## 4.1 - Acceleration

How do we describe speeding up or slowing down? What is the difference between slowing down gradually and hitting a brick wall? Both these questions have answers that involve *acceleration.* Acceleration describes how velocity changes. Any change in velocity, including speeding up, slowing down, or turning, creates acceleration.

### What is acceleration?

**How do we describe changes in velocity?**

Almost nothing moves at constant speed for very long in everyday life. Even a car on cruise control speeds up and slows down by small amounts to compensate for hills. How do we describe *changes* in velocity, such as going from rest to moving, or from moving to rest? The answer is the concept of **acceleration**. Acceleration is defined in equation (4.1) as the rate of change of velocity.

Interactive Equation

**Acceleration** definition

$$a = \frac{\Delta v}{\Delta t} \qquad (4.1)$$

$a$ = acceleration (m/s$^2$)
$\Delta v$ = change in velocity (m/s)
$\Delta t$ = change in time (s)

Acceleration is a crucial concept in the physics of motion because acceleration, not velocity, is the result of applied forces.

**What does acceleration mean?**

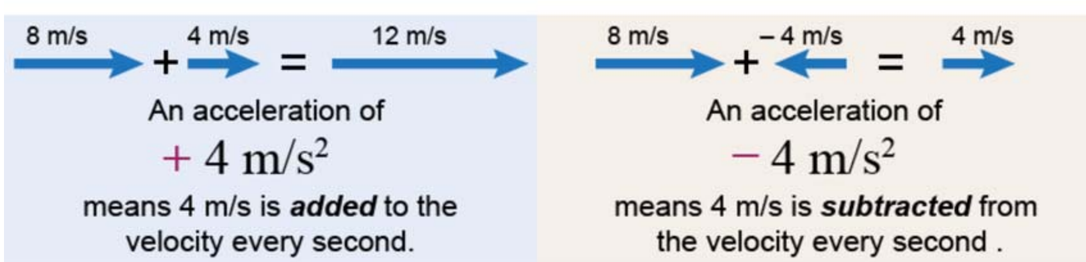
Equation (4.1) describes how acceleration is the change in velocity ($\Delta v = v_f - v_i$) divided by the change in time ($\Delta t = t_f - t_i$). For example, if a car starts at rest and is moving at 30 mph, then 10 s later the car's *acceleration* is 3 mph/s, or three miles per hour *per second.* The speedometer increases by 3 mph each second for 10 s. Acceleration is the *rate at which velocity changes.* more

**What are the units of acceleration?**

The units of acceleration are units of velocity divided by units of time. For a typical car a convenient unit would be miles per hour per second. A powerful sports car can accelerate from zero to 60 mph in 4 s. The change in velocity is 60 mph. The change in time is 4 s. The acceleration is 15 mph per second.

**What are m/s$^2$?**

Acceleration in this course will usually be expressed in SI units of m/s *per second*, or m/s$^2$. (This is sometimes written as m/s/s or m s$^{-2}$.) One meter per second squared means that the velocity changes by one meter per second each second.




**Positive and negative acceleration**

Acceleration can be positive or negative. For example, an acceleration of +4 m/s$^2$ adds 4 m/s of velocity each second. A car starting from rest would move at 4 m/s after one second, 8 m/s after two seconds, 12 m/s after three seconds, and so on. A *negative* acceleration of −4 m/s$^2$ *subtracts* 4 m/s every second. A car moving at +40 m/s would be moving at 36 m/s after one second, 32 m/s after two seconds, 28 m/s after three seconds, and so on. A negative acceleration is sometimes called a *deceleration*.

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**Acceleration of a car**

A car provides a good reference for developing an intuitive feel for acceleration. Suppose your speed goes from 0 to 29 m/s in 10 s. This is a typical acceleration for a small to midsize car. The change in speed is 29 m/s (29 m/s − 0 m/s). The change of 29 m/s divided by 10 s gives an acceleration of 2.9 m/s per second. The acceleration is 2.9 meters per second *per second* because your car *accelerated*, or gained 2.9 m/s of speed each second for 10 s. less

**What are the units of acceleration?**

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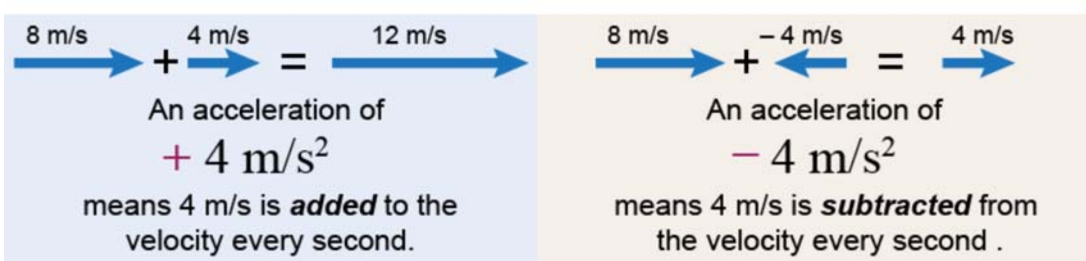
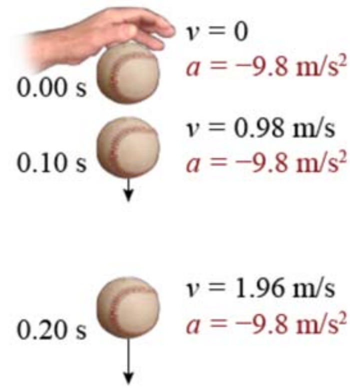


