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DELHI

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-Martha Graham

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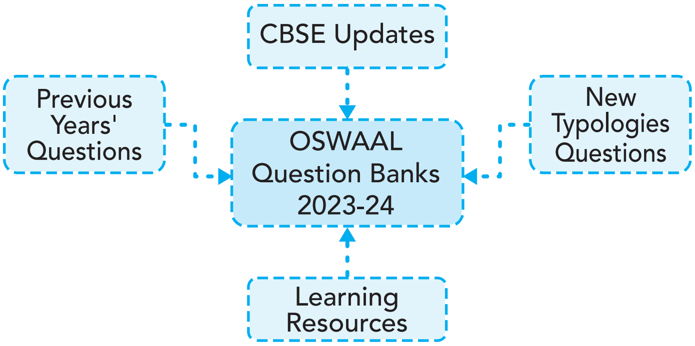

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This Question Bank would not have been made possible without the valuable contributions of the esteemed members of the Oswaal Editorial Board-Authors, Editors, Subject matter experts, Proofreaders & DTP operators who worked day and night to bring this incredible book to you. We are also highly grateful to our dear students for all their valuable and impeccable inputs in the making of this one-of-a-kind exam preparation tool.

All the best Students!! Be the perfectionist that you are!

Team Oswaal Books

Syllabus

Latest Syllabus

MATHEMATICS (Code No. 041)

CLASS–XII

One Paper **Max Marks : 80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit I : Relations and Functions

1. **Relations and Functions :** **15 Periods**

   Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. **Inverse Trigonometric Functions** **15 Periods**

   Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit II : Algebra

1. **Matrices** **25 Periods**

   Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Oncommutativity of multiplication of matrices, and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. **Determinants** **25 Periods**

   Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III : Calculus

1. **Continuity and Differentiability :** **20 Periods**

   Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, like $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives **10 Periods**

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals **20 Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2} \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx,$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals **15 Periods**

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only).

5. Differential Equations **15 Periods**

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q,$ where p and q are functions of x or constants.

$\frac{dx}{dy} + px = q,$ where p and q are functions of y or constants.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors **15 Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry **15 Periods**

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming **20 Periods**

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability **30 Periods**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

Syllabus

QUESTION PAPER DESIGN

Mathematics (Code No. 041)

Class XII

Time 3 Hours **Max. Marks : 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1.	**Remembering :** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding :** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2.	**Applying :** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3.	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating :** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating :** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	TOTAL	**80**	**100**

1. *No chapter wise weightage. Care to be taken to cover all the chapters.*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s) :

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections.

INTERNAL ASSESSMENT	20 MARKS
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note : For activities NCERT Lab Manual may be referred.

Conduct of Periodic Tests:

Periodic Test is a Pen and Paper assessment which is to be conducted by the respective subject teacher. The format of periodic test must have questions items with a balance mix, such as, very short answer (VSA), short answer (SA) and long answer (LA) to effectively assess the knowledge, understanding, application, skills, analysis, evaluation and synthesis. Depending on the nature of subject, the subject teacher will have the liberty of incorporating any other types of questions too. The modalities of the PT are as follows:

a) **Mode:** The periodic test is to be taken in the form of pen-paper test.

b) **Schedule:** In the entire Academic Year, three Periodic Tests in each subject may be conducted as follows:

Test	Pre Mid-term (PT-I)	Mid-Term (PT-II)	Post Mid-Term (PT-III)
Tentative Month	July-August	November	December-January

This is only a suggestive schedule and schools may conduct periodic tests as per their convenience. The winter bound schools would develop their own schedule with similar time gaps between two consecutive tests.

c) **Average of Marks:** Once schools complete the conduct of all the three periodic tests, they will convert the weightage of each of the three tests into ten marks each for identifying best two tests. The best two will be taken into consideration and the average of the two shall be taken as the final marks for PT.

d) The school will ensure simple documentation to keep a record of performance as suggested in detail circular no.Acad-05/2017.

e) **Sharing of Feedback/Performance:** The students' achievement in each test must be shared with the students and their parents to give them an overview of the level of learning that has taken place during different periods. Feedback will help parents formulate interventions (conducive ambience, support materials, motivation and morale-boosting) to further enhance learning. A teacher, while sharing the feedback with student or parent, should be empathetic, non- judgmental and motivating. It is recommended that the teacher share best examples/performances of IA with the class to motivate all learners.

Assessment of Activity Work:

Throughout the year any 10 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the respective class (XI or XII) which is available on the link: http://www.ncert.nic.in/exemplar/labmanuals.htmla record of the same may be kept by the student. An year end test on the activity may be conducted

The weightage are as under:

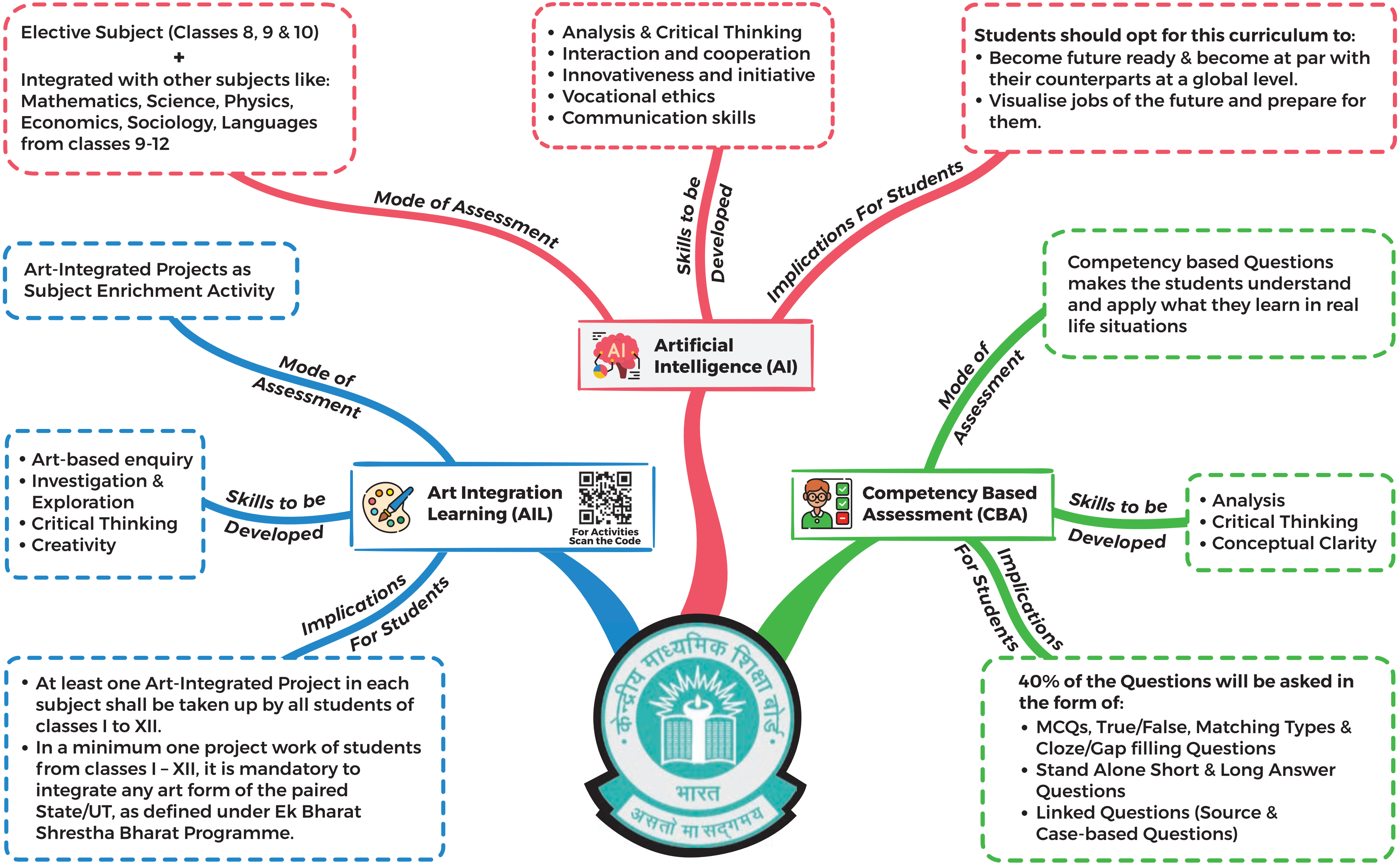
- The activities performed by the student throughout the year and record keeping : 5 marks
- Assessment of the activity performed during the year end test: 3 marks
- Viva-voce: 2 marks

