# The Teachers' Lounge Place Value and Division

Emphasizes place value and the distributive property as they relate to division. Partitive and quotative situations are examined, as well as division with remainders.



Instructional Support				
Unit	The Teachers' Lounge: Place Value and Division			
Additional Resources	Minilessons for Extending Multiplication and Division: A Yearlong Resource			
Teacher Supports	New Perspectives Online (P2S2): The Teachers' Lounge			
Assessment	New Perspectives on Assessment: Grade 4			

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Focus, Coherence, and Rigor Scope and Sequence of the Unit Dynamic Assessment Table 4: Lesson Summaries Focus, Coherence, and Rigor The focus of this unit is on division and the use of place value in supporting multiplication and division and it was designed with the understanding that division is challenging for many students. Research supports that through solving realistic problems students will develop a variety of methods for calculation. The story context of the vending machines is the teachers' lounge sets the stage for a series of investigations designed to support the development of a repertoire of strategies for multiplication and division (4.OA.A). As students investigate the vending machines, the open array emerges as a model of the situation, then as a model to represent students' thinking, and finally as a tool for student thinking. The array model is used in minilessons and in subsequent investigations to further develop strategies for division using partial products and the distributive law.

As students have previously used the array model for multiplication, setting a division problem in a context that uses an array enables a deeper understanding of the relationship between multiplication and division (4.NBT.B.5 and 4.NBT.B.6). In the opening investigation students consider two questions: How many sixes are in 156? (quotative division where in this context the sixes are six-packs and there are six columns in a vending machine holding 156 drinks), and, How many bottles are in each column? (partitive division, here the array is  $? \times 6 = 156$ .) The representation allows students to merge their thinking onto one diagram and in this way the investigation is well-suited for helping students understand the relationship between partitive and quotative division. In the subsequent investigation, by adding to the context that six-packs are delivered ten to a box, students begin to use ten sixes (60) as part of their strategies for division, noticing the place value patterns that occur when multiplying by 10, and thereby start using the distributive property of multiplication in solving division problems. The remainder of the unit contains more contexts that solidify these division strategies, including pulling out multiples of ten times a number when dividing, where the students can explain their strategies using equations, rectangular arrays and/or area models (4.NBT.B.6).

To more deeply understand how the big ideas, strategies and models developed in this unit lead to deep conceptual understanding, procedural

skill and fluency, and capacity to work with engaging applications, it is important to read the *Unit Overview* on pages 5 - 9 of the unit and complete the entire P2S2 module for this unit, <u>Intro to the Landscape</u>.

This unit builds upon work from previous units in grades 3 and 4 as follows:

- Grade 3 *Muffles' Truffles* develops the open array as a tool for multiplying (3.OA),
- Grade 4 *Puzzle Packing Companies* (4.NBT.A and 4.NBT.B.4) develops a generalized notion of place value understanding and fluency with addition and subtraction of multi-digit numbers, which then is applied within the investigations in *The Teachers' Lounge*, and
- Grade 4 *Number Detectives* utilizes the open number line, open array, and ratio table as students apply their knowledge of multiplication and division to solve problems (4.OA.A) and gain familiarity with factors and multiples (4.OA.B).

This unit sets the foundation for further development in subsequent units in grades 4 and 5 as follows:

- Grade 4 *Muffles Measurement* has students solving measurement problems (4.MD.A.1) that require some fluency with multiplication and division, and
- Grade 5 *The Box Factory* extends multiplication and use of factors to understand the foundations of volume using the 2-D and 3-D array as models (5.NBT.B.5).

Table 1. Focus of the reachers Lounge	Table	1:	Focus	of	The	Teachers'	Lounge
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Focus of the Unit	Landscape of Learning	<u>Grade 4 Focus MAP</u>
Core	The Landscape of Learning for Multiplication and Division	<ul> <li>4.OA.A Use the four operations with whole numbers to solve problems.</li> <li>4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic</li> </ul>
Supporting	The Landscape of Learning for Place Value	4.NBT.A Generalize place value understanding for multi-digit whole numbers



### Figure 1: The Teachers' Lounge Companies Coherence Map

# Scope and Sequence of the Unit

Table 2 provides the scope and sequence of the unit, including estimated class periods (assuming approximately 60 minutes each), alignment to clusters and common core state standards, the standards for mathematical practices, and a high level overview of differentiated supports. For more detail on each lesson click on the hyperlink for each Day. At the bottom of the table you will find related resources for additional opportunities to develop conceptual understanding, procedural skill, and fluency.

