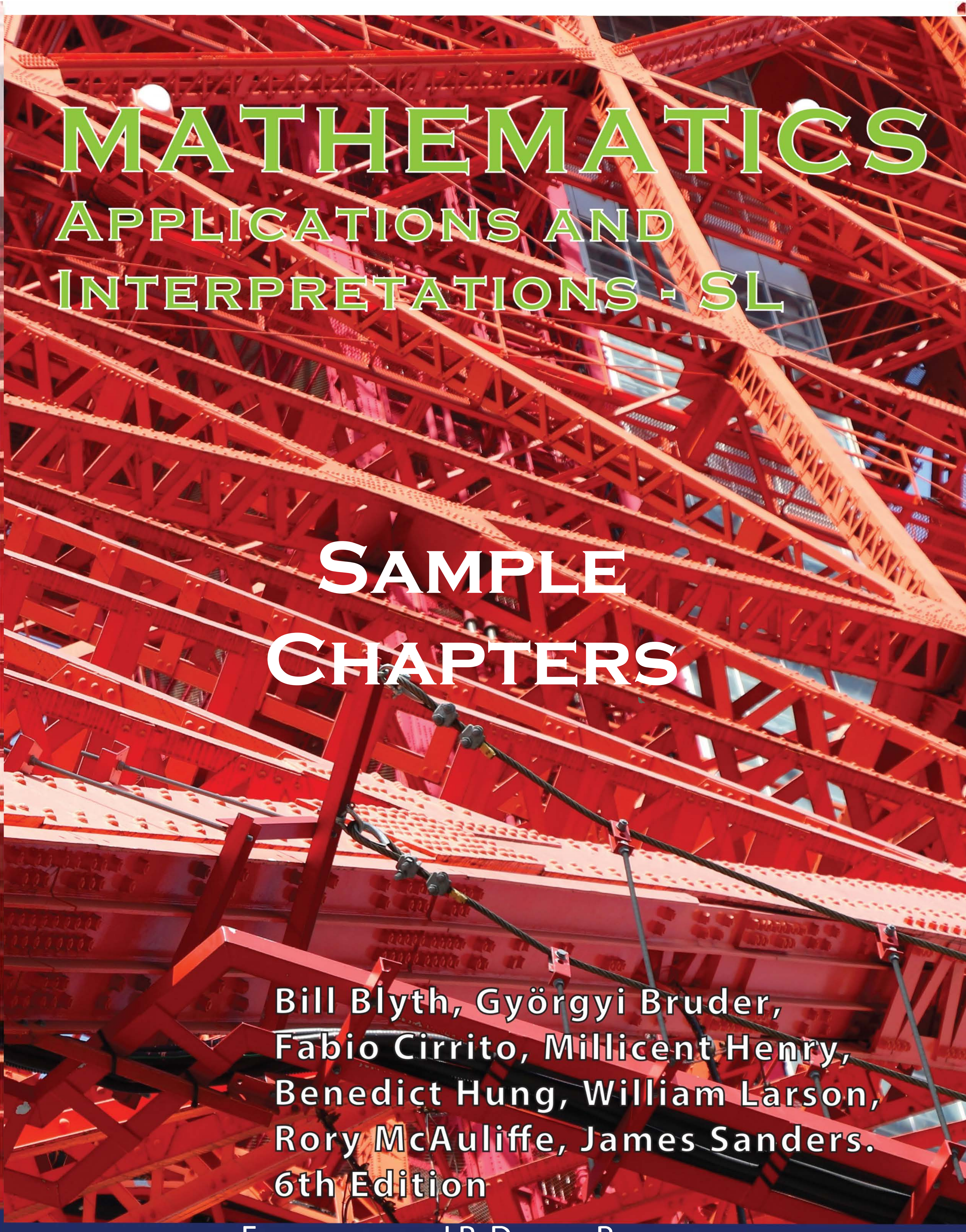

MATHEMATICS

Applications and Interpretations - SL

Sample Chapters

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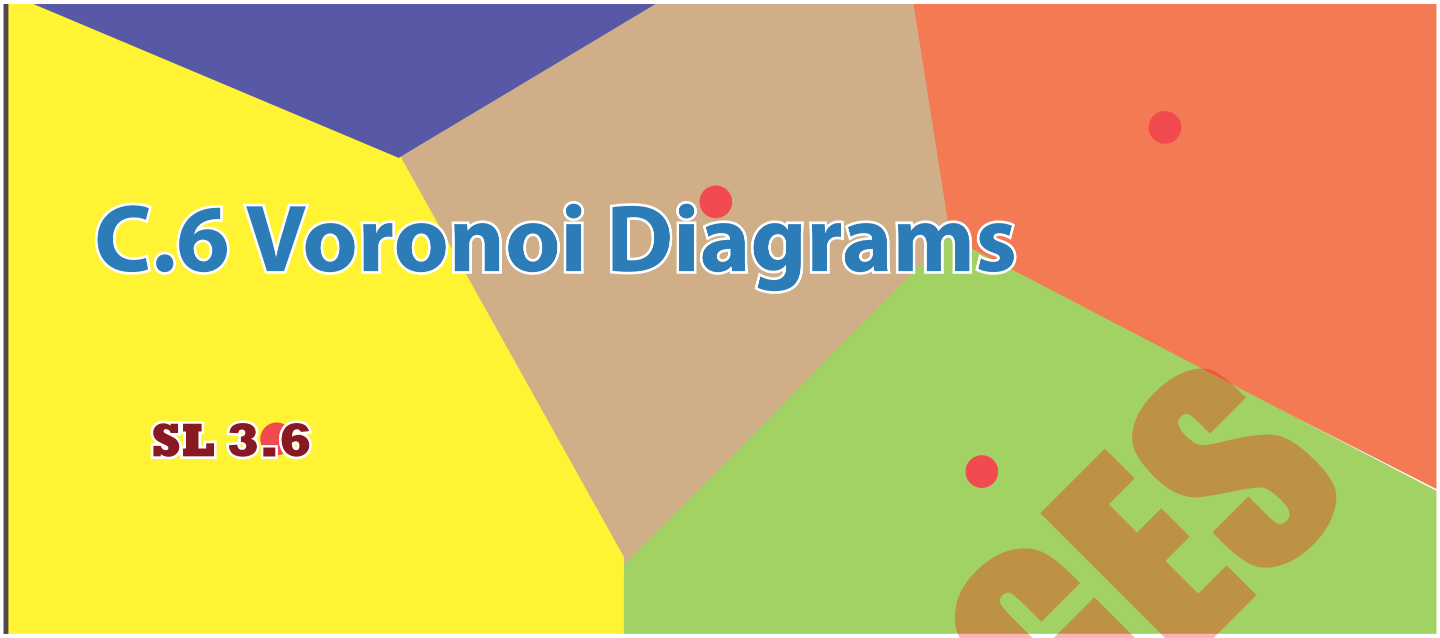
APPLICATIONS AND INTERPRETATION SL: TABLE OF CONTENTS