Prescribed Books:

1) Mathematics Textbook for Class XI, NCERT Publications
2) Mathematics Part I - Textbook for Class XII, NCERT Publication
3) Mathematics Part II - Textbook for Class XII, NCERT Publication
4) Mathematics Exemplar Problem for Class XI, Published by NCERT
5) Mathematics Exemplar Problem for Class XII, Published by NCERT
6) Mathematics Lab Manual class XI, published by NCERT
7) Mathematics Lab Manual class XII, published by NCERT

❑❑

NEP Derived Learning Resources Prescribed by CBSE for Year 2023-24



Hear it from our Happy Readers!

Good Book!
According to me, it is a brilliant book for CBSE students. It prepares students really well for the upcoming 2022-23 exams. Thankfully, Maths is no more a worry. Must buy!

Priyanka

All concepts have been explained with examples which simplifies the understanding of the concepts and makes practice very easy. It is worth the money.

Siddharth Gupta

This question bank is the best. It helps me to improve my skills and knowledge. My teacher recommended this book and it helps me a lot to increase my grades.

Avishake Kar

Great Book
This is really a nice book. It helps my son a lot in studies. Many of his friends recommended Oswaal Question Bank to him, so we bought it. It is good and helpful.

Amrik Singh Gujral

Very good book for 12th class preparation. This book contains Previous Years' Questions which is very helpful in exams. It also includes VSAQs, SAQs and one mark questions for exam practice. One must read this book to achieve high percentile in exams.

Priya J.

Outstanding Book!
It is really an outstanding book.
With this question bank, we are able to get 90+ % in CBSE Board.
It is really helpful.

Om Lingyat

I would definitely recommend this book for 12th Boards. It is covering the latest and updated syllabus with great-quality of questions.

Aryan

Fantastic book!
Along with Previous Years Questions & Board Marking scheme answers this book also includes new typology of questions: MCQs, Assertion-Reason, VSA ,SA , LA & case-based questions. Fantastic to study!!

Sumit

CHAPTER 1

RELATIONS AND FUNCTIONS

Syllabus *Types of relations : Reflexive, Symmetric, Transitive and Equivalence relations. One-to-one and onto functions.*

In this chapter you will study

- *Different types of relations- Reflexive, Symmetric, Transitive and Equivalence relations.*
- *Different types of functions - Injective, Surjective and Bijective functions.*

List of Topics

Topic-1: Relations
Page No. 1

Topic-2: Functions
Page No. 11

Topic-1 Relations

Concepts Covered • *Types of relations and their identification* • *Equivalence class*

Revision Notes

1. Definition

A relation R, from a non-empty set A to another non-empty set B is mathematically as an subset of $A \times B$. Equivalently, any subset of $A \times B$ is a relation from A to B.

Thus, R is a relation from A to B

$\Leftrightarrow \quad R \subseteq A \times B$

$\Leftrightarrow \quad R \subseteq \{(a, b) : a \in A, b \in B\}$

Illustrations:

(a) Let $A = \{1, 2, 4\}$, $B = \{4, 6\}$. Let $R = \{(1, 4), (1, 6), (2, 4), (2, 6), (4, 4) (4, 6)\}$. Here $R \subseteq A \times B$ and therefore R is a relation from A to B.

(b) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$, Let $R = \{(2, 3), (3, 5), (5, 7)\}$. Here $R \not\subset A \times B$ and therefore R is not a relation from A to B. Since $(5, 7) \in R$ but $(5, 7) \notin A \times B$.

(c) Let $A = \{-1, 1, 2\}$, $B = \{1, 4, 9, 10\}$ let $a \in A$ and $b \in B$ and $a\,R\,b$ means $a^2 = b$ then, $R = \{(-1, 1), (1, 1), (2, 4)\}$.