### Table 2: Scope and Sequence at a Glance

Class Periods	Lesson	<u>Cluster</u>	CCSSM	MPs	Differentiated Supports <u>Language Supports</u> <u>Universal Design for Learning</u>
2	<u>Day 1: The</u> <u>Teachers'</u> Lounge	4.OA.A 4.NBT.B	4.OA.A.3 4.NBT.B.5 4.NBT.B.6	MP1 MP5	<ul> <li>Language Supports</li> <li>SUPPORT SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning</li> <li>Universal Design for Learning</li> <li>REPRESENTATION: Activate or supply background knowledge.</li> </ul>
1	<u>Day 2: The</u>	4.0A.A	4.0A.A.3	MP3	Language Supports

	<u>Teachers'</u> Lounge	4.NBT.B	4.NBT.B.5 4.NBT.B.6		<ul> <li>OPTIMIZE OUTPUT: Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing</li> <li><u>Universal Design for Learning</u></li> <li>REPRESENTATION - Clarify Syntax and Structure.</li> </ul>
2	<u>Day 3: The</u> Iced Tea Machine	4.NBT.B	4.NBT.B.5 4.NBT.B.6	MP4 MP6	<ul> <li>Language Supports</li> <li>SUPPORT SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning.</li> <li>Universal Design for Learning Supports</li> <li>REPRESENTATION: Highlight patterns, critical features, big ideas, and relationships</li> </ul>
1	<u>Day 4: Fill the</u> <u>Machine</u>	4.NBT.B	4.NBT.B.5 4.NBT.B.6	MP7	<ul> <li>Understanding Language</li> <li>CULTIVATE CONVERSATION: Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class.)</li> <li>Universal Design for Learning Supports</li> <li>ENGAGEMENT: Vary demands and resources to optimize challenges.</li> </ul>
1	<u>Day 5:</u> Amusement <u>Park</u> <u>Conversation</u>	4.OA.A 4.NBT.B	4.OA.A.1 4.OA.A.2 4.OA.A.3 4.NBT.B.5 4.NBT.B.6	MP2 MP6	<ul> <li>Language Supports</li> <li>OPTIMIZE OUTPUT: Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing.</li> <li>Universal Design for Learning</li> <li>REPRESENTATION: Illustrate through multiple media.</li> </ul>
1	<u>Day 6:</u> <u>Amusement</u> <u>Park, Tickets</u> <u>for Rides</u>	4.OA.A 4.NBT.B	4.OA.A.1 4.OA.A.2 4.OA.A.3 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6	MP2	<ul> <li>Language Supports</li> <li>MAXIMIZE META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.</li> <li>Universal Design for Learning         <ul> <li>REPRESENTATION: Guide Information Processing and Visualization.</li> </ul> </li> </ul>

