Mathematics Bridge

CHIP Tutors

1st Edition

Contents

Number and Algebra

Functions

Geometry and trigonometry

Statistics and probability

Calculus

A6 Order of operations - BODMAS

Points to watch: Most modern calculators understand the order of operations implied by the mnemonic 'Brackets, Of, Divide, Multiply, Add, Subtract. Make sure that you are clear about how your model handles the issue.

1. Evaluate: $2 \times 3 - 5 \div 2$
   $2 \times 3 - 5 \div 2 = 2 \times 3 - 2.5$ (Of or ÷ first)
   $= 6 - 2.5$ (Multiply × next)
   $= 3.5$

2. Evaluate: $2.7 + 3.4 - 2.6 + 4.1$
   Add and subtract have equal priority. This expression can be evaluated from left to right.
   $2.7 + 3.4 - 2.6 + 4.1 = 6.1 - 2.6 + 4.1$
   $= 3.5 + 4.1$
   $= 7.6$

3. Evaluate: $(4.9 - 9.5) \times (-1.6 - 4.8)$
   $(4.9 - 9.5) \times (-1.6 - 4.8) = -4.6 \times -6.4$ Brackets
   $= 29.44$

4. Simplify: $(x + 2y - 3x) \times (3 - x)$
   $(x + 2y - 3x) \times (3 - x)$
   $= (-2x + 2y) \times (3 - x)$ Brackets simplified.
   $= -2x \times 3 + -2x \times -x + 2y \times 3 + 2y \times -x$
   $= -6x + 2x^2 + 6y - 2xy$
   No further simplification is possible as there are no 'like terms' - terms with exactly the same pattern of letters.

5. Simplify: $\dfrac{2x^2 - 5xy - 3y^2}{2x + y}$

$$\frac{2x^2 - 5xy - 3y^2}{2x + y} = \frac{(x - 3y)\times(2x + y)}{(2x + y)}$$
$$= x - 3y$$

Exercise

1. Evaluate: $5.9 \times 3.4 - 4.6 + 26.1 \div 3$

   $5.9 \times 3.4 - 4.6 + 26.1 \div 3$

   = [...........] – [...........] + [...........]

   = [...........] + [...........]

   = [...........]

2. Simplify: $\dfrac{x(2x-9)-35}{2x+5}$

$$\frac{x(2x-9)-35}{2x+5} = \frac{[\ldots\ldots\ldots\ldots]-35}{2x+5}$$
$$= \frac{[\ldots\ldots\ldots\ldots]}{2x+5}$$
$$= \frac{(\ldots\ldots)\times(\ldots\ldots)}{2x+5}$$
$$= [\ldots\ldots\ldots\ldots]$$

3. $12 \times 4 - 3 \times 5$
4. $12.5 \times 4.7 + 3.6 \times 2.9 - 1.6 \times 6.3$
5. $-4 \times 6 - 5 \times -2$
6. $2.3 \times 5.9 - 8.6 \times -5.4$
7. $45 + 29 \times 3 - 162 \div 9$
8. $3.6 \div 30 - 4.9 \div 7$
9. $4.7^2 + 5.6 \times 2.4$
10. $(3.4 - 5.5)(2.9 + 9.3)$
11. $(4^3/_4 - 2^2/_7) \div (3 - {}^5/_8)$
12. $(1 - {}^3/_5)^3$
13. $\dfrac{3.8 - 3.1}{1 + 2/5} - 6$
14. $\left(1.6^3 - 4.7\right) + \dfrac{1}{26}$ to 2 s.f.
15. $2.7^{0.25}$ to 2 s.f.
16. $\dfrac{3 + 7 \times 2}{4 - \dfrac{3+6}{9-4}} - 2$

Simplify:

17. $(2x + y - 1)^2$
18. $\left(x - \dfrac{1}{x}\right)\left(\dfrac{2}{x} + x\right)$
19. $(a + 2b)^3$
20. $\dfrac{1}{1 + \dfrac{2}{\dfrac{1}{1+x}}}$
21. $2x + 2 - \frac{7}{8}(x - 1)$
22. $\dfrac{3}{1 - \dfrac{5}{\dfrac{1}{2+x}}}$

Solve for x:

23. $1 + \dfrac{3}{2 - x} = -x$
24. $1 - \dfrac{5}{2 - x} = 3$
25. $2 + \dfrac{1}{\dfrac{1}{1 + \dfrac{1}{x+1}}} = 1$
26. $3 + \dfrac{2}{\dfrac{3}{4 - \dfrac{1}{x+5}}} = 1$

A10 Common Factors

Points to watch: When factorising, look for the Highest Common Factor.

Recall that:

$3x \times (5x^2 - 6) = 3x \times 5x^2 - 3x \times 6$

$= 15x^3 - 18x$

It follows that the reverse is true:

$15x^3 - 18x = 3x \times 5x^2 + 3x \times (-6)$

$= 3x(5x^2 - 6)$

This process is called 'factorisation'.

1. Factorise: $24x^2 + 16x$
   Numbers: the HCF of 24 & 16 is 8
   Pronumerals: the HCF if x^2 & x is x.
   Overall, the HCF is 8x, which can be factorised out of the expression.

$24x^2 + 16x = 8x(3x + 2)$

Having factorised, it is a good idea to check that the process will actually work in reverse.
$8x(3x + 2) = 24x^2 + 16x$

2. Factorise: $6a^3 - 18a$
   The HCF of $6a^3$ & $18a$ is $6a$.

   $6a^3 - 18a = 6a(a^2 - 3)$

3. Factorise: $24pq + 14p + 6$

   The HCF of: $24pq$, $14p$ & 6 is 2. There are no pronumerals common to all the terms.