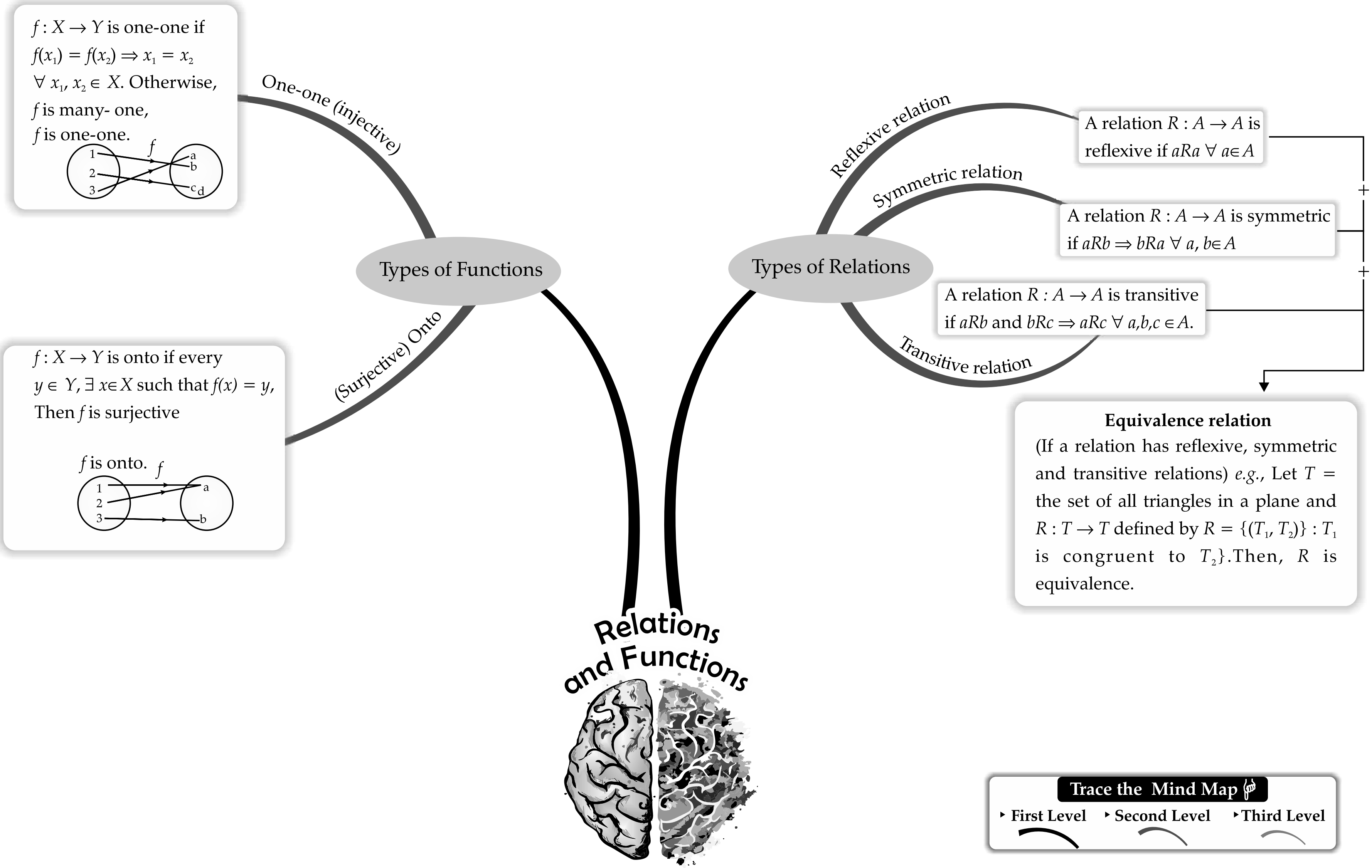
Note:

- A relation from A to B is also called a relation from A into B.
- $(a, b) \in R$ is also written as aRb (read as a is related to b).
- Let A and B be two non-empty finite sets having p and q elements respectively. Then $n(A \times B) = n(A)\cdot n(B) = pq$. Then total number of subsets of $A \times B = 2^{pq}$. Since each subset of $A \times B$ is a relation from A to B, therefore total number of relations from A to B will be 2^{pq}.

2. Domain & range of a relation

(a) **Domain of a relation:** Let R be a relation from A to B. The domain of relation R is the set of all those elements $a \in A$ such that $(a, b) \in R \ \forall \ b \in B$.

Thus, $\text{Dom.}(R) = \{a \in A : (a, b) \in R \ \forall \ b \in B\}$.


Relations and Functions
Types of Functions
One-one (injective)
$f : X \to Y$ is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $\forall\ x_1, x_2 \in X$. Otherwise, f is many- one, f is one-one.
(Surjective) Onto
$f : X \to Y$ is onto if every $y \in Y, \exists\ x \in X$ such that $f(x) = y$, Then f is surjective
f is onto.
Types of Relations
Reflexive relation
A relation $R : A \to A$ is reflexive if $aRa\ \forall\ a \in A$
Symmetric relation
A relation $R : A \to A$ is symmetric if $aRb \Rightarrow bRa\ \forall\ a, b \in A$
Transitive relation
A relation $R : A \to A$ is transitive if aRb and $bRc \Rightarrow aRc\ \forall\ a,b,c \in A$.
Equivalence relation
(If a relation has reflexive, symmetric and transitive relations) e.g., Let T = the set of all triangles in a plane and $R : T \to T$ defined by $R = \{(T_1, T_2)\} : T_1$ is congruent to $T_2\}$.Then, R is equivalence.
Trace the Mind Map
First Level
Second Level
Third Level

That is, the domain of R is the set of first components of all the ordered pairs which belong to R.

(b) Range of a relation: Let R be a relation from A to B. The range of relation R is the set of all those elements $b \in B$ such that $(a, b) \in R\ \forall\ a \in A$.

Thus, Range of $R = \{b \in B : (a, b) \in R\ \forall\ a \in A\}$.

That is, the range of R is the set of second components of all the ordered pairs which belong to R.

(c) Co-domain of a relation: Let R be a relation from A to B. Then B is called the co-domain of the relation R. So we can observe that co-domain of a relation R from A into B is the set B as a whole.

Illustrations: Let $a \in A$ and $b \in B$ and

(i) Let $A = \{1, 2, 3, 7\}$, $B = \{3, 6\}$. If aRb means $a < b$.

Then we have

$$R = \{(1, 3), (1, 6), (2, 3), (2, 6), (3, 6)\}.$$

Here, Dom.$(R) = \{1, 2, 3\}$,

Range of $R = \{3, 6\}$, Co-domain of $R = B = \{3, 6\}$

(ii) Let $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8\}$.

If $R_1 = \{(1, 2), (2, 4), (3, 6)\}$,

and $R_2 = \{(2, 4\}, (2, 6), (3, 8), (1, 6)\}$

Then both R_1 and R_2 are related from A to B because

$$R_1 \subseteq A \times B, R_2 \subseteq A \times B$$

Here, Dom

$(R_1) = \{1, 2, 3\}$, Range of $R_1 = \{2, 4, 6\}$;

Dom$(R_2) = \{2, 3, 1\}$, Range of $R_2 = \{4, 6, 8\}$

3. Types of relations from one set to another set

(a) Empty relation: A relation R from A to B is called an empty relation or a void relation from A to B if $R = \phi$.

For example, Let

$$A = \{2, 4, 6\}, B = \{7, 11\}$$

Let $R = \{(a, b) : a \in A, b \in B \text{ and } |a - b| \text{ is even}\}$.

Here R is an empty relation.

(b) Universal relation: A relation R from A to B is said to be the universal relation if $R = A \times B$.

For example, Let

$$A = \{1, 2\}, B = \{1, 3\}$$

Let $R = \{(1, 1), (1, 3), (2, 1), (2, 3)\}$.

Here, $R = A \times B$, so relation R is a universal relation.

Note:
- The void relation *i.e.*, ϕ and universal relation *i.e.*, $A \times A$ on A are respectively the smallest and largest relations defined on the set A. Also these are also called **Trivial Relations** and other relation is called a **Non-Trivial Relation**.
- The relations $R = \phi$ and $R = A \times A$ are two **extreme relations**.