| | |
|---|---|
| **Positive and negative acceleration** | Acceleration can be positive or negative. For example, an acceleration of +4 m/s$^2$ adds 4 m/s of velocity each second. A car starting from rest would move at 4 m/s after one second, 8 m/s after two seconds, 12 m/s after three seconds, and so on. A *negative* acceleration of −4 m/s$^2$ *subtracts* 4 m/s every second. A car moving at +40 m/s would be moving at 36 m/s after one second, 32 m/s after two seconds, 28 m/s after three seconds, and so on. A negative acceleration is sometimes called a *deceleration*. |
| **Test your knowledge** | When an object moving with positive velocity is slowing down, the acceleration that it experiences is ____________.<br>a. positive<br>b. negative<br>c. zero<br>d. infinite<br>show solution |
| | A man is driving his car north on the highway at a constant velocity of 50 mph for 100 s. What is the value of his acceleration?<br>a. 0.5 m/hr/s<br>b. 5,000 m/hr/s<br>c. 0 m/hr/s<br>d. not enough information<br>show solution |

## Positive and negative acceleration and velocity

**Can $v = 0$ while $a \neq 0$?**

Acceleration is the rate at which velocity changes. Even when an object has zero velocity, it can still have a nonzero acceleration. Consider dropping a ball from your hand. The ball is not moving initially, and the instant your fingers release the ball it still has zero velocity. One tenth of a second later the ball is moving downward at −0.98 m/s. That means that the ball was accelerating from the moment it was released, even though its velocity was zero at that instant.

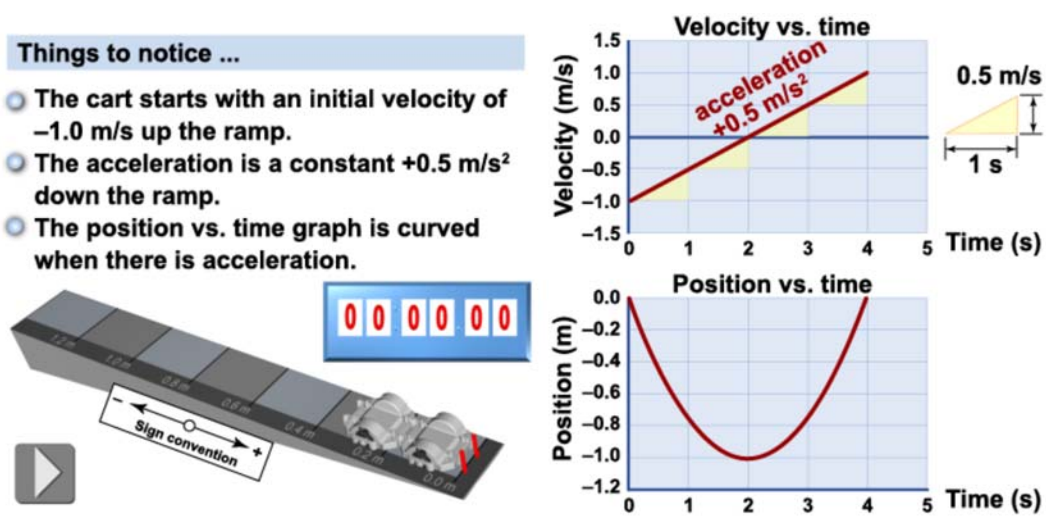



**Beware of signs!**

*Be very careful interpreting the signs of velocity and acceleration.* Many problems define positive as the expected direction of motion, such as *down* a ramp. In this case the acceleration of a car released from rest is also positive. Now think about what happens if the car is initially moving *up* the ramp. Is the acceleration still positive? How do we explain that the car moves upward, turns around, and rolls back down again?

**An uphill example**

The car in the diagram below has an initial upward velocity of −1 m/s. The constant downward acceleration adds +0.5 m/s to the velocity every second. The car's velocity starts negative then becomes 0.5 m/s *more positive* each second until $v = 0$. *At the car's highest point its velocity is zero.* After the turn-around, acceleration and velocity point in the same direction. Acceleration remains a constant +0.5 m/s$^2$ even though the sign of the velocity changes! more

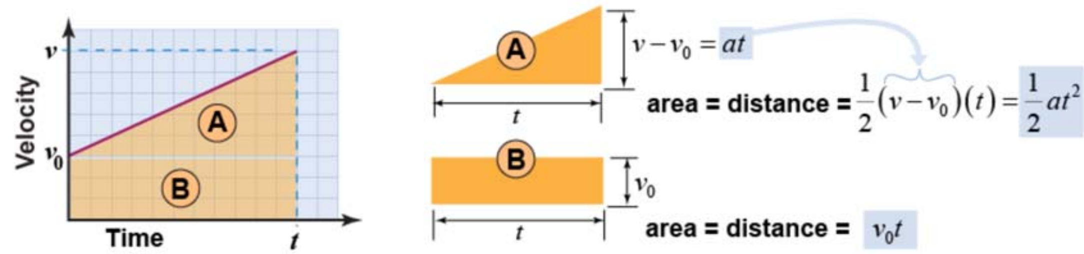



**When will motion reverse?**

Situations in which an object reverses direction always involve an acceleration that has the opposite sign from that of the velocity. For example, an acceleration of −1 m/s$^2$ adds −1 m/s to the velocity every second, so an initial positive velocity of +3 m/s becomes +2 m/s after one second, and then +1 m/s, 0 m/s, −1 m/s, etc. *Objects reverse direction when their velocity becomes zero and then change sign.* This fact is useful in solving many physics problems and in interpreting graphs.

