**.

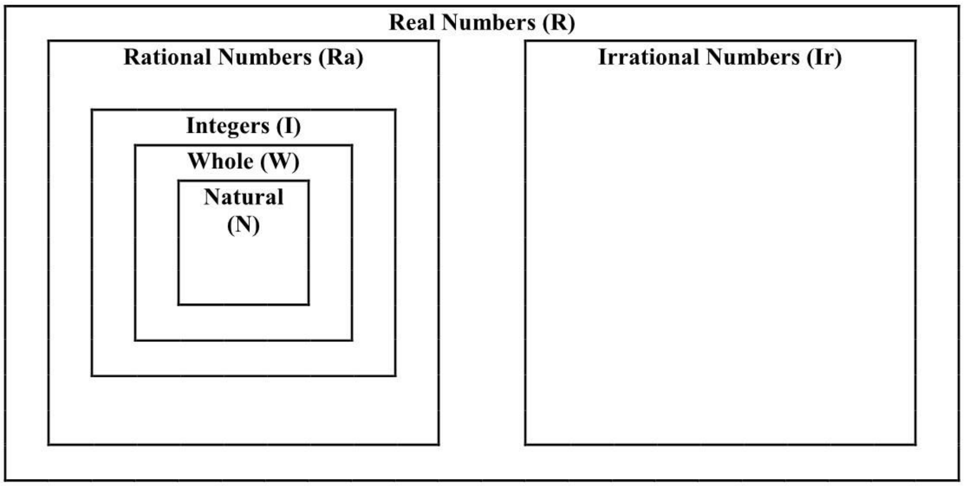
Examples of Irrational Numbers:

- Numbers that are roots of whole numbers that cannot be simplified to obtain a rational number
  e.g. $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$ (Note: $\sqrt{9}$ is rational since it is equal to 3.)
- Numbers whose decimal representation does not repeat in a pattern
  e.g. 0.1357421…… (Note: $0.33\bar{3}$ is rational since it repeats a pattern and is equal to $\frac{1}{3}$.)
- Special numbers such as π

Real Numbers

- **Real numbers** consist of the set of all rational and all irrational numbers.

  The diagram below shows the relationship among the sets of numbers discussed so far.



- Sets of rational numbers include the following:

  Natural numbers: N = $\{1,2,3,4,...\}$
  Whole numbers : W = $\{0,1,2,3,4,...\}$
  Integers: I = $\{...-3,-2,-1,0,1,2,3,4,...\}$
  Rational numbers: Ra = All of the above <u>plus</u> any other number that can be written in the form $\frac{a}{b}$, b $\neq 0$

Identifying Rational and Irrational Numbers

- Rational numbers can be shown in several different formats.
  1. Natural numbers, whole numbers, and integers
     Examples: 2, -23, 0, 5001, - 673
  2. Fractions, mixed numbers, or improper fractions
     Examples: $\frac{2}{7}$, $-\frac{3}{5}$, $1\frac{1}{4}$, $\frac{7}{5}$, $-2\frac{1}{10}$, $-\frac{8}{3}$
  3. Decimals (terminating or repeating)
     Examples: 0.8, -0.25, $0.2233\overline{3}$, $2.61\overline{61}$

Examples with Solutions

Put a check mark (√) if the number belongs to the set of numbers.

		Set of Numbers				
Question	**Number**	**N**	**W**	**I**	**Ra**	**R**
1.	3					
2.	-10					
3.	$\frac{11}{3}$					
4.	0.9					
5.	$0.\overline{7}$					
6.	π					
7.	$1\frac{5}{8}$					
8.	1.25					
9.	0					
10.	$\sqrt{9}$					

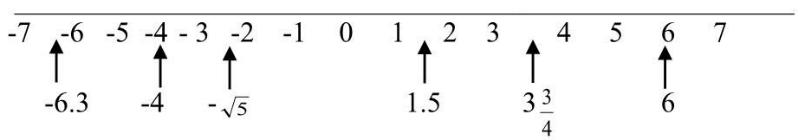
Note: N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, R = Real Numbers

Answers:

	Set of Numbers				
Question	***N***	***W***	***I***	***Ra***	***R***
1.	√	√	√	√	√
2.			√	√	√
3.				√	√
4.				√	√
5.				√	√
6.					√
7.				√	√
8.				√	√
9.		√	√	√	√
10.	√	√	√	√	√

Comparing and Ordering Rational Numbers

- Each rational number corresponds to a point on the number line. Several examples are shown next.