(c) Identity relation: A relation R defined on a set A is said to be the identity relation on A if

$$R = \{(a, b) : a \in A, b \in A \text{ and } a = b\}$$

Thus identity relation

$$R = \{(a, a) : \forall\ a \in A\}$$

Scan to know more about this topic

Types of Relations

The identity relation on set A is also denoted by I_A.

For example, Let $A = \{1, 2, 3, 4\}$,

Then $I_A = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

But the relation given by

$$R = \{(1, 1), (2, 2), (1, 3), (4, 4)\}$$

is not an identity relation because element of I_A is not related to elements 1 and 3.

Note:
- In an identity relation on A every element of A should be related to itself only.

(d) Reflexive relation: A relation R defined on a set A is said to be reflexive if $a\ R\ a\ \forall\ a \in A$ *i.e.*, $(a, a) \in R\ \forall\ a \in A$.

For example, Let $A = \{1, 2, 3\}$ and R_1, R_2, R_3 be the relations given as

$$R_1 = \{(1, 1), (2, 2), (3, 3)\},$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\} \text{ and}$$

$$R_3 = \{(2, 2), (2, 3), (3, 2), (1, 1)\}$$

Here R_1 and R_2 are reflexive relations on A but R_3 is not reflexive as $3 \in A$ but $(3, 3) \notin R_3$.

Note:
- The universal relation on a non-void set A is reflexive.
- The identity relation is always a reflexive relation but the converse may or may not be true. As shown in the example above, R_1 is both identity as well as reflexive relation on A but R_2 is only reflexive relation on A.

(e) Symmetric relation: A relation R defined on a set A is symmetric if

$(a, b) \in R \Rightarrow (b, a) \in R\ \forall\ a, b \in A$ *i.e.*, $aRb \Rightarrow bRa$

(*i.e.*, whenever aRb then bRa).

For example, Let $A = \{1, 2, 3\}$,

$$R_1 = \{(1, 2), (2, 1)\}, R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}.$$

$$R_3 = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}$$

$$R_4 = \{(1, 3), (3, 1), (2, 3)\}$$

Here R_1, R_2 and R_3 are symmetric relations on A. But R_4 is not symmetric because $(2, 3) \in R_4$ but $(3, 2) \notin R_4$.

(f) Transitive relation: A relation R on a set A is transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

i.e., aRb and $bRc \Rightarrow aRc$.

For example, Let $A = \{1, 2, 3\}$,

$$R_1 = \{(1, 2), (2, 3), (1, 3), (3, 2)\}$$

and $R_2 = \{(1, 3), (3, 2), (1, 2)\}$

Here R_2 is transitive relation whereas R_1 is not transitive because $(2, 3) \in R_1$ and $(3, 2) \in R_1$ but $(2, 2) \notin R_1$.

(g) Equivalence relation: Let A be a non-empty set, then a relation R on A is said to be an equivalence relation if

(i) R is reflexive *i.e.*,

$$(a, a) \in R\ \forall\ a \in A \text{ i.e., } aRa.$$

(ii) R is symmetric *i.e.,*

$$(a, b) \in R$$
$$\Rightarrow \quad (b, a) \in R \; \forall \, a, b \in A \text{ i.e., } aRb \Rightarrow bRa.$$

(iii) R is transitive *i.e.,*

$$(a, b) \in R \text{ and } (b, c) \in R$$
$$\Rightarrow \quad (a, c) \in R \; \forall \, a, b, c \in A$$

i.e., aRb and $bRc \Rightarrow aRc$.

For example, Let $A = \{1, 2, 3\}$

$R = \{(1,2), (1,1), (2,1), (2,2), (3,3)\, (1,3), (3,1), (3,2), (2,3)\}$

Here R is reflexive, symmetric and transitive. So R is an equivalence relation on A.

Equivalence classes: Let A be an equivalence relation in a set A and let $a \in A$. Then, the set of all those elements of A which are related to a, is called equivalence class determined by a and it is denoted by $[a]$. Thus, $[a] = \{b \in A : (a, b) \in A\}$

Mnemonics

Types of relation

RIPE STRAWBERRY TO EAT

Interpretations

Ripe – reflexive

Strawberry – Symmetric

To – transitive

Eat - Equivalence

Note:

- Two equivalence classes are either disjoint or identical.
- An equivalence relation R on a set A partitions the set into mutually disjoint equivalence classes.
- An important property of an equivalence relation is that it divides the set into pair-wise disjoint subsets called **equivalence classes** whose collection is called **a partition of the set**.

  Note that the union of all equivalence classes give the whole set.

e.g., Let R denotes the equivalence relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Then the equivalence class [0] is $[0] = [0, \pm 2, \pm 4, \pm 6, ...]$.

Key Words

Disjoint: These are sets which have no elements in common.

4. Tabular representation of a relation

In this form of representation of a relation R from set A to set B, elements of A and B are written in the first column and first row respectively. If $(a, b) \in R$ then we write '1' in the row containing a and column containing b and if $(a, b) \notin R$ then we write '0' in the same manner.

For example, Let $A = \{1, 2, 3\}$,

$B = \{2, 5\}$ and $R = \{(1, 2), (2, 5), (3, 2)\}$, then

R	2	5
1	1	0
2	0	1
3	1	0

5. Inverse relation

Let $R \subseteq A \times B$ be a relation from A to B. Then, the inverse relation of R, to be denoted by R^{-1}, is a relation from B to A defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Thus $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1} \; \forall \, a \in A, b \in B$.

Clearly, **Domain** (R^{-1}) = **Range** of R, Range of R^{-1} = **Domain** (R).

Key Words

Domain and Range: The set of x coordinate values is called domain and the set of y coordinate values is called range.

Also, $(R^{-1})^{-1} = R$.

For example, Let $A = \{1, 2, 4\}$, $B = \{3, 0\}$ and let $R = \{(1, 3), (4, 0), (2, 3)\}$ be a relation from A to B, then $R^{-1} = \{(3, 1), (0, 4), (3, 2)\}$.

Key Facts

1. **(i)** A relation R from A to B is an empty relation or void relation if $R = \phi$

   (ii) A relation R on a set A is an empty relation or void relation if $R = \phi$
2. **(i)** A relation R from A to B is a universal relation if $R = A \times B$.

   (ii) A relation R on a set A is an universal relation if $R = A \times A$.
3. A relation R on a set A is reflexive if $aRa, \; \forall \, a \in A$.
4. A relation R on a set A is symmetric if whenever aRb, then bRa for all $a, b \in A$.
5. A relation R on a set A is transitive if whenever aRb and bRc then aRc for all $a, b, c \in A$.
6. A relation R on A is identity relation if $R = \{(a, a) \; \forall \, a \in A\}$ *i.e.,* R contains only elements of the type $(a, a) \; \forall \, a \in A$ and it contains no other element.