2	<u>Day 7:</u> Amusement Park, Tickets for Rides	4.OA.A 4.NBT.B	4.OA.A.1 4.OA.A.2 4.OA.A.3 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6	MP3 MP8	Language Supports <ul> <li>MAXIMIZE META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.</li> </ul> Universal Design for Learning <ul> <li>REPRESENTATION: Highlight patterns, critical features, big ideas, and relationships.</li> </ul>
1	Day 8: <u>Tickets for</u> <u>the Teachers</u>	4.NBT.B	4.NBT.B.4 4.NBT.B.5	MP2 MP7 MP8	<ul> <li>Language Supports         <ul> <li>MAXIMIZE META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.</li> <li>Universal Design for Learning                 <ul> <li>REPRESENTATION: Guide Information Processing and Visualization.</li> </ul> </li> </ul> </li> </ul>
1	<u>Day 9: What</u> <u>to do with</u> <u>the</u> <u>Remainders</u>	4.NBT.B	4.NBT.B.4 4.NBT.B.5 4.NBT.B.6		<ul> <li>Language Supports</li> <li>SUPPORT SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning.</li> <li>Universal Design for Learning</li> <li>REPRESENTATION - Clarify Syntax and Structure.</li> </ul>
1	<u>Day 10:</u> <u>Opportunitie</u> <u>s for</u> <u>Assessment</u>	4.NBT.B	4.NBT.B.5 4.NBT.B.6	MP8	<ul> <li>Language Supports</li> <li>SUPPORT SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning.</li> <li>Universal Design for Learning         <ul> <li>ACTION AND EXPRESSION: Build fluencies with graduated levels of support for practice and performance.</li> </ul> </li> </ul>
			Additi	onal C	Dpportunities
Periods	Resou	rce	Model	l	Landscape of Learning <u>Cluster</u> <u>Standard</u>
2	Minilessons for Extending Multiplication and Division <sup>1</sup> • A4 -A7 • A13 - A20		The Open A	Array	The Landscape of Learning for Multiplication and Division4.NBT.B4.NBT.B.5 4.NBT.B.6

<sup>1</sup> Choose from the following minilessons based on the big idea or strategy you want to foster.

• A54 - A55	The Landscape of	
	Learning for Place	
	Value	

# Dynamic Assessment

CFLM's approach to assessment is <u>dynamic rather than static</u>. The goal is to capture students' emerging modeling, strategies, and big ideas in the moment, and formally, as students engage in solving routine and non-routine problems.

Hence, CFLM's assessment approach blurs the lines between assessment and learning where any opportunity for students to engage in solving routine and non-routine applications is also an opportunity for assessing. Three structures exist within every unit, through the <u>New Perspectives on</u> <u>Assessment application</u>.

- A Progress Monitoring Sheet can be used throughout the unit as the teacher "listens in the moment" during conferrals, or examines student artifacts. You can find guidance throughout the unit and on P2S2 (e.g., *Tips for Assessment, Reflections on the Day,* and *Supporting the Investigation.*)
- Open Ended routine and/or non-routine problems capture students developing ideas rather than only assessing right or wrong answers. See the last page of each assessment for guidance on Strategies and Teaching Implications.
- Two-Pen assessments are designed to assess procedural skill and fluency through the conceptual connections they make among problems. See the last page of each assessment for guidance on Strategies and Teaching Implications.

Teachers are highly encouraged to re-administer these assessments to measure any growth or progress from their original administration.

### Table 3: Assessments

Class Periods	Structures for Assessing	Landscape of Learning	Alignment to Cluster
N/A	Operations and Algebraic Thinking, Progress Monitoring Sheet, Grade 4	The Landscape of Learning for Multiplication and Division	4.0A.A 4.0A.B
N/A	Numbers in Base Ten, Progress Monitoring Sheet, Grade 4	The Landscape of Learning for Multiplication and Division The Landscape of Learning for Place Value	4.NBT.A 4.NBT.B
0.5	<ul><li>A. Grade 4 Open Ended NBT #3</li><li>B. Grade 4 Two Pen NBT #3</li></ul>	The Landscape of Learning for Multiplication and Division The Landscape of Learning for Place Value	4.NBT.A 4.NBT.B
0.5	<ul><li>C. Grade 4 Open Ended OA #2</li><li>D. Grade 4 Two Pen OA #2</li></ul>	The Landscape of Learning for Multiplication and Division	4.0A.A
1	Additional Days of Targeted Instruction (see <i>Implications for</i> <i>Teaching</i> in the Assessment discussion.)	The Landscape of Learning for Multiplication and Division The Landscape of Learning for Place Value	4.NBT.A 4.NBT.B 4.OA.A

#### Table 4: Lesson Summaries

Lesson	<ul> <li>Day 1: The Teacher's Lounge</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> <li>Preparing for the Math Congress</li> </ul>
Common Core State Standards	In the first investigation, the students investigate two division contexts involving vending machines (4.OA.3) (4.NBT.5, 6). Note that although the context is division, the way they work with the investigation will engage them in all four operations pertaining to the division calculation.
Standards for Mathematical Practice	As students use a variety of strategies to solve the two problems in this investigation, they engage in MP1 and MP5. See some examples on page 14 of the unit and the <u>Supporting</u> the Investigation page on P2S2 for this unit.

