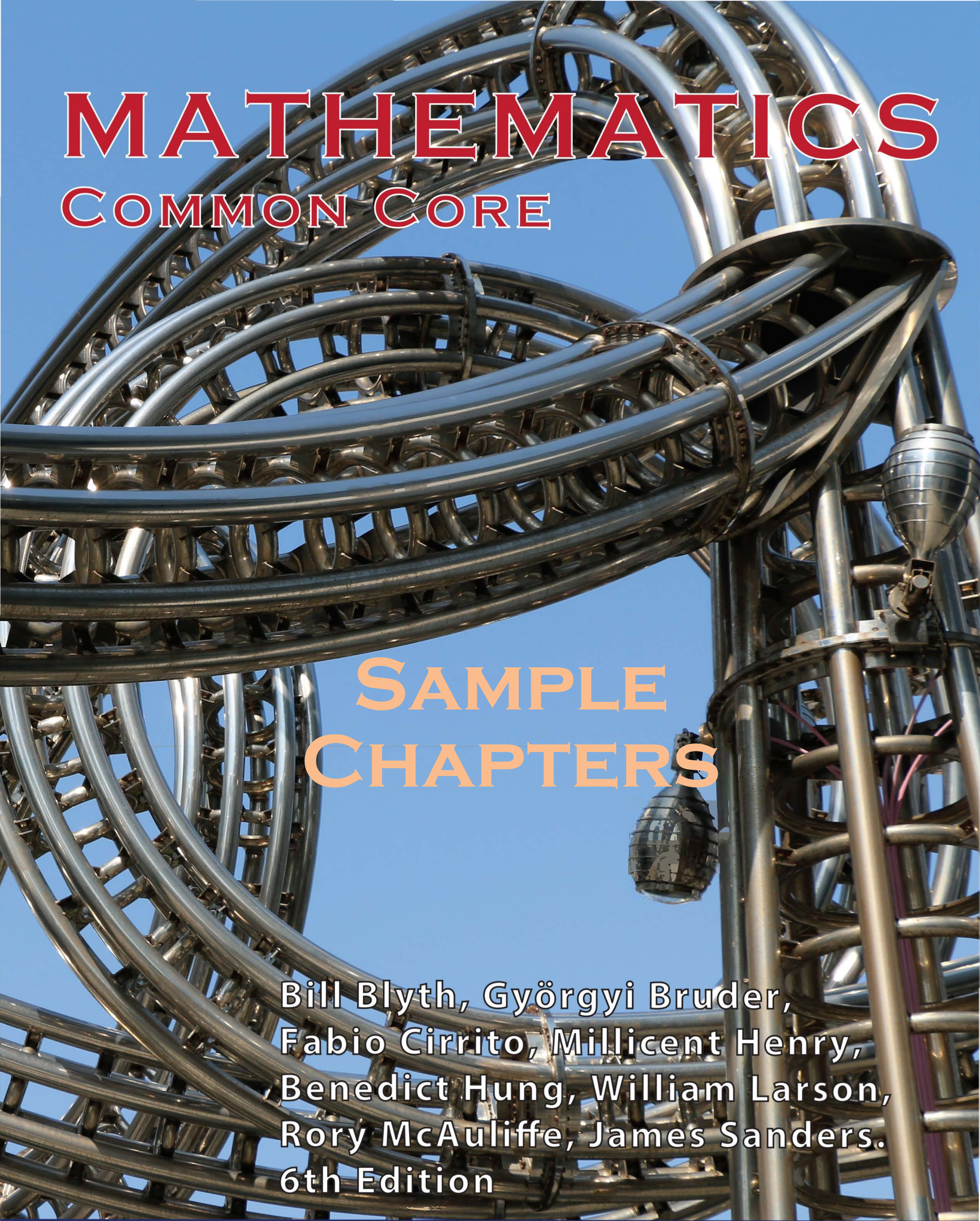

MATHEMATICS
Common Core

Sample Chapters

Bill Blyth, Györgyi Bruder, Fabio Cirrito, Millicent Henry, Benedict Hung, William Larson, Rory McAuliffe, James Sanders.

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B.3 Graphs

SL 2.3
SL 2.4

In the previous section, we dealt with the graphing of functions that are mathematically defined. In this section we will deal with the use of graphs in other contexts such as the processing of experimental data. It is also important to be able to construct graphs that illustrate real situations and to interpret graphs that you encounter in books, magazines, scientific literature etc.

We will begin with a famous old problem.

Example B.3.1

The Monk's Staircase Problem.

A monk intends to climb a very long staircase so that he can meditate at a temple at the top of a hill.

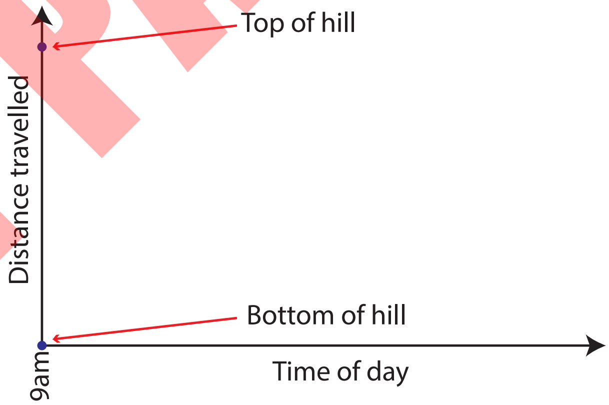
He starts the climb at 9am on Monday. During the day, he climbs, occasionally stopping to rest, admire the view etc. He arrives at the temple at around sunset.

He starts his descent on Tuesday at 9am. Again, on his descent, he stops from time to time. He does not descend at the same speed as he ascended. He arrives at the bottom of the hill in the late afternoon.

Is there a point on the staircase that he reaches at exactly the same time of day on both Monday and Tuesday?

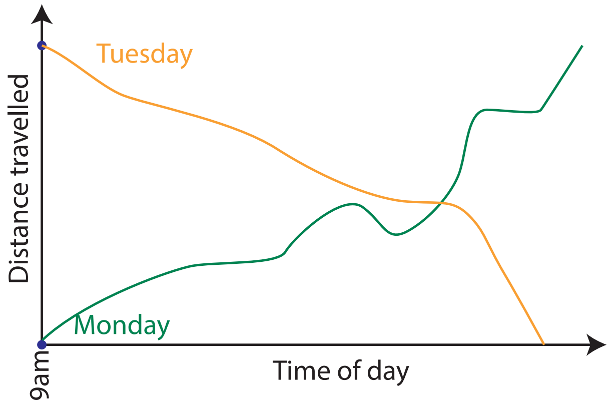
The difficulty with this problem is the lack of data. There is no information about the length of the staircase, the speed at which the monk moves or when and where he rests.

A well chosen graph, however, will clarify things. The axes represent the time of day (horizontal) and the distance travelled along the staircase (vertical).



On Monday (green curve) the monk goes from the bottom to the top across the course of the day. The curve shows variable speed and even one occasion on which the monk went back down a bit.

Tuesday's curve (orange) starts at the top and finishes at the bottom - not necessarily at the same time as Monday's curve.



It is impossible to draw the orange curve without crossing the green one at least once.

At the intersection, the monk is at exactly the same step at exactly the same time of day on both days.

Other situations also suggest the use of graphs.

Example B.3.2

Tank A initially contains 0.5 litres. Water is poured in at a rate that doubles the amount in the container every 100 seconds.

Tank B initially contains 10 litres. Water is added at the rate of 2 litres every 100 seconds.

When will the two tanks contain the same amount of water?

Unlike the first example, this is numeric. We need to construct a model for each tank.

If we label the time (measured in units of 100 seconds) as t and the contents of the tanks as $C_A(t)$ (tank A) and $C_B(t)$ (tank B).

Tank A

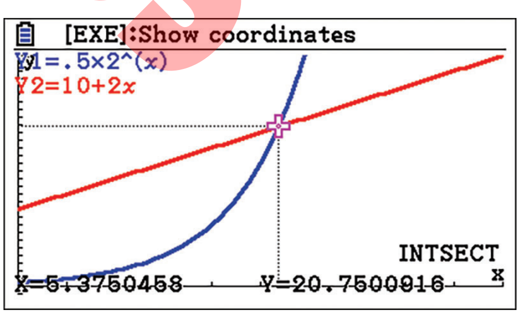
We need an exponential model. The volume needs to start at 0.5, become 1 at $t = 1$, 2 at $t = 2$, 4 at $t = 3$ etc.

The function $C_A(t)=0.5\times 2^t$ will do this.

Tank B

This is a linear model $C_B(t)=10+2t$.

The problem can now be solved using a calculator.



The solution point occurs at t = 5.37 which is 537 seconds. The tanks contain 20.75 litres.

Example B.3.3

The Ångstrom family have borrowed one million Swedish Krona to buy a house. The interest rate is 6% per annum calculated monthly on the current balance.

Construct a spreadsheet that will track the loan. Show the results on a graph.

The Ångstroms intend to pay off the loan in ten years. What monthly repayment will secure this?

There is not a single way of achieving this. One implementation is:

	A	B	C	D
1	Loan Repayment			
2		Monthly payment		5600
3				
4	Month	Balance	Interest	
5	0	1000000	5000	
6	1	999400	4997	

Enter =A5+1

Enter =B5+C5–D$2

Enter =B5*0.005 and copy to cell C6

Column A records the month, column B the balance (the current level of the debt) and column C the monthly interest (which depends on the balance at the start of the month.

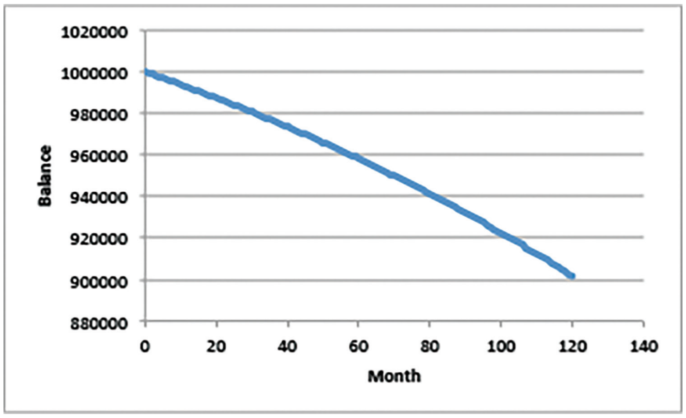
The formulas are:

In cell A6: =A5+1. This increases the month by 1 for each row.

In cell B6: =B5+C5–D$2. This takes the last balance (B5), adds the interest (C5) and subtracts the payment (D$2). The $ sign allows the formula to be copied down the rows with the D$2 element unchanged.

Use 'Fill Down' to copy A6 to C6 down another 120 rows. Look at the way the cells are copied. You should find that the formula in B7 has 'copied' as =B6+C6–D$2. The first two elements have been updated, but the last one (payment) has not. This is known as relative cell addressing.

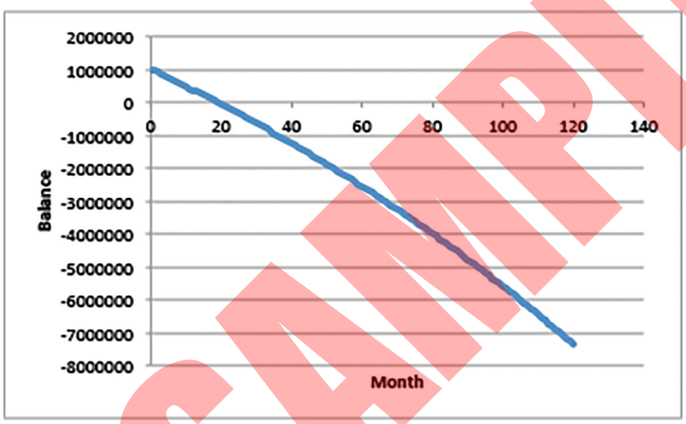
Next, select the relevant data that you want to graph. The question talks about 10 years so you need to go from 0 to 120 months. You need the balances in column B as well. We have chosen an XY scatter diagram with the line only option.



Note that the result is not linear. The debt is also not going to get paid off in ten years because the montly payment of 5600 is too low.

We next alter the value in B2 to try to get the balance at 120 months to be zero. Unless you have exceptional eyesight or a giant screen, you will not be able to see the two relevant cells at once. A way around this is to enter =C125 in cell B3. This enables you to see the payment and final balance next to each other.

Now use 'guess and check' to home in on the solution. As our first guess was way too small, it is a good idea to make the next guess a lot bigger. If you make the payment 10 time bigger, you will get a negative final balance. If you leave the graph active, you will also see the effect graphically.



If you are too timid and make a small increase to, for example, 5700, you will take a long time, even with the technology.

We have bracketed the answer between 5 600 and 56 000 with the likely answer closer to the lower figure. We made our next guess 12 000 and got a final balance of –736. You are probably thinking 'lucky'. Well, yes, but not entirely. A good final answer to the correct payment is 11 100.

Exercise B.3.1

1. The arch of a 500m bridge stretches across a valley and has a shape that can be modelled by the equation of a parabola. It suspends a straight and level roadway.

   a. Determine the equation of the arch if the height of the overpass 50m from one end of the bridge is 36m.

   b. Find the distance from the top of the arch to the roadway 45m from one end of the bridge.

2. In chemistry, the pH measure of acidity is based on a logarithmic scale and it is given by the formula $\mathrm{pH} = -\log_{10}\left[\mathrm{H}^+\right]$, where $\left[\mathrm{H}^+\right]$ is the concentration of hydrogen ions in moles per litre.

   a. Determine the concentration of hydrogen ions for household bleach if its pH is 12.6.

   b. The pH for milk is about 6.5. How much stronger in acidity is milk than household bleach?

3. The Richter scale is used to compare the intensity of earthquakes and it is given by:

$$M = \log_{10}\left[\frac{I}{S}\right]$$

   where M is the magnitude reader on the Richter scale, I is the intensity of the earthquake, and S is the intensity of a standard earthquake.

   If an average earthquake measures 4.5 on the Richter scale, how much more intense is an earthquake that measures 6?

4. The formula that is used to measure sound is:

$$L = 10 \times \log_{10}\left[\frac{I}{I_0}\right]$$

   where L is the loudness measured in decibels (dB), I is the intensity of the sound being measured, and I_0 is the intensity of sound at the threshold of hearing. Determine the change in intensity of sound between a normal conversation measuring at 60dB with the sound produced by a jet engine at 140dB.

5. A canoeist takes 2 hours longer to go 38km up a river than to return. Determine the speed of the canoe in still water if the speed of the current is 5km/hr.

6. A swimming pool can be filled in 4 hours by two pipes working simultaneously. The smaller pipe takes 3 hours longer than the bigger pipe, if each pipe is to pump the water into the swimming pool alone. Determine the exact amount of time required by bigger pipe to pump the water into the swimming pool.

7. A rope footbridge across a river is modelled by the function:

$$h(x)=6\left(e^{-0.3x}+e^{0.3x}\right), -3\le x\le 3$$

where h is the height of the bridge above the mean river level at a point x across the bridge. Both are measured in metres. Find:

i the maximum height of the bridge.

ii the minimum height of the bridge.