   $24pq + 14p + 6 = 2(12pq + 7p + 3)$

4. Factorise: $6abc - 2ac - 8bc$

   The HCF of $6abc$, $2ac$ & $8bc$ is $2c$.

   $6abc - 2ac - 8bc = 2c\,(3ab - a - 4b)$

   What we are saying in this last statement is that, whatever values we choose for a, b & c, $6abc - 2ac - 8bc$ will give the same answer as $2c\,(3ab - a - 4b)$.

Exercise

Factorise:

1. $21x^2 - 14x$

   The HCF of $21x^2$ & $14x$ is: ....

   $21x^2 - 14x$ = ...(...... – ......)

2. $2abc - 6ac$

   The HCF of $2abc$ & $6ac$ is: ....

   $2abc - 6ac$ = ...(...... – ......)

3. $2x^2y + 2xy^2 - 2xy$

   The HCF of $2x^2y$, $2xy^2$ & $2xy$ is: ....

   $2x^2y + 2xy^2 - 2xy$ = ...(...... + ... – ......)

4. $\frac{6}{x} - \frac{8}{x^2}$

   The HCF of $\frac{6}{x}$ & $\frac{8}{x^2}$ is: ....

   $\frac{6}{x} - \frac{8}{x^2}$ = ...(...... – ......)

5. $6x^2 - 18x$
6. $7a^2 - 49a$
7. $5p^2q + 20pq^2$
8. $3x^4 + 3x^3y$
9. $6abc + 42ab$
10. $4x^2y^2 - 4x^2y$
11. $6x^2y - 3xy$
12. $12a^3b - 60ab^2$
13. $2xe^x - e^x$
14. $3ab - 2xy$
15. $a^2 + ab + ac + a$
16. $12x^3y - 6x^2y^2 - 3x^2y$
17. $ab + 2 - c^2$
18. $21xy - 35x$
19. $26a^2b - 39ab^2 - 13ab$
20. $e^{2x} - e^x$
21. $x2^x + 2^{x+1}$
22. $10^{x+1} - 10$
23. $\frac{3}{x} - \frac{6}{x^2}$
24. $\sin x + \cos x$
25. $\frac{3}{ab} - \frac{5}{ac} + \frac{1}{a}$
26. $2^{2x} - 2^x$
27. $x^{3/2} - 2\sqrt{x}$
28. $1 - \frac{a}{b}$
29. $a^2\sqrt{b} + ab\sqrt{b} - a\sqrt{b}$
30. $ab - \frac{a^2b}{c} + \frac{2ab}{c}$

A14 Rearranging Formulas

Points to watch: Treat the formula as if it were an equation - a balance.

The formula $C = 2\pi r$ gives the circumference (C) of a circle of radius r. C is on its own on one side of the formula and is said to be the 'subject' of the formula. In this arrangement, the formula is able to calculate the circumference if the radius is known.

If we change the subject to be the other variable (r), then it is arranged to calculate the radius if the circumference is known. This is done using the 'balance method' that should be familiar from your experience of solving equations.

The balance method means that the same processes should be applied to both sides of the formula. This preserves the balance of the equality.

In this case, we divide both sides of the formula by 2π.

$$C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = r$$

$$r = \frac{C}{2\pi}$$

The formula $T = 2\pi\sqrt{\frac{l}{g}}$ gives the period (time for one complete swing) of a simple pendulum of length l. g is the acceleration due to gravity (approx. 9.8 ms^{-2} on Earth and a sixth of this figure on the Moon).

If we want the formula with l as the subject - so that it is arranged to calculate the length of a pendulum with a given period:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 2^2\pi^2\frac{l}{g} \text{ square both sides}$$

$$\frac{T^2}{2^2\pi^2} = \frac{l}{g} \text{ divide both sides by } 2^2\pi^2$$

$$l = \frac{T^2 g}{2^2\pi^2} \text{ multiply both sides by } g$$

Exercise

Change the subject of the following formulas to the pronumerals given in the square brackets:

1. $A = 4\pi r^2 \quad [r]$

   $\sqrt{A} = ...r$

   $r = \frac{........}{........}$

2. $s = ut + \frac{1}{2}at^2 \quad [a]$

   $s - ...... = \frac{1}{2}at^2$

   $...(s - ......) = at^2$

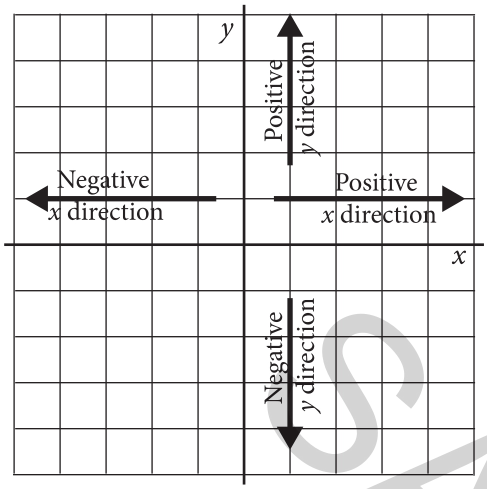
   $a = \frac{..................}{.........}$