Differentiated	Language SupportsToday's context (see pg. 12) was purposeful in that that a soda machine can be visually represented as an array. However, allow time for students to think deeply and converse about the context so that they will be able to better represent it - <a href="Support">Support</a> SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning.• Given that students are working in pairs, consider the following resource when making a decision about partnerships - <a href="Strategic Grouping for Home Language Supports">Strategic Grouping for Home Language</a>
Supports	<ul> <li><u>Universal Design for Learning Supports</u></li> <li>Today's context, the vending machines, provide a familiar context upon which students can access to further learning. See pg. 14 for the variety of ways students are making sense of mathematics - <u>REPRESENTATION: Activate or supply background knowledge</u>.</li> <li>Consider beginning your conferrals with students needing more support. See the excerpt on pg. 13, <i>Conferring with Students at Work,</i> as an example of how to support student development through inquiry - <u>Promoting Understanding through Discourse: Use questions, prompts, and hints</u>.</li> </ul>

Lesson	<ul> <li>Day 2: The Teachers' Lounge</li> <li>Preparing for the Math Congress (continued from Day One)</li> <li>Facilitating the Math Congress</li> </ul>
Common Core State Standards	Students review their work, create posters, have a gallery walk and congress on the investigation from Day 1(4.OA.3) (4.NBT.5, 6).
Standards for Mathematical Practice	During the gallery walk and congress, students engage in MP3. See the <u>Planning Math</u> <u>Congresses page on P2S2 for this unit</u> .
	Language Supports The gallery walk is an opportunity for students give and receive written feedback prior to engaging in the discussion of the congress - <u>OPTIMIZE OUTPUT: Strengthen the</u> <u>opportunities and supports for helping students to describe clearly their mathematical</u> <u>thinking to others, orally, visually, and in writing</u>
Differentiated Supports	• If you feel students need more opportunities to prepare for the congress, consider embedding <u>Mathematical Language Routine 1: Stronger and Clearer</u> <u>Each Time</u> .

<u>Universal Design for Learning Supports</u> The guidance on pgs. 19 - 20, <i>Introducing Division Notation,</i> suggests that now that students have some understanding of division, the teacher should discuss the various symbols used for division - <u>REPRESENTATION - Clarify Syntax and Structure</u> .
<ul> <li>Some students may need a larger or more structured sticky note to provide written feedback - <u>Helping Students Manage Tasks and Organization: Reformat</u> <u>handouts to provide more workspace</u>.</li> </ul>

Lesson	<ul> <li>Day 3: The Iced Tea Machine</li> <li>Minilesson: A String of Related Problems</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> <li>Preparing for the Math Congress</li> <li>Facilitating the Math Congress</li> </ul>
Common Core State Standards	The minilesson involves a string of related problems where students are multiplying numbers using partial products (4.NBT.5). Students then investigate how many six packs and boxes of ten six packs are needed to fill the iced tea machine (4.NBT. 5, 6) and a congress is held.
Standards for Mathematical Practice	The minilesson allows students to attend to calculate accurately and efficiently and formulate clear explanations of their thinking as they discuss partial products (MP6). The investigation also promotes the use of MP6 for the same reasons, but also is designed to model with mathematics (MP4).
Differentiated Supports	<ul> <li>Language Supports</li> <li>Today's context (see pg. 12) was purposeful in that it maintains the array model with an iced teach machine. Allow time for students to think deeply and converse about the context so that they will be able to better represent it - <u>SUPPORT SENSE-MAKING</u>: <u>Scaffold tasks and amplify language so students can make their own meaning</u>.</li> <li>For a more structured approach to unpacking today's context consider embedding, <u>Mathematical Language Routine 6</u>: <u>Three Reads</u>.</li> </ul>
	<ul> <li>Universal Design for Learning Supports         The string from today's <i>Minilesson</i> (pg. 22) is crafted to support understanding of the         distributive property and partial product strategies modeled through the open array -         <u>REPRESENTATION: Highlight patterns, critical features, big ideas, and relationships</u>         For today's investigation, some students may not yet have constructed the fluency         needed to solve the problem. Encourage them to use ideas in the open array and         provide connecting cubes only as needed (see pgs. 23 - 24, <i>Differentiating         Instruction</i>) - Helping Students Access Math in Varied Ways: Offer manipulatives.     </li> </ul>