- It should be noted that numbers **increase** in magnitude as you go from left to right on the line.
  e.g. $1 < 3$, $2.1 < 4$; $-7 < -6$; and $2 > 1.8$; $-3 > -5$; $-1 > -10.5$

Examples with solutions:

To compare the magnitudes of rational numbers where one is written in decimal and the other in common fraction form, write both either in decimal or else in common fraction form and then compare.

1. Compare 0.1 with $\frac{3}{20}$ (convert both to fractions first).
   - Change 0.1 to $\frac{1}{10}$.
   - The common denominator is 20, $\therefore \ \frac{1}{10} = \frac{2}{20}$.
   - $\frac{2}{20} < \frac{3}{20}$ or $0.1 < \frac{3}{20}$

2. Compare 3.15 with $3\frac{1}{11}$ (convert both to decimals first).
   - Change $3\frac{1}{11}$ to a decimal $\rightarrow 3.09\overline{09}$.
   - $\therefore \ 3.15 > 3.09\overline{09}$ or $3.15 > 3\frac{1}{11}$.

Exercises 1.1

Basic Level

1. Put a check mark (√) if the number belongs to the set of numbers.

		Set of Numbers				
Question	**Number**	**N**	**W**	**I**	**Ra**	**R**
1.	7					
2.	-18					
3.	$\frac{7}{8}$					
4.	0.36					
5.	$0.\overline{5}$					
6.	$\sqrt{6}$					
7.	$2\frac{3}{4}$					
8.	-5.6					
9.	$\frac{0}{8}$					
10.	$\sqrt{16}$					

2. Locate the following numbers on the number line.

$3.1,\ 2\frac{5}{8},\ -\frac{13}{12},\ -\sqrt{6},\ -\sqrt{16}$

-4 -3 -2 -1 0 1 2 3 4

Mid-level

3. Arrange the following numbers from smallest to largest.

Student Work

a. –0.57, -0.507, -5.07, -5.70

b. $3.4,\ -\frac{11}{3},\ -3.4,\ -3.5$

c. $-\frac{3}{8},\ -\frac{2}{3},\ -0.6,\ -0.4$

4. Put the correct symbol (>, =, <) between each pair of numbers.

a. $0.15 \square \frac{7}{40}$ b. $-1.8 \square -\frac{9}{5}$ c. $-2.8 \square -\frac{13}{5}$

Extra for Experts

5. Express each term in common fraction form (as a quotient of two integers)

a. 0.17

b. $-0.\overline{5}$

c. $-1\frac{2}{3}$

d. 3.07

6. Which rational number is greater?

Student Work

a. $-0.\overline{6}$ or -0.6?

b. -0.25 or $-\frac{1}{3}$?

c. $-\frac{2}{3}$ or $-\frac{4}{5}$?

1.2 Ratios – Two and Three Term

Ratios and Proportions

- When you make orange juice from concentrate, the instructions usually ask you to combine 3 cans of water with 1 can of frozen concentrate. In this case, the ratio of cans of water to cans of concentrate is 3 to 1. Conversely, the ratio of cans of concentrate to cans of water is 1 to 3.