3. $a^2 = b^2 + c^2 \quad [b]$
4. $I = \frac{Prt}{100} \quad [t]$
5. $v = \sqrt{2gs} \quad [s]$
6. $V = \frac{1}{3}\pi r^2 h \quad [h]$
7. $V = \frac{1}{3}\pi r^2 h \quad [r]$
8. $V = \frac{4}{3}\pi r^3 \quad [r]$
9. $A = \pi r(r+s) \quad [s]$
10. $E = \frac{1}{2}mv^2 \quad [v]$
11. $E = mc^2 \quad [c]$
12. $C = \frac{5}{9}(F-32) \quad [F]$
13. $A = \frac{1}{2}r^2\theta \quad [\theta]$
14. $v^2 = u^2 + 2as \quad [s]$
15. $S = \frac{a(1-r^n)}{1-r} \quad [a]$
16. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad [v]$
17. $V = \frac{\pi}{3}h^2(3a-h) \quad [a]$
18. $y = e^{2x+1} \quad [x]$
19. $y = 2^{2x} \quad [x]$
20. $y = x^2 + 2x + 1 \quad [x]$

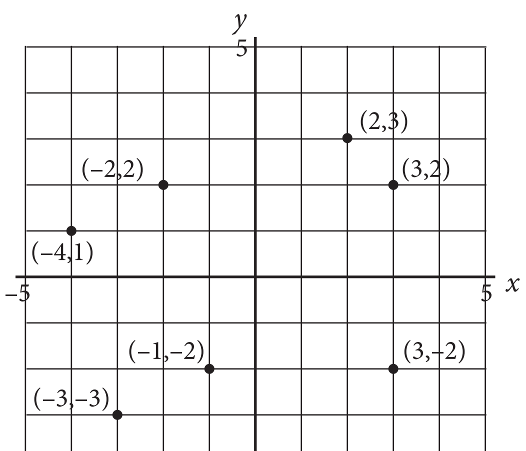
C2 Cartesian Diagrams

Points to watch: Not that coordinates are ORDERED pairs.

Cartesian Diagrams are a very important part of mathematics. They are used to represent geometric figures, functions etc. They are named after the French mathematician Rene Descartes. Conventions such as using x & y as axis labels m for gradient etc are due to him.

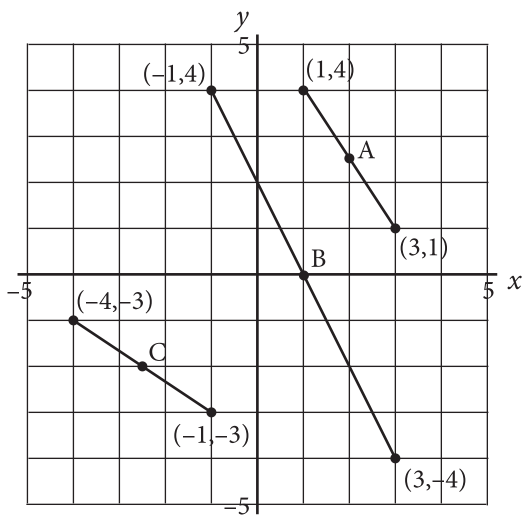


Points on the diagram are identified by ordered pairs. Again, by convention, the horizontal coordinate is always given first and the vertical coordinate is given second. Every coordinate pair is linked to a unique point on the diagram and every point on the diagram is linked to a unique coordinate pair. Here are some examples:



Fractional points such as (⅓,−⅜) also have their unique place on the diagram. Similarly, points with irrational coordinates such as (−√2,π) can be plotted in a unique position.

Mid-Points.
Example: Find the mid-point (A) between the points (1,4) & (3,−4), the mid-point (B) between the points (−1,4) & (3,1),the mid-point (C) between the points (−4,−3) & (−1,−3).



The mid-point (A) between the points (1,4) & (3,−4) is found by finding the number that is midway between the points in the x-direction - 2 is midway between 1 & 3. Likewise, in the y-direction, the number that is midway between 1 & 4 is 2½. A is the point (2, 2½). A common way of doing this is to find the mean of the two coordinates:

For B:

$$x\text{-coordinate of B} = \frac{-1+3}{2} = \frac{2}{2} = 1$$

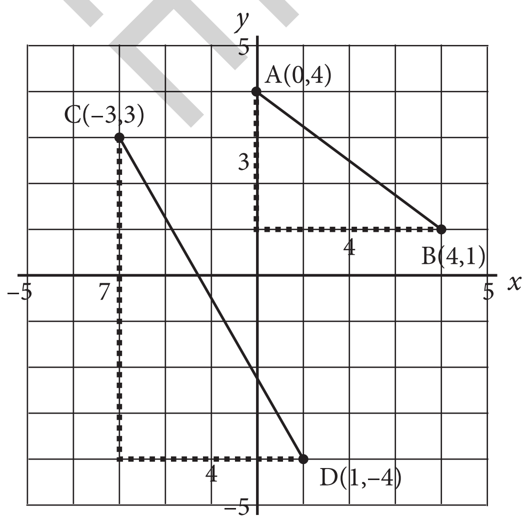
$$y\text{-coordinate of B} = \frac{4+(-4)}{2} = 0 = 0$$

and for C:

$$x\text{-coordinate of C} = \frac{-4+(-3)}{2} = \frac{-7}{2}$$

$$y\text{-coordinate of C} = \frac{(-3)+(-3)}{2} = \frac{-6}{2} = -3$$

Distances between points
These are found by using the Pythagoras' Theorem.