8. Use the data in Example B.3.3 to answer these questions:

a How much interest do the family pay over the course of the loan?

b If the family receive a legacy at the start and reduce the starting balance to 800 000, what is the new payment and how much interest will they save?

Graphs from Experimental Data

Frequently, you will use graphs to analyse the results of experiments.

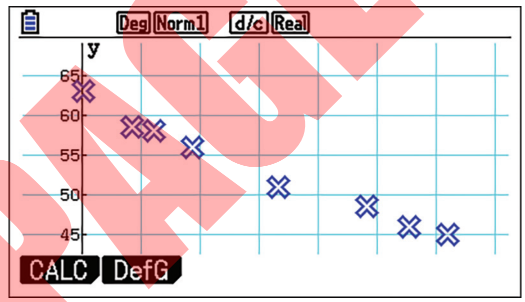
This video shows an experiment on cooling using very rudimentary equipment. The video has been very considerably speeded up. You should pause it periodically in order to take readings of time and temperature.

Cooling video:

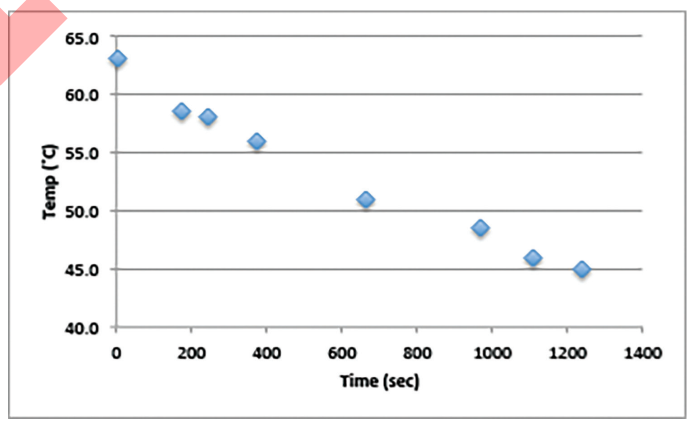
Data can be collected from the video (or preferably your own data) using freeze frame. Here are some.

Time			Temp
Min	Sec	Total Seconds	˚C
0	6	6	63.0
2	52	172	58.5
4	4	244	58.0
6	13	373	56.0
11	5	665	51.0
16	7	967	48.5
18	29	1109	46.0
20	40	1240	45.0

These can now be transferred to a graph using graph paper, a calculator,



or a spreadsheet.



From here on, it is possible to make qualitative comments that relate to the shape of the graph and hence to the experiment.

It is, for example, possible to make comments such as:

"The coffee cools most quickly when it is hot"

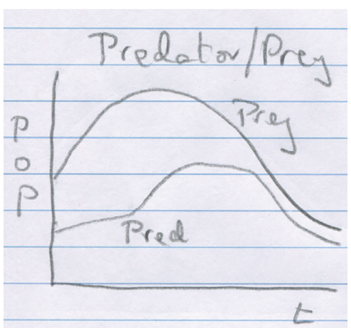
and feel that the graphs supports it (just!).

Tidying Graphs

Most of this text concentrates on graphs for which we have an algebraic rule. However, there are circumstances in which we will want to draw a graph that illustrates a point rather than gives accurate information.

For example, suppose you have attended a lecture on animal populations. You have made pencil and paper notes, but want to record the main points in a word processed document. During the lecture, the link between the populations of prey animals and the predators that feed off them was discussed. The point was made that an increase in the population of prey is often followed, after a delay, by an increase in the population of predators. This is because of the increased food supply.

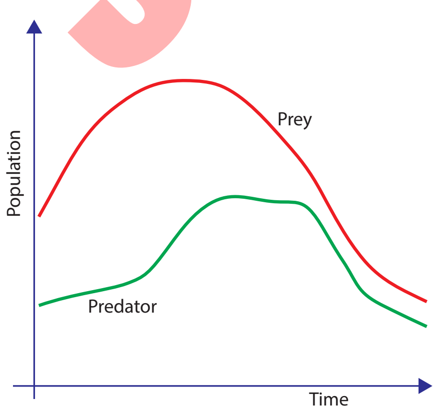
You have made the following hurried sketch and want to convert it into an electronic document. The first step is to scan it.



This can be neatened using a variety of methods. We will use the drawing package *Illustrator*. However, there are many alternatives. We find it helps to use layers. Our sample file uses four layers:

1. The scan
2. The axes
3. Text
4. Graphs

Sample file: Video:



Sums and Differences

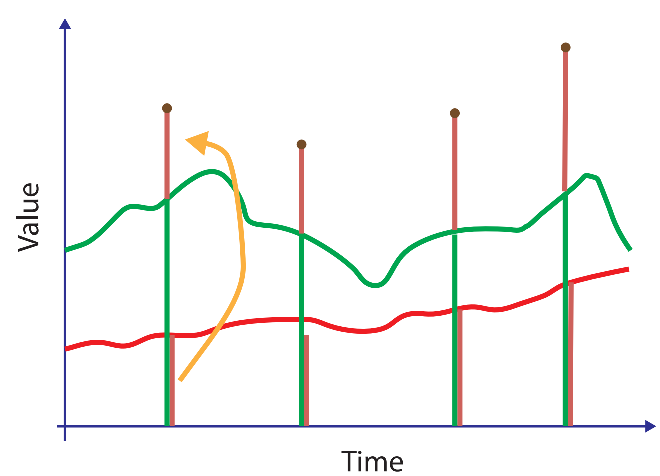
Sometimes we are presented with graphical data without the underlying data. If we are asked to plot the sum and or differences of the data

The diagram is typical of the sort of graph you will see in magazines. In this case, the performance of share market investments is being compared with property investments. The complete absence of any numbers makes arithmetic impossible.

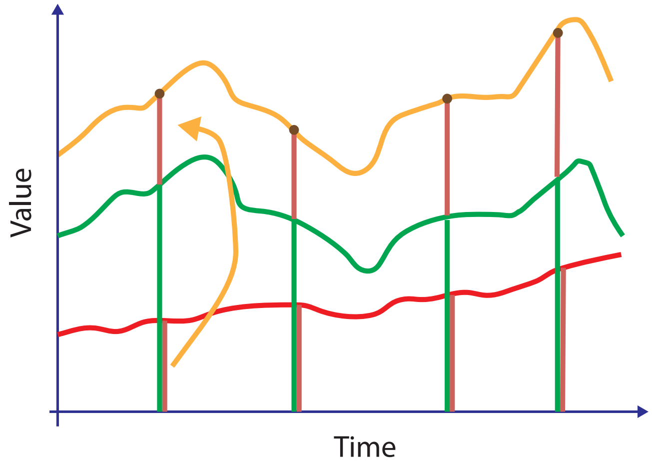
Suppose we are interested in the performance of a portfolio that contains both types of investment. We need to 'add' the two graphs.

Before doing that, we will compress the vertical scale. As we are working in a drawing package (Illustrator), we can use select and compress this graphs.

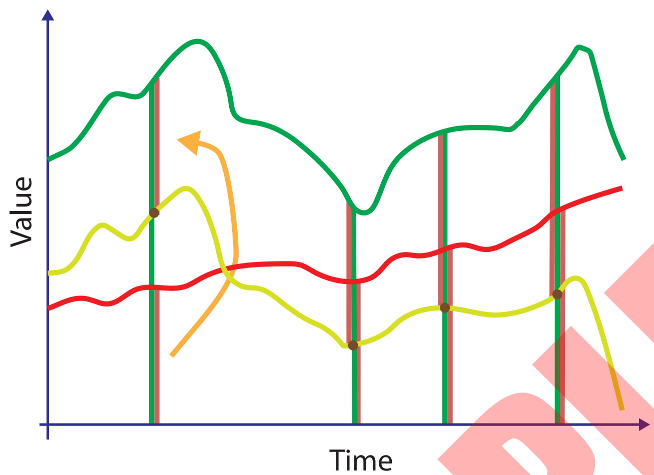
Following this, we proceed point by point across the graph. Use the line tool to measure the two heights and then stack them. We show four examples:



The result (orange) has some of the features of both graphs.



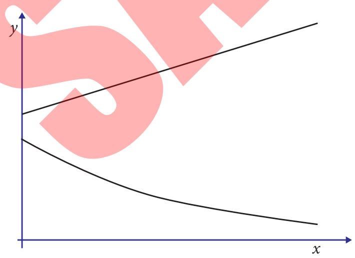
If, instead, we are interested in the difference between the two types of graph, we do a similar thing, but this time subtracting the heights. This time there is no need to compress the vertical scale.



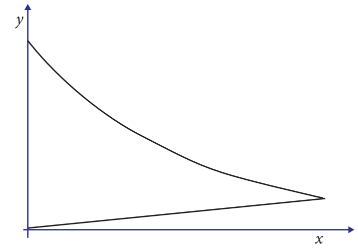
Exercise B.3.2

1. Sketch these graphs and show the sum of the functions and their difference.

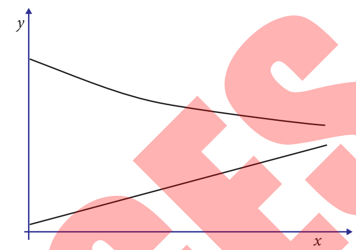
a



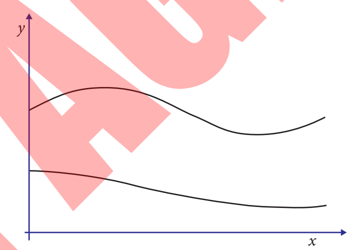
b



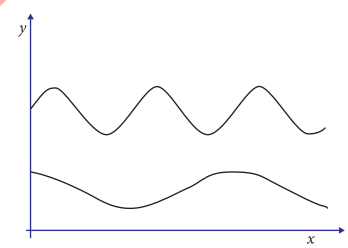
c



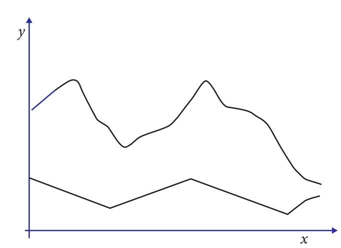
d



e



f



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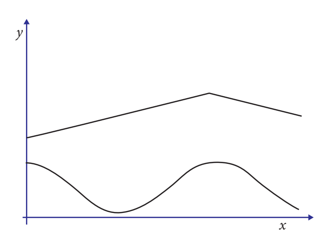
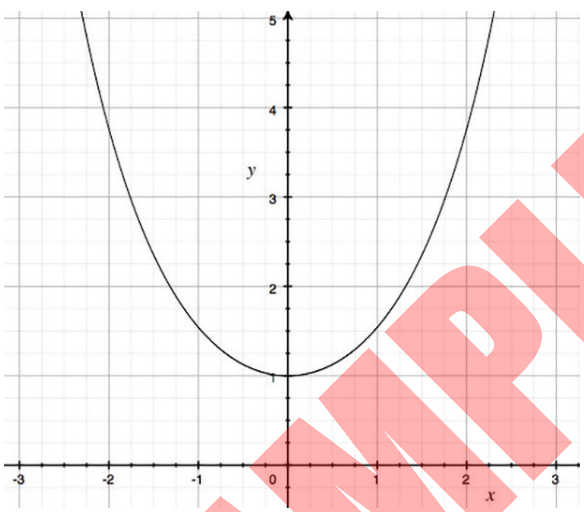


Features of Graphs

A curve followed by a freely hanging wire is known as a **catenary**.

The function that describes this curve is known as the hyperbolic cosine (or cosh for short). This function is not included in this course, but you will find it on most calculators and graphing software.

The graph is shown below.



Note that this is not all engineers need to know about the catenary if they are designing a cable lift of the type shown in our title photograph. The passenger car will deform the curve significantly. There is a lot more work to do to ensure that the cable lift can operate safely.

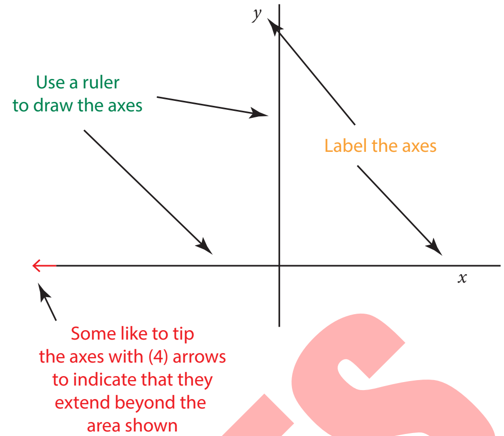
This means remaining clear of the ground, trees etc. under all operating circumstances.

The diagram on the previous page is often known as a 'plot'. The points have been put on the graph with as much accuracy as the computer can muster.

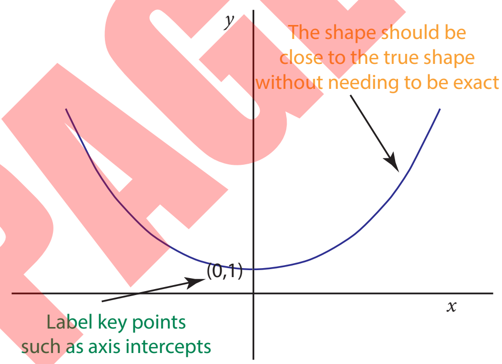
When graphs are drawn in tests and exams, they will almost always be 'sketches'.

What is the difference between a plot and a sketch?

Here is a sketch of the catenary:



Note that the axes labels will not always be x & y. They may be v (for volume) and C (for cost).

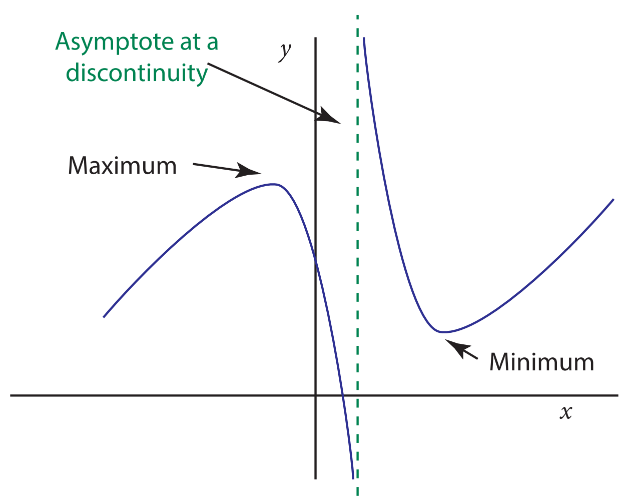


When labelling points on a sketch, it is a good idea to get used to using coordinates. In the present case, the label is (0,1) rather than just 1.

Exam questions often use the phrase 'showing the coordinates of all...' in which case the coordinate pair becomes obligatory.

This example also shows a minimum point at (0,1). Be aware that such points can be important in applications.

Other features that often appear on graphs are maxima/minima and discontinuities.