- Equivalent ratios are pairs of numbers, written as ratios, that are equal to each other.

  e.g. The ratio of 3 to 1 is equivalent to the ratio of 6 to 2 since $\frac{3}{1}=\frac{6}{2}$. This could also be written as 3 : 1 = 6:2.
- Equivalent ratios can be formed by multiplying or dividing the terms by the same non-zero number.

  e.g. $\frac{2}{5}=\frac{6}{15}$ (multiplying by 3); $\frac{2}{5}=\frac{-4}{-10}$ (multiplying by –2); $\frac{2}{5}=\frac{1}{10}$ (dividing by 2)
- When one ratio is equal to another, it forms a proportion.

Examples with solutions:

1. Write a ratio equivalent to $\frac{3}{4}$.

   - $\frac{3}{4} = \frac{top}{bottom}$ *, multiply the top and the bottom by the same non-zero number.*
   - $\therefore \; \frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{12} \; ...$

2. Write a ratio equivalent to $\frac{36}{20}$ in lowest terms.

   - *Divide the top and the bottom by the greatest common factor (GCF), which is 4.*
   - $\therefore \; \frac{36}{20} = \frac{9}{5}$**,** *in lowest terms.*

Finding Missing Terms in Proportions

- A **proportion** consists of one ratio that is set equal to another.
  e.g. $\frac{3}{4}$, $\frac{15}{20}$, and 18 : 24 are all equivalent ratios.
- When one ratio is set equal to an equivalent ratio, it forms a proportion.
  e.g. $\frac{3}{4} = \frac{15}{20}$; $\frac{5}{x} = \frac{30}{4}$
- Note that $\frac{3}{4} = \frac{15}{20} \rightarrow 3 \text{ x } 20 = 4 \text{ x } 15$ (the numerator of the 1st ratio times the denominator of the 2nd is equal to the denominator of the 1st times the numerator of the 2nd.
  e.g. $\frac{2}{5} = \frac{x}{15} \rightarrow 2(15) = 5x$. Solving, $x = 6$.
- If more than one term is missing in a proportion consisting of three ratios, break the question into two simpler ones. Sometimes these are shown as numerator ratios equal to denominator ratios as shown below.
  e.g. 3 : 6 : y = x : 14 : 28

  $\frac{3}{x} = \frac{6}{14} = \frac{y}{28} \rightarrow \frac{3}{x} = \frac{6}{14}$ and $\frac{6}{14} = \frac{y}{28}$

  $\rightarrow 6x = 3(14)$ and $14y = 6(28)$

  $\rightarrow x = 7$ and $y = 12$

Examples with solutions:

1. Find the missing term in the following proportion $\frac{x}{4} = \frac{15}{12}$.

   12x = 4(15)

   $x = \frac{4(15)}{12} = 5$

2. Find the missing term in the following proportion $\frac{8}{x} = \frac{5}{6}$.

   6(8) = 5x

   $x = \frac{6(8)}{5} = \frac{48}{5} = 9.6$

3. Solve for x and y in the following proportion $\frac{3}{x} = \frac{9}{12} = \frac{y}{60}$.

   9x = 3(12) and 12y = 9(60)

