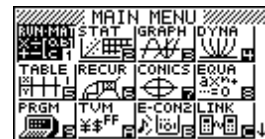# LCM and GCD in RUN-MAT mode.

This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.

Select **RUN** mode from the **Main Menu** by using the arrow keys to highlight the RUN icon or pressing 1.

## LCM (Least Common Multiple) and GCF (Greatest Common Divisor)

To find either the Least Common Multiple (LCM) or Greatest Common Divisor (GCD) of two numbers, you always start out the same way. Find the prime factorisations of the two numbers. Then compare and take what you need.

RUN-MAT   OPTN   F6   F4

                        F2   F3
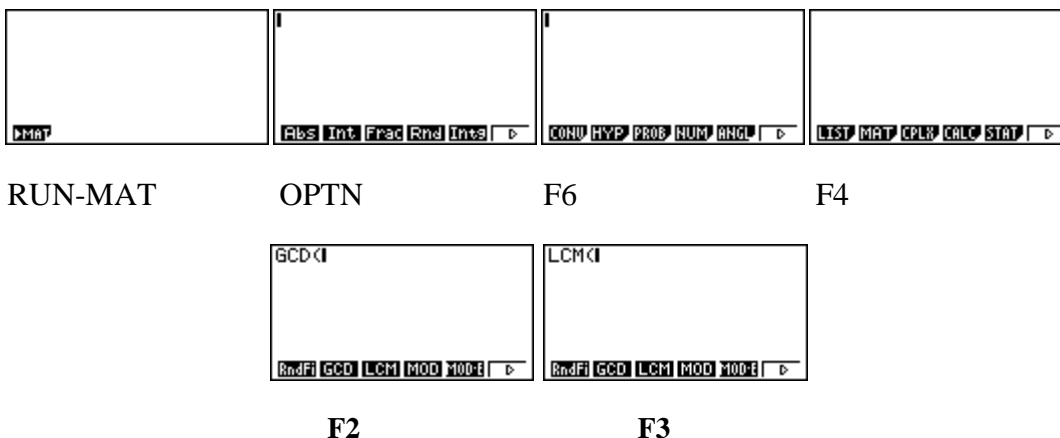
Example 1:   Find the GCD and LCM of 2940 and 3150.

Answer: The prime factorisations are: **2940** $= 2 \times 2 \times 3 \times 5 \times 7 \times 7$

                                                   3150 $= 2 \times 3 \times 3 \times 5 \times 5 \times 7$

The Greatest Common Divisor, (GCD) is the biggest number that will divide into (is a factor of) both 2940 and 3150. It is the number that contains all the factors common to both numbers. In this case, the GCF is the product of all the factors that 2940 and 3150 have in common. That is: The GCF $= 2 \times 3 \times 5 \times 7 = 210$
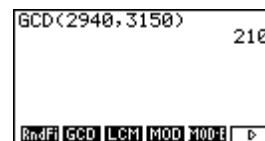
The Least Common Multiple (LCM) is the smallest number that both 2940 and 3150 will divide into. It is the smallest number that contains both 2940 and 3150 as factors, the smallest number that is a multiple of both these values. Then it will be the smallest number that contains one of every factor in these two numbers.

3150 has one copy of the factor of 2; 2940 has two copies. Since the LCM must contain all factors of each number, the LCM must contain both copies of 2.
The LCM of 2940 and 3150 must contain both copies of the factor 2. By the same reasoning, the LCM must contain both copies of 3, both copies of 5, and both copies of 7.
That is: LCM $= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7 = 44\ 100$.
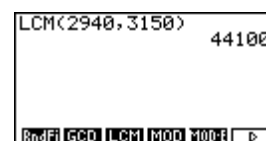
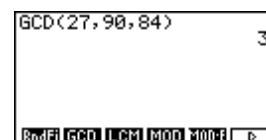**Example 2:**   Find the LCM and GCF of 27, 90, and 84.