**How does position change with acceleration?**

The last step in building a model for motion is to develop a single equation that relates position, velocity, time, and acceleration. Consider a moving object with initial velocity $v_0$ that undergoes constant acceleration. At time $t$, the velocity has increased from $v_0$ to $v$. The distance the object travels between time $t = 0$ and time $t$ is the area shaded on the graph.




The total distance traveled is the sum of the two areas. $d = v_0 t + \frac{1}{2}at^2$

**How do we calculate the distance?**

The $v$ vs. $t$ graph breaks down into two shapes. The area of a triangle is ½ base × height. For triangle A this is $\frac{1}{2}(v - v_0)t$. We also know that the change in velocity is acceleration × time, so $v - v_0 = at$. Therefore, the area of triangle A is $\frac{1}{2}at^2$. The area of rectangle B is $v_0 t$. Area on a $v$ vs. $t$ graph equals distance, and adding the triangle to the rectangle gives us this result

$$d = v_0 t + \frac{1}{2}at^2$$

**Deriving the position from the distance**

One last step remains. The distance traveled is $d = x - x_0$. Substituting this expression yields equation (4.3), which relates position $x$ at any time $t$ to initial position $x_0$, initial velocity $v_0$, and acceleration $a$.

Interactive Equation

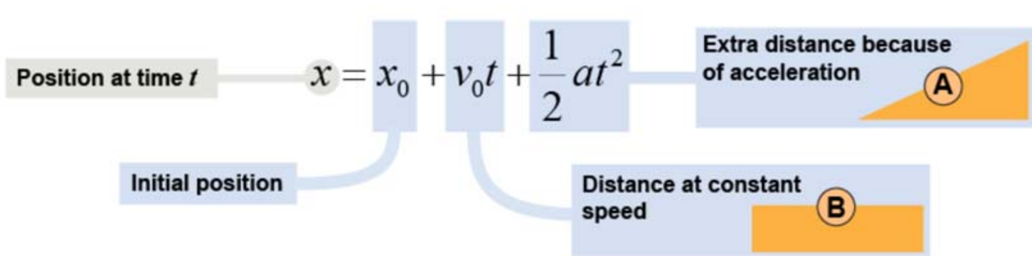
**(4.3)** $$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$x$ = position (m)
$x_0$ = initial position (m)
$v_0$ = initial speed (m/s)
$a$ = acceleration (m/s$^2$)
$t$ = time (s)

**Position**
accelerated motion

**What does the equation mean?**

Equation (4.3) has three terms on the right-hand side, and each term has its own meaning. The first term is the initial position. The second term is the change in position the object would have had if it continued at constant initial speed $v_0$. The third term is the additional change in position resulting from changes in speed that come from acceleration. Note that, if the acceleration is zero, we get back $x = x_0 + vt$, the equation for constant velocity from the last chapter!



more

| | |
|---|---|
| **Test your knowledge** | A baseball is thrown straight upward and returns to the point from which it was thrown after 5.0 s. Find the baseball's original speed. The acceleration of the baseball is 9.8 $m/s^2$ downward.<br>a. 49 m/s<br>b. 4.9 m/s<br>c. 123 m/s<br>d. 25 m/s<br>show solution |

## Section 1 review

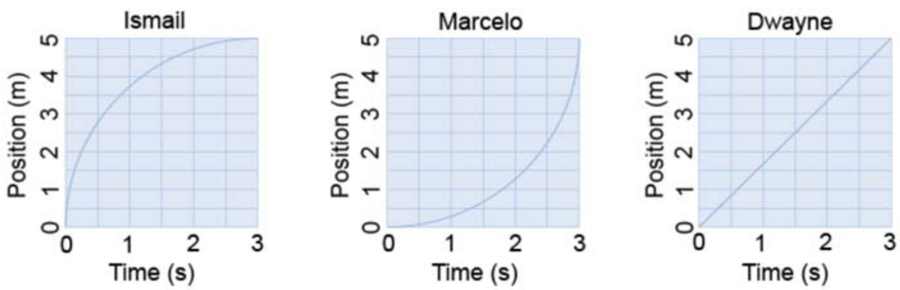
Acceleration is the rate at which velocity changes. Its SI unit is meter per second squared, or m/s$^2$. Acceleration is a vector. In one-dimensional motion, acceleration can be positive or negative (as well as zero). The slope of the velocity–time ($v$ vs. $t$) graph is the acceleration. The area under a velocity–time graph equals an accelerating object's displacement.

| | |
|---|---|
| **Vocabulary words** | acceleration, quadratic |

| | |
|---|---|
| **Key equations** | $a = \dfrac{\Delta v}{\Delta t}$ $\quad v = v_0 + at \quad x = x_0 + v_0 t + \dfrac{1}{2}at^2$ <br> $x = x_0 + \frac{1}{2}(v_0 + v)t \quad v^2 = v_0^2 + 2a(x - x_0)$ |

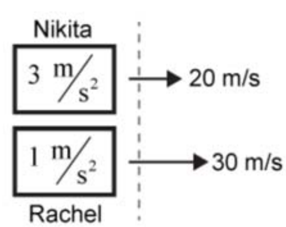
### Review problems and questions

1. Two friends are each driving a car down Highway 101. During a particular 10 s interval, Jodi's car moves at a constant 60 mph. During that same time interval, Julie's car transitions from a speed of 20 mph to a speed of 30 mph.
   a. Whose car has the greater acceleration?
   b. Do you have to convert the given quantities into SI units in order to figure out which car has the greater acceleration? show solution