   x = 4 and y = 45

Exercises 1.2

Basic Level

1. Write an equivalent ratio in lowest terms.

Student Work

a. 8 : 24

b. $\frac{14}{35}$

c. 24 to 9

d. 100 : 50 : 25

e. $\frac{0.5}{0.4}$

2. Solve for the missing term in each proportion.

Student Work

a. 4 : 5 = x : 25

b. $\frac{20}{16} = \frac{5}{x}$

c. $\frac{x}{5} = \frac{7}{1}$

d. x : 2 = 2 : 1

e. $\frac{x}{2.5} = \frac{4}{10}$

Mid Level

3. Determine the missing term.

Student Work

a. $\frac{48}{72} = \frac{x}{9}$

b. x : 7 = 192 : 84

c. 1.2 : 2.0 = x : 3.0

d. $\frac{0.9}{0.6} = \frac{x}{2}$

e. $\frac{6}{11} = \frac{20}{x}$

4. Find the unknown values in each proportion.

Student Work

a. $\frac{3}{x} = \frac{9}{15} = \frac{y}{45}$

b. $\frac{x}{4} = \frac{5}{y} = \frac{20}{16}$

c. 9 : 8 : 2 = x : 40 : y

d. 15 : 10 : 25 = 45 : x : y

e. x : y : 49 = 4 : 5 : 7

Extra for Experts

5. Find the unknown values in each proportion.

Student Work

a. $x : 3 : 5 = 6 : y : 15$

b. $x : 6 : 8 = 36 : y : 24$

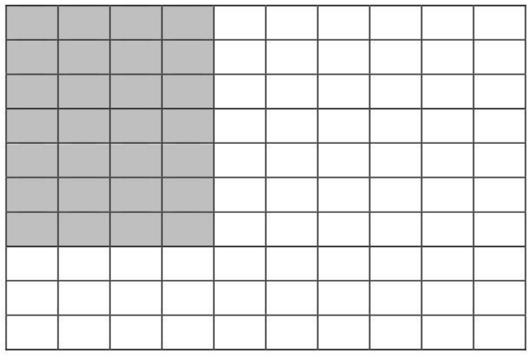
c. $3x : 5 = 6 : 10$

d. $7 : 4 : 3 = x : 18 : y$

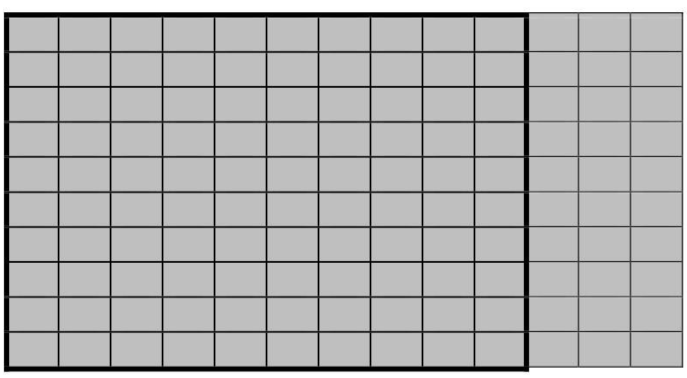
e. $9 : 7 : x = 15 : y : 2$

1.3 Percent

- In the following diagram of 100 squares a portion, 28 of them are shaded.



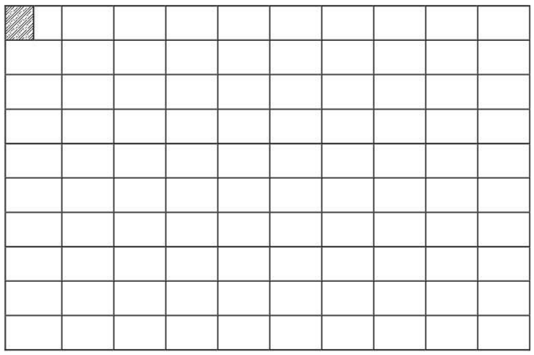
  - Since 28 out of 100 squares are shaded, we can say that 28 percent of them are shaded. (percent means out of 100). The symbol % means □/100 or *out of 100*.
  - We could have written this as a fraction such as $\frac{28}{100}$ (again, 28 out of 100, with 100 being the whole part).
  - We also could have written this as the decimal 0.28 (with the 28 representing the number of hundredths).
  - So the diagram represents the following versions of the same number: 28%, 0.28, or $\frac{28}{100}$.
- The following diagram illustrates a number that is greater than 100%.



  - Here we have 130 squares shaded and when this is compared to 100 squares it becomes 130%.

- Written as a fraction, it would be $\frac{130}{100}$, or 130 out of 100.
- As a decimal it would be 1.30.

- The next diagram illustrates a number between 0 and 1 percent.



- If one half of a rectangle is shaded it is one half out of 100 or $\frac{1}{2}$%.
- This could also be written as 0.5%.