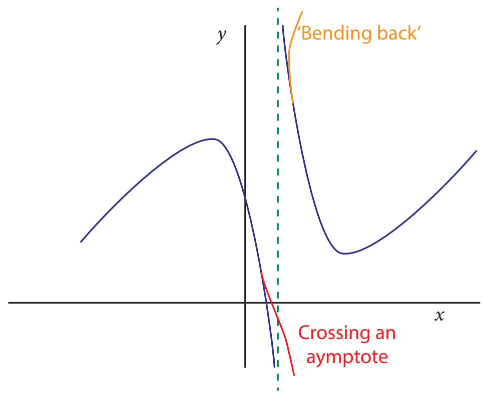


We will see a number of these in the examples.

Pitfalls

There are some common errors in curve sketching that can lead to lost marks in exams. Most of these refer to missing labels. However, there are a number of 'wrong shape errors' that should be avoided.

Some, but not all, are shown below.



Always remember that you can find the general shape of a graph by generating a table of values.

Our first example looks at this method.

Example B.3.4

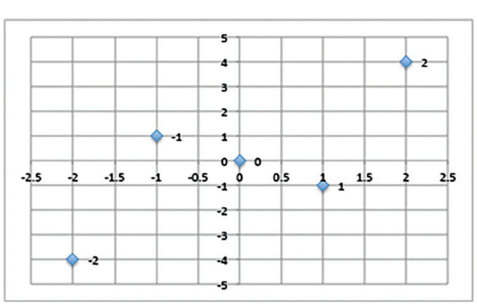
Sketch the graph of $y = x^3 - 2x$, $\quad -2<x\leq 2$.

Note that there is a specified domain ($-2<x\leq 2$). This needs to be reflected in the table:

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$2x$	-4	-2	0	2	4
y	-4	1	0	-1	4

If these are transferred to a cartesian graph, the result can be confusing.

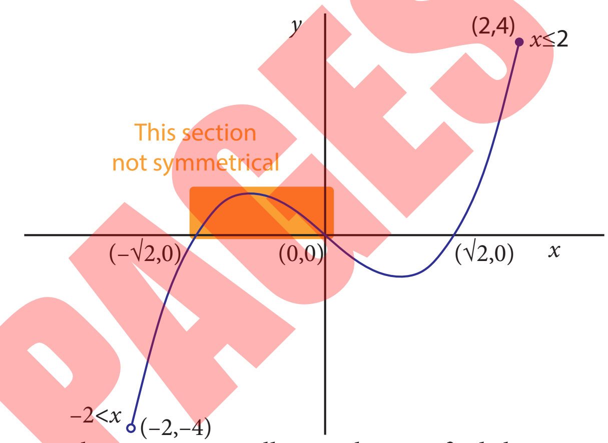
This graph has been produced using the Excel chart option. This is one of many technological avenues that can be used.



The three points on a straight line are a bit of a mystery. If you are using the table of values method, you will need to clarify matters by choosing some extra points in the region of concern.

x	-1	-0.5	0	0.5	1
x^3	-1	-0.125	0	0.125	1
$2x$	-2	-1	0	1	2
y	1	0.875	0	-0.875	-1

The sketch should look like this:



Later in the course we will cover how to find the maximum and minimum points of this graph.

The 'loops' are not symmetrical and the sketch should show this.

Example B.3.5

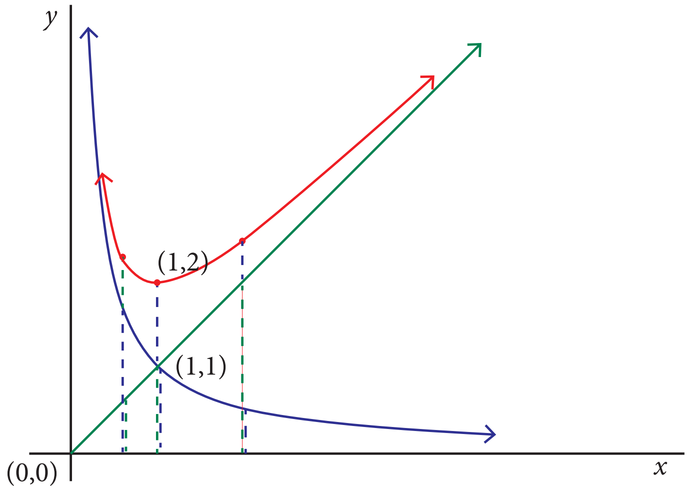
Sketch the graph of $y = x + x^{-1}$, $0<x$.

We will approach this solution from a slightly different viewpoint. The function is the sum of two parts. The first part is $y = x$, a straight line through the origin with gradient 1.

The other function: $y = x^{-1} = \dfrac{1}{x}$ is a hyperbola.

The full function (red) can be found by adding the heights of the blue curve to those of the green line, point by point (as discussed earlier). We have shown three examples of this - the dotted lines show the stacking of the two heights of the graph to produce the sum of the functions. This is not as time consuming as it seems as the green line has small y values for small x. This means that the red function behaves like the blue curve for small x. For large x, the position is reversed because the reciprocal of a large number is small. The red curve approaches the green line for large x.

We have used arrows on the graph to show that the graph extends beyond what is shown on the diagram.



This is an example of a curve (red) being asymptotic to another curve (blue).

Analysing Graphs - Intersections

You should be thoroughly familiar with the graphing capabilities of your model of calculator. This video discusses the connection between drawing graphs and solving equations. We will be looking at finding the positive solution of the equation:

$$x^5 - x^2 - 2 = \frac{1}{x+1} + 5$$

Finding a solution of this by rearranging presents difficulties - try it! The solution we are looking for is $x \approx 1.581$

The method is to draw the graphs of

$y = x^5 - x^2 - 2$ and $y = \frac{1}{x+1} + 5$

Exercise B.3.3

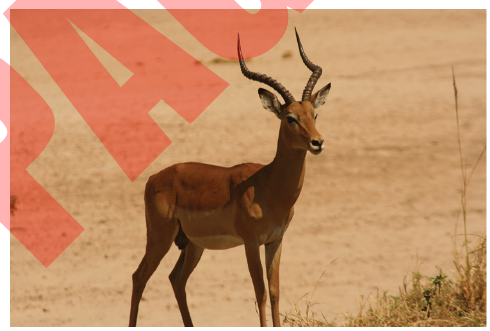
Sketch the graphs of the following functions showing the coordinates of all axis intercepts, maxima and minima.

1. $y = 2 - x$
2. $y = x^2 - x - 2$
3. $y = \frac{1}{x^2}$
4. $y = x^2 - 4$
5. $y = 2^{-x}$
6. $y = 1 - \frac{1}{x}$
7. $y = x^x$
8. $y = \sqrt{x-1}$
9. $y = \sqrt{\frac{1}{2-x}}$
10. $y = 2^{\frac{1}{x}}$

Applications

Mathematical models and their graphs are crucial to many scientific and technical activities.

The first deals with the modelling of animal populations.



Example B.3.6

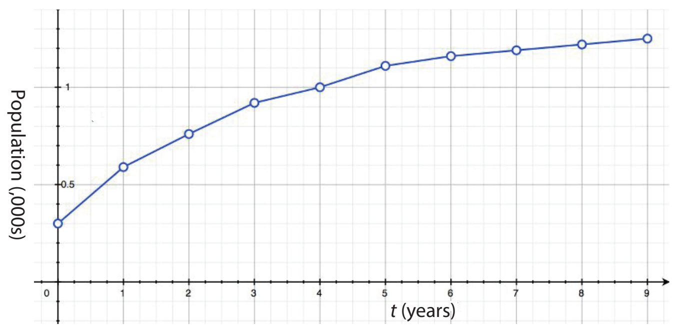
A new game park is stocked with a small population of gazelles. The population of animals is counted annually with the results shown in this table.

Year	Population (1,000s)
0	0.30
1	0.59
2	0.76
3	0.92
4	1.00
5	1.11
6	1.16
7	1.19
8	1.22
9	1.25

It is thought that the population is modelled by the function: $P(t) = 1.3 - 0.2^{0.2t}$

Discuss this graphically and explain what this means for the future of the population.

Graphing the raw data gives:



The straight line segments between the data points are not that useful and are produced by the graphing package used.

The general shape of the graph shows (amongst other things):

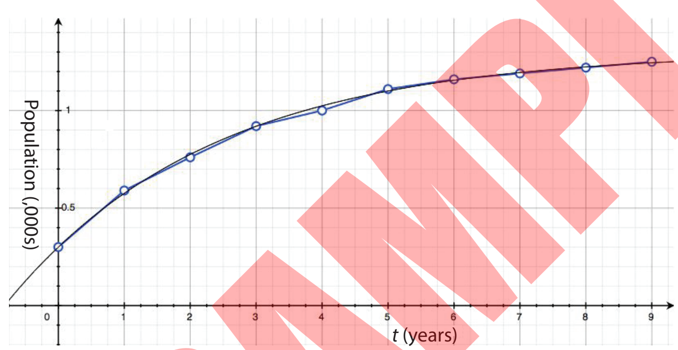
1. The initial 'seeding' population was 300 animals.

2. In the early years, the population rises quite quickly.

3. As time passes, the rise in population slows down.

The third point is probably due to the limited supply of food suitable for the gazelles.

Adding the given function to the graph gives:



The graph of the function fits the data points closely. This means that the modelling function works quite well.

One feature of the mathematical function is that, as t gets large, P approaches 1.3. This is asymptotic behaviour and suggests that the population will stabilise at around 1,300.

Of course, we always have to be careful about predicting the future.

Example B.3.7

A manufacturer spends \$1.2 million in research and development on a new washing machine.

The machines sell for \$350 each and cost \$70 each to manufacture

Find a mathematical function that models the profit the manufacturer will make.

At what level of sales will the manufacturer make a profit?

Let n be the number of thousands of machines made/sold.

Let P be the profit in \$1,000s

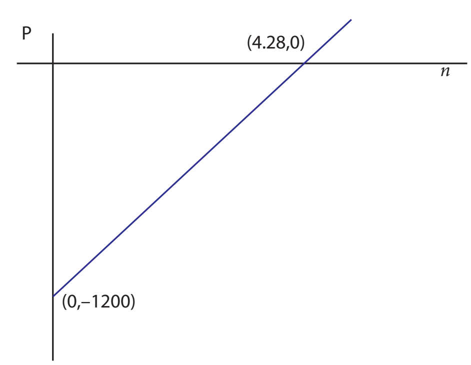
The costs are fixed and, as we are working in thousands of dollars, this is 1,200.

The income is \$350 per machine but there is also a cost of \$70 per machine. This means that the income is \$280 per machine. This is 0.28 thousands. However, we are counting our machines in thousands so, for each thousand machines, we make \$280 thousand.

The profit function is:

$$P = -1200 + 280n$$

This is linear:



The point at which the profit changes from negative (ie. a loss) is at approximately $n = 4.28$.

This means that the company will make a profit after it has sold about 4,280 machines.

Example B.3.8

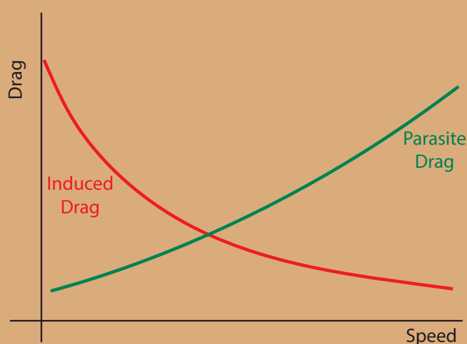
Aeroplanes experience drag due to air resistance. This drag resists the forward motion and is overcome by the engine(s).

There are two types of drag:

Lift induced drag that results from the need to provide lift to keep the aircraft flying. This decreases as the speed increases and the wings work more efficiently.

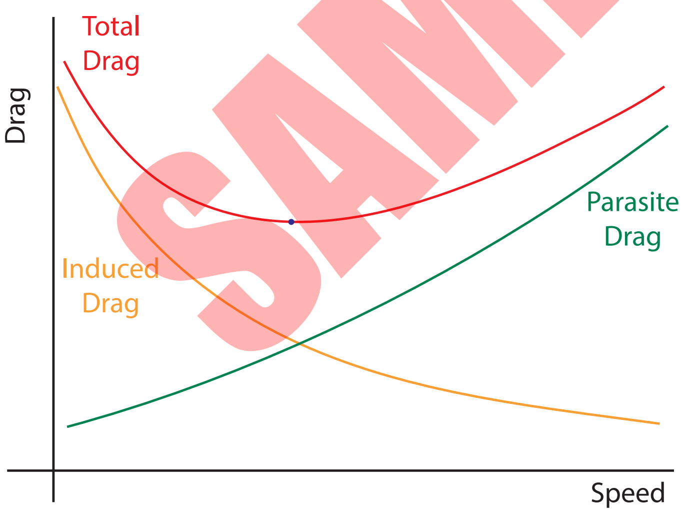
Parasite drag caused by friction as the air passes over the aeroplane. This increases with speed.

This is shown below:



Show the graph of the total drag and comment on its shape.

The total drag is obtained by adding the two functions in the same manner as we did in the shares and property example.



The total drag is at a minimum at the point marked by the blue dot. This is all that is required for comment in an examination. The following is for information.

This minimum is termed the 'glide speed' of the aeroplane.

If pilots experience an engine failure, they first adjust the airspeed to the correct glide speed for the aeroplane type. This maximises the range and the choice of sites for a forced landing.

Example B.3.9

An enzyme catalysed reaction proceeds at a rate that is affected by the temperature.

The table shows some measured rates at a range of temperatures:

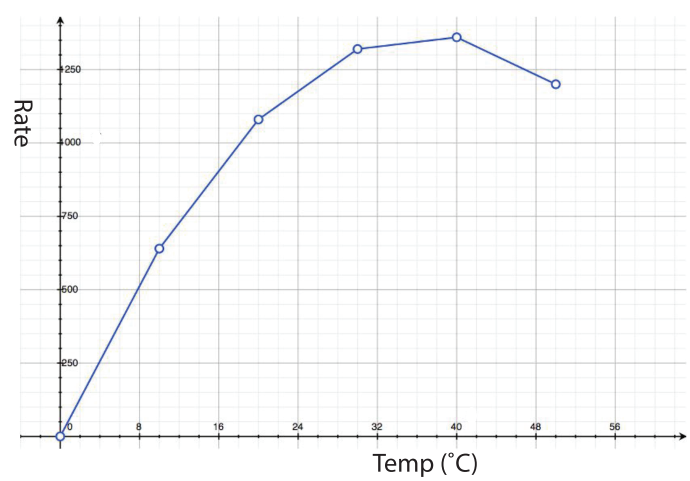
Temperature (˚C)	Rate
0	0
10	640
20	1080
30	1320
40	1360
50	1200

Show these data graphically.

It is hypothesised that the data can be modelled by a function of the form:

$$R(t) = at^2 + bt + c$$

where R is the rate, t is the temperature and a, b & c are constants. Find this function and test its fit to the data.



Note that the straight line segments joining the points are not very informative about this example.

As the graph passes through the origin, we can conclude immediately that $c = 0$.