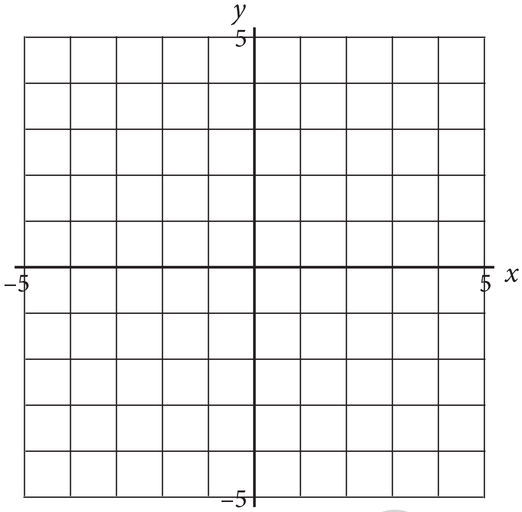


$$AB = \sqrt{(0-4)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

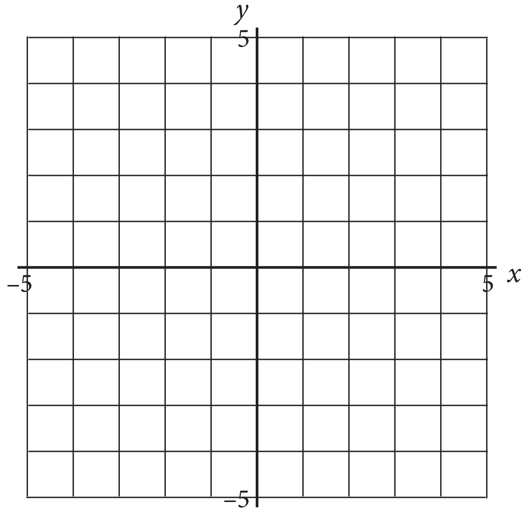
$$CD = \sqrt{(-3-1)^2 + (3-(-4))^2} = \sqrt{(-4)^2 + 7^2} = \sqrt{16+49} = \sqrt{65}$$

Exercise

1. Show the points: (2,1), (3,–4), (–2,–3) & (–3,3) on a Cartesian Diagram.



2. A quadrilateral has vertices: A(–3,3), B(–2,–3), C(3,–4) & D(2,1). Show these on a Cartesian Diagram. Find the perimeter of the quadrilateral.



$$AB = \sqrt{(\ldots-\ldots)^2+(\ldots-\ldots)^2} = \sqrt{\ldots^2+\ldots^2} = \sqrt{\ldots}$$

$$BC = \sqrt{(\ldots-\ldots)^2+(\ldots-\ldots)^2} = \sqrt{\ldots^2+\ldots^2} = \sqrt{\ldots}$$

$$CD = \sqrt{(\ldots-\ldots)^2+(\ldots-\ldots)^2} = \sqrt{\ldots^2+\ldots^2} = \sqrt{\ldots}$$

$$AD = \sqrt{(\ldots-\ldots)^2+(\ldots-\ldots)^2} = \sqrt{\ldots^2+\ldots^2} = \sqrt{\ldots}$$

Perimeter = ...

3. Show the points A(2,4), B(4,–2), C(–3,–4) & D(–2,1) on a Cartesian Diagram.

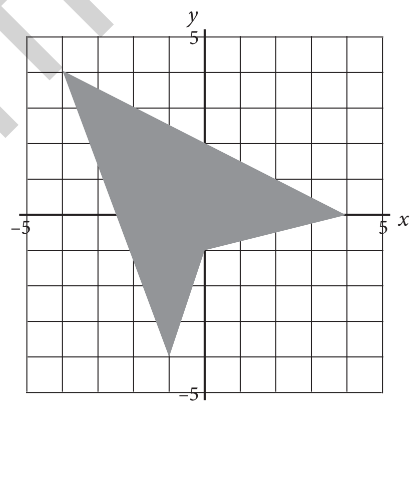
4. Find the mid-points between these pairs of points:

I	(9,5)	(–6,8)
II	(–1,–5)	(–4,–3)
III	(–2,–5)	(2–6)
IV	(0,0)	(–1,–10)
V	(–8,–4)	(6,8)
VI	(–7,1)	(2,9)
VII	(7,1)	(6,–7)
VIII	(7,–5)	(–10,–2)
IX	(–2,–5)	(7,7)
X	(5,2)	(6,–9)
XI	(–8,–8)	(7,1)
XII	(6,0)	(4,5)
XIII	(5,9)	(1,9)
XIV	(1,6)	(–3,0)
XV	(–7,–9)	(–1,–4)
XVI	(–8,9)	(8,–8)
XVII	(–9,–3)	(–4,–1)
XVIII	(9,8)	(4,2)
XIX	(–4,3)	(–4,–4)
XX	(–6,–2)	(–6,–9)
XXI	(–5,3)	(–3,–2)
XXII	(1,9)	(8,4)
XXIII	(4,–8)	(8,7)
XXIV	(5,–5)	(–6,1)
XXV	(5,–4)	(–1,–4)
XXVI	(3,–6)	(–7,6)
XXVII	(0,–5)	(–1,–3)
XXVIII	(–1,–6)	(–10,–9)
XXIX	(–5,–10)	(3,2)
XXX	(9,–4)	(–8,7)

5. Find the distances between these pairs of points, giving your answers correct to two decimal places:

I	(4,6)	(–2,–4)
II	(8,–6)	(–4,–6)
III	(–2,9)	(–9,–5)
IV	(–4,1)	(5,–8)
V	(–8,6)	(–8,8)
VI	(–5,–3)	(–7,–2)
VII	(8,9)	(–9,3)
VIII	(–3,9)	(–6,–3)
IX	(5,–5)	(–1,0)
X	(–2,–10)	(8,7)
XI	(–3,–10)	(–9,–5)
XII	(–4,–4)	(–10,–9)
XIII	(–9,7)	(–7,–4)
XIV	(2,–6)	(6,7)
XV	(2,2)	(–8,–4)
XVI	(2,–7)	(–6,9)
XVII	(4,8)	(4,7)
XVIII	(3,6)	(–4,7)
XIX	(–2,–9)	(–1,3)
XX	(–7,–4)	(6,2)
XXI	(–5,8)	(9,8)
XXII	(7,–9)	(–7,9)
XXIII	(2,–5)	(3,5)
XXIV	(–6,9)	(–3,–6)
XXV	(7,6)	(0,0)
XXVI	(0,7)	(8,–7)
XXVII	(–10,–10)	(–9,–10)
XXVIII	(–9,–4)	(–8,7)
XXIX	(2,3)	(4,–10)
XXX	(–7,–2(	(–6,–8)