- When a number is written in the form $\frac{a}{b}$, such as $\frac{2}{3}$, it can represent different meanings, depending on the context or situation it is used in. Several examples follow.

As a fraction:	▪ 2 parts out of 3 ▪ 2 is the number of parts and 3 is the whole.
As a rate:	▪ $2 earned for every 3 hours worked ▪ The unit would be dollars per hour.
As a ratio:	▪ 2 units of rock for every 3 units of sand ▪ So the ratio of rock to sand would be 2 to 3.
As a quotient:	▪ 2 would be divided by 3 ▪ So if 2 pies are to be divided among 3 friends, each of them would get $\frac{2}{3}$ of a pie.
As a probability:	▪ 2 wins for every 3 games played ▪ So the chance of winning a particular game would be 2 out of 3 or $\frac{2}{3}$.

- As shown, numbers can be written in different forms (e.g. integer, ratio, common fraction, decimal fraction, percent) depending on their use in specific situations. Numbers have the same value when written in equivalent forms.

e.g.

Ratio	1 : 2
Fraction	$\frac{1}{2}$
Decimal	0.5
Percent	50%

- These all have the same value but, depending on their use, one form is more suitable than another.

Examples: Writing Numbers in Different Forms

Ratio to a Fraction

1. 2 : 5 $\rightarrow \frac{2}{5}$
2. 7 : 5 $\rightarrow \frac{7}{5}$

Fraction to a Ratio

3. $\frac{2}{3} \rightarrow$ 2 : 3
4. $\frac{10}{6}$ = reduced to $\frac{5}{3} \rightarrow$ 5 : 3
5. $1\frac{7}{8}$ = written as an improper fraction $\frac{15}{8} \rightarrow$ 15 : 8

Fraction to a Decimal

6. $\frac{2}{5} = 0.4$ (divide 2 by 5)
7. $\frac{7}{5} = 1.4$ (divide 7 by 5)
8. $\frac{2}{3} = 0.6\bar{6}$ (divide 2 by 3 to get a repeating decimal)
9. $\frac{5}{3} = 1.6\bar{6}$ (divide 5 by 3 to get a repeating decimal)
10. $1\frac{7}{8} = 1.875$ (divide 7 by 8)

Decimal to a Fraction

11. $0.7 = \frac{7}{10}$ (since 7 is in the tenths place)

12. $2.37 = 2\frac{37}{100}$ or $\frac{237}{100}$ (since 37 is in the hundredths place)

13. $0.125 = \frac{125}{1000} = \frac{1}{8}$ (since 125 is in the thousandths place and fraction reduced)

14. $0.\bar{3} = \frac{1}{3}$ (since $0.\bar{1} = \frac{1}{9}$, $0.\bar{2} = \frac{2}{9}$ and $\bar{3} = \frac{3}{9}$ to be reduced)

15. $0.004 = \frac{4}{1000} = \frac{1}{250}$ (since 4 is in the thousandths place to be reduced)

16. 0.25 = 25% (To make a decimal a portion out of 100, move the decimal 2 to the right.)

17. 0.375 = 37.5% or $37\frac{1}{2}\%$

18. 1.6 = 160%

19. 8.875 = 887.5% or $887\frac{1}{2}\%$

20. $4.\bar{3} = 433.\bar{3}\%$ or $433\frac{1}{3}\%$

Percent to a Decimal

21. 35% = 0.35 (Since percent is out of a 100, move the decimal 2 to the left.)