Choosing two other points: (10,640) & (20,1080):

$$10^2 a + 10b = 640$$
$$20^2 a + 20b = 1080$$
$$100a + 10b = 640 \quad [1]$$
$$400a + 20b = 1080 \quad [2]$$

Halving [2] and subtracting from [1] gives:

$$100a - 200a = 640 - 540$$
$$-100a = 100$$
$$a = -1$$

Substituting this in [1] gives:

$$-100 + 10b = 640$$
$$10b = 740$$
$$b = 74$$

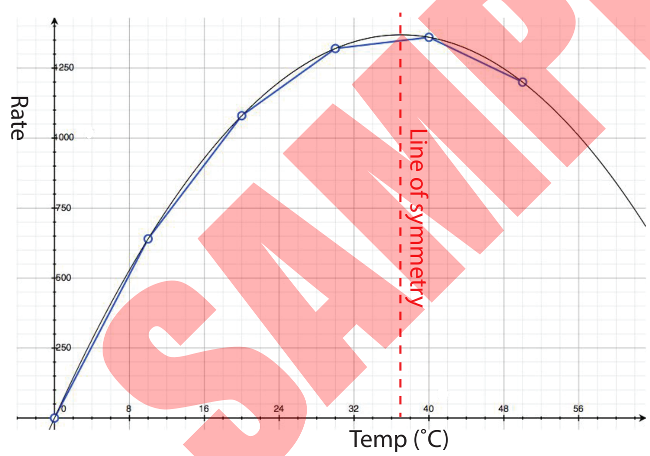
The modelling function is:

$$R(t) = -t^2 + 74t$$

Since this is a quadratic, it is symmetrical (it is a parabola).

The line of symmetry is temperature = 37˚C

This means that the optimum temperature for conducting this reaction is 37˚C.



The calculated function fits the data very well.

Exercise B.3.4

Use appropriate technology to draw graphs to illustrate these situations.

1. A population of antelope is small initially. Following good rains, the population grows rapidly until it reaches a new stable level.

2. Heat is lost from a cup of coffee most rapidly when it is freshly made.

3. Daylight hours over a year in Berlin.

4. A drug company has just developed a new treatment for asthma. Legislation guarantees the a period of time during which they alone can manufacture the drug. Following that, other manufacturers are able to copy the drug and market substitutes.

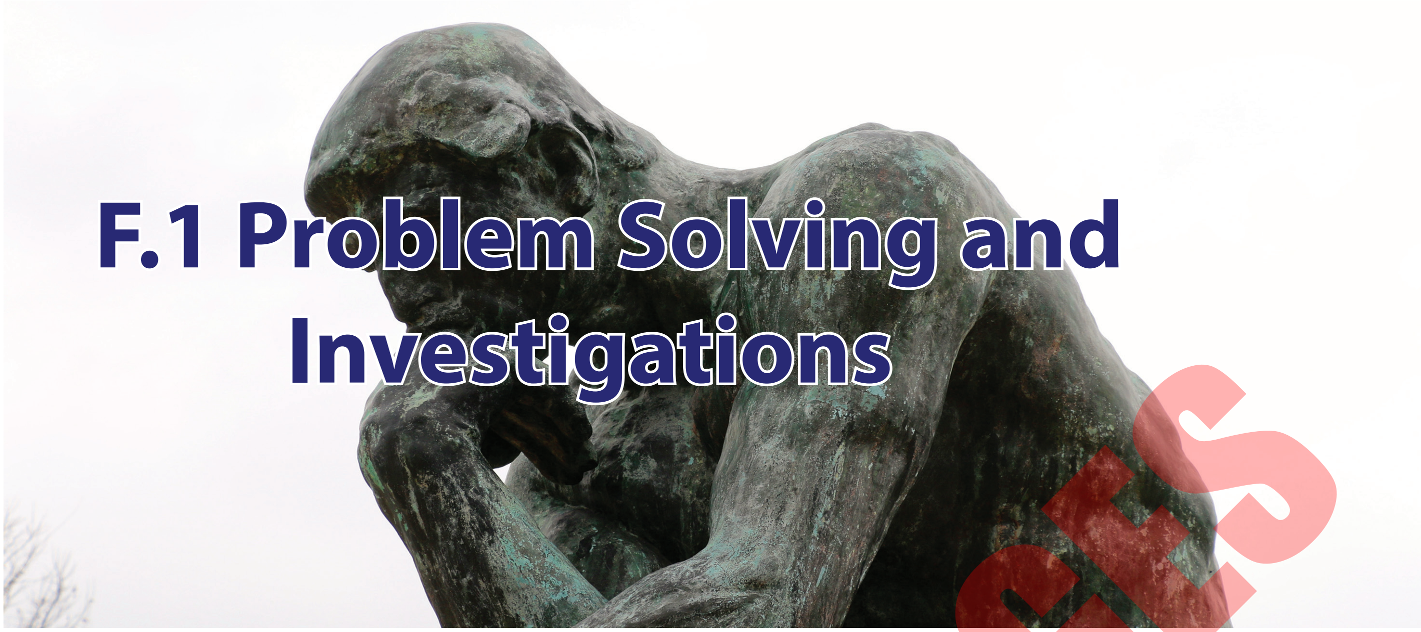
5. A long haul jet flight takes off and climbs to an initial cruise altitude. After being lightened by fuel burnoff, it climbs to long-term cruise before descent and landing. Live data can be found at https://www.flightradar24.com

6. Ice-cream sells better when the weather is hot. A multinational sells ice-cream in the southern hemisphere, the northern hemisphere and in the tropics. Show sales in these three regions and worldwide.

7. A sky-diver jumps from an aircraft and free falls before opening her parachute and descending to a safe landing. Show her vertical speed.

8. When scuba diving, nitrogen dissolves in the tissues of the diver. This rate gets bigger the deeper the diver descends. A diver descends rapidly to explore a wreck at a depth of 35 metres. He explores the wreck and then ascends slowly to a depth of 10 metres where he performs a decompression stop for 10 minutes. The purpose of this is to allow nitrogen to disperse slowly and hence avoid decompression sickness ('the bends') Finally he performs a 3 minute safety stop at 5 metres before returning to the surface. Show on a graph the depth profile and the nitrogen concentration.

Answers



F.1 Problem Solving and Investigations

This Chapter has been included to help students and teachers complete the 'Toolbox' component of each course. As this comprises 20% of the assessment (externally moderated) and 20% for SL & 12.5% for HL of the teaching time, it is a component that needs to be seriously addressed.

We divide this Chapter into two main sections.

1. **Problem Solving.** We will look at the meaning of this rather puzzling term. We will cover some of the common strategies used to solve problems and offer examples and practice questions.

2. **Investigations.** These require a written report that is externally moderated by IBO staff. We offer advice as to choice of topic, collection and analysis of data and preparation of a report.

Throughout this series of texts, students will find 'Toolbox' sections that provide further suggestions.

Problem Solving

In everyday life we encounter problems that we try to solve as best we can. The kettle has stopped working - use a saucepan on the cook-top. And so on, seemingly endlessly.

Mathematical problem solving is a bit different.

When you are learning the techniques of mathematics, it is usual to practise each technique by completing related examples. In the chapter on sequences and series, there was a section on summing arithmetic series. It was followed, surprise-surprise, by an exercise on summing arithmetic series.

Problem solving exercises tend to offer you a problem with no hints as to the techniques needed to solve it. This is probably the most challenging aspect of the task.

1. Don't Panic!

Just because you cannot see how to do a problem, don't just give up. Beware of staring at the ceiling hoping for the solution to drop into your head.

So what can you do instead?

Try to identify the information in the question. Once you have done that, attempt to get a version of it on paper, a table, a diagram, a graph, etc.

Identify the information that you are asked to find (ie. the question at the end).

Look for any information that you might like to have been told but were not. Above all, put it on paper.

Example F.1.1

Alice, Bella and Candice hold a series of races. They have a whole numbered scoring system with first place scoring more than second which scores more than third which does not score zero. There were no ties.

The first race was 100m. At the end of the competition, Alice had 20 points, Bella with 10 and Candice with 9. If Alice did not win the 200m, who did?

Try to solve this before turning the page.

There is not a lot of information in the question, so getting it on paper is vital.

We do know there are three girls and that there are at least two races. We also know the number of points gained. A table suggests...

	100m	200m	??	??	Total
Alice	Not 1st				20
Bella					10
Candice					9

That is all we know.

Next, what are we asked? - 'Who won the 200m?'

This means we have to work out all the results. At this stage, it seems impossible.

The next question is what don't we know? This is a really important question as it may tip us off as to an intermediate step.

Whilst we do know that there are three girls, we don't know how many races they ran. Hence the question marks in the table.

Importantly also, we don't know the scoring system. A table beckons here too:

	100m	200m	??	??	Total
1st					
2nd					
3rd					

We don't know any of the entries. Neither the number of races nor the scoring system.

So we now focus on what we do not know (the number of races and the scoring system) and compare it with what we do know. This is the points scored by the three girls (20, 10, 9).

What information is hidden here that will lead to our missing data?

If you have not already solved the problem, break off here and think again.

Welcome back!

The total points scored is 20 + 10 + 9 = 39. This has exactly one factorisation: 3 × 13 (other than 1 × 13). Prime Numbers!

What does this mean for the problem?

Either there are 13 races and the scoring system adds to 3 or there are 3 races and the scoring system adds to 13.

The scoring system cannot add to 3 so:

Major Breakthrough: there are 3 races and the scoring system adds to 13.

	100m	200m	??	Total
Alice	Not 1st			20
Bella				10
Candice				9
Total	13	13	13	39

Again, if you have not solved this problem, try again.

From here on, we are looking at one of the stalwarts of problem solving 'guess and check'. We will deal with this method in more detail later.

In essence, we are looking for three numbers that add to 13. Rather like a Sudoku, they must be filled into the above table to give the right totals for each row and column.

The only constraint is that Alice did not win every race.

The key with guess and check is to be systematic. This means that you need to check the possibilities in an ordered manner.

This is one possibility: 3rd: 1 pt, 2nd 2 pt 1st 10 pt. Having chosen the points for 3rd and 2nd (1+2) the points for 1st has to be 10 as the scoring system totals 13. Can this work? No. Just look at Alice who must come 1st twice and in another place in the third race.

How about 3rd: 1 pt, 2nd 3 pt 1st 9 pt? You cannot make 20 out of a set of 3 of these.

Work systematically through all of these (there are not as many as you may first have thought!) and you should arrive at the unique solution:

	100m	200m	??	Total
Alice	8	4	8	20
Bella	1	8	1	10
Candice	4	1	4	9
Total	13	13	13	39

See the Proof Chapter for the meaning of **Exhaustion**. There are not that many cases to check to establish that Bella won the 200m race. After you have solved the problem, it is wise to check back to the question. Have you answered the actual question asked?

2. Simplify the Problem

This is a close cousin to the 'Prime Directive' of 'Don't Panic'. Just to mix Sci Fi metaphors!

Many problems ask you to look at 'A Right Angled Triangle' or a 'Prime Number'. They then ask you to prove that some condition always holds. It can help to, in the example of the right angled triangle, look at a particular right angled triangle such as a 3,4,5. Can you prove it for that? If you can, does your proof generalise to every right angled triangle? This can be an easier proof as you have a strategy that worked for the particular case.

Our example works in a slightly different way. This technique is multi-faceted.

Example F.1.2

Find seven different positive integers a to f such that:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}+\frac{1}{f}=1$$

As before, try to solve this without looking at what follows.

As we have said, one of the challenges of this part of Mathematics is that the appropriate techniques may not be that obvious. In this case, it seems that addition of fractions using LCMs is unavoidable. But the LCM here is horrid.

One way of doing this is to look at a simpler version of the same problem.

For example: $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$

Suppose we look at choosing $a = 2$

It follows that $\frac{1}{2}+\frac{1}{b}+\frac{1}{c}=1 \Rightarrow \frac{1}{b}+\frac{1}{c}=\frac{1}{2}$

It does not seem so difficult to find two fractions in the 'one-over' form that add to one half.

For example since $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$ it follows that: $\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$

and so we have the next level of complication of the problem (three one-over fractions that add to one):

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$$

Continue to decompose the fractions until you have a (non-unique) solution to the problem.

3. Guess and Check

There are two golden rules here:

1. Be systematic
2. Learn from your mistakes.

The first of these means that you should choose your guesses sensibly, keep a record of the results and not find yourself checking solutions that fail more than once.

The second means that, when a guess fails, as it almost certainly will, why does it fail? Is it too big? Too small? Is it close or way off?

Our example illustrates this.

Example F.1.3

Five loaves of bread are weighed two at a time in all possible combinations. The weights in grams are 310, 312, 313, 314, 315, 316, 317, 318, 320 and 321. How much does each loaf weigh?

The first step is to make sure you understand the problem and the data you have been given. The key is to understand that the five loaves are weighed in pairs. Why does this lead to the 10 results?

Suppose the loaves are called A, B, C, D & E with the corresponding weights being a, b, c, d & e.

The weighing pairs are AB, AC, AD, AE, BC, BD, BE, CD, CE & DE. There are ten of these, corresponding to the data.

Can we simplify the problem? Since all the weights are 'three hundred and something', we could subtract 300 from each and then add 150 back in at the end. Remember that the loaves are weighed in pairs and all the answers will be 'one hundred and fifty something'.

This means that the pair weights become: 10, 12, 13, 14, 15, 16, 17, 18, 20 and 21.

Now ask what we do and do not know. There are 5 loaves, but are there two that weigh the same? Think about this and the consequences. Suppose C & D weigh the same. AC and AD would also weigh the same (as well as CE & DE etc.). As there are no repeated numbers in the pair weights, we conclude that all the weights are different.

Since all the weights are different, it may help to think of them arranged from lightest to heaviest:

$$a < b < c < d < e$$

From here on, it looks like guess and check will need to be employed. As we have said, it is vital to be systematic. Don't just have a wild guess, find it fails and follow up with another unrelated guess. Keep a record of your work so you don't end up going round in circles. For example, record your work in a table.

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1						
2						
3						
4						
5						
6						

Suppose we begin with $a = 1$ (ie. a loaf with weight 151). We now know that $b = 9$ because the two smallest numbers add to 10. Allowing the smallest possible numbers for the other loaves we get the first row as:

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No

The answer 'No' means that the given solution is not correct. This can be verified by looking at the two largest numbers 11 + 12 = 23 (not 21). Rather like we know the smallest numbers add to 10, we also know that the two largest numbers add to 21.

This means that there is no need to consider any more solutions with $a = 1$. What about $a = 2$, $b = 8$?

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No
2	2	8	9	10	11	

This does not fail the largest pair test as 10 + 11 = 21 which is correct. However, it is not possible to make 12 so this is another 'No'. Also, there is no need to check any more $a = 2$ solutions. Next, $a = 3$.

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No
2	2	8	9	10	11	No
3	3	7	8	9	12	

Note that the last pair is 9, 12 in order to make 21. However, this is not correct as we still cannot make 12. Another 'No'.

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No
2	2	8	9	10	11	No
3	3	7	8	9	12	No

However, we are not yet finished with $a = 3$.