C.6 Voronoi Diagrams

SL 3.6

In 1854, London, England was the centre of the largest political entity that has ever existed. The British Empire ruled a quarter of the land area of the world and all its oceans. Despite all this political power, people still fell victim to infectious diseases, many of which were often fatal. During 1854 an outbreak of cholera struck central London that was killing people in the hundreds.

At the time, the 'science was settled' on the subject of what cased infectious disease. It was 'bad air' - end of story. This bad air went under various names such as 'miasma' or 'malaria' (literally bad air). The bad smells were caused by the absence of proper sewerage systems. It was not poor garbage disposal as people of the time threw almost nothing out and particularly not food scraps. A well run house put nothing but dust into what the English still call dustbins.

Faced with a seemingly endless series of such outbreaks, a local physician, Dr. John Snow (1813-1858) decided to try to get to the bottom of the causes of cholera. In doing so, he was taking on the scientific establishment.

Central to Snow's method was careful record keeping on the outbreak and in particular his 'map of cases' - see opposite.

Snow noticed that the cases occurred in geographic clusters like the one shown in the map above. Snow reasoned that, if cholera was spread by bad air, the cases would be evenly spread across the city because air is so mobile.

The cluster shown is centred on a water pump in Broad Street. Very few houses had water piped into them. Most people collected water from public wells and pumps like the one in Broad Street. They obviously sought out the closest pump as water is heavy and full buckets are difficult to carry.

Snow observed that catching cholera might depend on which pump you lived closest to. The implication of this is that cholera is a water borne disease. Clean the water and you prevent the disease. Sadly, Snow did not live to see his theory accepted and the end of cholera - at least in London.

John Snow's cholera map is an early example of what are now known as Voronoi diagrams after Georgy Feodosevich Voronoi (1868 – 1908) - pictured.

We have already looked at some small 'proximity diagrams'. In the previous chapter we looked at dividing Iceland into two parts for the purpose of minimising response times for emergency services.

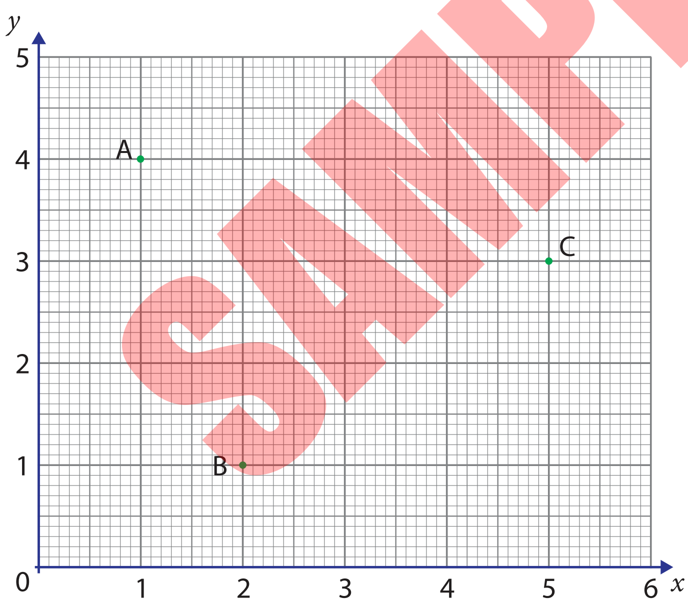
We will now look at complicating the problem by adding some more 'bases'.

Example C.6.1

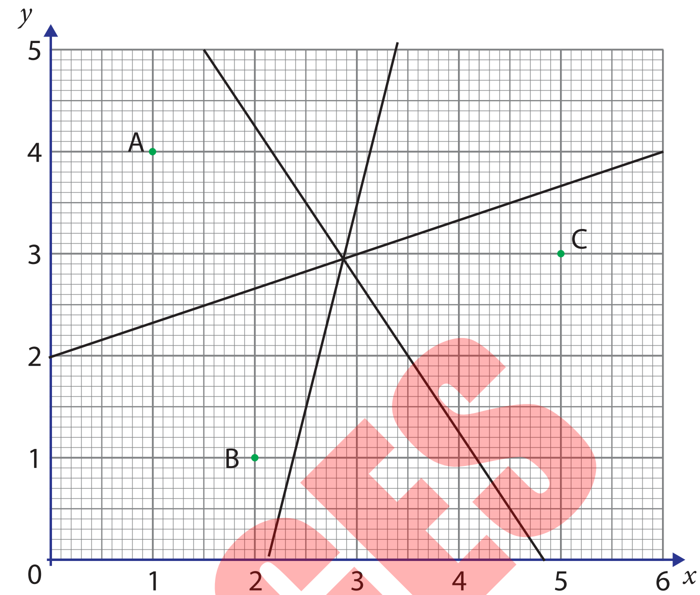
Show the points A(1,4), B(2,1) & C(5,3) on a Cartesian Diagram for which $0 \le x \le 6$ and $0 \le y \le 5$.

Show the region which is closest to A in red, the region closest to B in blue and the region closest to C in yellow.