6. Find the perimeter of this quadrilateral giving your answer correct to two decimal places:

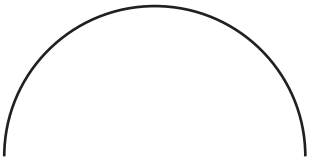


C13 Circles (2)

Points to watch: Pats of circles are common in many designs.

In C12, we dealt with circles as whole shapes. We will now look at the mathematics of parts of circles.

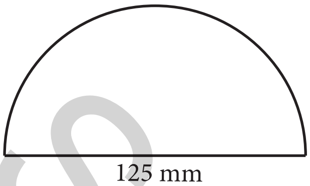
The semi-circle (half circle) has found many uses. For example, the Roman Arch.



Semi-circle

Roman Arches
Hungary

Example 1
Find the area and the perimeter of the figure shown. Give answers to the nearest whole number.



The radius is ½ × 125 = 62.5 mm (do not round yet!)
Area of full circle = $\pi r^2 = \pi \times 62.5^2 \approx 12\ 271.846$
Area of semi-circle ≈ ½ × 12 271.846 ≈ 6 135.92

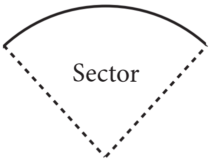
Now it is appropriate to round the answer and add units:

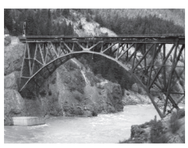
Area of semi-circle = 6 136 mm^2 to the nearest mm^2.

The perimeter is made up of the curved part which is half the circumference of the full circle plus the diameter.

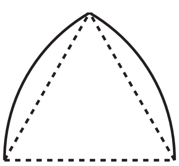
Perimeter = ½πd + d
= ½π × 125 + 125
≈ 321.35
= 321 mm to the nearest mm.

A part of a circle other than a half is known as a sector. Sectors are also common in architecture.





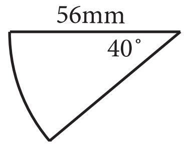
Arch on
Candian Railways



Gothic
Arch

Cologne Cathedral
Germany

Example 2
Find the area and perimeter of this sector. Give answers to the nearest whole number.



Both calculations depend on the fact that this sector is a fraction of a full circle.

The fraction is: $\frac{40}{360}=\frac{1}{9}$ th.

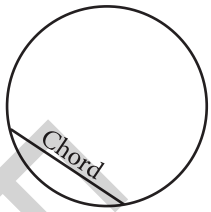
Area = $\frac{1}{9}\pi r^2=\frac{1}{9}\pi\times 56^2\approx 1094.67$.

Area = 1 095 mm^2 to the nearest mm^2

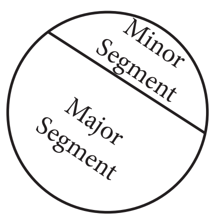
$$\text{Perimeter}=\frac{1}{9}\pi\times 2r+2r$$
$$=\frac{1}{9}\pi\times 2\times 56+2\times 56$$
$$\approx 151.095$$
$$=151 \text{ mm to the nearest mm}$$

Other divisions of circles.

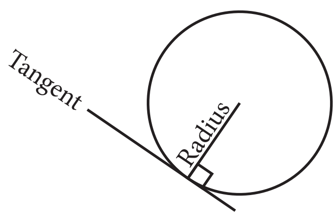
A line that cuts a circle in two paces is known as a chord.



A chord divides a circle into two segments. These are sometimes called the major segment (the bigger one) and the minor segment (the smaller one).

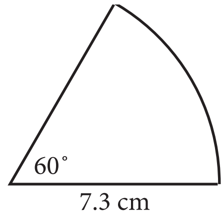


A line that just touches a circle is known as a tangent. A tangent makes a right angle with the radius.



Exercise

1. Find the area and perimeter of this sector.



Perimeter = arc of circle + two radii

$$= \frac{....}{....} \times \pi d + 2d$$

$$= \frac{....}{....} \times \pi .... + 2....$$

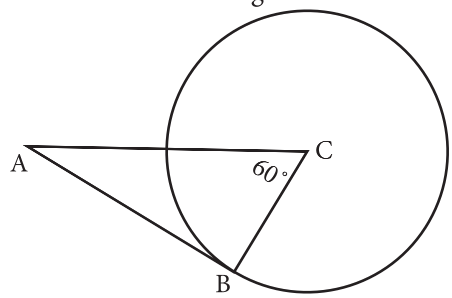
$$= ....\text{cm}$$

$$\text{Area} = \frac{....}{....} \times \pi r^2$$

$$= \frac{....}{....} \times \pi ....^2$$

$$= ....\text{cm}^2$$

2. Find the angle CAB in this diagram. C is the centre of the circle and AB is a tangent



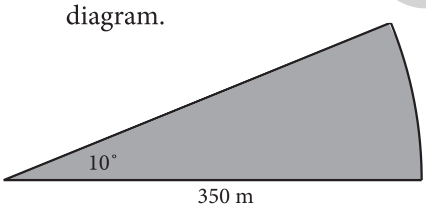
Angle ABC = ...˚

The sum of the angles of a triangle = ...˚

Angles CAB + BCA + ABC = ...