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No
2	2	8	9	10	11	No
3	3	7	8	9	12	No
4	3	7	9	10	11	
5						
6						

This is another 'No' as there is still no way of making 12. Proceeding in this manner:

Guess	*a*	*b*	*c*	*d*	*e*	OK?
1	1	9	10	11	12	No
2	2	8	9	10	11	No
3	3	7	8	9	12	No
4	3	7	9	10	11	No
5	4	6	7	8	13	No
6	4	6	8	9	12	Yes

The sixth guess is correct: 4+6=10, 4+8=12, 4+9=13, 6+8=14, 6+9=15, 4+12=16, 8+9=17, 6+12=18, 8+12=20 & 9+12=21.

There is more work needed (not much) to establish that this solution is unique.

The weights of the five loaves are 154, 156, 158, 159 & 162 g.

4. Look for the Pattern

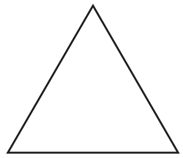
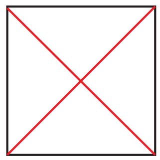
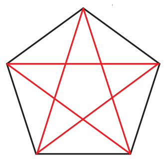
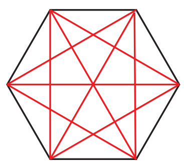
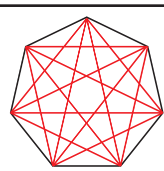
Pattern finding is possibly the most potent of all generally applied mathematical techniques. Some Mathematicians actually describe their job as 'pattern searching'.

Example F.1.4

Find the number of diagonals in a 100 sided convex polygon.

Actually drawing this is, of course possible.

However, a pattern search is easier.

Sides	Shape	Diagonals
3		0
4		2
5		5
6		9
7		14

Do we have enough data to look for a pattern?

One technique is to make a differences table:

Sides	Diagonals	1st diff	2nd diff
3	0		
		2–0=2	
4	2		3–2=1
		5–2=3	
5	5		4–3=1
		9–5=4	
6	9		5–4=1
		14–9=5	
7	14		

The difference in the third column is calculated by subtracting the upper brown cell from the lower orange cell. The same pattern is used throughout the table.

We have a constant second difference. What does this suggest?

If the first differences had been constant we would look for a linear relationship between the number of diagonals (d) and the number of sides (s).

This would mean a relationship of the form $d = a{\times}s{+}b$ where a and b are constants that we would need to determine.

Because we have a constant second difference, we should look for a quadratic relationship:

$$d = a{\times}s^2 + b{\times}s + c$$

There are three parameters a, b, c to determine. There are a number of ways of going about this. One is simultaneous equations

Triangle:	$0 = 9a + 3b + c$	[1]
Square:	$2 = 16a + 4b + c$	[2]
Pentagon:	$5 = 25a + 5b + c$	[3]
[2] – [1]	$2 = 7a + b$	[4]
[3] – [2]	$3 = 9a + b$	[5]
[5] – [4]	$1 = 2a$	

Thus $a = ½$

Substitute in [4] $2 = 7{\times}½ + b$

Thus $b = -1½$

Finally, using [2] $2 = 8 - 6 + c$

Thus $c = 0$

The rule is: $d = ½s^2 - {}^{3}/_{2}\, s$

It is a good idea to check the rule for a couple of cases:

Triangle:	$d = ½{\times}9 - {}^{3}/_{2}{\times}3 = 0$ correct
Pentagon:	$d = ½{\times}25 - {}^{3}/_{2}{\times}5 = 5$ correct
Hexagon:	$d = ½{\times}36 - {}^{3}/_{2}{\times}6 = 9$ correct

An alternative is to look for patterns in the diagrams.

You may have noticed that the number of vertices is the same as the number of sides (s). Each vertex 'sprouts' $s - 3$ diagonals. The product of these is $s(s - 3)$. But this counts each diagonal twice so:

$$d = ½s(s - 3) = ½s^2 - {}^{3}/_{2}\, s$$

Finally, to answer our question, if $s = 100$, $d = 50(100 - 3) = 4850$. Note that we have not had to work out all the answers up to the 100 side case but have gone straight to the answer.

These are not the only techniques used to solve problems, however, they are frequently useful.

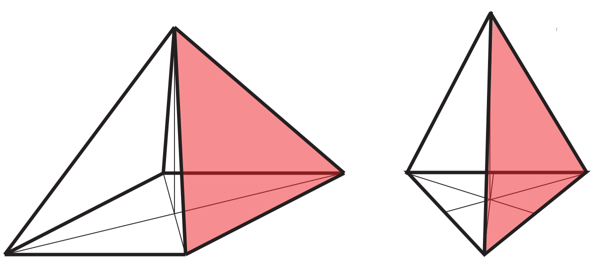
Now try some of these problems. We have made suggestions as to necessary prior knowledge, but these are only suggestions. As our last example showed, there can be more than one solution to a problem and some may be more 'elegant' than others.

Problem 1

A Sticky One

Assumed Knowledge: Core Chapter C1.

A square based pyramid and a regular tetrahedron are such that their triangular faces are congruent equilateral triangles.

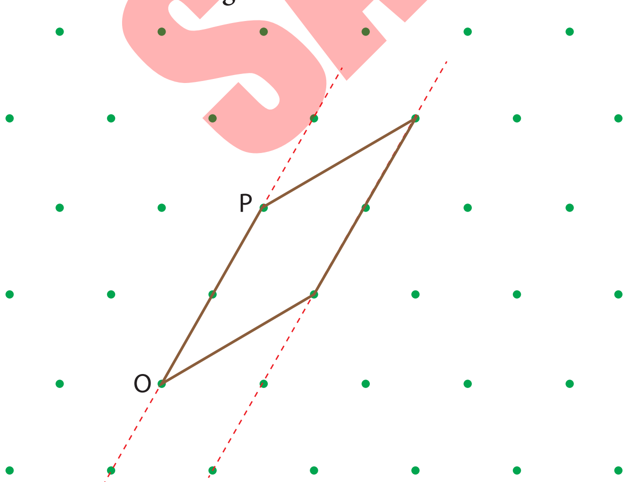


The red shaded faces are identical. If they are glued together, how many faces does the resulting solid have? The answer is not 7. Paul Halmos (one of the great writers on Mathematics - Bing/Google him) relates that this problem was set in a College Entrance Exam with the answer 7. A student with an alternative answer took the matter to court and won. This becomes (probably) the only Mathematics Problem whose solution has legal backing!

Problem 2

Lattice Parallelograms

Assumed Knowledge: MYP.



The diagram shows lattice points from a portion of a triangular lattice which is made up of successive equilateral triangles with sides of unit length.

A line is drawn from the origin O to a lattice point P.

Another line is drawn parallel to the line OP, so that there are no lattice points between the two parallel lines.

A parallelogram is formed so that the four corners of the parallelogram are at lattice points lying on the two parallel lines, and no lattice points, other than those at the corners, are on the boundary of the parallelogram.

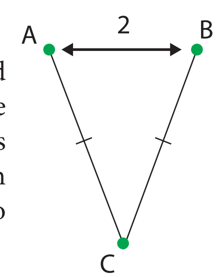
Calculate the area of any parallelogram which can be drawn in this way.

Problem 3

Fibre Optics

Assumed Knowledge: Core Chapters C1-C3.

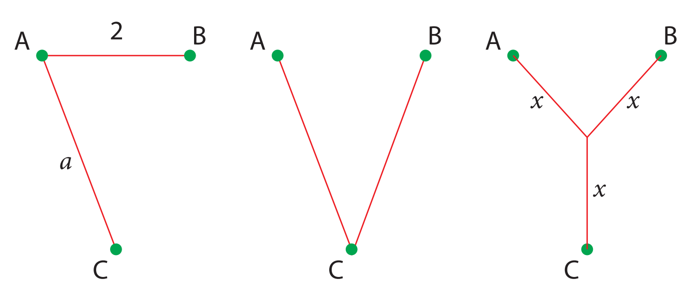
Part A



Engineers from a data link company need to provide a fibre optics link between three towns A, B and C which form an isosceles triangle with AB = BC. The engineers wish to use the shortest possible cable length to join the three towns.

Instead of just solving the problem for the particular towns being considered, the engineers would like to obtain a more general result which could be used if a similar situation arises in the future. To do this they propose the following model: fix the distance between A and C at 2 units and change the isosceles triangle by moving B along a line perpendicular to AC and through the mid-point of AC.

The engineers have put forward three plans which they believe will be useful in solving the problem. These plans, showing the cable links, are shown below.

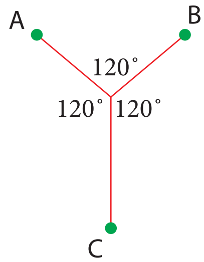


In the 'C-plan', cable is run from B to A to C. In the 'V–plan', cable is run from A to B to C. In the 'Y-plan', cables of equal length are run to A, B and C from a point between the towns.

For different allowable positions of B, when should a given plan be used to obtain the shortest cable length?

Part B

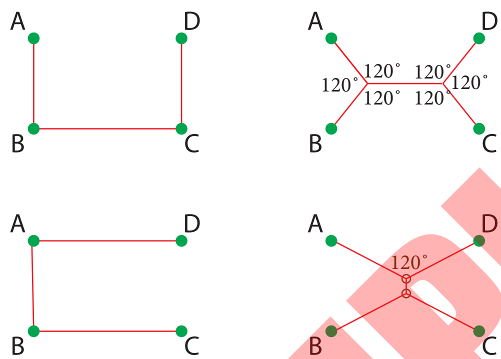
A consulting engineer suggests another option: to link the three towns using a modified Y-plan in which the cables are joined so that the angles between them are each 120°. Is this plan better than any of the three plans in part A?



Explain your answer fully.

Part C

The consulting engineer is working on the problem of connecting four towns A, B, C and D with cable. She is considering four plans which are shown below:



Suppose the distance between A and B is fixed at 2 units and we change the rectangle by adjusting the distance between B and C.

If the engineers want to obtain the shortest cable length, which plan should be used (consider all possible values for part A?

Problem 4

Buried Treasure

Assumed Knowledge: Chapter C2.

You have found a map and instructions to find treasure buried on an island on which are located a water tower, a large oak tree and a peppercorn tree. The instructions include the following directions.

‘When you stand at the water tower you should be able to see the large oak tree ahead of you and somewhat to the left, and the peppercorn tree ahead of you and somewhat to the right.

Starting from the water tower, walk directly to the large oak tree, counting the number of steps taken. Turn left through an angle of 90° and walk the same distance away from the oak tree. Mark this spot.

Do this again by starting at the water tower, walking in a straight line to the peppercorn tree and counting the number of steps taken, turning right through an angle of 90°, walking the same distance away from the peppercorn tree and marking the spot.

Half-way along a straight line joining the two marked spots lies the treasure.’

Unfortunately, when you get to the island, the oak tree and the peppercorn tree are plainly visible, but the water tower is nowhere to be seen. It appears that the tower has been demolished and all traces removed. Where is the treasure buried?

You calculate the spot where you think the treasure is buried and have been digging unsuccessfully for some time when you remember that your compass was out of alignment and was not measuring angles accurately.

The angle at the oak tree through which you had turned left was only 85° and the angle at the peppercorn tree through which you had turned right was really 95°. Your compass has now broken down completely and you have no way of accurately determining a right-angle. What can you now do in your attempt to find the treasure?

In your concern to find the treasure you become exhausted and confused and you lose the map in a sudden gust of wind. You think that the instructions were as follows.

‘Starting from the water tower, walk directly to the large oak tree, counting the number of steps taken. Turn right through an angle of 90° and walk twice the same distance away from the oak tree. Mark this spot. Do this again by starting at the water tower, walking in a straight line to the peppercorn tree and counting the number of steps taken, turning left through an angle of 90°, walking twice the same distance away from the peppercorn tree and marking the spot.

Halfway along a straight line joining the two marked spots lies the treasure.’

If you were able to find this mid-point, how far from the treasure would you be if the oak tree and the peppercorn tree are 50 metres apart?

Problem 5

Easter Sunday

Assumed Knowledge: MYP.

In theory, Easter Sunday occurs on the first Sunday after the Paschal full moon, which is the first full moon in Jerusalem after 21 March. In practice, the scheduled date of Easter Sunday in each year is determined by a formula specified in Christian literature.

A simpler formula was derived by the mathematician C. F. Gauss (1777–1855), which gives the same date as the scheduled date for every year this century except for 1954 and 1981. The formula derived by Gauss involves the use of the symbol a mod b which means the remainder when a is divided by b. For example, 18 mod 7 is equal to 4 since 18 divided by 7 gives 2 with a remainder of 4.

Gauss's calculation of the date of Easter Sunday is as follows.

For the year which is x years after 1900, for example the year 1931 has $x = 31$, the first full moon occurs c days after 22 March where:

$$c = [19\,(x \bmod 19) + 24] \bmod 30$$

The following Sunday, Easter Sunday, occurs d days after the full moon where:

$$d = (2a + 4b + 6c + 3) \bmod 7$$

with $a = x \bmod 4$, $b = x \bmod 7$, and c defined as before.

You will need to make use of the following information to answer the questions below.

1. In 1900 the full moon occurred 24.07 days after 22 March. This was a Sunday.

2. The time between two full moons is 29.53059 days.

Use the Gauss formula to calculate the date of Easter Sunday for each year in the period 1990–1999.

Explain the reasoning underlying the formula for c relating it to the full moon cycle.

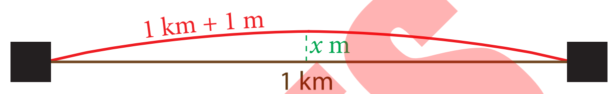
Now let c be any number of days, from 0 to 29 inclusive, after 22 March. Show that the first Sunday after this date occurs in a further d days, as given by the Gauss formula. In the cases for which this date is already a Sunday, show that the Gauss formula gives $d = 0$.

Problem 6

Pipes

Assumed Knowledge: Estimation and Geometry

A straight length of pipeline is fixed at both ends and is 1 km long. On a very hot day, the pipe expands by 0.1% (1 metre). As a result it bows into the arc of a circle. See the diagram below, which is not drawn to scale.



1. Guess the size of x.

2. Use a straight line approximation and Pythagoras to estimate the size of x.

3. Calculate the size of x.

Problem 7

Sparse Matrices

Assumed Knowledge: Matrices - Mathematics: Applications and Interpretation syllabus topic AHL 1.14.

Find:

1. $\sqrt{\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}$

2. $\sqrt{\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}$

Problem 8

Watermelons

Assumed Knowledge: MYP.

Watermelons are 99% water by weight.

500 kg of watermelons are stored over the weekend. On Monday, they are 98% water.

How much do they weigh on Monday?

Problem 9

Tilings

Assumed Knowledge: MYP.

Roman mosaic floor, Morocco

The tiling pattern on this mosaic floor contains triangles, squares, hexagons and rhombuses.