The basic diagram is:



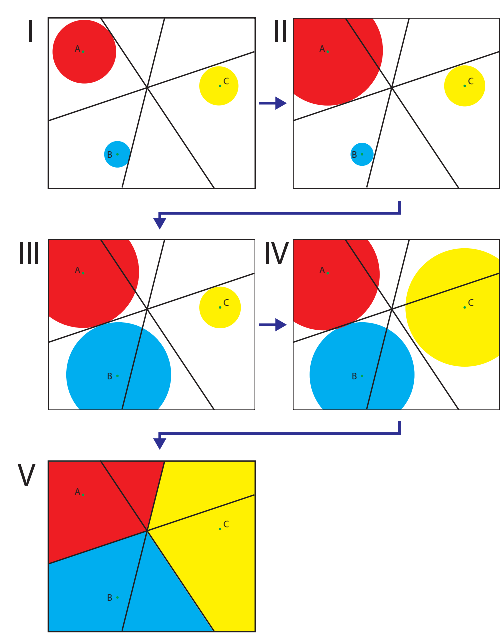
As we discussed in the previous chapter, the important line in determining whether we are closer to one point than another is the perpendicular bisector of the line joining the two points. There are three of these in this problem.

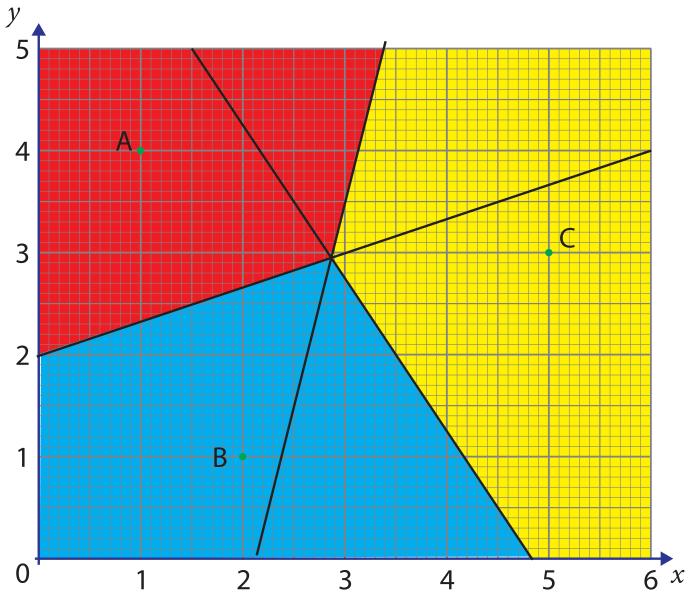


Even though this diagram has been kept uncomplicated by having all three boundary lines intersect in one point, we still have to decide where to place the boundaries of our three proximity regions - the diagram has six regions.

One method is to start with circles on each 'base' that reach as far as possible without crossing any boundaries - Diagram I.

Next, can we expand the circle about base A? If we do so, we will encroach on the triangle at the middle at the top. As this is not yet 'owned' by any of the other bases, let's expand the red circle (II). Next look at base B. Its area can be expanded as shown in (III). It cannot encroach on the red area. The yellow area surrounding C can now be expanded as shown in IV. Every region is now coloured and we arrive at the final diagram (V).





Does this always work? We have the right answer, but surely, in working through the bases in order, we are favouring base A. Is this so?

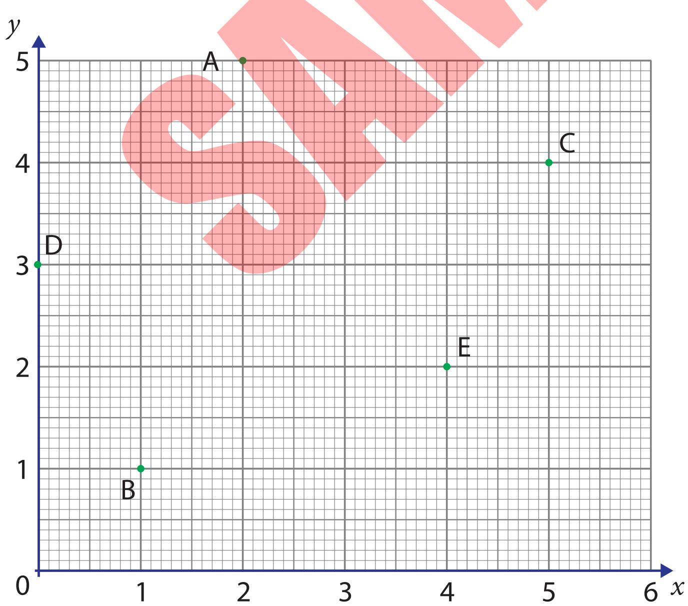
We now look at a more complex example.

Example C.6.2

Show the points A(2,5) B(1,1) C(5,4) D(0,3) & E(4,2) on a Cartesian Diagram for which $0 \le x \le 6$ and $0 \le y \le 5$.

Display this information on a Voronoi Diagram.

The basic coordinate diagram is:

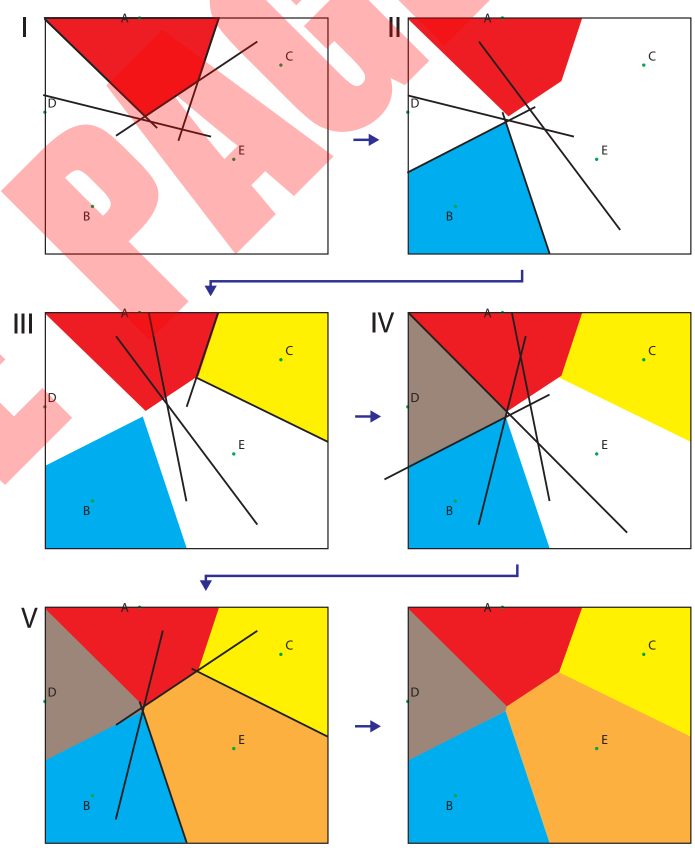


We will adopt a slightly different approach to this more complex example. This will look at each point in turn and attempt to draw its region by looking at the boundaries (perpendicular bisectors) of the point with each of the other points.