Angle CAB = ...

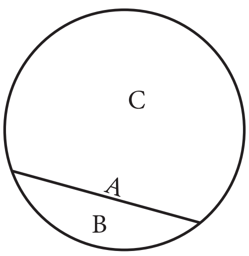
3. Find the shaded area in this diagram.



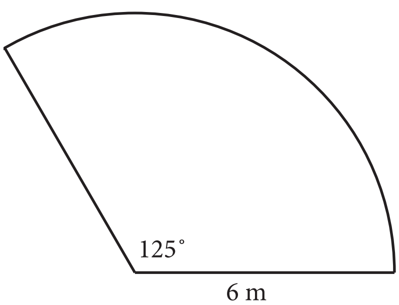
Answer correct to the nearest square metre.

4. Find the perimeter of a quarter circle of radius 25 mm.

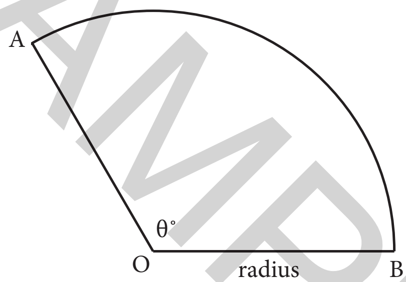
5. What are the correct names of the features labelled A, B & C.



6. Find the perimeter of this figure, correct to the nearest mm.



Questions 7 to 15 refer to this diagram.



7. Find the perimeter if the radius is 12 cm and the angle is 57˚.

8. Find the area if the radius is 27 cm and the angle is 29˚.

9. Find the perimeter if the radius is 27.1 cm and the angle is 117˚.

10. Find the area if the radius is 49.6 cm and the angle is 156˚.

11. Find the angle if the area is 577 m^2 and the radius is 23 m.

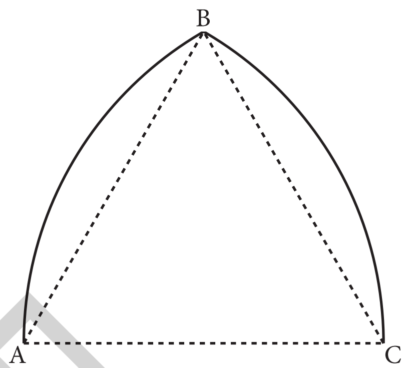
12. Find the angle if the area is 415.6 cm^2 and the radius is 23 cm.

13. Find the angle if the area is 49.8 m^2 and the radius is 4.4 m.

14. Find the angle if the area is 89.03 m^2 and the radius is 5.7 m.

15. Find the perimeter if the area is 18.8 m^2 and the radius is 7 m.

16. The classic gothic arch has the structure shown in this diagram



A is the centre of the arc BC and C is the centre of the arc AB.

a What type of triangle is ABC?

b What size are the angles BAC & ACB?

Consider an arch for which the width is 6.00 metres.

c Find the arc length BC.

d Find the perimeter of the arch.

e By considering a perpendicular from the point B to AC, find the height of the arch.

f Find the area of the arch.

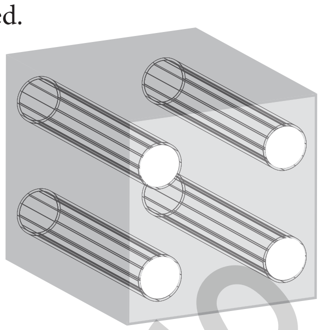
C20 Solids (3)

Points to watch: Be careful to identify all the parts of any solid.

In this section, we will look at compound solids

Examples

1. Find the volume and surface area of this engineering component. The component is based on a 20 mm steel cube through which four 6 mm diameter circular holes have been drilled.



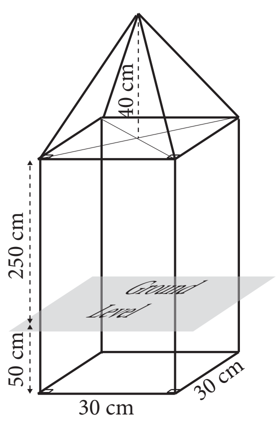
Volume.
The cube without the holes has volume 20^3 mm^3.
Each hole is a cylinder of radius 3 mm and length 20 mm. Remember there are four of them and their volume is subtracted.

$$\begin{aligned} \text{Volume} &= 20^3 - 4 \times \pi \times 3^2 \times 20 \\ &= 8000 - 720\pi \\ &\approx 5738 \text{ mm}^3 \end{aligned}$$

The surface area needs even more care. We start with the six square sides 6×20^2 mm^2.
From this we have removed 8 circles ($8\pi \times 3^2$). But we have added the curved surfaces of 4 cylinders ($2\pi rh = 4 \times 2\pi \times 3 \times 20$).

$$\begin{aligned} \text{Surface area} &= 6 \times 20^2 - 8\pi \times 3^2 + 4 \times 2\pi \times 3 \times 20 \\ &= 2400 - 72\pi + 480\pi \\ &\approx 3682 \text{ mm}^2 \end{aligned}$$

2. A gatepost consists of a cuboid capped by a right pyramid - as shown. 50 cm of the post is buried in the ground.



a Find the volume of the post.
i The cuboid

$$\begin{aligned} V_{\text{cuboid}} &= 30 \times 30 \times (50 + 250) \\ &= 270000 \text{ cm}^3 \end{aligned}$$

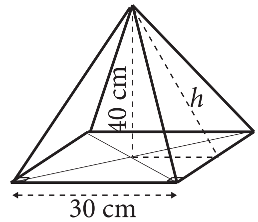
ii The pyramid

$$\begin{aligned} V_{\text{pyramid}} &= \frac{1}{3} \times 30 \times 30 \times 40 \\ &= 12000 \text{ cm}^3 \end{aligned}$$

Total volume = 270 000 + 12 000
= 282 000 cm^3.

b The above ground parts of the post are to be painted. Find this area.

i There are four rectangular faces. The above ground parts are:
30 × 250 (×4) = 30 000.
There are four triangles and we need to calculate the height (h).

