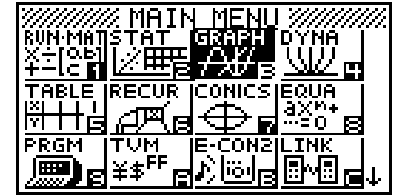


Linear Programming

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Linear Programming is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. A technique for the optimization of an objective function, subject to linear equality and/or linear inequality constraints. The feasible region is a 2-dimensional region, which is a set defined by the intersections of finite points (vertices), each of which is defined by the intersecting linear inequalities. The objective function is defined on this region. Linear programming finds a point (or series of points) in the region where this function has the smallest (or largest) which minimises (or maximizes) the objective function.



A linear function to be maximized:

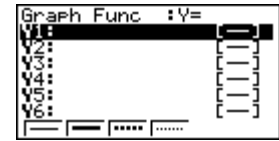
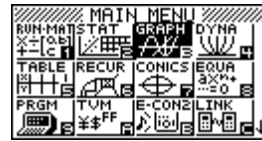
$$f(x_1, x_2) = c_1x_1 + c_2x_2$$

Problem constraints of the following form:

$$a_{11}x_1 + a_{12}x_2 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 \leq b_3$$



STYL [F4] Changing the style of the line drawn.

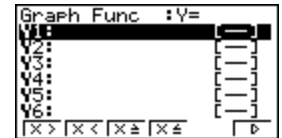
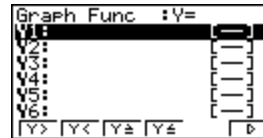
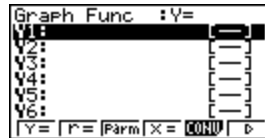


Non-negative variables:

$$x_1 \geq 0$$

$$x_2 \geq 0$$

TYPE [F3]



MORE CHOICES [F6]

[F1] [F2] [F3] [F4] ... [F6]
Y> Y< Y≥ Y≤

[F1] [F2] [F3] [F4]
X> X< X≥ X≤

Example:

A farmer wishes to put in no more than 100 hectares of land into two types of crops, peas and beans. The pea crop requires twice as many bags as the bean crop per hectare. The farmer is able to afford no more than 160 bags in total. Other expenses include \$3 000 per hectare of peas and \$1 000 per hectare for the bean crop. The farmer cannot afford more than \$24 000 in total, in expenses. The farmer estimates that the pea crop will return \$6 000 per hectare and the bean crop return is \$8 000 per hectare. How should the farmer plant the land to maximize the return from the two crops.

Solution: Setting up the constraints.

	Land (Hectares)	Bags (Count)	Expenses (\$)
Pea crop	1	1	3 000
Bean crop	1	2	1 000
Constraints	≤ 100	≤ 160	$\leq 240\,000$
Equations	$x + y \leq 100$	$x + 2y \leq 160$	$3\,000x + 1\,000y \leq 240\,000$

Together with $x \geq 0$ and $y \geq 0$

To maximize the profit equation: $f(x, y) = 6\,000x + 8\,000y$

$x + y \leq 100$ becomes $y \leq 100 - x$

$x + 2y \leq 160$ becomes $y \leq (160 - x) \div 2$

$3\,000x + 1\,000y \leq 240\,000$ becomes $y \leq (240\,000 - 3\,000x) \div 1\,000$

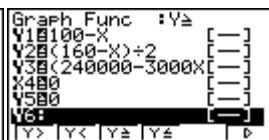
Enter the equations:



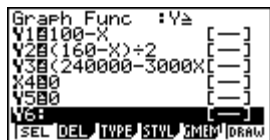
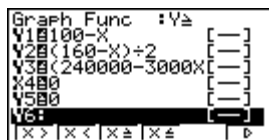
[F3] for Type



[F6] for Y inequalities

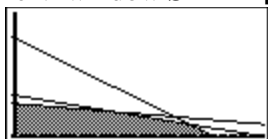


[F6] again for X inequalities



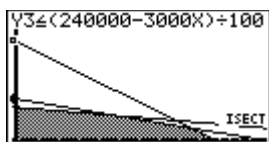
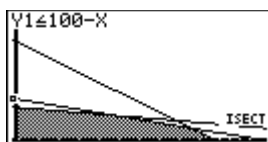
Set up the V-window **SHIFT** [F3]:

then **[EXIT]** followed by **[F6]** or **[EXE]** to draw.

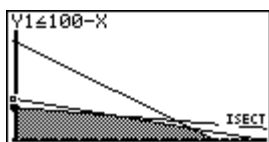
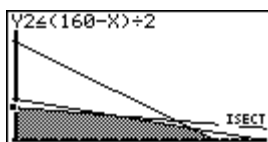
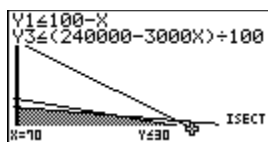


Result:

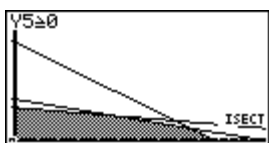
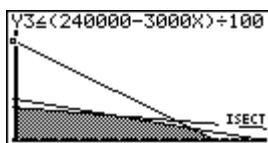
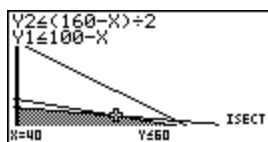
Now, using **G-Solve** **[SHIFT]** **[F5]** and to find the intersection points **[F5]** for **ISCT** (Intersection). Selecting two equations at a time, select one then **[EXE]**, then the other followed by **[EXE]**.



Result: (70 , 30)



Result: (40 , 60)



Result: (80 , 0)

[Note: The calculator cannot find the point of intersections for vertical line $x = c$.]

Maximise the equation is $f(x, y) = 6\,000x + 8\,000y$.

x	y	$f(x, y) = 6\,000x + 8\,000y$.	Profit (\$)
70	30	$6\,000 \times 70 + 8\,000 \times 30 = 660\,000$	660 000
40	60	$6\,000 \times 40 + 8\,000 \times 60 = 720\,000$	720 000
80	0	$6\,000 \times 80 + 8\,000 \times 0 = 480\,000$	480 000
0	80	$6\,000 \times 0 + 8\,000 \times 80 = 640\,000$	640 000

Maximum profit occurs for the farmer when there are 40 hectare of pea crops and 60 hectares of Bean crops planted.

For further tips, more helpful information and software support visit our websites
www.casio.edu.monacocorp.co.nz or <http://graphic-technologies.co.nz>