Diagram I shows point A and its boundary with each of the other four points. The inside of this polygon can now be coloured in as region A.

Diagram II shows point A and its boundary with each of the other four points (including A). The inside of this polygon can now be coloured in as region B.

Diagram III shows the procedure being repeated for point C. and so on until the diagram is complete.



There are some objective methods for generating Voronoi Digrams. You may like to research Fortune's algorithm. There are some online visualisations of how this algorithm works but they are not really practical at this level.

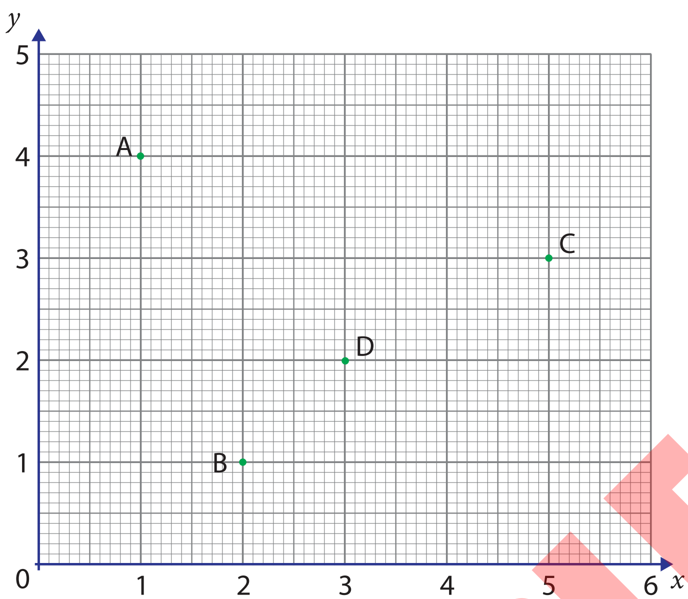
We will continue to use perpendicular bisectors and a stage-by-stage approach.

Example C.6.3

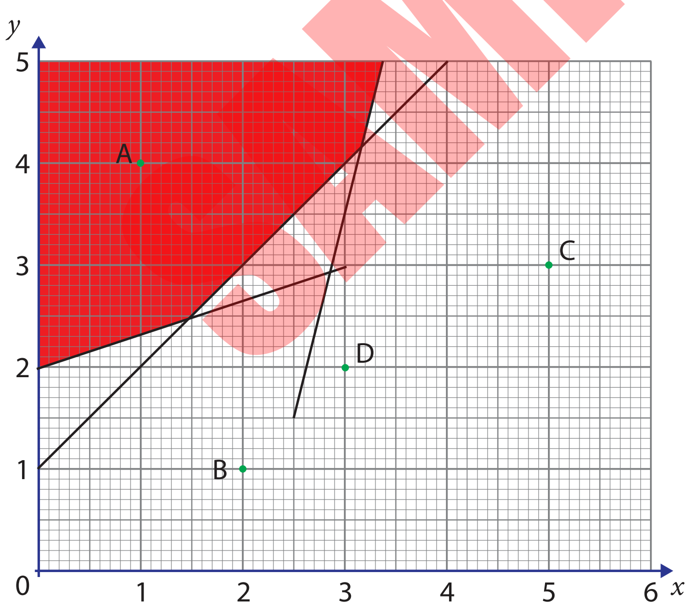
Show the points A(1,4) B(2,1) C(5,3) & D(3,2) on a Cartesian Diagram for which $0 \le x \le 6$ and $0 \le y \le 5$.

Display this information on a Voronoi Diagram.

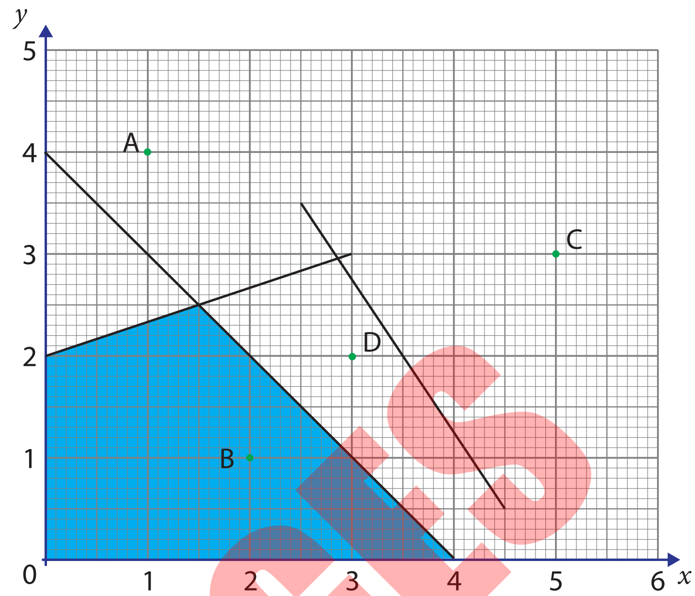
The basic coordinate diagram is:

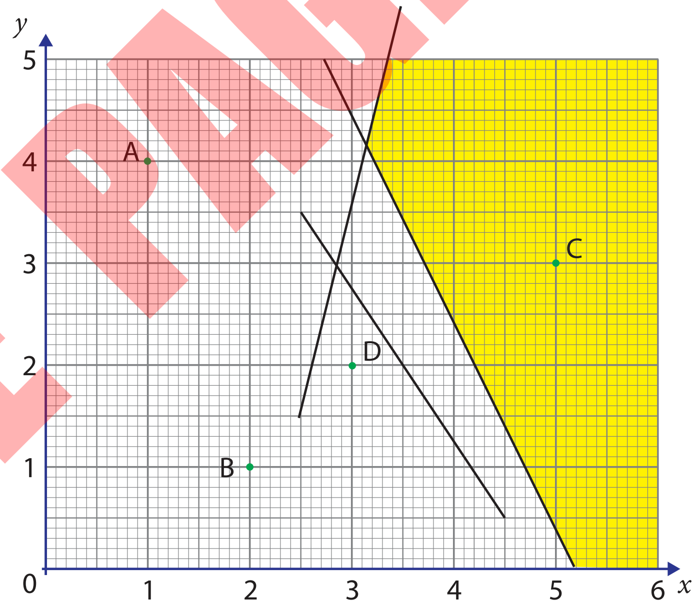


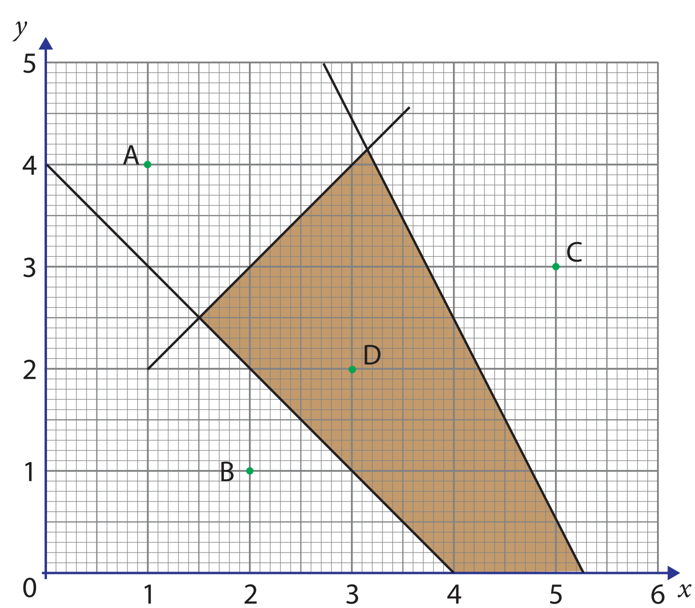
We now construct the three perpendicular bisectors relating to point A. This enables us to construct A's region.



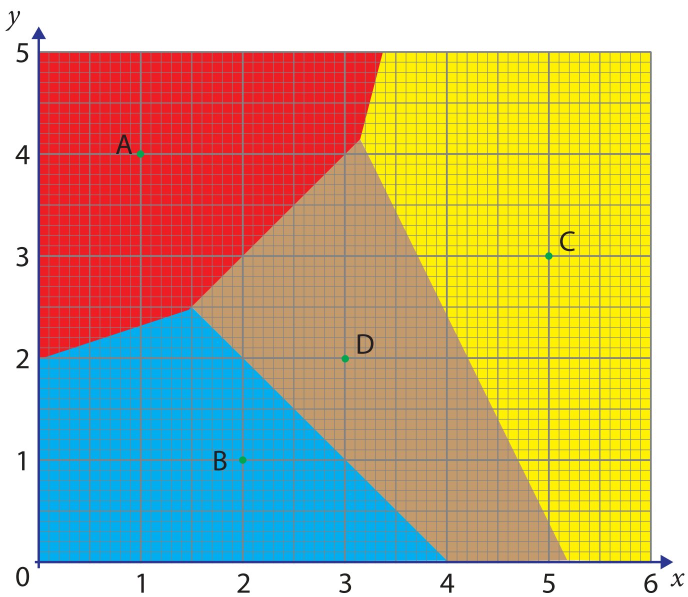
The process is repeated for the other points.







And the diagram is competed.



Before passing on to our final example, this diagram is also the basis for solving another sort of problem.

The Toxic Waste Dump

Still using the data of Example C.6.3.

Suppose the four points represent towns that want to set up a toxic waste dump. Fearing 'NIMBY' (not in my backyard) protests, the towns decide to site the dump at a point at which nobody can argue "This dump is closer to me than it is to anyone else".

Where on the diagram are places that meet their requirement?

Actually there are a lot as every point on the boundaries of two coloured region is an equi-distance point.

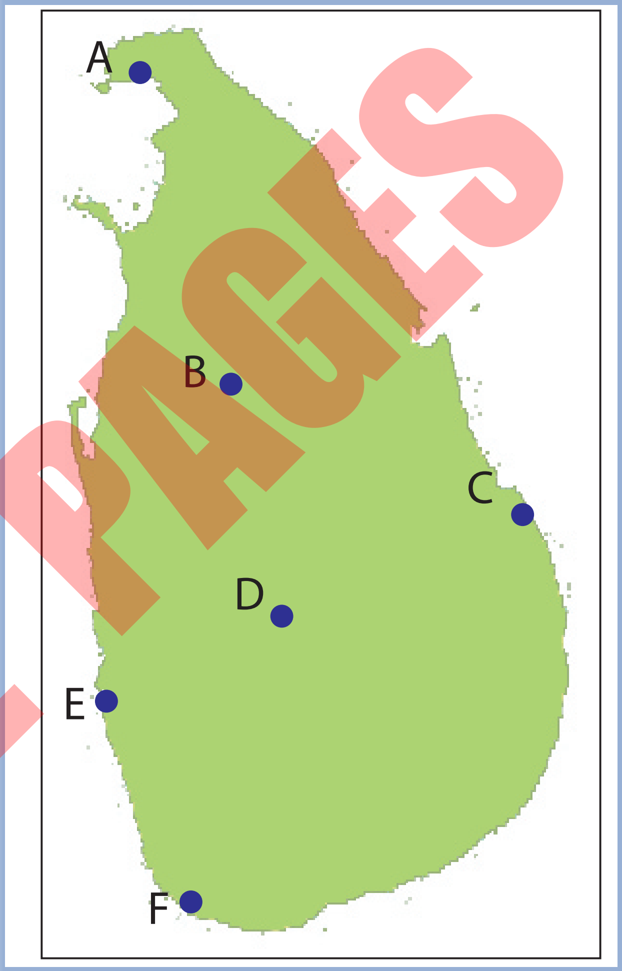
All points on the red/blue boundary are the same distance from A and B and further from C and D. So these all meet the requirement.