22. 0.4% = 0.004

23. 513% = 5.13

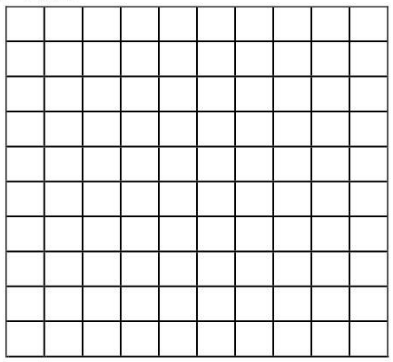
24. 2.7% = 0.027

25. $66.\bar{6}\% = 0.66\bar{6}$

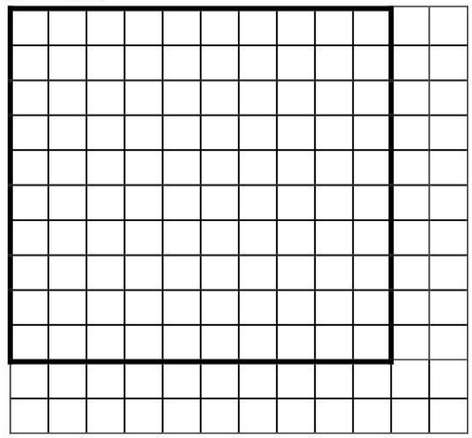
Exercises 1.3

1. Shade the correct number of squares for each of the following percents.

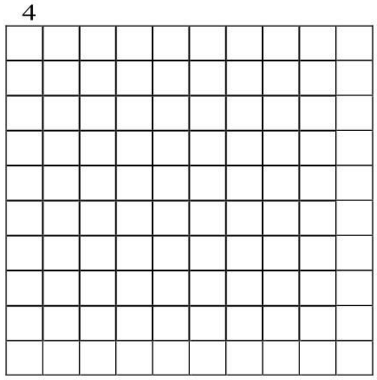
a. 78%



b. 110%

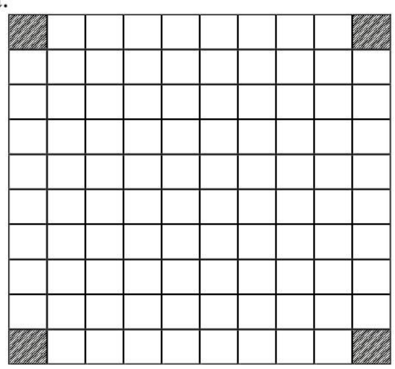


c. $\frac{3}{4}$%

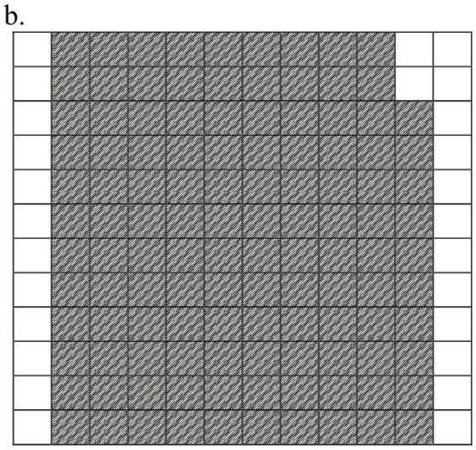


2. Determine the percent represented by each shaded region.

a.



b.



3. Complete the following chart by filling in the missing equivalent forms for each number.

Number	Ratio	Fraction	Decimal	Percent	STUDENT WORK AREA
Basic level a.	3 : 8				
b.		$\frac{6}{5}$			
c.			0.39		
d.				7%	
Mid-level e.	11 : 3				
f.		$2\frac{5}{6}$			
g.			5.1		
h.				375%	
i.				0.05%	
j.				$\frac{1}{4}\%$	
k.				5167%	
l.			$0.\bar{5}$		
m.		$5\frac{1}{11}$			
n.	3 : 11				
o.		$\frac{17}{9}$			
Expert level p.			$0.\overline{36}$		
q.				$45.\overline{45}\%$	
r.	2 : 99				
s.			$0.\overline{05}$		
t.				$0.\bar{3}\%$	

1.4 Square Root Concept

A list of perfect squares that are useful to remember is shown below. Most of these should look familiar.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	25
Square of Number	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	400	625