2. The position–time graphs of three sprinters are shown here. Which sprinter best matches each of the following statements?
   a. "I underwent positive acceleration during the three-second interval shown."
   b. "I underwent negative acceleration during the three-second interval shown."
   c. "I underwent zero acceleration during the three-second interval shown." show solution

3. Nikita and Rachel are driving their new cars westward on the highway. At the very same instant, they cross the Weston town line (call it $t = 0$). Nikita's velocity is 20 m/s westward at that moment, while Rachel is going faster at 30 m/s (in the same direction). Each driver is accelerating westward: Nikita at 3 m/s$^2$, and Rachel at 1 m/s$^2$. They maintain these accelerations for 10 s (until $t = 10$ s).




   a. At what time $t$ are Nikita and Rachel equally fast?
   b. At that time (when they are equally fast), who has gone farther from the town line?
   c. After 10 s has passed (that is, at $t = 10$ s ), who is farther from the town line? show solution

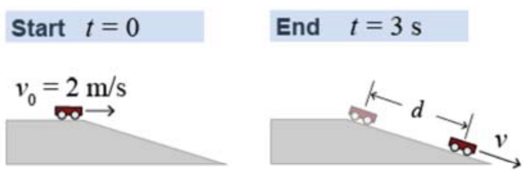
Take a Quiz

# Chapter 4 review

## Standardized test practice

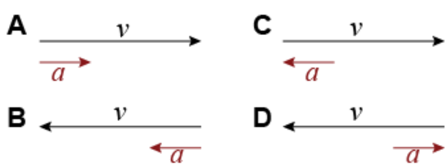
60. If the slope of a position versus time graph is decreasing, what does this indicate about velocity over time?
    A. Velocity is increasing.
    B. Velocity is decreasing.
    C. Velocity remains constant.
    D. Velocity is zero.

61. Ruth throws a baseball straight up at 20 m/s. What is the ball's velocity after 4 s?
    A. 10.2 m/s
    B. −19.2 m/s
    C. 16 m/s
    D. −76.8 m/s

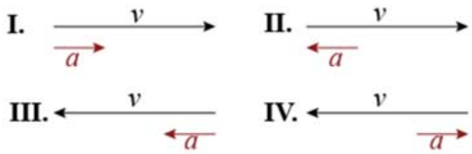



62. A cart is rolling along at a steady 2 m/s when it meets a slope. The cart accelerates at 1 m/s$^2$ as it rolls down the hill. How far will the cart move in 3 s?
    A. 6 m
    B. 5 m
    C. 15 m
    D. 10.5 m

63. If an object starts accelerating from rest at a rate of 34 m/s$^2$, how far does it travel during the first second?
    A. 20 m
    B. 17 m
    C. 34 m
    D. 26 m

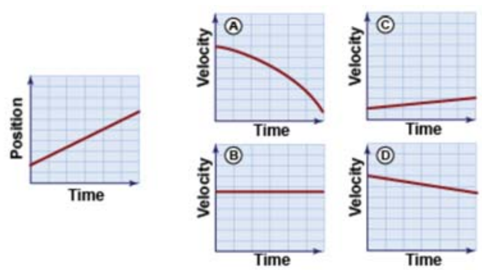



64. Which of the four diagrams best represents a negative velocity with a positive acceleration? Assume the positive direction is to the right.
    A. A
    B. B
    C. C
    D. D




65. Four different objects are moving along a horizontal axis, each with a different combination of velocity and acceleration as shown here. How many of these objects are slowing down?
    A. one
    B. two
    C. three
    D. four

66. Jed drops a 1 kg box off of the Eiffel Tower. After 3 s, how fast is the box moving?
    A. 44.1 m/s
    B. 29.4 m/s
    C. 9.8 m/s
    D. 3.3 m/s




67. Only one of the four velocity–time graphs (above right) can possibly correspond to the single position–time graph (above left). Which is it?
    A. Graph A
    B. Graph B
    C. Graph C
    D. Graph D

68. You are driving a metric car that has a speedometer that displays in meters per second. When you look down at your speedometer, you see that it reads 15 m/s. Four seconds later, you look down and it reads 23 m/s. What is your average acceleration over those 4 s?
    A. 5.75 m/s$^2$
    B. 2 m/s$^2$
    C. 3.75 m/s$^2$
    D. 2.5 m/s$^2$

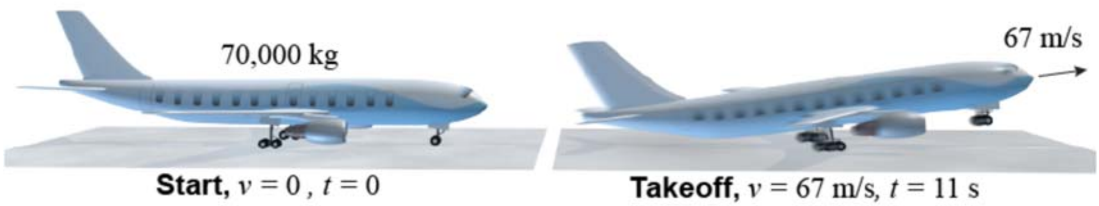
# Determining forces from motion

**How is the second law used to find a force?**

A good example of determining force from motion is the specification of aircraft engines. An aircraft must reach a minimum speed to take off and fly. The mass of the plane is known. The runway length is also known, as is the maximum comfortable acceleration for passengers. Newton's second law is applied to calculate how much thrust force must be supplied by the engines.