Answer: First find the prime factorizations:

                        27 $= 3 \times 3 \times 3$

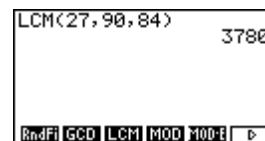   **90** $= 2 \times 3 \times 3 \times 5$

                        84 $= 2 \times 2 \times 3 \times 7$

The GCD (being the product of the *shared* factors) = 3
The LCM (being the product of *all* factors) are given by: $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3\,780$

```
LCM(27,90,84)          3780



RndFi GCD LCM MOD MOD-E ▷
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Integer Division and Remainders

Integer division is concerned with the division of whole numbers (or *integers*) into a *quotient* and a *remainder*. For example, 7 divided by 3 is 2 (the quotient) with a remainder of 1. 12 divided by 3 is 4 with a remainder 0. **Modular arithmetic** (sometimes called **clock arithmetic**) is a system of arithmetic of integers, where numbers "wrap around" after they reach a certain value—the **modulus (MOD)**.
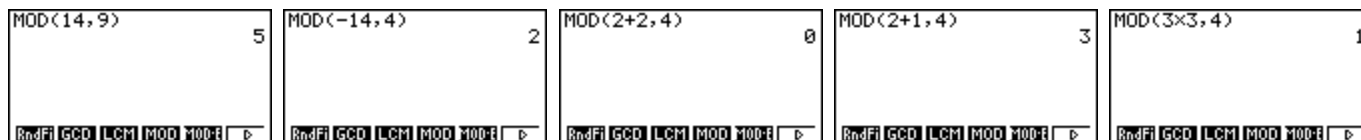
The Swiss mathematician Leonhard Euler, about 1750, introduced the idea of the modulo a number N.[ ]
Modular arithmetic was further advanced by Carl Friedrich Gauss in 1801. Time uses modulo 12. A familiar use of modular arithmetic is in the 12-hour clock, in which the day is divided into two even 12-hour periods. If the time is 7:00 now, then 8 hours later it will be 3:00. Usual addition would suggest that the later time should be $7 + 8 = 15$, but this is not the answer because clock time "wraps around" every 12 hours; in 12-hour time, there is no "15 o'clock". (Unless you are using 24 hour time!). Likewise, if the clock starts at 12:00 (noon) and 21 hours elapse, then the time will be 9:00 AM the next day, rather than 33:00. Since the hour number starts over after it reaches 12, this is arithmetic *modulo* 12. 12 is congruent not only to 12 itself, but also to 0, so the time called "12:00" could also be called "0:00", since $0 \equiv 12 \bmod 12$.

To use some examples: 7 MOD 3 = 1 and 34 MOD 5 = 4.
For a more complicated example 1996 MOD 19 is 1 (because 1996 divided 19 is 105 with a remainder of 1).

A subtle point is that the remainder is always a positive number. So -7 divided by 3 is -3, remainder 2 (rather than -2, remainder -1). This is important for the calculation of Easter dates because we must calculate the remainder when dividing into a negative number. Since in this case we are not interested in the quotient, it is sufficient to calculate the negative remainder and then simply add to it the number we were dividing by. So, -11 mod 3 may be calculated as: -11 divided by 3 is -4, remainder 1 (i.e. $3 \times$ -4 + 1).

```
MOD(14,9)        5   MOD(-14,4)       2   MOD(2+2,4)       0   MOD(2+1,4)       3   MOD(3×3,4)       1



RndFi GCD LCM MOD MOD-E ▷   RndFi GCD LCM MOD MOD-E ▷   RndFi GCD LCM MOD MOD-E ▷   RndFi GCD LCM MOD MOD-E ▷   RndFi GCD LCM MOD MOD-E ▷
```

Complete the addition and multiplication table for modulo 4.

$+_4$	0	1	2	3
0				
1				
2				
3				

$\times_4$	0	1	2	3
0				
1				
2				
3				

Note: $+_4$ indicates addition and $\times_4$ indicates multiplication modulo 4.

Extension Activity:
Find out how Easter dates are calculated using the Julian calendar.
Try: http://www.oremus.org/liturgy/etc/ktf/app/easter.html