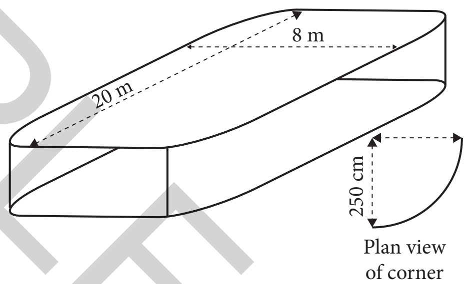


$$\begin{aligned} h &= \sqrt{15^2 + 40^2} \\ &= \sqrt{1825} \end{aligned}$$

$$\begin{aligned} \text{Area of triangles} &= \frac{1}{2} \times 30 \times \sqrt{1825} \times 4 \\ &\approx 2563 \end{aligned}$$

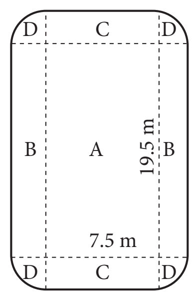
Total surface area = 2 563 + 30 000 = 32 563 cm^2.
This is probably best rounded to 33 000 cm^2 to match the accuracy of the data.

3. A swimming pool has rounded corners and dimensions as shown.



Find the number of litres of water necessary to fill the pool to a depth of 1.6 m.

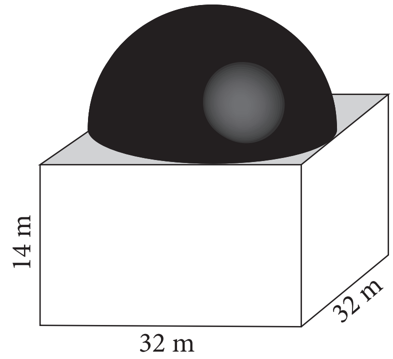
We must find the area of the water surface. This can be done using the dissection shown at right.



Area A = 7.5 × 19.5
= 146.25
Areas B = 0.25 × 19.5 × 2
= 9.75
Areas C = 0.25 × 7.5 × 2
= 3.75
The corners are 4 quarter circles.
Together they make a full circle of radius 0.25 m.
Areas D = $\pi \times 0.25^2$
≈ 0.196
Total A = 146.25 + 9.75 + 3.75 + 0.196
≈ 159.964 (ie. not much different from a rectangle)
Volume ≈ 159.964 × 1.6
≈ 256 m^3
≈ 256 000 litres.

Exercise

1. The figure shows a design for an exhibition hall.



The vertical walls are to be brick. There is to be one entrance, 5 metres wide and 3 metres high. There are to be no windows as the natural lighting will be provided by the hemispherical glass dome. The horizontal portion of the roof (grey) will be metal cladding.

a Find the volume of the lower (cuboid) part of the structure.

$V_{cuboid} = ... \times ..... \times .....$

$= .... m^3$

b Find the volume of hemispherical dome.

$V_{dome} = \frac{4}{3}\pi \times .....^3 \times \frac{.....}{.....}$

$\approx .... m^3$

c Find the total volume of the hall.

$V_{total} = ... + ...$

$\approx .... m^3$

d Find the total area of the brickwork.

$A_{brick} = 4 \times ..... \times ..... - ..... \times .....$

$\approx .... m^2$

e Find the area of the g;ass dome.

$A_{dome} = \frac{....}{....} \times 4\pi \times .....^2$

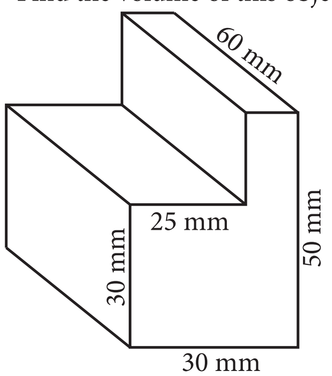
$\approx .... m^2$

f Find the area of the roof cladding.

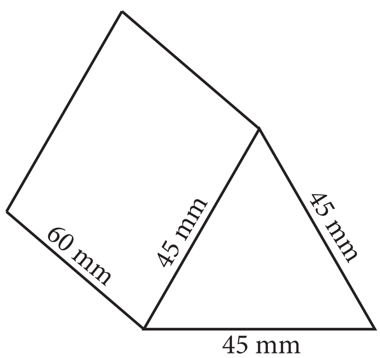
$A_{cladding} = ... \times ..... - \pi \times .....^2$

$\approx .... m^2$

2. Find the volume of this object.

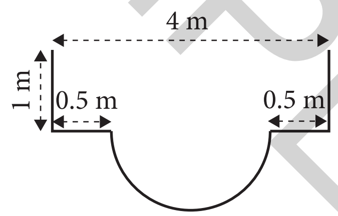


3. Find the surface area of this object, correct to the nearest square cm.



4. A solid block of aluminium is a cuboid measuring 34 by 66 by 79 mm. A circular hole of radius 20 mm is drilled straight from the 34 by 66 mm face to the opposite one. What is the volume of the resulting solid, correct to 2 significant figures?