**Solution strategy for typical problems**

| Problem | Solution strategy |
|---|---|
| Find force when given mass and acceleration. | $F = ma$ |
| Find force when given mass, a change in velocity, and a change in time. | $a = \dfrac{\Delta v}{\Delta t} \Rightarrow F = ma$ |
| Find force when given mass, a change in position, and a change in time. | $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$ $\Rightarrow a \Rightarrow F = ma$ |




**A problem giving velocity and time**

A 70,000 kg aircraft reaches a takeoff velocity of 67 m/s (150 mph) in 11 s. Calculate the minimum force required from the engines.

| | |
|---|---|
| *Asked:* | force $F$ |
| *Given:* | $m$ = 70,000 kg, $v_f$ = 67 m/s, and $t$ = 11 s |
| *Relationships:* | $F = ma$, $a = \Delta v/\Delta t$ |
| *Solution:* | Find acceleration first: $a = \Delta v/\Delta t = (67\text{ m/s}) \div (11\text{ s}) = 6.1\text{ m/s}^2$<br>Next, find force: $F = ma = (70{,}000\text{ kg})(6.1\text{ m/s}^2) = 427{,}000\text{ N}$ |
| *Answer:* | 427,000 N |

**A problem giving position and time**

A 65 kg sprinter reaches a maximum speed of 11 m/s at a distance of 10 m from the start of a race. What average force must the sprinter's muscles create?


| | |
|---|---|
| *Asked:* | force $F$ |
| *Given:* | $m$ = 65 kg, $v_f$ = 11 m/s, and $x$ = 10 m |
| *Relationships:* | $F = ma$, $x = x_0 + v_0 t + ½at^2$, $v = v_0 + at$ |
| *Solution:* | Use both motion equations to find acceleration first:<br>$v = v_0 + at \rightarrow t = v/a$<br>$x = x_0 + v_0 t + ½at^2 \rightarrow x = ½at^2$<br>$x = ½a(v/a)^2 \rightarrow x = v^2/2a$<br>$a = v^2/2x = (11\text{ m/s})^2 \div (2)(10\text{ m}) = 6.05\text{ m/s}^2$<br>Next, find force:<br>$F = ma = (65\text{ kg})(6.05\text{ m/s}^2) = 393\text{ N}$ |
| *Answer:* | 393 N |

## Equations of projectile motion

**Gravity pulls downward**

How can we simplify these equations of motion for projectiles? We use the fact that *gravity accelerates down*; therefore, $a_y = -g$ and $a_x = 0$! The equations of motion simplify a great deal because all the terms that include $a_x$ become zero. Also, to make the math simpler, we choose the initial position to be zero, so $x_0 = 0$ m and $y_0 = 0$ m.

**Equations of motion in the *x*-direction**

Let's first examine the motion in the *x*-direction. Since there is no acceleration in *x*, the component of velocity in the *x*-direction *is constant*. The projectile moves with constant velocity along *x*! The *x* position changes linearly, as shown in equation (6.6).

**Projectile motion** *x*-component

$$x = v_{x0}t \qquad (6.6)$$
$$v_x = v_{x0}$$

$x$ = position in *x*-direction (m)
$v_{x0}$ = initial velocity in *x*-direction (m/s)
$v_x$ = velocity component in *x*-direction (m/s)
$t$ = time (s)

**Equations of motion in the *y*-direction**

Now look at the equations of motion in the *y*-direction. The acceleration in *y* has a constant value of $-g$. As we learned in Chapter 4, constant (nonzero) acceleration results in velocity that changes linearly, as shown in equation (6.7). Position in the *y*-direction is a function of $t$ and $t^2$, as also shown in equation (6.7).

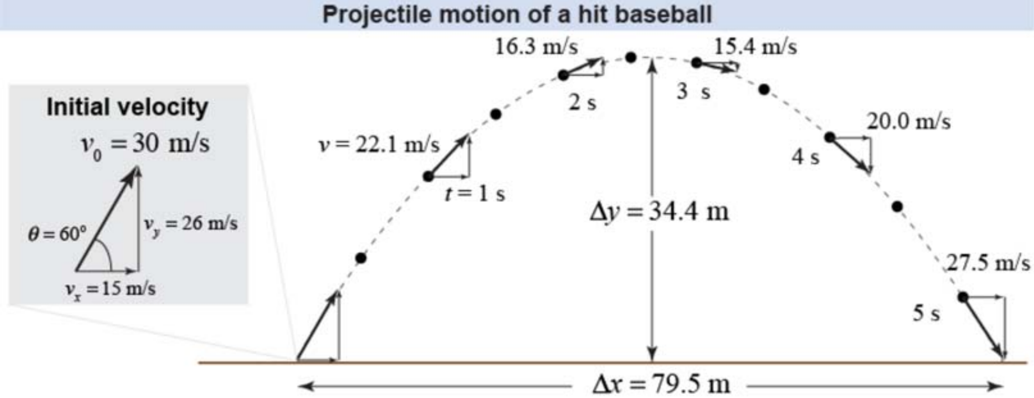
**Projectile motion** *y*-component

$$y = v_{y0}t - \frac{1}{2}gt^2 \qquad (6.7)$$
$$v_y = v_{y0} - gt$$

$y$ = position in *y*-direction (m)
$v_{y0}$ = initial velocity in *y*-direction (m/s)
$v_y$ = velocity component in *y*-direction (m/s)
$t$ = time (s)
$g$ = acceleration due to gravity = 9.8 m/s$^2$

Equations (6.6) and (6.7) are the equations of projectile motion when gravity is acting in the negative *y*-direction. The initial velocity given to the projectile—soccer ball, cannonball, etc.—has components $v_{x0}$ in the *x*-direction and $v_{y0}$ in the *y*-direction.