Consider the set of regular shapes:

{equilateral triangles, squares, regular pentagons, regular hexagons, rhombuses}

The set {equilateral triangles, regular hexagons, rhombuses} is said to be a subset of the original set. There are $2^5 = 32$ subsets (including the empty set and the complete set).

Investigate these and classify them according to whether they will or will not form a tessellation if all shapes in the subset are used in the pattern.

Another floor from the same site, Morocco

Problem 10

Tablets

Assumed Knowledge: Probability.

Carol has a bottle containing 18 tablets. She has been told by her doctor that, each day before breakfast, she should take one and a half tablets. On the first day she takes two tablets from the bottle, breaks one in half and puts the unused half back in the bottle. On a subsequent day, perhaps she might take two whole tablets and break one of them, or she might take a whole and a half or three halves so as not to have to break any tablets that day.

Suppose that on day d, before Carol takes her daily dose, there are W whole tablets and H half tablets in the bottle.

The full bottle corresponds to day zero.

a i On day zero there is only one possibility for the mixture of whole and half tablets, namely $W = 18$, $H = 0$.

How many possibilities are there on day d?

ii More generally, if there were N tablets in the bottle initially, find a rule or set of rules which will tell how many possibilities for the mixture of whole and half tablets there are on day d. The case $N = 18$, $d = 10$ is a good one to test your rule. Give the answer in this case, and similarly give several more cases which for one reason or another you think are good ones to test your rule.

b In going from the starting situation of N whole tablets to the final situation where a daily dose cannot be taken, the sequence of possible values for (W, H) shall be called a 'course'.

For example, starting with 14 tablets, one possible course is: $(14, 0) \to (12, 1) \to (10, 2) \to (9, 1) \to (7, 2) \to (5, 3) \to (5, 0) \to (3, 1) \to (2, 0) \to (0, 1)$.

How many possible courses are there starting with:

i 12 tablets?

ii 13 tablets?

iii 14 tablets?

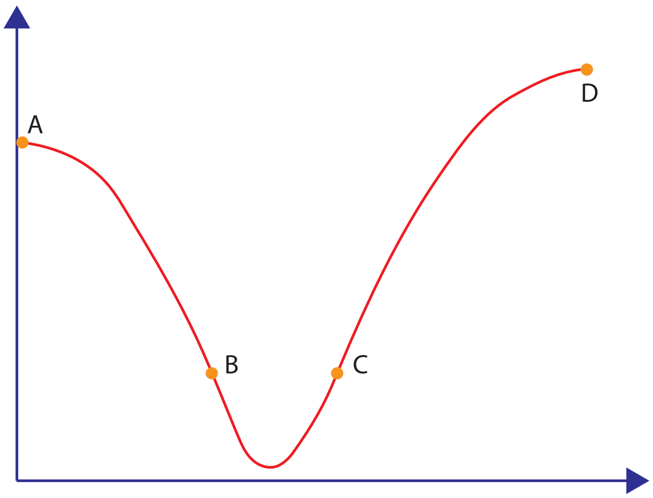
iv 15 tablets?

Problem 11

Wild Ride

Assumed Knowledge: Functions and Calculus.

The track of a section of a switchback railway at an amusement park is in disrepair and needs replacing. Viewed from the side, the track has the shape shown in the following diagram.



The profile of each of the segments AB, BC and CD is a parabola. The gradients of the profiles at the points B and C, where the segments meet, match so that the track is smooth.

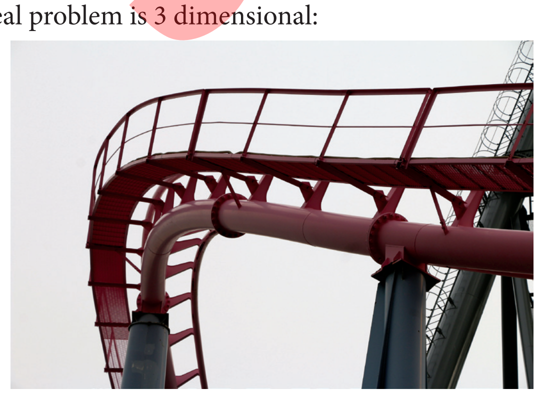
The track at point A is 30 metres above ground level and inclined at an angle of 10˚ below the horizontal, while the track at point B is 10 metres above ground level and 8 metres horizontally across from A. The point C is at the same height as B and 11 metres across horizontally from A. The track at point D is 35 metres high and has zero slope.

The new track costs \$100 per metre. Devise a method to estimate the cost of the new track.

Draw diagrams to scale.

Consider where the curved ramps meet the ground or other obstacles. How do the designers of these circuits ensure a ‘smooth’ ride for the riders?

The real problem is 3 dimensional:

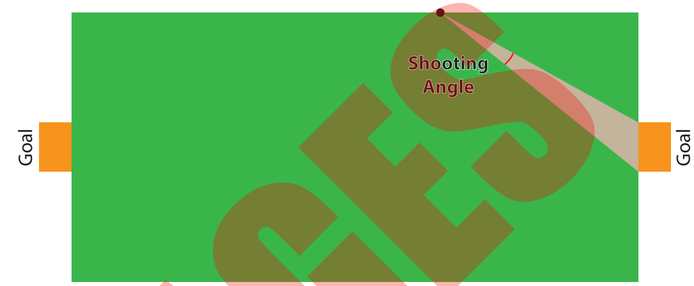


Problem 12

Hockey

Assumed Knowledge: Geometry, Functions.

Hockey is played on a rectangular playing field with a goal area at each of the shorter sides of the rectangle. A player on the long boundary, as shown in the diagram below, wants to shoot a ball through the goal.



What position on the boundary will give the player the biggest shooting angle through the goal?

If the field was circular instead of rectangular, what position on the boundary would give the player the best shooting angle through the goal?

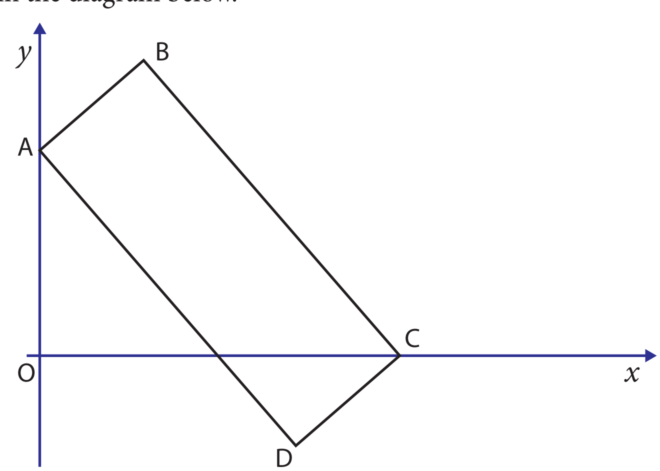
What is the best position on the boundary to shoot from if the field is elliptical?

Problem 13

Sliding Rectangles

Assumed Knowledge: Coordinate Geometry.

A rectangle ABCD is placed on the Cartesian plane as shown in the diagram below.



The rectangle is free to slide so that A is always somewhere on the positive y-axis and C is always somewhere on the positive x-axis.

As the rectangle moves, what path will the point B follow? Explain your results.

What will be the path of the point D?

If the point A can move over both the positive and negative parts of the y-axis, and the point C can move over both the positive and negative parts of the x-axis, what paths will the points B and D follow?

What effect will changing the dimensions of the rectangle have on the paths of points B and D?

Problem 14

Epidemic

Assumed Knowledge: Functions, Sequences & Series.

One Sunday evening, five people with infectious influenza arrive in a large city with a population of about two million. They go to different locations in the city and thus the disease begins to spread throughout the population.

At first, when a person becomes infected he or she shows no sign of the disease and cannot spread it. About one week after first catching the disease, this person becomes infective and can spread the disease to other people.

This infective phase also lasts for about one week. After this time the person is free from the influenza, although he or she may catch it again at some later time.

Epidemiologists are trying to model the spread of the influenza. They make a simplifying assumption that the infection progresses in one week units. That is, they assume that everybody who becomes infected does so on a Sunday evening, becomes infective exactly one week later, and is free of infection exactly one week after that. People free of the disease are called susceptibles.

The epidemiologists also assume that the city population is large enough so that the population size is constant for the duration of the disease. That is, they ignore births, deaths and other migrations into or out of the population.

Finally, they assume that each infective person infects a fixed fraction f of the number of susceptibles, so that:

1. the number of infectives at week $n+1$ is equal to $f \times$ (the number of susceptibles at week n) $\times$ (the number of infectives at week n).

and

2. the number of susceptibles at week $n+1$ is equal to (the number of susceptibles at week n) + (the number of infectives at week n) – (the number of infectives at week $n+1$).

Why must the number of infectives, plus the number of susceptibles, be constant from week to week?

Choose values of f between $\frac{1}{10^6}$ and $\frac{2}{10^6}$ and use this model to show how the number of infectives changes from week to week from the Sunday when the five infective people arrived.

What limiting values does the model predict for the number of infectives?

Will there always be a limiting value for the number of infectives? If so, how are the limiting values related to the population size and to f?

For what values of f will it be possible to have a situation where the number of infectives eventually oscillates between two values?

Problem 15

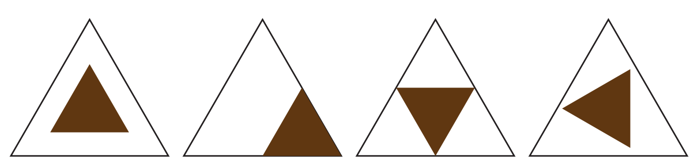
Chocolate Wrapping

Assumed Knowledge: Geometry.

A triangular slice of chocolate has to be wrapped in a triangular piece of paper, which cannot be cut or torn. The chocolate is sliced so thinly that you can ignore its thickness.

Both the chocolate and paper are equilateral triangles. The paper can be folded along an edge of the chocolate. The edges of the wrapping do not have to overlap, they can just meet.

The chocolate can be positioned in various ways on the paper. Some examples appear in the diagram below:



When the chocolate has a side length of 4 centimetres, what is the smallest side length of the paper that will allow the chocolate to be wrapped?

How does the solution differ if the chocolate and paper are both rectangular (not necessarily similar rectangles)?

What optimization issues arise when dealing with such a situation?

Problem 16

Area and Perimeter

Assumed Knowledge: Arithmetic, Geometry, Functions.

The following question was posed to a group of mathematics students.

> 'Are there shapes for which the numerical value of the length of the perimeter is the same as the numerical value of the area?'

One student quickly saw that a square is a shape with this property because a square which has a side length of 4 units has a perimeter of 16 units and an area of 16 square units. The student could also easily show that there could only be one square with this property.

After looking at families of shapes like triangles, circles, rectangles and other polygons the students made the following conjecture.

> 'For every family of shapes there is at least one of these shapes for which the numerical value of the area and the numerical value of the perimeter are the same.'

By a 'family of shapes' the students meant all shapes which are similar to a given shape. For example there is only one family of squares, but there is an infinite number of families of rectangles.

You are required to find the following.

For which shapes does the conjecture hold?

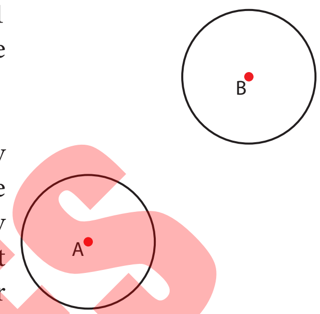
For each class of shapes for which the conjecture holds, give a method for finding an actual shape for which the numerical value of the area is the same as the numerical value of the perimeter.

Problem 17

Circles upon Circles

Assumed Knowledge: Geometry.

Here are two circles of radius 1 with centres A and B, which are 4 units apart.



For a given pair of circles, any circle that intersects each of the original circles twice, with every intersection forming a right angle, we will call perpendicular to that pair.

For the pair of circles pictured:

a

i give a precise description of a circle, perpendicular to the pair, with centre on the line joining A and B.

ii give a precise description of any other circles perpendicular to the pair.

iii comment on any noteworthy features of the family of all circles perpendicular to the pair.

b Consider the circles, perpendicular to the pair, with centres above the line joining A and B.

Each such circle has a point furthest from the line joining A and B, which we will call 'the top'. Suppose that the spacing between two such circles is defined to be the distance between their tops. Find and describe a family of 'equispaced' circles, all of which are perpendicular to the original pair of circles.

c. Now consider the following. Each of the circles considered in part b has a counterpart obtained by reflection about the line joining A and B. If we now take one of the circles from part b and its reflected counterpart to form a pair of circles, describe the family of circles perpendicular to this pair and comment on any noteworthy features.

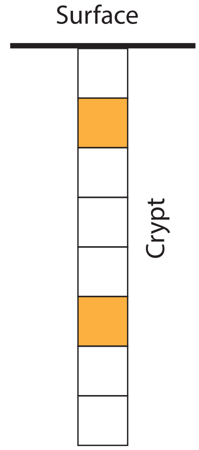
Generalize your results in part a to the case where the original pair of circles has unequal radii.

Problem 18

Crypts

Assumed Knowledge: Statistics.

A crypt is found in the human body. It is like a tube of cells which descends from the surface of the colon. The figure opposite shows a highly simplified picture of a crypt which in this case is 10 cells long.



In this picture the shaded boxes show cells which are undergoing cell division. Such cells are found by a staining technique which makes them readily apparent under a microscope.

Each clear box shows a cell which is not stained and thus is not undergoing cell division.

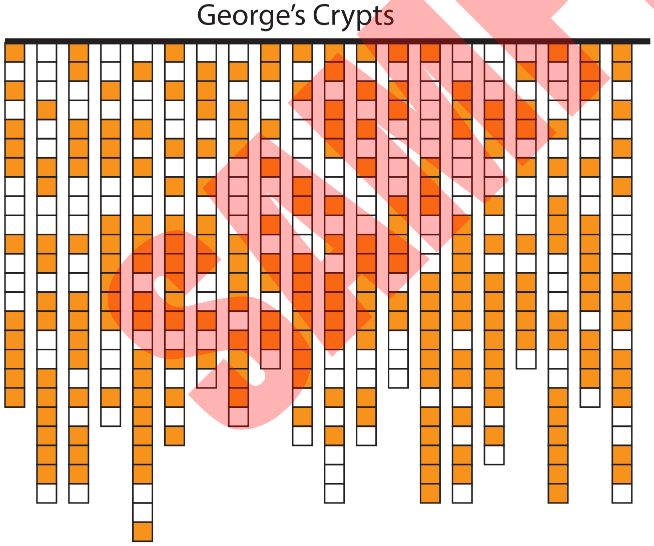
It is thought that a high-fibre diet may alter the way in which cell division occurs within a crypt.

The diagrams below show 20 crypts selected randomly from the colon of George, who has a normal diet.