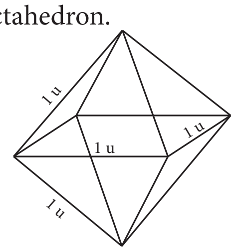
5. The diagram shows a vertical section through a four person spa. The upper portion is a cuboid (4 by 4 by 1 metre). There is a hemispherical foot well as shown.



a Find the maximum amount of water (to the nearest 10 litres) that the spa can contain.

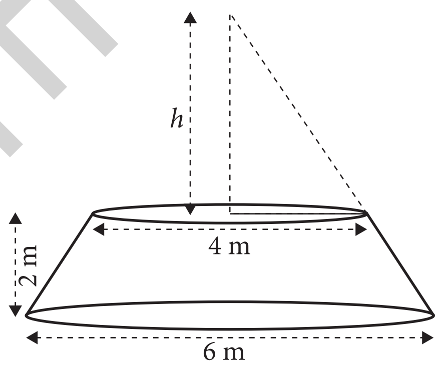
b The average adult displaces 70 litres of water if immersed to the neck. What is the maximum depth that the spa should be filled to if it is not to overflow when in use by 4 average adults.

6. A regular octahedron consists if two pyramids placed base to base. Find the surface area and the volume of a unit regular octahedron.



7. A hemispherical bowl has an outer diameter 350 mm and glass of thickness 8 mm. Find the volume of glass used in its manufacture expressed in cubic centimetres, correct to the nearest whole number.

8. The diagram shows a pedestal based on a cone with the top cut off.



a Use similar triangles to calculate the value of h.

b Find the volume of the pedestal.

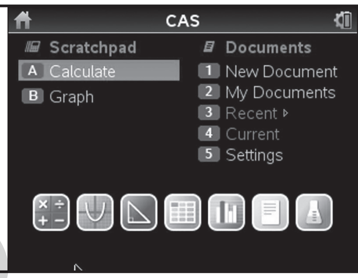
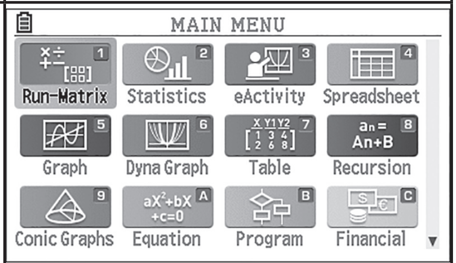
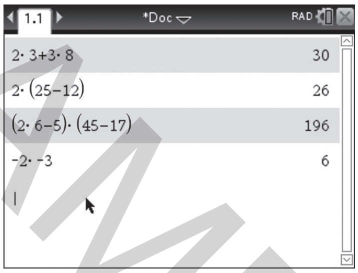
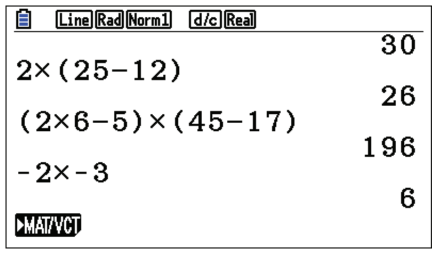
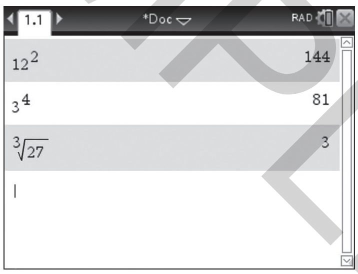
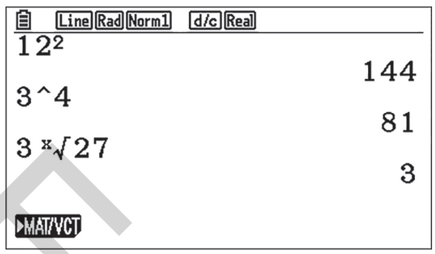
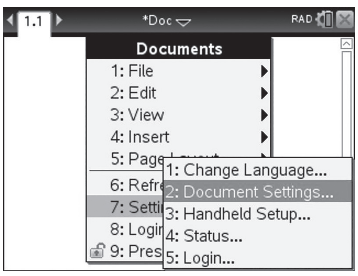
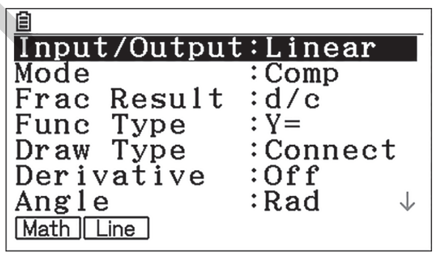
c Find the total area of the three surfaces of the pedestal.

Answer b & c correct to the nearest whole number.

Calculator Supplement

This section contains some basic advice. It is important that all students master these techniques on their own models. We are assuming that students will by this stage of their course be using graphic calculators. This in not a complete list of important calculator skills!

In the first column, we suggest a principle that you should understand and be able to implement on your model.

Skill/Principle	TI Series	Casio Series
Understand the overall structure of your calculator. Most allow you to open different types of document		
The most frequently used type of document is the basic calculator screen. Access this on your model and check that it uses BODMAS. Learn to use brackets and note the difference between the negative key (-) and subtraction.		
Understand the use of powers and roots. Use examples that you know the answers to such as 'the cube root of 27 is 3'.		
Document settings. Usually, these can be set from a special panel.		
Know the meaning of number formats such as 'Float' and 'Fix. We are doing the same calculation here, first with the 'Fix2' format and second with 'Float2' (TI) or 'Sci2' (Casio). Note that neither is an exact answer. Above all, do not be surprised by these formats in an examination situation when it is easy to panic!