**Projectile motion of a hit baseball**




**Trajectory of a baseball**

How do these equations work in a real-world example, such as a fly ball hit in a baseball game? Let's assume the batter hits the ball somewhat upward with a velocity of 30 m/s and a projection angle of 60°. The ball's initial velocity has components of 15 and 26 m/s in the *x*- and *y*-directions, respectively. Plugging these values into equations (6.6) and (6.7) results in the trajectory illustrated above—where we calculated the magnitude of the velocity using $v^2 = v_x^2 + v_y^2$.

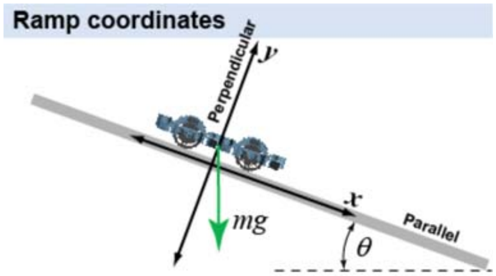
**Particle model**

In the illustration of the fly ball above, the position of the ball is depicted at fixed time intervals (every 0.5 s) to create a *particle model* of its motion. The ball positions are closest to each other at the top of the trajectory, which means that the speed is slowest there. The ball positions are furthest apart near the ground where the speed is fastest.

## Forces along a ramp

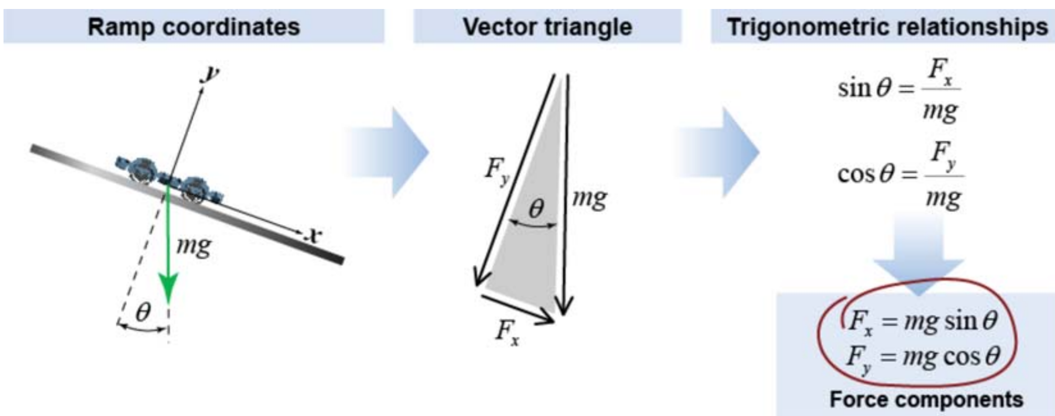
**Ramp coordinates**

Consider a car (or ErgoBot) moving on a ramp that makes an angle $\theta$ relative to the horizontal direction. There are forces both parallel and perpendicular to the ramp surface. It is convenient to define **ramp coordinates** in which the $x$-axis is along the direction of motion and parallel to the ramp. The $y$-axis is then the direction of the normal force.




**Acceleration along the ramp**

The car's motion depends on the net force in the $x$-direction (along the ramp). Because the ramp is at an angle, the weight force is not vertical in ramp coordinates but is tilted at the ramp angle. A component of the car's weight equal to $mg\sin\theta$ lies parallel to the ramp surface. This component causes the car to accelerate down the ramp at $a_x = g\sin\theta$.




Interactive Equation

$$(6.12)\quad a_x = g\sin\theta$$

$a_x$ = acceleration along ramp (m/s$^2$)
$g$ = acceleration of gravity = 9.8 m/s$^2$
$\theta$ = ramp inclination angle (degrees)

**Acceleration** in ramp coordinates

**Acceleration versus angle**

The table on the right shows how the acceleration of the car varies with the angle of the ramp. The acceleration varies as the sine of the ramp angle. The sine of 0º is 0, and the acceleration of a horizontal ramp is also zero. The sine of 90º is 1, and a vertical ramp has an acceleration of 9.8 m/s$^2$, the same as free fall.

**Acceleration vs. angle (no friction)**

| Angle | sin $\theta$ | Acceleration (m/s²) |
|---|---|---|
| 0° | 0.000 | 0 |
| 5° | 0.087 | 0.85 |
| 10° | 0.174 | 1.70 |
| 20° | 0.342 | 3.35 |
| 30° | 0.500 | 4.90 |
| 45° | 0.707 | 6.93 |
| 60° | 0.866 | 8.49 |
| 90° | 1.000 | 9.80 |

**Ramp as a simple machine**

Notice that for shallow angles the acceleration is small. This means that the force along the ramp is also small. This works to great advantage, because you can push a load up a ramp using a force smaller than the weight of the load. The ratio $F_x/mg$ is the sine of the inclination angle. At an angle of 10º the sine has a value of 0.174, so it only takes 0.174 N of force applied parallel to the ramp to raise each newton of weight. Used this way, the ramp is a *simple machine*, which you will learn more about on page 345.