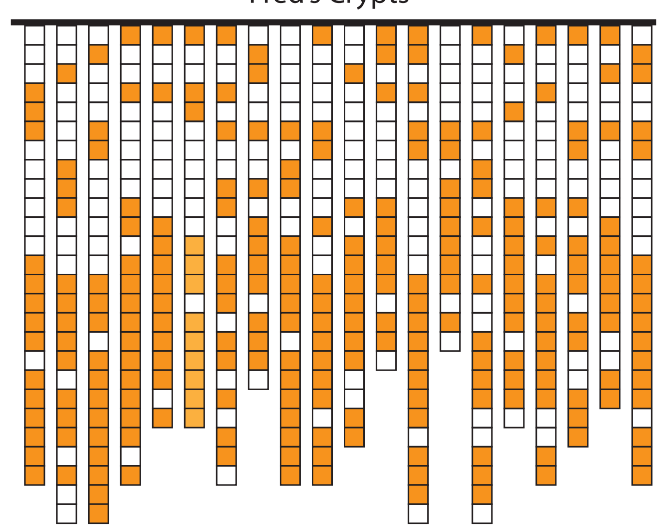
They also show 20 crypts selected randomly from the colon of Fred, who has a high-fibre diet.

Find a good descriptive variable which indicates that statistically George's crypts are in some way rather different from Fred's crypts. Show also your analysis of other descriptive variables which do not indicate any significant difference between George and Fred.

George's Crypts



Fred's Crypts



Investigations

We begin this section by observing that the capabilities we are trying to develop are important life skills. We hope that you will use them as you face challenges in your personal and working lives.

Questions that face many of us at some stage in our lives: which job to apply for, which car to buy, which course to take, whether to buy or rent a home etc. may seem simple. Some are, but most are not. Some of these carry penalties if you make a poor decision.

We are suggesting that, when faced with an important decision, you should follow the process that we hope you will learn in this part of the course. That said, many of history's most valuable investigations (into, for example, the causes of disease, navigation, atomic structure etc.) have been undertaken more out of interest than in the expectation of personal gain.

The main steps in the process are:

1. **Define the problem**. Should I buy or rent a flat? How should I finance it? Where should my company buy its paper, cars, iron ore, legal advice …?

2. **Identify the parameters** that define a 'good decision'. These may be all sorts of things such as price, status value, fast service etc.

3. **Research appropriate data**. What are the fees? How powerful is the engine? etc.

4. **Analyse the data**. This is where maths generally comes in!

5. **Arrive at a decision** and be prepared to justify it to an employer, partner, voter, etc. A good analysis works wonders here! This is known as evaluation. You will probably end up living with the decision (occupying the house, paying for the car, using the software) for some time, and so it is very much in your interest to remember this step!

There are many ways to conduct a successful investigation. This chapter is intended to give you ideas. It is not intended to provide a recipe or set of 'how to do it' instructions. Nor does it provide a complete investigation!

We will look at investigation ideas in the areas of probability, functions and statistics.

To understand these you will need to have covered the relevant chapters:

Idea 1: Monte Carlo methods (Probability).

Idea 2: Damped oscillations (Trigonometric and Exponential Functions, Calculus).

Idea 3: Weights in lifts and transport (Statistics).

Choice of Topic (defining the problem)

Try not to spend too much time on this! We have provided investigations suggestions in two forms:

1. A few suggestions at the end of each chapter, but you should not feel bound by these.

2. A list of investigation 'themes' at the end of this chapter. We have adopted a 'themes' approach so that the investigations would not be prescriptive but, rather, allow you to use them as a springboard to topics of rich mathematical content and of interest to you.

However, some of the best investigations that we have seen have arisen from quite simple questions such as:

Is it true that petrol prices go up at the weekend?

OR

Will a coin that is spun on its edge fall equally heads and tails?

Try to avoid questions that are of no interest to you personally. If you are an artist, consider looking at the mathematics of perspective. If you are keen on a sport such as sailing, look at the mathematics of navigation.

Equally, avoid questions that are pointless. Collecting the sizes of the feet of a group of students is pointless. However, imagining that you run a shoe shop and want to know how many of each size of shoe to stock is not!

So, try to pick a simple, relevant question that is in an area that interests you. Then discuss your choice with your teacher.

Of course, your investigation will also be assessed and graded. So, it is important that you are familiar with the 'Internal Assessment Criteria' that will be used to grade your investigation work.

What will define a good decision?

This will depend heavily on your choice of problem and may be as simple as 'the ship will not run aground', but will probably be complicated by a requirement such as 'will be cheaper than my competitor's ship whilst still not running aground'. Do not get over-complicated here as you may make the next step too difficult!

Collecting Data

If you want real data (and we suggest that you do!), the internet is an excellent starting point. Our three case studies give examples in which data came from this source as well as other possibilities. There is nothing wrong with collecting your own data. Remember, however, that the data that you collect must be relevant to the problem you are working on.

Also, there are some dangers to the collection of data. Here are a few:

1. It is a general principle that the process of collecting data affects the results that you get.

A simple question such as

"How many people are in the room at the moment?"

may seem to be easy to answer exactly. However, do you know if there is a person hiding in a cupboard?

2. Measurement also affects the quantity that it is trying to measure.

The act of putting a cold thermometer into a hot cup of coffee lowers its temperature. The only way to prevent this is to know the temperature of the coffee and to heat the thermometer to that before putting it into the cup.

But, if we know the temperature to start with, why are we wasting time measuring it? The sharp eyed amongst you may also see the small bubble in the fluid. This makes this thermometer read about 1˚C too much all the time. This is known as a **systematic error**.

The size of these errors depend on a variety of factors.

It will be very small if we use a thermometer to measure the temperature of a swimming pool, particularly if we guess that it is likely to be about 25°C and warm the thermometer up to this temperature before using it.

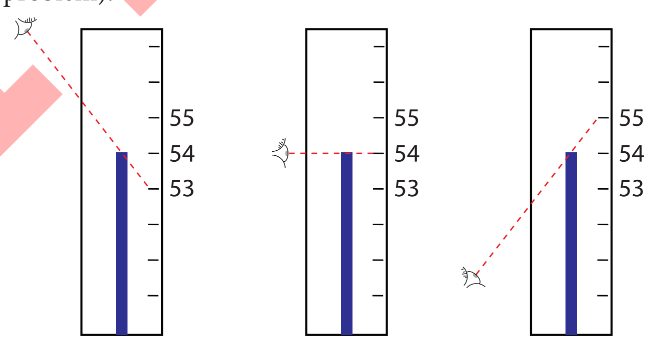
Of course, in the case of the swimming pool the 'cooling error' will be much less than the other main source of error: instrument inaccuracy. Thermometers cannot measure temperature exactly. Some are better than others. The thermometer shown cannot be relied on to measure to better than 1°C. More accurate instruments are more expensive and none are EXACT.

3. Users can also introduce errors by misreading an instrument.

The photograph in the previous column shows a view of the thermometer from above. It suggests that the coffee is at about 78°C.

However, the scale lies behind the liquid column which contains the possibility of a PARALLAX error.

Parallax error is illustrated next (the diagram exaggerates the problem).



View from above, 53°, answer too small. | Level view, 54°, correct answer | View from below, 55°, answer too big.

The second of our three investigation ideas measures the position of an object using a fairly crude method that is subject to the parallax error. It would be important to recognize that this limits the accuracy of the measurements made.

4. Problems can also arise when we ask people questions. There are two main reasons for this. The first is the possibility that people may lie to us.

Suppose you are approached by a researcher in the street and asked,

"Did you watch the 9 o'clock news on ABC last night?",

you will probably tell the truth.

If, however, this stranger asks:

> "How much do you weigh?" or "How many friends do you have?",

you may well be less likely to tell the truth by 'massaging' the answer to be closer to what you would like it to be.

The other problem is that your questions may inadvertently affect the answers people give.

The question:

> "Will you be going to cheer on the school basketball team in their final on Saturday?"

may well get a different answer from the question:

> "Will you be going to the Rock-Concert or the basketball on Saturday?"

5. Sequences of questions can also affect answers:

Suppose you are leading up to the question:

> "Will you be voting for President Clover in the election on Saturday?"

and you lead up to this question with:

> "Are you aware that President Clover has increased the health care budget by 8%?"
>
> "Do you approve of President Clover's new measures to protect the environment?"
>
> "Will you be voting for President Clover in the election on Saturday?"

You will probably get a higher proportion of positive replies than if you use this sequence:

> "Are you aware that President Clover has decreased the police budget by 8%?"
>
> "Do you approve of President Clover's frequent trips to resorts in the Caribbean?"
>
> "Will you be voting for President Clover in the election on Saturday?"

If you intend to use a questionnaire, you should be aware of these problems and your report should underline the steps that you have taken to minimize their effects.

During the Episode *The Ministerial Broadcast* of the BBC comedy show *Yes Prime Minister*, Sir Humphrey illustrates the issue of leading questions in a brilliant sequence.

Many questionnaires include 'lie detector questions'. One of the best of these is: "Have you ever told a lie?" Would you trust anyone who replied "no" to this question?

Many investigators make use of information derived from the internet. Whilst much of this has been collected by professional researchers who are skilled at minimizing the problems described above, the information should not be treated as completely reliable.

Analysing the data

Following this, you should use mathematics to analyse your data. Analyse means 'pull apart and find how it works'.

This is a mathematics project and you should use mathematics contained in the course. Your teacher should help you determine if your problem is beyond the scope of the course.

Remember to relate the mathematics that you use to your original question. If you are looking at the question, "Do petrol prices go up at the weekend?", you will have already collected information on 'petrol prices' and you will have noted that since you, presumably, have not visited every petrol retailer in the World, there are limits to the validity of your data. What mathematics is it appropriate to use here? There is a temptation to, for example, work out the average price as this is reasonably easy. But does this help you address your question? Far more relevant would be a price/time graph with the weekends clearly identified. Then, you can start to look at whether you have a pattern of steady increase (a trend) or whether the price is stable and 'spikes' at the weekend (or has both a trend and spiking).

It is also important to know what your mathematical results are telling you. If you, for example, calculate a trend line for petrol price data, make sure that you have understood the implications of the calculation before drawing any conclusions from it.

Conclusion

Finally, you should reach a conclusion, related to the question that forms the subject of your investigation. This should be evaluated. In some respects, this is the hardest part of the project.

What do we mean by 'evaluation'? Some decisions have a right answer. If you are playing 'noughts and crosses' there is always a best move that you should make, whatever the game situation. Chess, and most of life, is different. Some moves are better than others but we do not currently (and probably never will) know the best move in every situation. Your project will probably be like this. Your conclusion may be wrong for a variety of reasons including incomplete information. Evaluation means that you have looked at these possibilities!

Try also to avoid looking for the data to confirm your previous opinion. If you ask a motorist "Do petrol prices go up at the weekend?", you may well get the answer "Sure, everyone knows that!". This means that, if your evidence points to the truth being that prices do not go up at the weekend, you may feel under pressure to go along with what everyone thinks. That is not the scientific method! Whilst you will not be held up to public ridicule because you have written something in a school-based investigation that goes against public perceptions, many scientists have been vilified and occasionally imprisoned for insisting on the truth of their (unpopular) findings – the fate of Galileo Galilei (1564–1642) is the best-known case in point.

We will now look at three case studies to expand these ideas. We have taken some ideas further than others and none can be considered as even approaching complete. They are to be taken as skeletons on which a good report might be built!

Idea 1: Monte Carlo methods

(Background theory: Probability).

If you type 'Monte Carlo' into an internet search engine, you will get a lot of information about the small and glamorous Mediterranean state that is almost synonymous with 'high rolling'.

The third site listed by my search engine was a mathematical paper on 'Monte Carlo Methods'.

The name is no accident as it is a mathematical method based on gambling.

1. The question

The first question was raised by looking at this aerial photograph. It emphasizes that most real objects are not bounded by straight lines, circles etc. as they are in mathematics texts. This coastline is very 'crinkly'.

This coastline is smoother.

This suggests the problem: how do we find the areas of irregular shapes?

2. A good answer?

This will be a numerical answer to the problem. Given that there is no 'correct' answer with which we can compare ours, we will be looking at an approximation. Try to set a realistic target such as 'to the nearest 100 km^2'.

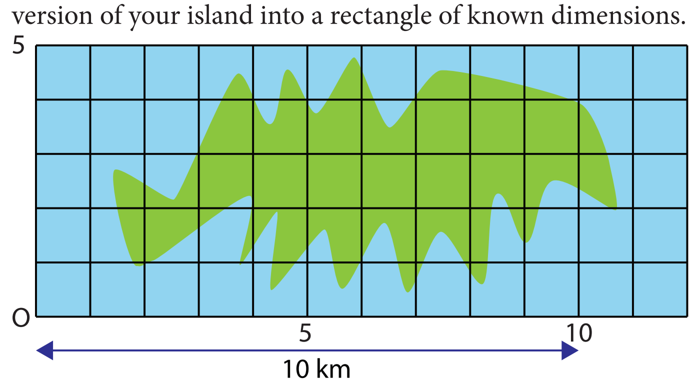
3. Collect information

Even though Monte Carlo methods are not specifically a part of the course, they depend on the concepts of probability that you study. They have many variants. At its simplest, the Monte Carlo method is like throwing darts at a map whilst wearing a blindfold. You will obviously need to research what this might have to do with area!

It would also be a good idea to set yourself the task of finding the area of a real island such as Hokkaido. You will need to obtain an accurate map. As part of your report, you should note the accuracy and scale of the map. A mariner's chart is made to very different standards from those you see on tourist brochures that encourage you to 'Holiday in Happy Hokkaido' (the ones that have cartoon fish leaping from cartoon lakes and cartoon waves rolling onto cartoon beaches).

4. Analysis

The simplest version of the method involves copying a scale version of your island into a rectangle of known dimensions.



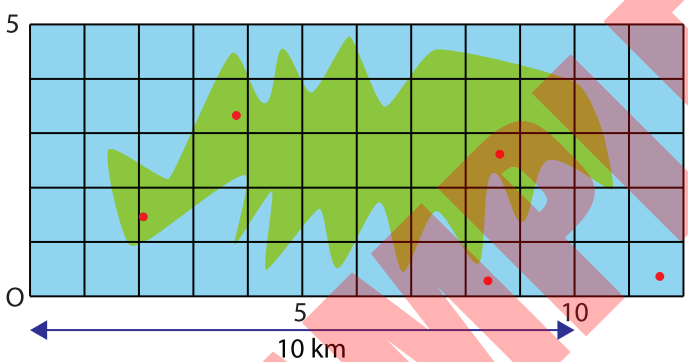
This rectangle measures 12 km by 5 km and so has an area of 60 km^2. But what proportion of this is inside the island?

This is where we don our mathematical blindfold and start throwing darts!

To blindfold ourselves, we can use the random number generator of a computer or calculator. These vary but, commonly, produce a random (what does this mean?) number between 0 and 1. If we multiply a random number in this range by 12, we will get a random number in between 0 and 12. If we take a second random number and multiply it by 5, we will get a random number in the range 0 to 5. If we now view these two numbers as a coordinate pair on the map, we will have, effectively, thrown a blind dart at the map.