Lesson	<ul> <li>Day 4: Fill the Machine</li> <li>Minilesson: A Multiplication String with Associative and Distributive Properties</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> </ul>
Common Core State Standards	The minilesson involves a string of related problems to support multiplying numbers using associative and distributive properties (4.NBT.5). The game <i>Fill the Machine</i> develops division strategies using partial products and the array model (4.NBT. 5, 6).
Standards for Mathematical Practice	In the minilesson, students notice patterns appearing in the answers and use the open array to explore why the patterns occur (MP7). See the Minilesson section on pages 27-28 of the unit. During the game, students are provided opportunities to think about large friendly chunks of a number – to use the ten-times strategy in making partial products (MP7).
Differentiated Supports	<ul> <li>Language Supports         The excerpt, A Portion of the Minilesson (pgs. 27 - 28) illustrates talk moves the teacher             utilizes to ensure students are engaged in the discussion (e.g., questioning, pair talk. and             modeling student thinking) - <u>CULTIVATE CONVERSATION: Strengthen the opportunities             and supports for constructive mathematical conversations (pairs, groups, and whole             class.)         For more guidance on how to facilitate the discussion to support language             development, see <u>Mathematical Language Routine 8: Discussion Supports</u>.         </u></li> </ul>
	<ul> <li><u>Universal Design for Learning Supports</u></li> <li>In today's game, <i>Fill the Machine</i> (pgs. 28 - 29) students are expected to use the strategies and models from the minilesson (associative and distributive property with use of ten-times) to support the computations in today's game. The guidance on pg. 29 under <i>Differentiated Instruction</i>, suggests making available arrays of multiplication facts for students who have not yet automatized them - <u>ENGAGEMENT: Vary demands and resources to optimize challenge</u>.</li> <li>Consider a more structured and larger recording sheet for students to draw the open array associated with the cards from Appendix D - <u>Helping Students Manage Tasks and Organization: Reformat handouts to provide more workspace</u>.</li> </ul>

	<ul> <li>Day 5: Amusement Park Conversation</li> <li>Minilesson: A Multiplication String Using the Distributive Property of Multiplication over Subtraction</li> </ul>
Lesson	<ul> <li>Developing the Context</li> </ul>
	<ul> <li>Supporting the Investigation</li> </ul>
	<ul> <li>Preparing for the Math Congress</li> </ul>
	<ul> <li>Facilitating the Math Congress</li> </ul>

Common Core State Standards	During the minilesson, students work with a string of related problems that involves multiplying numbers using the distributive property of multiplication over subtraction (4.NBT.5). The students investigate partitive division of 100 divided by 13 and have a congress on their work (4.NBT. 5, 6). The context is given as a word problem and the way the operations are used involves multiplicative comparisons (4.OA.1,2,3).
Standards for Mathematical Practice	Students attend to precision during the minilesson as they describe relationships between the problems (MP6). During the investigation, students engage in MP2 as they represent the problem and make connections between the symbols and the context. See the section titled Conferring with Students at Work on page 32 of the unit for an example.
Differentiated Supports	<ul> <li>Language Supports</li> <li>After sufficient time is devoted to the problem, have students make posters of their to for a gallery walk - <u>OPTIMIZE OUTPUT</u>: Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing.</li> <li>For more opportunities to clarify thinking and communicating mathematically about how a student solved the problem, consider embedding <u>Mathematical Language Routine 1: Stronger and Clearer Each Time</u> after students have solved the problem.</li> <li>Universal Design for Learning Supports</li> <li>Today's minilesson (pg. 31) utilizes a visual model, the open array, to highlight the associative and distributive properties - <u>REPRESENTATION</u>: Illustrate through multiple media.</li> <li>For students who struggle with working memory or today's computation, consider providing clip or white boards upon which they can process the problems - <u>Helping Students Access Math in Varied Way: Offer alternative ways for students to show what they know</u>.</li> </ul>
Lesson	<ul> <li>Day 6:Amusement Park, Tickets for Rides</li> <li>Minilesson: Connecting Multiplication and Division</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> <li>Preparing for the Math Congress</li> </ul>
Common Core State Standards	The minilesson uses a string of related problems to support students in using the distributive property and connecting multiplication and division (4.NBT.4,5). Then, students investigate a division context involving ride tickets and have a congress on their