**Test your knowledge**

Calculate the acceleration of a car on a ramp when the ramp's inclination angle is either 0° or 90°, and explain physically why you got those answers. show solution

## Kinetic energy

**Thinking about kinetic energy**

Video

**Kinetic energy** ($E_k$ or KE) is the energy of motion. Any object that has mass and is moving has kinetic energy *because* it is moving. When you catch a ball, your hand applies a force over some distance (your hand recoils a bit) to stop the ball. That force multiplied by the distance represents the transfer of the ball's kinetic energy to your hand by doing work on your hand. Now think about catching balls of different masses and speeds. You can probably guess that a massive or a fast-moving ball has more kinetic energy—it is harder to stop!—than a lighter, slower ball.

Interactive Equation

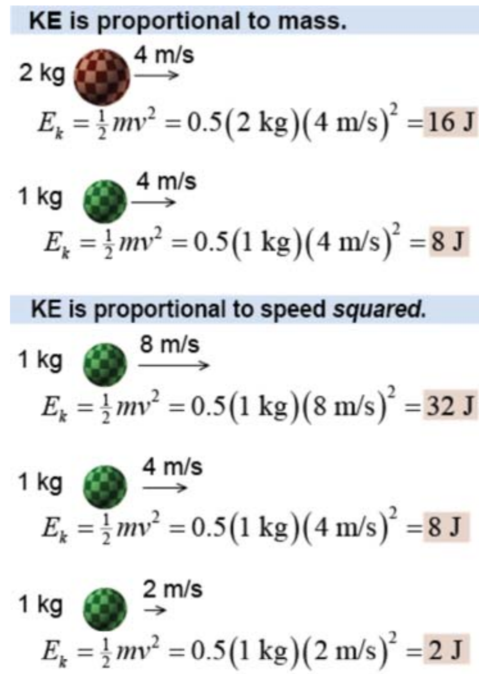
**Kinetic energy**

$$E_k = \frac{1}{2}mv^2 \qquad (9.2)$$

$E_k$ = kinetic energy (J)
$m$ = mass (kg)
$v$ = speed (m/s)

**How is kinetic energy related to mass?**

Kinetic energy is calculated using equation (9.2) above. The kinetic energy of a moving object is proportional to mass. If you double the mass, you double the kinetic energy. For example, a 2 kg ball moving at a speed of 1 m/s has 1 J of kinetic energy according to the equation. A 4 kg ball moving at the same speed has 2 J of kinetic energy, or twice as much. This is a *linear* relationship.

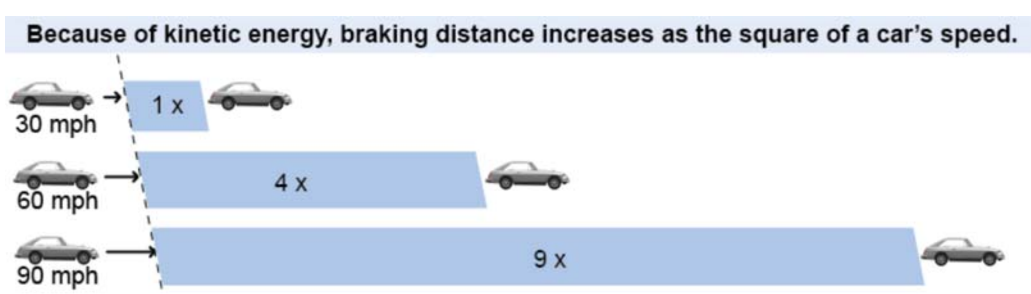



**Kinetic energy increases as speed squared**

According to equation (9.2) kinetic energy depends on the *square* of the speed of a moving object. Consider a 2 kg ball traveling at 1 m/s with 1 J of kinetic energy. The same ball moving at 3 m/s has 9 J of kinetic energy. If you multiply the speed by 3, then the kinetic energy is multiplied by a factor of $3^2 = 9$. This is an example of a *nonlinear* relationship. more

**Kinetic energy and braking distance**

The fact that kinetic energy increases with the square of the speed has implications for the stopping distance of a car. As a car brakes, work is done to transform the car's kinetic energy into thermal energy. Work is force multiplied by distance. Assuming that the braking and road conditions result in a relatively constant stopping force, then the distance it takes the car to stop is proportional to the initial kinetic energy. At 30 mph a car can stop in about 15 m. When the speed is doubled to 60 mph (and the kinetic energy increases by a factor of 4) it takes four times as much distance to stop.



# Investigation 11A: Conservation of momentum

| **Essential questions** | *How does momentum change for objects in an isolated system?*<br>*What is momentum conservation?* |
|---|---|

Newton's third law states that for every force there is an equal and opposite reaction force. The law of conservation of momentum is a powerful generalization of Newton's third law. For an isolated system, the total momentum of all the objects inside is constant. In this investigation, you will explore the conservation of momentum for two carts that are subject to no outside net force. The only catch is that the carts have a compressed spring inserted between them! Is momentum conserved for this system?

## Conservation of momentum for spring-loaded carts

The interactive model simulates two carts with a compressed spring between them. When they are released, the spring causes the carts to move in opposite directions. These are a type of *ballistic cart*.

1. Select a mass for each cart.
2. Press [Run] to start the simulation.
3. Run the simulation for different combinations of masses for the two carts. Use your data table to record the mass and velocity for each combination.