Here are ten random numbers and the five 'darts' that they throw at the map:

Random 1	Random 2	12×Rand 1	5×Rand 2
0.723018492	0.529733397	8.68	2.65
0.310504197	0.673043069	3.73	3.37
0.167302964	0.293733395	2.01	1.47
0.96699594	0.078452756	11.60	0.39
0.695841987	0.075482986	8.35	0.38



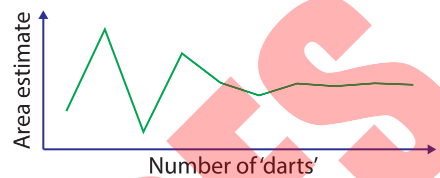
As things stand, 3 out of the 5 'darts' have landed in the island. This suggests that its area is three-fifths of that of the rectangle or $^3/_5 \times 60 = 36$ km^2.

However, if one of the 'darts' that 'hit' was recorded as a 'miss', the area estimate becomes $^2/_5 \times 60 = 24$ km^2. As this is quite different from the previous answer, we cannot be very confident about our current measure of the area.

How do we improve matters? More darts would seem to be indicated, but just how many do we need to achieve accuracy to a particular level (such as 'nearest 1 km^2')? This is where the 'evaluation' stage of the investigation can begin.

5. Evaluation and conclusion

There are several ways to go about this. If you read further into this subject, you should discover that it is possible to estimate the accuracy of an answer obtained by a Monte Carlo method by using a formula. However, if you calculate your area every 10 'darts' and plot a graph of area estimate against number of darts, you should see that the result starts to stabilize after early fluctuations. The extent of the fluctuations in the estimate can give you a measure of accuracy:



Whatever method you use to assess the accuracy of your method, you should comment on this in your report.

Idea 2: Damped Oscillations

(Background theory: Trigonometric and Exponential Functions and Calculus).

1. The question

This idea came from a very rough ride on the back of a truck along a dirt road! Why do we get a much smoother ride in a modern saloon car than on the back of a truck or tractor?

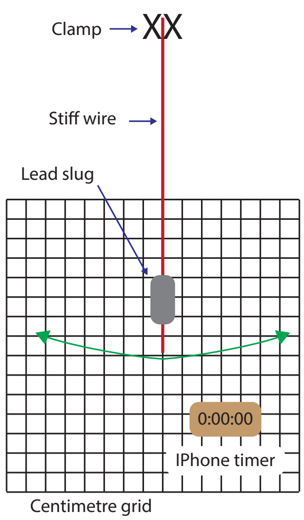
So why do we 'bounce around' more in some vehicles than others?

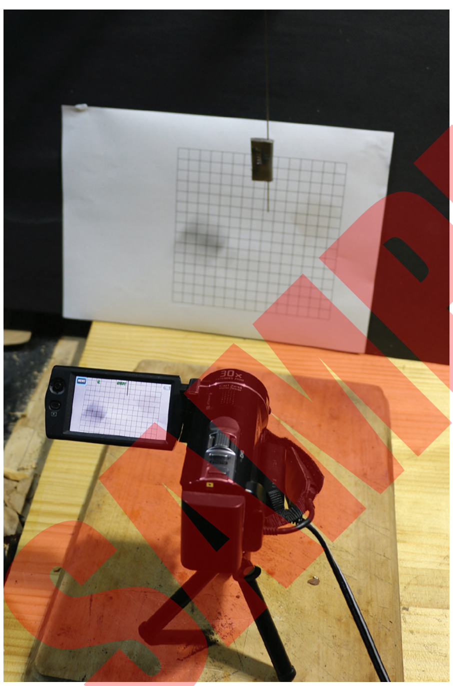
There are several answers to this question, not least the different surfaces over which they usually travel! One of the keys is the quality of the wheel springs and what are known as shock absorbers. The idea also came from the slightly worrying feelings that you get in tall buildings when they 'sway' and in aircraft when the wings 'flap'.

Real vehicle suspensions, buildings and aircraft wings are complex structures that are difficult to observe. What we decided to do was make our measurements on a 'home-built' structure that had similarities to the 'real thing'.

In setting up an experiment to investigate this, we have decided to simplify the problem to look at a simple damped vibrating system using a piece of stiff garden wire, a lead slug (weight) and a digital video camera. The structure is shown in the photograph. Apart from the camera, we used items that were to hand in a domestic garage.

The intention is to move the slug sideways and use pictures taken from the video recording and the grid to measure the position of the slug as it varies over time. To assist in measuring this position, we made and printed a 1 cm square grid using a computer. Note that this setup introduces some parallax error. There is also an error introduced by measuring the horizontal rather than the actual deflection of the tip of the wire.





2. Define the objectives and criteria for success

It is our hope that, working from measurements from the 'structure', we can set up a mathematical model that can be used to predict how it will perform under conditions different from the original experiment (a different initial deflection).

3. Collect the data

The complete data set is a digital video.

Use the pause button to collect data pairs: (time, deflection).

Here are some of ours - obtained by pausing the video. We chose the stopwatch reading 13.15 sec as the zero of time. The second row of the table was taken at the clock reading of 13.15 + 0.06 = 13.21 sec.

Time (sec)	Deflection (cm)
0	1
0.06	5.8
0.19	8.5
0.39	-0.7
0.68	-1.2
0.8	3.1
0.87	6.5
0.99	7
1.05	4.4

It is an important part of the data collection section of an investigation that you should report on the limitations of your method. In this case, we have used a 1 cm grid and have estimated to one decimal place. It is unlikely that we have done better than an accuracy level of ±0.2 cm, particularly in the frames where there is 'motion blur'. Also, there is the effect of parallax, although we have tried to minimize this. Note also that we are ignoring the fact that the tip moves vertically as well as horizontally (it moves in the approximate arc of a circle).

If you choose to do a similar modelling exercise, you will find it easier if you choose a slower moving event. Use a heavier weight or a less stiff wire. Or a different setup altogether.

We have included an example of a practical investigation to illustrate the fact that these may not be as difficult as they at first appear. However, we strongly advise that you discuss a proposal of this sort with your teacher before undertaking any such experiment. There are many reasons for this, the main ones being practical difficulties and safety. We have also seen proposals for this type of investigation which have been too complex to be carried out in a reasonable time frame.

It is important to avoid investigations that expose people or animals to danger or may produce anti-social side-effects such as loud noise etc.

4. Analysis

As a first step, we have entered the data given in the table above onto a spreadsheet. This enables us to generate a graph of the data set for the early part of the motion.

We will look at the two parts to modelling the motion separately.

1. Periodicity (trigonometric)

2. Decay (negative exponential)

Periodicity

If D = deflection from the zero point (the vertical gridline 2 cm to the left of the phone) and t is the time in seconds after our arbitrary start time of 13.15 on the clock, then we are looking for a model of the form:

$$D = A \times \sin(n \times t + c)$$

See the Chapter on the Trigonometric Functions to recall that the three parameters A, n, c control different features of the function:

A determines the amplitude. The video shows that 8 is a reasonable estimate for this.

n determines the period. Looking at the data table it looks like two successive maximum deflections to the right occur at about $t = 0.19$ & 0.87. This suggests a period of about 0.68.

Using $\tau = \frac{2\pi}{n} \Rightarrow n = \frac{2\pi}{\tau} \approx \frac{2\pi}{0.68} \approx 9.2$

c represents the left/right displacement or phase of the model. For no particular reason, we start with a value of 1.

Video discussion of the use of a spreadsheet to model the periodic component of these data.

Link to the spreadsheet.

The periodic component is modelled by:

$$D = 8 \times \sin(8.6t)$$

To model the decay aspect, we use a similar method. Here are some data points selected to be the maximum deflection to the right.

Time	Maximum Deflection
0	7.3
20	6
30	5.2
50	4.2
80	3
100	2.6

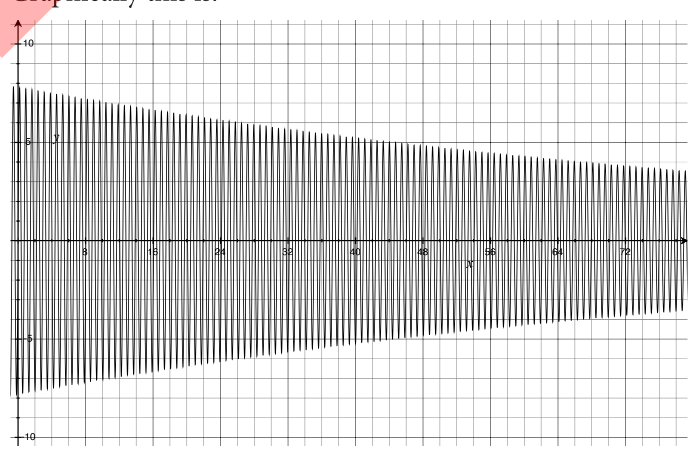
The second page of the spreadsheet shows how we arrive at the model:

$$D_{max} = 7.8e^{-t/100}$$

The two models do not agree exactly as to the correct value of A. We will compromise with $A = 7.9$

The final model is: $D = 7.9 \times e^{-t/100} \times \sin(8.6t)$

Graphically this is:



5. Evaluation and conclusion

We have succeeded in taking measurements from a real structure that was set in motion. We have used mathematics to find a model that can be used to predict the behaviour of the structure in circumstances different from those that we actually tried. The model performed much better in the short term than later in the motion. A good investigation will explain that this is much more likely to be due to errors in the trigonometric part of the model than in the exponential part.

Idea 3: Statistics (weights in lifts and transport)

The question here was raised by this safety plaque in an elevator (lift).

The manufacturers seem to be assuming an average weight of

$$\frac{2030}{30} \approx 68 \text{ kg per person.}$$

Further research on the internet indicated that airlines assume standard weights for passengers. These are under review (people are getting heavier) and vary from country to country (as do people) but in the 1990s approximate values were:

Seating capacity	Adult Male (kg)	Adult Female (kg)	Teenage Male (kg)	Teenage Female (kg)	Child (kg)	Infant (kg)
7–9	86	71	65	56	44	17
40–59	83	68	63	57	42	16
100–149	82	66.9	61.1	55.2	41	16
300–499	81.4	66.3	60.6	54.8	41	16

There seem to be many questions that could be followed here and it is important to define what you choose to investigate.

If you accept the airline figures, what chance is there that the elevator will be overloaded?

If you accept the implied elevator figures, what chance is there that an aeroplane will be overloaded?

If you accept none of these figures, you will need to get data on the actual weights of people. This can be a real problem as 'body weight' is very definitely in the area of 'personal sensitivity'. Ask someone their weight and they may well tell you their preferred weight rather than their actual weight. Ask them to get on a set of bathroom scales and they will probably refuse. This seems to be a line of enquiry best avoided!

2. Define the problem

After a variety of internet searches on 'body weight', we failed to come up with any definitive data on this distribution. The outcome of a search for useful data on human weight distributions was to produce a flood of offers for diet programs! You may, of course, be more successful in forming your search questions than were the authors!

This became an investigation in which 'definition' emerged as a major feature.

We have airline data on the 'standard weights' of various classes of humans. We also have a safety plaque that indicates the safe loading of elevators.

One option is to accept the most conservative weights assumed by airlines (the row for 7–9 seat aeroplanes). Why is this the best assumption and why do the airlines assume that everyone gets lighter when they get onto large aeroplanes? There is a good investigation in this question alone!

However, we will accept the most pessimistic data on weights assumed by the airlines and ask ourselves:

What are the chances (mathematicians call this probability) that the elevator whose plaque was shown earlier will accept a load of 30 persons or fewer and yet still be overloaded?

This is now a clearly defined problem. We were not able to arrive at this definition until we looked at the available data. This may well be the case with the investigation that you choose and underlines the fact that the stages identified earlier in the chapter may well over-lap.

3. Collect data

Our problem definition has now assumed that we accept these data on human body weight:

Adult Male (kg)	Adult Female (kg)	Teenage Male (kg)	Teenage Female (kg)	Child (kg)	Infant (kg)
86	71	65	56	44	17

The elevator is restricted to 30 persons and a weight of 2030 kg.

If 30 adult males get into the elevator they will weigh: $30 \times 86 = 2580$ kg and the elevator will be overloaded. It appears that the manufacturers are assuming that the normal load will seldom consist of 30 adult males. So what is a normal load?

There are two questions to investigate here:

1. What is the distribution of the numbers of people who enter elevators?

2. What is the distribution of the types of people who enter elevators?

Both these questions can be answered by collecting real data. Observe an elevator and make a frequency distribution of:

The number of people riding in the elevator (not the number who get into it)

AND

The distribution of the types of people (adult females, infants etc. as defined by the weight data) who ride in elevators.

4. Analyse the data

Having collected the data described above, you now need to display it mathematically. This can mean graphs, tables etc.

However, remember that the issue is to look at the probability that the elevator will be overloaded and your analysis should reflect this.

For example, if you observed a family consisting of a mother, father, two female teenagers and an infant riding in the elevator, you can assume that they weigh: $86 + 71 + 2 \times 56 + 17 = 286$ kg which is well inside the weight limit. If, however, you observe a full load of adult males, you will record an overload.

Taking into account both these variables (number of occupants and composition of the load) what is the probability that the elevator will be overloaded?

By the way, you do not need to flee in panic if you find yourself in an elevator full of heavy people. Many have weight sensors that prevent them moving if overloaded. In common with all structures, the designers will have built in a safety factor that they are not keen to advertise. Imagine if the plaque had said '30 persons, but will take 35'. Everyone would work on 35 and it is 'goodbye to the safety factor'. Also, though this will not be a part of a mathematics project, most elevators include a truly ingenious 'fail safe' device that ensures that, even if the cable snaps, the 'cage' will not plummet to the bottom of the shaft.

5. Conclusions

This topic has made many assumptions with its data. Your conclusions should reflect on the difficulties that were experienced in acquiring reliable data. These, of course, follow through into the reliance that can be placed on the conclusions.

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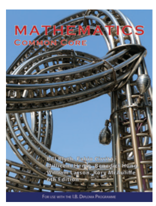
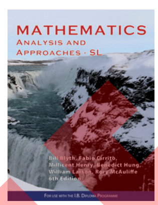
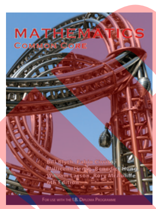
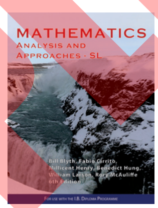
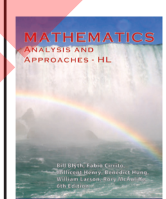
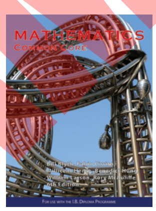
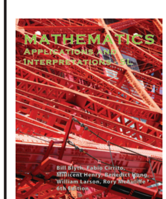
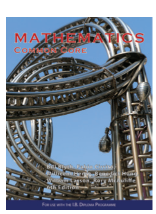
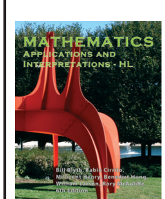
Mathematics: Analysis and Approaches (SL)						Pure(SL)
Mathematics: Analysis and Approaches (HL)						Pure(HL)
Mathematics: Applications and Interpretations (SL)						Applied(SL)
Mathematics: Applications and Interpretations (HL)						Applied(HL)

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