7. A relation R on a non-empty set A is an equivalence relation if the following conditions are satisfied :

(i) R **is reflexive** *i.e.,* for every $a \in A$, $(a, a) \in R$ *i.e.,* aRa.

(ii) R **is symmetric** *i.e.,* for $a, b \in A$, $aRb \Rightarrow bRa$ *i.e.,* $(a, b) \in R \Rightarrow (b, a) \in R$.

(iii) R **is transitive** *i.e.,* for all $a, b, c \in A$, we have, aRb and $bRc \Rightarrow aRc$ *i.e.,* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

TYPES OF INTERVALS

(i) Open Intervals: If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying strictly between a and b is called an open interval. It is denoted by $]a, b[$ or (a, b) *i.e.,* $\{x \in R : a < x < b\}$.

(ii) Closed Intervals: If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes both a and b as well is known as a closed interval. It is denoted by $[a, b]$ *i.e.,* $\{x \in R : a \le x \le b\}$.

(iii) Open Closed Interval: If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it excludes a and includes only b is known as an open closed interval. It is denoted by $]a, b]$ or $(a, b]$ *i.e.,* $\{x \in R : a < x \le b\}$.

(iv) Closed Open Interval: If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes only a and excludes b is known as a closed open interval. It is denoted by $[a, b[$ or $[a, b)$ *i.e.,* $\{x \in R : a \le x < b\}$.

Example 1

Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R(c, d) if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

Sol.

Step I : Given (a, b) R (c, d) as $ad(b + c) = bc(a + d)$

$\therefore \quad \forall a, b \in \text{N}$

or $\quad ab(b + a) = ba(a + b)$

or $\quad (a, b)$ R (a, b)

$\therefore$ R is reflexive. ...(i)

Step II : Let (a, b) R (c, d) for $(a, b), (c, d) \in \text{N} \times \text{N}$

$\therefore \quad ad(b + c) = bc(a + d)$ ...(ii)

Also, $\quad (c, d)$ R (a, b)

$\because \quad cb(d + a) = da(c + b)$

[By commutation of addition and multiplication on N]

$\therefore$ R is symmetric. ...(iii)

Step III : Let (a, b) R (c, d) and (c, d) R (e, f) for $a, b, c, d, e, f \in \text{N}$

$\therefore \quad ad(b + c) = bc(a + d)$ ...(iv)

and $\quad cf(d + e) = de(c + f)$ ...(v)

Dividing eqn. (iv) by $abcd$ and eqn. (v) by $cdef$

i.e., $\quad \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$

and $\quad \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

On adding, we get

$$\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

or $\quad af(b + e) = be(a + f)$

Hence, (a, b) R (e, f)

$\therefore$ R is transitive. ...(vi)

From equations (i), (iii) and (vi), R is an equivalence relation.

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. Let set X = {1, 2, 3} and a relation R is defined in X as: R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are

(A) {(1, 1), (2, 3), (1, 2)}

(B) {(3, 3), (3, 1), (1, 2)}

(C) {(1, 1), (3, 3), (3, 1), (2, 3)}

(D) {(1, 1), (3, 3), (3, 1), (1, 2)}

[CBSE TERM-I 2021-22]

Ans. Option (C) is correct.

Explanation:

(i) R is reflexive if it contains {(1, 1), (2, 2) and (3, 3)}.

Since, $(2, 2) \in R$. So, we need to add (1, 1) and (2, 2) to make R reflexive.

(ii) R is symmetric if it contains {(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)}.

Since, $\{(2, 2), (1, 3), (3, 2)\} \in R$. So, we need to add (3, 1) and (2, 3).

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are {(1, 1), (3, 3), (3, 1), (2, 3)}.

Q. 2. If $R = \{(x, y); x, y \in Z, x^2 + y^2 \le 4\}$ is a relation is set Z, then domain of R is

(A) {0, 1, 2} **(B)** {–2, –1, 0, 1, 2}
(C) {0, –1, –2} **(D)** {–1, 0, 1}

[CBSE TERM-I 2021-22]

Ans. Option (B) is correct.

Explanation: Given, $R = \{(x, y) : x, y \in Z, x^2 + y^2 \le 4\}$

Let $y = 0$, then $x^2 \le 4 \Rightarrow x = 0, \pm1, \pm2$

Thus, domain of $R = \{-2, -1, 0, 1, 2\}$

Q. 3. A relation R in set A = {1, 2, 3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

(A) (1, 1) **(B)** (1, 2)
(C) (2, 2) **(D)** (3, 3)

[CBSE TERM-I SQP 2021-22]

Ans. Option (B) is correct.

Q. 4. Let the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4}. Then [1], the equivalence class containing 1, is :

(A) {1, 5, 9} **(B)** {0, 1, 2, 5}
(C) ϕ **(D)** A

[CBSE TERM-I SQP 2021-22]

Ans. Option (A) is correct.

Explanation: Equivalence class [1] is the set of elements related to 1 = {1, 5, 9}

Q. 5. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then :

(A) $(2, 4) \in R$ **(B)** $(3, 8) \in R$
(C) $(6, 8) \in R$ **(D)** $(8, 7) \in R$

[CBSE TERM-I SQP 2021-22]

Ans. Option (C) is correct.

Explanation: 6 = 8 – 2

(6, 8) is an element of R.

Q. 6. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b $\forall$ $a, b \in T$. Then R is

(A) reflexive but not transitive
(B) transitive but not symmetric
(C) equivalence relation
(D) None of these

Q. 7. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is

(A) symmetric but not transitive
(B) transitive but not symmetric
(C) neither symmetric nor transitive
(D) both symmetric and transitive

Q. 8. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

(A) 1 **(B)** 2
(C) 3 **(D)** 5

Ans. Option (D) is correct.

Explanation: Given that, $A = \{1, 2, 3\}$

Now, number of equivalence relations are as follows:

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$
$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$
$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$
$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$

$\therefore$ Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. How many reflexive relations are possible in a set A whose $n(A) = 3$ A **[CBSE SQP 2020-21]**

Sol. 2^6 reflexive relations. **1**

[CBSE Marking Scheme, 2020-21]

Detailed Answer:

Given, $n(A) = 3$

Total number of reflexive relations $= 2^{n(n-1)}$

$= 2^{3(3-1)} = 2^{3 \times 2} = 2^6$

Commonly Made Error

► Since the reflexive relation should contain $(x, x) \in A$, mostly students write the answer as 3.

Answering Tip

► Number of reflexive relations on a set containing n elements is 2^{n^2-n}.

Q. 2. An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3.

What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$.