*a. Describe the velocities when the masses of the two carts are equal.*

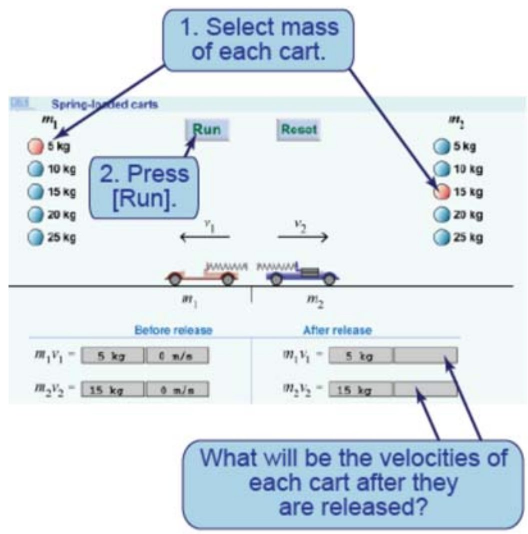
*b. Describe the velocities when the red cart has more mass than the blue cart.*

*c. Describe the velocities when the blue cart has more mass than the red cart.*

*d. Evaluate the data in your table. What quantity can you construct or calculate that is equal and opposite for the two carts after they are released? How is this the most logical conclusion to draw from your data?*

*e. Why are ballistic carts useful in studying conservation of momentum? Explain.*

*f. If the two carts together are considered a closed system, what is the net force on the system? What is the change in the system's momentum after being released? Use appropriate equations to explain how these two questions are related to each other.*




| $m_1$ (kg) | $v_1$ (m/s) | $p_1$ (kg m/s) | $E_{k1}$ (J) | | $m_2$ (kg) | $v_2$ (m/s) | $p_2$ (kg m/s) | $E_{k2}$ (J) |
|---|---|---|---|---|---|---|---|---|
| 5 | | | | | 5 | | | |
| 10 | | | | | 5 | | | |
| 15 | | | | | 5 | | | |
| 5 | | | | | 10 | | | |
| 10 | | | | | 10 | | | |
| 15 | | | | | 10 | | | |
| 5 | | | | | 15 | | | |
| 10 | | | | | 15 | | | |
| 15 | | | | | 15 | | | |

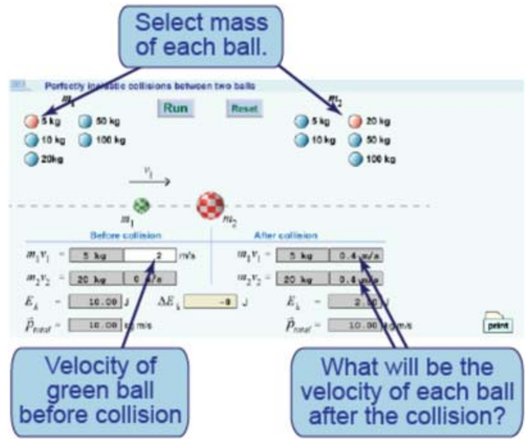
| Interactive Simulation | In this interactive simulation, you will investigate the conservation of momentum using two carts that are initially in contact with each other and have a compressed spring between them. When the carts are released, the spring expands and the two carts are shot out in opposite directions. |
|---|---|

# Investigation 11B: Collisions

**Essential questions** *How can we predict the outcome of a collision?*

In an elastic collision, both kinetic energy and momentum are conserved. This means that we can predict the outcome of a collision if we know the energy and momentum of the system before the collision. In an inelastic collision, some or all of the kinetic energy is transformed into other forms of energy (called losses). Momentum, however, is still conserved in an inelastic collision, just as it is in any collision. In this investigation, you will predict the outcome of collisions involving one moving ball and one stationary ball (the target).

## Part 1: Perfectly inelastic collisions

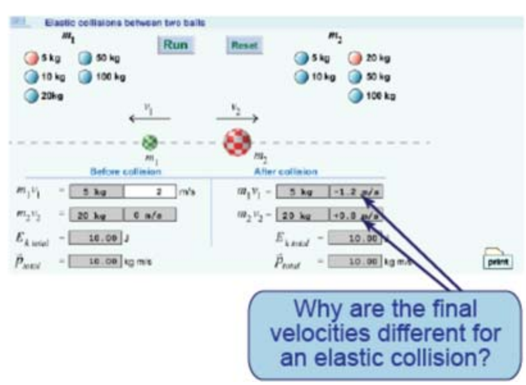



1. The interactive model simulates a *perfectly inelastic* collision between two balls.
2. [Run] starts the simulation. [Stop] stops it without changing values. [Repeat] resets the final values to zero and runs the simulation again.
3. Select an initial velocity for the moving ball.
4. Run the simulation for different combinations of masses for the red and green balls. For each combination, tabulate the masses and velocities.
5. Examine the table for patterns in the data.

a. *Describe the velocities before and after the collision when masses are equal.*
b. *Describe the velocities (before and after) when the red target ball has more mass.*
c. *Describe the velocities (before and after) when the green ball has more mass.*

**Interactive Simulation** In this simulation the green ball hits the stationary red ball. *Both balls stick together after impact,* which is a perfectly inelastic collision. Investigate the different combinations of mass for the two balls. Use your data table to record the initial and final velocities for each combination.

## Part 2: Elastic collisions




1. The interactive model simulates the collision of two *elastic* rubber balls.
2. Run the simulation for different combinations of masses for the red and green balls. For each combination, tabulate the masses and velocities.

a. *Describe the velocities before and after the collision when masses are equal.*
b. *Describe the velocities (before and after) when the red target ball has more mass.*
c. *Describe the velocities (before and after) when the green ball has more mass.*
d. *Describe the measurements that must be made with a collision apparatus, such as this simulation, to distinguish between elastic and inelastic collisions.*