The towns may also want the dump to meet a second requirement: "The distance from the dump to the closest town must be as large as possible".

Where would you place the dump in this case?

Example C.6.4

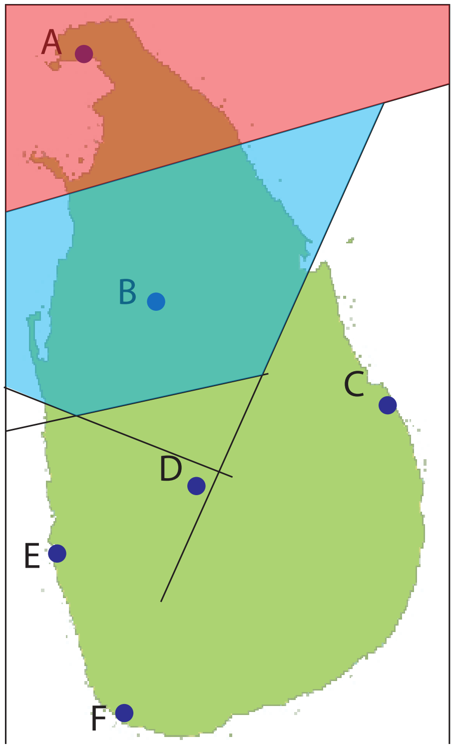
The diagram below shows an outline of Sri Lanka and some of its major cities and towns. Construct Voronoi regions for this map.



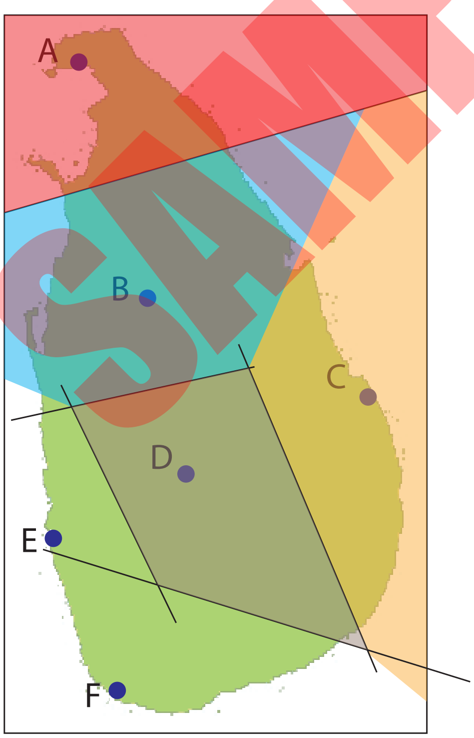
An A4 printable version may be downloaded here.

The worksheet is downloadable at this URL.

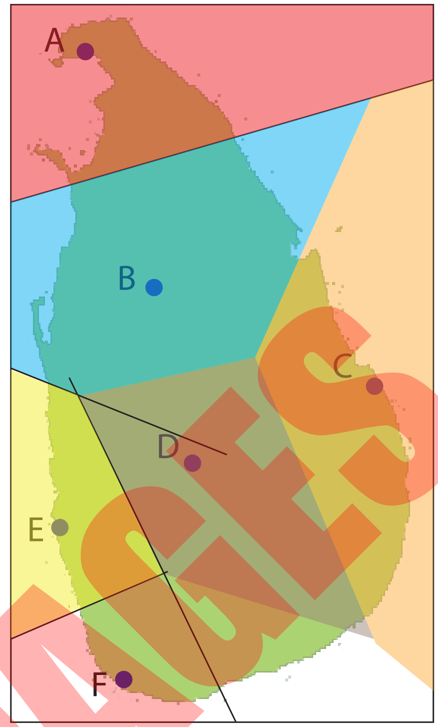
The construction of region A is comparatively straightforward and our first diagram shows it and the construction lines for region B.



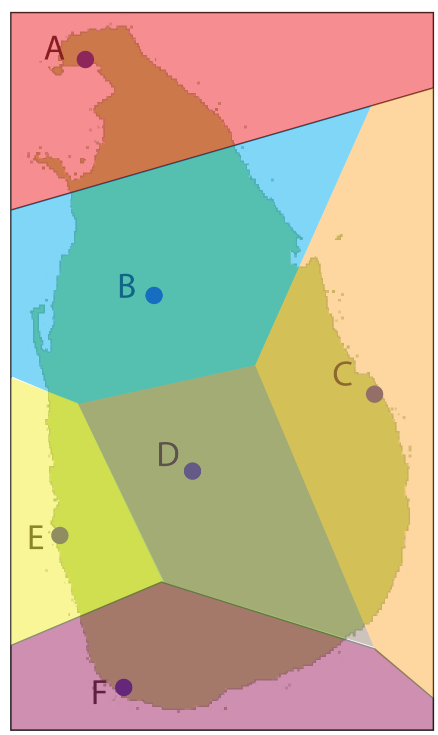
The construction of region C is also comparatively straightforward though you need to remember the proximity to point F. Region D needs care. All its boundaries with B, C, E & F are shown as well as the appropriate shading,



The E region is found: in this diagram.