	work (4.NBT. 5, 6). The context is given as a word problem and the way the operations are used involves multiplicative comparisons (4.OA.1,2,3).
Standards for Mathematical Practice	The minilesson provides opportunities for students to look for and make use of structure as they examine relationships between multiplication and division (MP7).
	In the investigation, students make connections between the context and the quantities (MP2) as they solve the problem.
Differentiated Supports	<ul> <li>Language Supports</li> <li>Today's context (pg. 35) has many quantities described with phrases such as "nine groups of eight children", "three and five tickets each", and "each chaperone". Spend some time dissecting this language so students can understand the context - MAXIMIZE</li> <li>META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.</li> <li>For a more formal protocol of deconstructing the text, consider using Mathematical Language Routine 6: Three Reads.</li> </ul>
	<ul> <li><u>Universal Design for Learning Supports</u></li> <li>The guidance in <i>Supporting the Investigation</i> (pg. 36) suggests the teacher focus on strips of 10 during the conferral - <u>REPRESENTATION: Guide Information Processing and</u></li> <li><u>Visualization</u>.</li> <li>For students needing more concrete support, consider including actual sets of 10 from a ticket roll, only as needed - <u>Helping Students Access Math in Varied Ways:</u> <u>Move from concrete to representational to abstract.</u></li> </ul>
Lesson	<ul> <li>Day 7: Amusement Park, Tickets forRides</li> <li>Preparing for the Math Congress</li> <li>Facilitating the Math Congress</li> <li>Minilesson: A Division String Using the Distributive Property of Multiplication over Addition.</li> </ul>
Common Core State Standards	The students have a congress on the investigation from day six. (4.NBT. 5, 6), (4.OA.1,2,3). The day closes with a division string using the distributive property and connecting multiplication and division (4.NBT.4,5).
Standards for Mathematical Practice	The gallery walk and congress provide opportunities to engage in MP3. Then, the minilesson encourages students to look at partial quotients to consider how they can be combined to determine larger quotients of related numbers (MP8).

Differentiated Supports	<ul> <li>Language Supports         Today's congress (pgs. 39 - 40) focuses on efficiency and the use of ten-times. Note the many talk moves (see A Portion of the Math Congress) from the teacher to encourage reflection and discussion - MAXIMIZE META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.     </li> <li>For students needing more language support to participate fully in the congress consider using Bounce Cards for Primary Grades in English or Spanish and other L1's as needed.</li> </ul>
	<ul> <li>Universal Design for Learning Supports         Today's minilesson (pg. 41) utilizes is crafted to examine how partial quotients can be         combined to find larger quotients of related numbers. Understanding ten-times (eg, 60 ÷ 3         is the same as 6-tens ÷ 3, which equals 2-tens or 20) - <u>REPRESENTATION: Highlight         patterns, critical features, big ideas, and relationships.         </u></li> <li>For students who struggle with working memory or today's computation, consider         providing clip or white boards upon which they can process the problems - <u>Helping         Students Access Math in Varied Way: Offer alternative ways for students to show         what they know.</u></li> </ul>
Lesson	<ul> <li>Day 8:Tickets for the Teachers</li> <li>Minilesson: A Division String Using the Distributive Property of Multiplication over Addition or Subtraction</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> <li>Preparing for the Math Congress</li> <li>Facilitating the Math Congress</li> </ul>
Common Core State Standards	The minilesson involves a string of related problems using the distributive property and connecting multiplication and division (4.NBT.4,5). Then, students investigate a division context involving tickets that come in strips of ten. A congress on the problem is held (4.NBT. 5, 6).
Standards for Mathematical Practice	The minilesson continues the development of MP7 and MP8 as students look for relationships and use shortcuts in dividing larger numbers.
	During the investigation, students are reasoning abstractly and quantitatively (MP2) as they use mathematical models and strategies to investigate the three questions about the context.