A **[CBSE SQP 2020-21]**

These questions are for practice and their solutions are available at the end of the chapter

Sol. $A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$ 1

[CBSE Marking Scheme 2020-21]

Q. 3. Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$. Find the equivalence class [(1,3)]. R&U [SQP 2017-18]

Q. 4. State the reason why the Relation $R = \{(a, b) : a \leq b^2\}$ on the set R of real numbers is not reflexive. R&U [NCERT SQP 2016-17]

Sol. $$\frac{1}{2} > \left(\frac{1}{2}\right)^2 \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R.$$

Hence, R is not reflexive. 1

[CBSE Marking Scheme 2016]

Q. 5. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive. R [NCERT]

Sol. We know that, for a relation to be transitive,

$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Here, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

$\therefore$ R is not transitive. 1

Q. 6. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. R&U [O.D. Set I, II, III, 2014]

Sol. $R = \{(2, 3), (4, 2), (6, 1)\}$

Range $= \{3, 2, 1\}$ 1

Q. 7. Let $R = \{(a, a^3) : a$ is a prime number less than 5$\}$ be a relation. Find the range of R. R&U [Foreign Set I, 2014]

Sol. Given $R = \{(a, a^3) : a$ is a prime number less than 5$\}$

$\Rightarrow$ $R = \{(2, 8), (3, 27)\}$ ½

$\therefore$ Range $= \{8, 27\}$ ½

Q. 8. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2$ divides $(a - b)\}$. Write the equivalence class [0]. R&U [Delhi Comptt. Set I, II, III, 2014]

Sol. Given $R = \{(a, b) : 2$ divides $(a - b)$

$\forall\, a, b \in A = \{0, 1, 2, 3, 4, 5\}$

Equivalence class $[0] = \{0, 2, 4\}$ 1

Short Answer Type Questions-I (2 marks each)

Q. 1. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2$ divides $a - b\}$. Show that the relation R transitive ? Write the equivalence class [0]. A I R&U [CBSE SQP 2020-21]

Sol. Let 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$. 1

So 2 divides $[(a - b) + (b - c)]$

2 divides $(a - c)$: Yes relation R is transitive 1

$[0] = \{0, \pm 2, \pm 4, \pm 6, ...\}$

[CBSE SQP Marking Scheme 2020]

Q. 2. Check if the relation R in the set R of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive. A I R&U [Delhi Set I, II, III - 2020]

Q. 3. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y$ is divisible by $x\}$ is (i) symmetric (ii) transitive. A I R&U [O. D Set I, II, III - 2020]

Sol. (i) As $(2, 4) \in R$ but $(4, 2) \notin R \Rightarrow R$ is not symmetric, 1

(ii) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow b = \lambda a$ and $c = \mu b$

Now, $c = \mu b = \mu(\lambda a) \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive 1

[CBSE Marking Scheme 2020]

Detailed Answer:

$A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y$ is divisible by $x\}$

(i) Symmetric

Let $(x, y) \in R$

y is divisible by x

$\therefore$ x is not necessarily divisible by y

$(y, x) \notin R$

e.g., $(1, 2) \in R$

2 is divisible by 1

but 1 is not divisible by 2

$(2, 1) \notin R$

Hence, Given Relation is not symmetric

(ii) Transitive

Let $(x, y) \in R$

y is divisible by x ...(i)

and $(y, z) \in R$

z is divisible by y ...(ii)

From eq(i) and eq(ii)

z is divisible by x

$\therefore$ $(x, z) \in R$

e.g., $(1, 2) \in R$

2 is divisible by 1 ...(i)

$(2, 4) \in R$

4 is divisible by 2 ...(ii)

From eq(i) and eq(ii)

4 is divisible by 1

$(1, 4) \in R$

Hence, Given Relation is transitive.

These questions are for practice and their solutions are available at the end of the chapter

Commonly Made Error

- Some students take the relation as "is a factor of" and go wrong.

Answering Tip

- 'is divisible by' should be taken as 'is a multiple of'

Q. 4. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all ? Justify your answer. U [CBSE SQP 2016-17]

Sol. Equivalence relations could be the following :

{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)} and **1**

{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)} **1**

So, only two equivalence relations.

[CBSE Marking Scheme 2016]

Q. 5. Let $f : X \to Y$ be a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is a transitive relation. A

Short Answer Type Questions-II (3 marks each)

Q. 1. Check whether the relation R in the set Z of integers defined as R = {$(a, b) : a + b$ is "divisible by 2"} is reflexive, symmetric or transitive. Write the equivalence class containing 0 *i.e.* [0]. R&U [CBSE SQP 2020-21]

Sol. (i) Reflexive:

Since, $a + a = 2a$ which is even

$\therefore (a, a) \in R \ \forall \ a \in Z$

Hence R is reflexive. ½

(ii) Symmetric:

If $(a, b) \in R$, then $a + b = 2\lambda$

$\Rightarrow \quad b + a = 2\lambda$

$\Rightarrow (b, a) \in R$. Hence R is symmetric. **1**

(iii) Transitive:

If $(a, b) \in R$ and $(b, c) \in R$

then $\quad a + b = 2\lambda$ ...(i)

and $\quad b + c = 2\mu$ ...(ii)

Adding (i) and (ii) we get

$$a + 2b + c = 2(\lambda + \mu)$$

$\Rightarrow \quad a + c = 2(\lambda + \mu - b)$

$\Rightarrow \quad a + c = 2k$

where $\quad \lambda + \mu - b = k$

$\Rightarrow (a, c) \in R$

Hence R is transitive

$[0] = \{...-4, -2, 0, 2, 4...\}$ **1½**

[CBSE SQP Marking Scheme 2020]

Commonly Made Error

- Equivalence class of 0 is the set of all elements related to 0.

Answering Tip

- Mostly students go wrong in finding the equivalence class. Some students forget to write 0 in the equivalence class.

Q. 2. Prove that the relation R on Z, defined by $R = \{(x, y) : (x - y)$ is divisible by 5$\}$ is an equivalence relation. A I R&U [CBSE O.D SET I - 2020]

Sol. For reflexive

$x - x = 0$, for every $x \in Z$ is divisible by 5 $\Rightarrow (x, x) \in R$

For symmetric ½

$(x, y) \in R \Rightarrow x - y$ is divisible by 5 $\Rightarrow y - x$ is divisible by 5 ½

$\Rightarrow (y, x) \in R \Rightarrow R$ is symmetric

For transitive

Let $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R \Rightarrow x - y = 5\lambda$ ...(i)

$(y, z) \in R \Rightarrow y - z = 5\mu$ ...(ii)

adding (i) and (ii), $x - z = 5(\lambda + \mu) = 5k$

$\Rightarrow (x, z) \in R \Rightarrow R$ is transitive

Hence R is an equivalence relation. 1

[CBSE Marking Scheme 2020 (modified)]

Q. 3. Show that the relation R on R defined as $R = \{(a, b) : a \le b\}$, is reflexive, and transitive but not symmetric. U [CBSE Delhi Set III-2019]

Sol. Clearly $a \le a \ \forall \ a \in R \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. ½

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in R$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. **1½**

For non-symmetric:

Let $a = 1, b = 2$, As $1 \le 2 \Rightarrow (1, 2) \in R$

but $2 \not\le 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow R$ is non-symmetric. **1**

[CBSE Marking Scheme 2019] (Modified)

These questions are for practice and their solutions are available at the end of the chapter

Detailed Solution:

Topper Answer, 2019

$R = \{(a, b) : a \le b\}$

REFLEXIVE :

Every element $a \in \mathbb{R}$ is equal to itself

$\Rightarrow a = a$

$\Rightarrow a \le a$ is true

$\therefore (a, a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of Real nos

The relation is REFLEXIVE $R \rightarrow$ Relation.