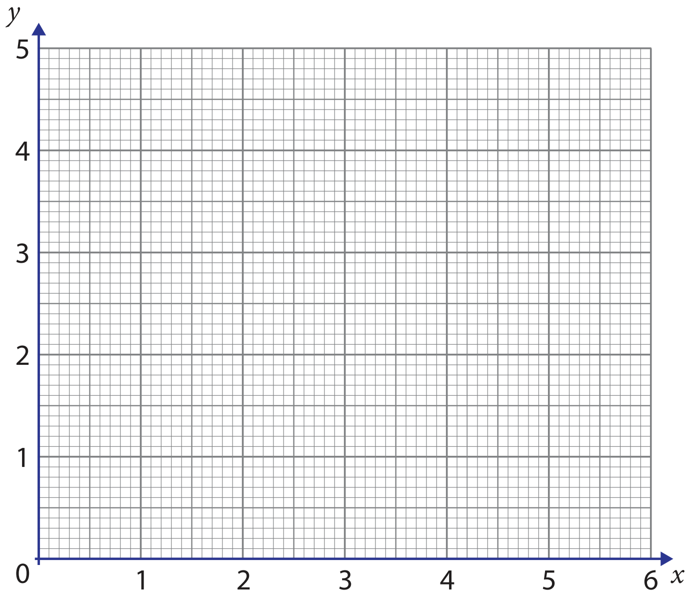


The complete diagram is:



Exercise C.6.1

1. Use a Cartesian grid similar to:



to show the Voronoi regions appropriate the these point sets:

a (2,1), (2,4), (4,2) & (4,4).

b (2,1), (4,1) & (3,4).

c (2,1), (2,5), (3,3), (4,1) & (4,5).

d (1,1), (2,4), (3,3), (5,2) & (5,5).

e (1,1), (2,2), (2,4), (4,1), (5,2) & (5,5).

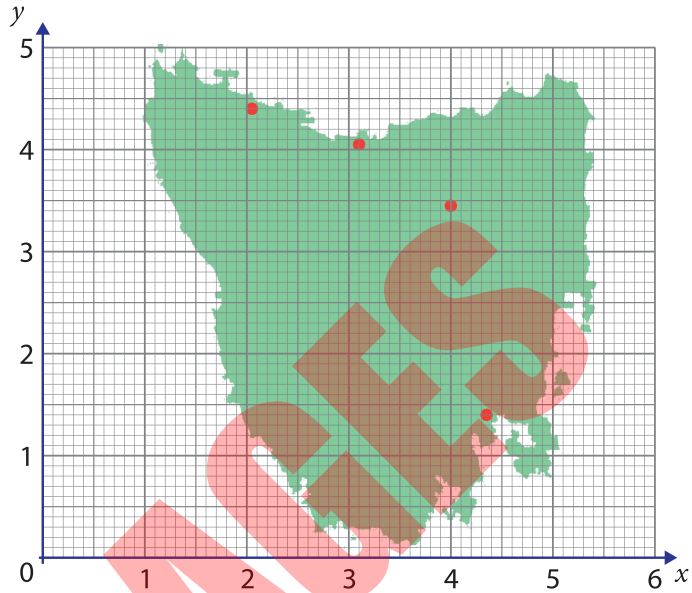
f (1,1), (2,4), (3,1), (3,3), (4,2) & (5,4).

Use your answers to question 1 to answer the next two questions.

2. Which point is equidistant from each of these points: (2,1), (2,4), (4,2) & (4,4)?

3. Which point is equidistant from each of these points: (2,1), (4,1) & (3,4)?

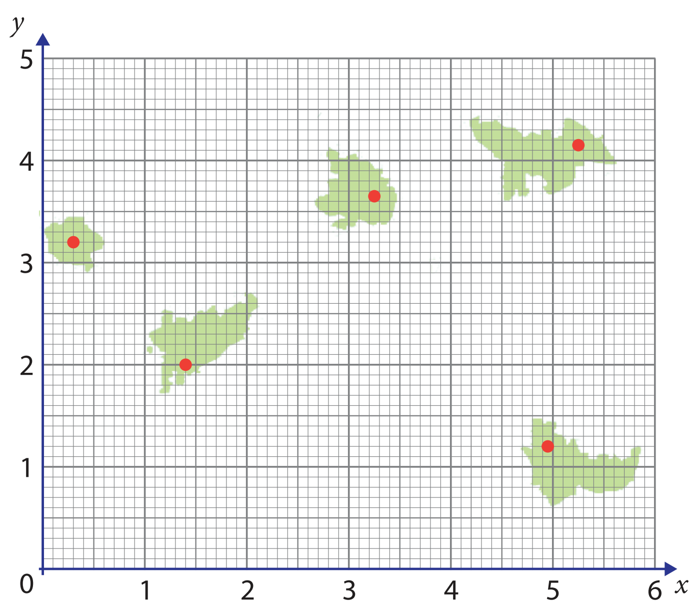
4. The diagram shows an outline map of Tasmania with its four major airports.



a Transfer the positions of the airports to a coordinate diagram.

b Draw regions on the diagram that will assist pilots in choosing the nearest airport to which they might divert in case of emergency.

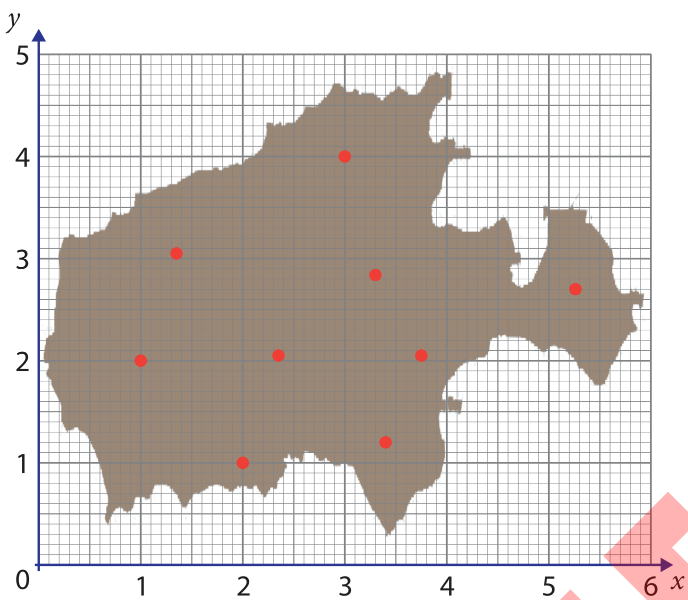
5. The islands shown each have a communications tower that transmit on different frequencies. Mariners are advised to tune their radios to the frequency of the closest tower.



a Transfer the positions of the beacons to a coordinate diagram.

b Draw regions on the map that will inform mariners at to which beacon they should tune into.