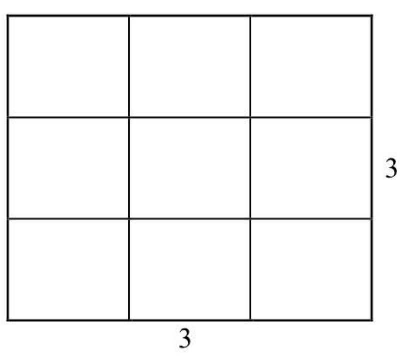
- The process of squaring a number involves multiplying it by itself.
  e.g. $5^2 = 5(5) = 25$, or $(\frac{3}{7})^2 = (\frac{3}{7})(\frac{3}{7}) = \frac{9}{49}$

- The process of finding the square root of a number is to determine one of its two equal factors. e.g. $\sqrt{36} = \sqrt{(6)(6)} = 6$ (It should be noted that $\sqrt{36} = \sqrt{(-6)(-6)} = -6$ as well). However, in most cases, when finding the square root of a number we will take only the positive square root of a number which is called the **principal square root**.

- When we are solving an equation, then both the positive and the negative values of the square root are taken. e.g. If $x^2 = 121$, then $x = \pm\sqrt{121} = \pm 11$.

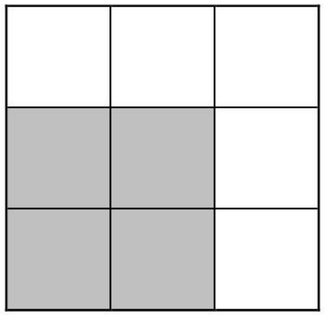
To inspect the square root of a number pictorially, we can examine a perfect square using a grid.

<u>Example 1:</u> The square root of 9



$$(\text{what})(\text{what}) = 9$$
$$(?)\ (?) = 9$$
$$\sqrt{(?)(?)} = ?$$
$$\text{side of square} = 3$$
$$\therefore \sqrt{9} = 3$$

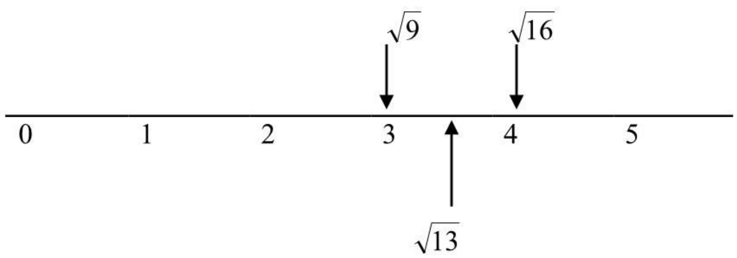
<u>Example 2:</u> The square root of four ninths ($\sqrt{\frac{4}{9}}$)



Side of shaded square = 2
Side of larger square = 3
$\therefore \sqrt{\frac{4}{9}} = \frac{2}{3}$

If a number is not a perfect square, we can estimate its approximate value by locating it between two close perfect squares on the number line.

e.g. Find $\sqrt{13}$



$\sqrt{13}$ is closer to 4 than to 3 and $\sqrt{13} \cong 3.6$.

Exercises 1.4

Question	Student Work
<u>Basic Level</u> 1. Find the following square roots. a. $\sqrt{81}$ b. $\sqrt{121}$ c. $\sqrt{400}$ d. $\sqrt{625}$ e. $-\sqrt{144}$	

2. Draw $\sqrt{16}$ pictorially and then find $\sqrt{\frac{9}{16}}$.

Mid-Level

3. Find:
   a. $\sqrt{\frac{1}{49}}$
   b. $\sqrt{\frac{81}{25}}$
   c. $\sqrt{0.09}$
   d. $\sqrt{1.21}$

4. Without a calculator, find the approximate values of each of the following to one decimal.
   a. $\sqrt{7}$
   b. $\sqrt{28}$
   c. $\sqrt{70}$
   d. $\sqrt{200}$