TRANSITIVE :

For all $(a, b) \in R$ and $(b, c) \in R$

$a \le b$ and $b \le c$ where, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le c$

$\Rightarrow (a, c) \in R$

$\therefore$ The ~~set is tr~~ relation is TRANSITIVE

$\because$ for $(a, b), (b, c) \in R$, $\cancel{(a, c) \in R}$ $(a, c) \in R$

SYMMETRIC : For relation to be symmetric,

for all $(a, b) \in R$, (b, a) should also exist in R

$\cancel{a \le b \not\Rightarrow}$ $a \le b$

$b \nleq a \rightarrow$ This relation is true only $a = b = 1$.

For eg: $\frac{1}{2} \le 1 \Rightarrow \left(\frac{1}{2}, 1\right) \in R$

but $1 \nleq \frac{1}{2}$ $\therefore \left(1, \frac{1}{2}\right) \notin R$

$\therefore$ Relation is NOT SYMMETRIC

Commonly Made Error

- Some students use numerical examples to show that a reflexive, symmetric or transitive which is wrong.

Answering Tip

- Counter examples can be used only to show that a relation is not reflexive, symmetric or transitive.

Q. 4. Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

$\boxed{\text{R\&U}}$ [CBSE Comptt. Set I, II, III 2018]

Sol. $(x - x) = 0$ is divisible by 3 for all $x \in Z$. So, $(x, x) \in R$

$\therefore R$ is reflexive **½**

So $(x, y) \in R$ implies $(y, x) \in R$, $\forall x, y \in Z$

$\Rightarrow$ R *is* symmetric, **1**

$(x - y)$ is divisible by 3 and $(y - z)$ is divisible by 3.

So $(x - z) = (x - y) + (y - z)$ is divisible by 3. **1½**

$\therefore (x, z) \in R \Rightarrow R$ is transitive

Hence, R is an equivalence relation.

[CBSE Marking Scheme 2018 (modified)]

Q. 5. Check whether the relation R in the set R of real numbers, defined by $R = \{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

R&U [SQP 2018-19]

Q. 6. Show that the relation R in the set $N \times N$ defined by $(a, b)\ R\ (c, d)$ if $a^2 + d^2 = b^2 + c^2\ \forall\ a, b, c, d \in N$, is an equivalence relation. R&U [SQP 2015-16]

Sol. Let $(a, b) \in N \times N$

then,

$\because \quad a^2 + b^2 = a^2 + b^2$

$\therefore \quad (a, b)\ R\ (a, b)$

Hence R is reflexive. ½

Let $(a, b), (c, d) \in N \times N$ be such that

$(a, b)\ R\ (c, d)$

$\Rightarrow \quad a^2 + d^2 = b^2 + c^2$

$\Rightarrow \quad c^2 + b^2 = d^2 + a^2$

$\Rightarrow \quad (c, d)\ R\ (a, b)$

Hence, R is symmetric. 1

Let $(a, b), (c, d), (e, f) \in N \times N$ be such that $(a, b)\ R\ (c, d), (c, d)\ R\ (e, f)$.

$\Rightarrow \quad a^2 + d^2 = b^2 + c^2 \quad$ ...(i)

and $\quad c^2 + f^2 = d^2 + e^2 \quad$ ...(ii)

Adding eqn. (i) and (ii),

$\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$

$\Rightarrow \quad a^2 + f^2 = b^2 + e^2$

$\Rightarrow \quad (a, b)\ R\ (e, f)$ 1½

Hence, R is transitive

Since, R is reflexive, symmetric and transitive. Therefore, R is an equivalence relation.

[CBSE Marking Scheme 2015 (Modified)]

Commonly Made Error

- Students go wrong in solving problems involving ordered pairs.

Answering Tip

- Practice more problems involving relations with ordered pairs.

Long Answer Type Questions (5 marks each)

Q. 1. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1. Write the equivalence class [2].

A I R&U [CBSE Delhi & OD Set 2018]

Sol. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall\ a \in A$ 1

$\therefore (a, a) \in R, a \in A \therefore R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 4

$\Rightarrow |b - a|$ is divisible by 4 $\quad (\because\ |a - b| = |b - a|)$ 1

$\Rightarrow (b, a) \in R \therefore R$ is is symmetric

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow |a - b|\ \&\ |b - c|$ are divisible by 4

$\Rightarrow a - b = \pm 4m, b - c = \pm 4n, m, n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$ 2

$\Rightarrow (a - c)$ is divisible by 4.

$\Rightarrow |a - c|$ is divisible by 4 $\therefore (a, c) \in R$

$\Rightarrow R$ *is* transitive

Hence R is an equivalence relations in A

Set of elements related to 1 is (1, 5, 9),

Equivalence class [2]=$\{2,6,10\}$ 1

[CBSE Marking Scheme 2018 (modified)]

OR

Topper Answer, 2018

To show : R is an equivalence relation

Solution.

For Reflexive :-

$(a, a) : a, a \in A$

$a R a \quad \forall\ a \in A$

$|a - a|$ is divisible by 4

0 is divisible by 4

which is true

$\Rightarrow$ R is a reflexive relation.

For Symmetric relation :-

let $(a, b) \in R \quad \forall\ a, b \in A$

$a R b \quad \forall\ a, b \in A$

$|a-b|$ is divisible by 4

$\Rightarrow$ $|b-a|$ is divisible by 4

$\Rightarrow$ $b\,R\,a \;\forall\; b, a \in A$

$\Rightarrow$ $(b, a) \in R$

$\Rightarrow$ R is an symmetric relation since $(a, b) \in R$ also $(b, a) \in R$

For transitive relation :-

let $(a, b) \in R$ and $(b, c) \in R \;\forall\; a, b, c \in A$

$a\,R\,b$ and $b\,R\,c \;\forall\; a, b, c \in A$

$|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$|a-b| = 4\lambda$ —(i)

$|b-c| = 4\mu$ —(ii)

(i) + (ii)

$|a-b+b-c| = 4(\lambda+\mu)$

$|a-c| = 4(\lambda+\mu)$

$\Rightarrow$ $a\,R\,c \;\forall\; a, c \in A$

$\Rightarrow$ $(a, c) \in R$

R is an transitive relation since $(a, b) \in R$, $(b, c) \in R$ and also $(a, c) \in R$

Hence R is reflexive, symmetric and transitive so it is a equivalence relation.