Differentiated Supports	Language Supports Today, students prepare posters for a gallery walk (pg. 44) focusing on only the important ideas and strategies they came up with - MAXIMIZE META-AWARENESS: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language. • Now that your students have had several opportunities to develop how place value supports division, consider using the posters or conversation to assess students' mathematics language development. See <u>Assessment of Student Writing and Oral</u> Language Production for guidance.
	<ul> <li><u>Universal Design for Learning Supports</u></li> <li>Students are provided with graph paper as they set off to answer the questions from Appendix H. See pgs. 43 - 44, <i>Supporting the Investigation</i>, for guidance on how to use the visuals to support understanding - <u>REPRESENTATION: Guide Information Processing and Visualization</u>.</li> <li>For students still needing more concrete support, consider including actual sets of 10 from a ticket roll, only as needed - <u>Helping Students Access Math in Varied Ways: Move from concrete to representational to abstract.</u></li> </ul>

<ul> <li>Day 9: What to Do with the Remainders</li> <li>Minilesson: A Division String</li> <li>Supporting the Investigation</li> </ul>
The minilesson involves a string of related division problems using the distributive property and connecting multiplication and division (4.NBT.4,5). The students then work on a collection of division problems with remainders using Appendix I (4.NBT. 5, 6).
The minilesson continues the development of MP7 and MP8 as students look for relationships and use shortcuts in dividing larger numbers. During the investigation, students are reasoning abstractly and quantitatively (MP2) as they use mathematical models and strategies to investigate what to do with the remainder.
<ul> <li>Language Supports         Today's investigation uses real world contexts to support sense making of remainders -             SUPPORT SENSE-MAKING: Scaffold tasks and amplify language so students can make their             own meaning.         </li> <li>For a more structured approach to unpacking today's context consider         embedding, <u>Mathematical Language Routine 6: Three Reads</u>.</li> </ul>

### Universal Design for Learning Supports

Guidance is provided within *Supporting the Investigation* (pg. 47- 48) to introduce syntax or more formal notation to represent a remainder - <u>REPRESENTATION - Clarify Syntax and</u> <u>Structure</u>.

 For students who are struggling, to solve the problems without teacher support, consider, teach how to represent the contexts of the problems visually before solving it (e.g., drawing tables with 8 chairs) - <u>Building Student Independence:</u> <u>Teach and model strategies for: Organization, Self-questioning and self-monitoring</u>, and Problem-solving.

<ul> <li>Day 10: Opportunities for Assessment</li> <li>Developing the Context</li> <li>Supporting the Investigation</li> <li>Math Journals and Building a Learning Scroll</li> </ul>
The students then work on a collection of division problems with remainder as part of an assessment of their learning in the unit using Appendix J (4.NBT. 5, 6).
This investigation allows students to engage in MP8 as they consider the variety of strategies that have emerged in the unit and use strategies that work well for the numbers and/or context provided.
Language SupportsThe assessment (Appendix J) is to be completed independently so you can assess students individually. The assessments contain both bare number problems, and word problems within familiar contexts that should evoke familiar language (eg., 6 packs) - <a href="SUPPORT SENSE-MAKING">SUPPORT</a> SENSE-MAKING: Scaffold tasks and amplify language so students can make their own meaning.• For students who struggle with language, consider encouraging them to use a structured routine such as Mathematical Language Routine 6: Three Reads to support comprehension of the word problem.
<ul> <li><u>UDL Supports</u></li> <li>The guidance on pg. 50, <i>Assessment Tips</i>, states that the goal of the assessment is "to capture students' thinking", as opposed to right or wrong answers. This approach then allows the teacher to understand where the student is on the landscape of learning with the knowledge to provide feedback that support further development - <u>ACTION AND</u></li> <li><u>EXPRESSION: Build fluencies with graduated levels of support for practice and performance</u>.</li> <li>Consider providing a larger workspace for students to use as a tool to support</li> </ul>

handouts to provide more workspace.