Extra for Experts

5. Without a calculator, solve for x.
   a. $x^2 = 36$
   b. $x^2 = 50$
   c. $x^2 = 0.81$
   d. $x^2 = 0.0625$
   e. $x^2 = 40\ 000$

APPENDIX

MATHEMATICS GRADE 8

ANSWERS TO EXERCISES AND UNIT TESTS

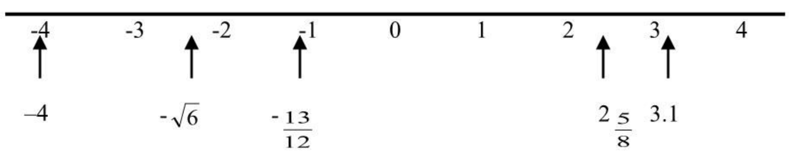
UNIT 1 – ANSWERS

Page 6: Exercises 1.1

1. Put a check mark (√) if the number belongs to the set of numbers

		Set of Numbers							Set of Numbers				
#	No.	N	W	I	Ra	R	#	No.	N	W	I	Ra	R
1.	7	√	√	√	√	√	6.	$\sqrt{6}$					√
2.	-18			√	√	√	7.	$2\frac{3}{4}$				√	√
3.	$\frac{7}{8}$				√	√	8.	-5.6				√	√
4.	0.36				√	√	9.	$\frac{0}{8}$		√	√	√	√
5.	$0.\overline{5}$				√	√	10.	$\sqrt{16}$	√	√	∧	√	√

2. First, change $-\sqrt{6}$ to -2.449 and $-\sqrt{16}$ to -4.
Next, plot locations on the number line as shown below.



3. a. -5.70, -5.07, –0.57, -0.507; b. $-\frac{11}{3}$, -3.5, -3.4, 3.4; c. $-\frac{2}{3}$, - 0.6, -0.4, $-\frac{3}{8}$

4. a. < ; b. = ; c. <

5. a. $\frac{17}{100}$; b. $-\frac{5}{9}$; c. $-\frac{5}{3}$; d. $3\frac{7}{100} = \frac{307}{100}$

6. a. –0.6 (it is to the right of $-0.\overline{6}$ on the number line) ; b. –0.25 ; c. $-\frac{2}{3}$ (change to $-\frac{10}{15}$ and $-\frac{12}{15}$)

Page 10: Exercises 1.2

1. a. 1 : 3; b. $\frac{2}{5}$; c. 8 to 3; d . 4 : 2 : 1; e. $\frac{5}{4}$

2. a. x = 20; b. x = 4; c. x = 35; d. x = 4; e. x = 1

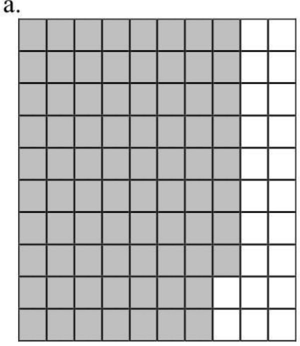
3. a. x = 6; b. x = 16; c. x = 1.8; d. x = 3; e. x = $36\frac{2}{3}$

4. a. x = 5, y = 27; b. x = 5, y = 4; c. x = 45, y = 10; d. x = 30, y = 75; e. x = 28, y = 35

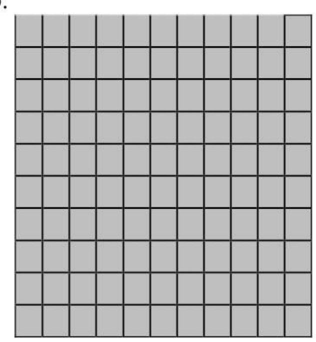
5. a. x = 2, y = 9; b. x = 12, y = 18; c. x = 1; d. x = 31.5, y = 13.5; e. x = 1.2, y = $11\frac{2}{3}$

Page 17: Exercises 1.3

1a.



b.



c.