6. The positions of group of towns are shown on this diagram:



The towns are looking for a place to site a toxic waste dump. Their criteria are:

1. The dump must not be closer to one town than it is to any of the others.

2. The dump must be as far as possible from the nearest town.

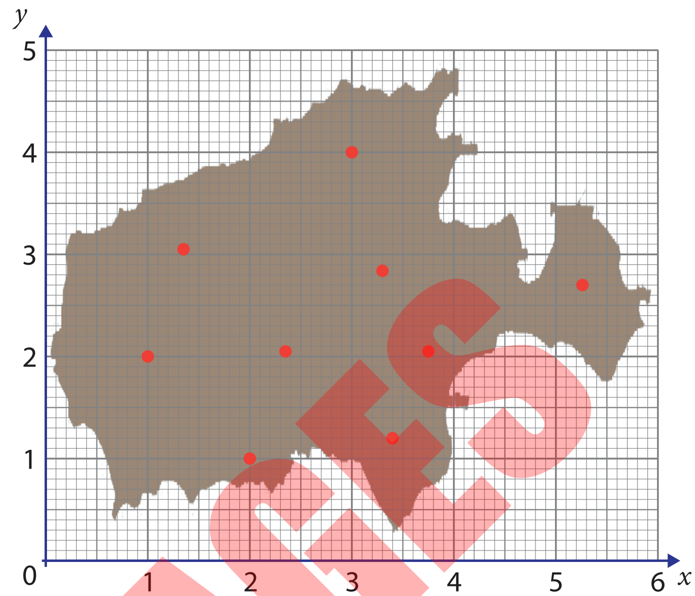
3. The dump must have coordinates $1 \leq x \leq 6$ and $1 \leq y \leq 4$.

Define the position(s) of suitable dump sites in coordinate terms.

7. A triangle has vertices at the points (1,1), (2,4) & (5,2). Find the coordinates of the point that is equidistant from each of these points.

8. Prove that the point at which the diagonals of a square intersect is equidistant from each of the corners.

9. The diagram shows the locations of the emergency facilities in a city. The scale is in units of kilometres.

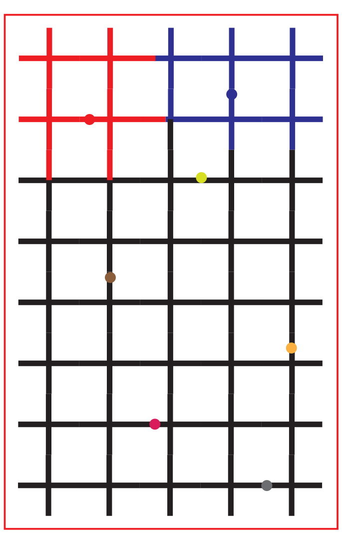


a Show the Voronoi regions on the diagram.

b If the city wishes to discover which areas are more than 1 km from the nearest emergency facility, show these areas on a second map.

10. This question uses the so called 'Manhattan metric' in which distances have to be measured along a grid of roads similar to that in Manhattan. We started this chapter with the work done by John Snow on cholera. Look again at his map. How would he have measured distance?

We have started this proximity diagram. Can you finish it?



At the time of writing, there was a splendid example of the regions proximate to the World's major airports at this URL:

https://www.jasondavies.com/maps/voronoi/airports/

This diagram has considerable significance for aviation safety.

Fuel economy pressures have driven a recent move from 4 engined jets such as the Boeing 747 to two engined options such as many Airbus types, the B777 etc.

As we discuss in our various Probability sections, 4 engines has a safety advantage over 2 engines.

If you wish to pursue this as an investigation, the relevant seach is 'ETOPS'. This will tell you how far twin jets are allowed to fly from an alternate airport.

ETOPS is an acronym for 'Extended Twin Operations' not 'Engines Turning or Passengers Swimming' as one wag described it.

Emergency services are driven by a complex set of parameters. Ambulance services, for example, face a continuously varying set of demands. Their assets - ambulances - are mobile and do not behave in the way our fixed assets do. Also, the have to respond to incidents that occur at unpredictable times at unpredictable places. Optimising this sort of service is an immensely complex problem.

Sea Lanes.

Rather like the World's flight routes, the immensely complex business of sea cargo presents a networking problem similar to the ones we have been discussing in this chapter.

An fascination picture of this trade is presented in this website:

https://www.ppsp.com.au/ais/ais-vessel-positions.aspx

Answers

Using the 6th Editions of IBID Press Mathematics Texts

This series of texts has been written for the IB Courses Mathematics: Analysis and Approaches and Mathematics: Applications and Interpretations that start teaching in August 2019.

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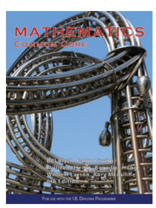
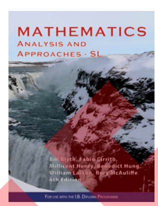
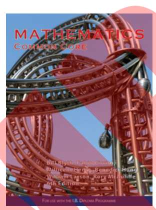
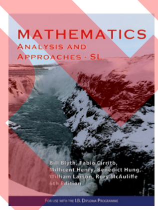
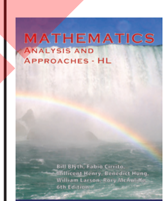
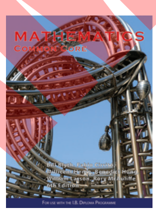
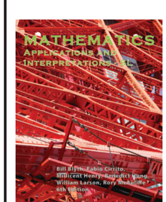
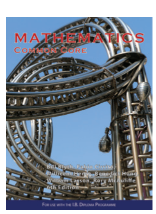
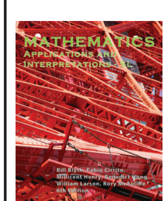
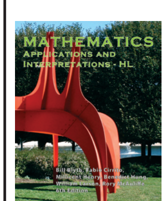
Mathematics: Analysis and Approaches (SL)						Pure(SL)
Mathematics: Analysis and Approaches (HL)						Pure(HL)
Mathematics: Applications and Interpretations (SL)						Applied(SL)
Mathematics: Applications and Interpretations (HL)						Applied(HL)