The set of all elements related to A are :-

$R = \{(1,5), (5,1), (1,9), (9,1), (1,1)\}$

Equivalence class $[2] = \{2, 6, 10\}$

Q. 2. Show that the relation R in the Set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is divisible by 2$\}$ is an equivalence relation. R&U **[O.D. Set I, II, III Comptt. 2015]**

Topic-2 Functions

Concepts Covered • *Types of functions and their identification*

Revision Notes

1. **Domain:** If a function is expressed in the form $y = f(x)$, then domain of f means set of all those real values of x for which y is real (*i.e.*, y is well-defined).

   Remember the following points:

   (a) Negative number should not occur under the square root (even root) *i.e.*, expression under the square root sign must be always ≥ 0.

   (b) Denominator should never be zero.

   (c) For $\log_b a$ to be defined, $a > 0$, $b > 0$ and $b \neq 1$. Also note that $\log_b 1$ is equal to zero *i.e.*, 0.

Scan to know more about this topic

Types of Functions

2. **Range:** If a function is expressed in the form $y = f(x)$, then range of f means set of all possible real values of y corresponding to every value of x in its domain.

   Remember the following points:

   (a) At first find the domain of the given function.

   (b) If the domain does not contain an interval, then find the values of y putting these values of x from the domain. The set of all these values of y obtained will be the range.

   (c) If domain is the set of all real numbers R or set of all real numbers except a few points, then express x in terms of y and from this find the real values of y for which x is real and belongs to the domain.

3. **Function as a special type of relation:** A relation f from a set A to another set B is said be a function (or mapping) from A to B if with every element (say x) of A, the relation f relates a unique element (say y) of B. This y is called f - image of x. Also x is called pre-image of y under f.

These questions are for practice and their solutions are available at the end of the chapter

4. **Difference between relation and function:** A relation from a set A to another set B is any subset of $A \times B$; while a function f from A to B is a subset of $A \times B$ satisfying following conditions:

   (a) For every $x \in A$, there exists $y \in B$ such that $(x, y) \in f$.

   (b) If $(x, y) \in f$ and $(x, z) \in f$ then, $y = z$.

S. No.	Function	Relation
(i)	Each element of A must be related to some element of B.	There may be some elements of A which are not related to any element of B.
(ii)	An element of A should not be related to more than one element of B.	An element of A may be related to more than one element of B.

5. **Real valued function of a real variable:** If the domain and range of a function f are subsets of R (the set of real numbers), then f is said to be **a real valued function of a real variable** or **a real function.**

6. Some important real functions and their domain & range

S. No. Function	Representation	Domain	Range
(i) Identity function	$I(x) = x \ \forall \ x \in R$	R	R
(ii) Modulus function or Absolute value function	$f(x) = \lvert x \rvert = \begin{cases} -x, \text{if } \ x < 0 \\ x, \ \text{if } \ x \geq 0 \end{cases}$	R	$[0, \infty)$
(iii) Greatest integer function or Integral function or Step function	$f(x) = [x] \ \forall x \in R$	R	Z
(iv) Smallest integer function	$f(x) = [x] \ \forall \ x \in R$	R	Z
(v) Signum function	$f(x) = \begin{cases} \dfrac{\lvert x \rvert}{x}, \text{if } x \neq 0 \\ 0, \text{ if } \ x = 0 \end{cases}$ *i.e.,* $f(x) = \begin{cases} 1, \ x > 0 \\ 0, \ x = 0 \\ -1, x < 0 \end{cases}$	R	$\{-1, 0, 1\}$
(vi) Exponential function	$f(x) = a^x, \ \forall \ a > 0, a \neq 1$	R	$(0, \infty)$
(vii) Logarithmic function	$f(x) = \log_a x, \ \forall \ a \neq 1, a > 0$ and $x > 0$	$(0, \infty)$	R

7. Types of Function

(a) One-one function (Injective function or Injection): A function $f : A \to B$ is one-one function or injective function if distinct elements of A have distinct images in B.

Thus, $f : A \to B$ is one-one $\Leftrightarrow f(a) = f(b)$

$\Rightarrow \quad a = b, \forall a, b \in A$

$\Leftrightarrow \quad a \neq b \Rightarrow f(a) \neq f(b) \ \forall \ a, b \in A.$

- If A and B are two sets having m and n elements respectively such that $m \leq n$, then total number of one-one functions from set A to set B is ${}^nC_m \times m!$ *i.e.,* ${}^nP_m$.
- If $n(A) = n$, then the number of injective functions defined from A onto itself is $n!$.

ALGORITHM TO CHECK THE INJECTIVITY OF A FUNCTION

STEP 1: Take any two arbitrary elements a, b in the domain of f.

STEP 2: Put $f(a) = f(b)$.

STEP 3: Solve $f(a) = f(b)$. If it gives $a = b$ only, then f is a one-one function.

(b) Onto function (Surjective function or Surjection): A function $f : A \to B$ is onto function or a surjective function if every element of B is the f - image of some element of A. That implies $f(A) = B$ or range of f is the co-domain of f.

Thus, $f : A \to B$ is onto $\Leftrightarrow f(A) = B$ *i.e.,* range of f = co-domain of f.

ALGORITHM TO CHECK THE SURJECTIVITY OF A FUNCTION

STEP 1: Take an element $b \in B$, where B is the co-domain of the function.

STEP 2: Put $f(x) = b$.

STEP 3: Solve the equation $f(x) = b$ for x and obtain x in terms of b. Let $x = g(b)$.

STEP 4: If for all values of $b \in B$, the values of x obtained from $x = g(b)$ are in A, then f is onto. If there are some $b \in B$ for which values of x, given by $x = g(b)$, is not in A. Then f is not onto.

Mnemonics

Types of functions
Indian Syndicate Bank
Interpretations
Indian – injective
Syndicate – surjective
Bank - Bijective

Also note that a bijective function is also called a one-to-one function or one-to-one correspondence.

If $f : A \rightarrow B$ is a function such that,

(i) f is one-one $\Rightarrow n(A) \leq n(B)$.

(ii) f is onto $\Rightarrow n(B) \leq n(A)$.

For an ordinary finite set A, a one-one function $f : A \rightarrow A$ is necessarily onto and an onto function $f : A \rightarrow A$ is necessarily one-one for every finite set A.

(d) Identity function: The function $I_A : A \rightarrow A$; $I_A(x) = x, \forall x \in A$ is called an identity function on A.

Note:
- Domain (I_A) = A and Range (I_A) = A.

(e) Equal function: Two functions f and g having the same domain D are said to be equal if $f(x) = g(x)$ for all $x \in D$.

8. Constant and Types of Variables

(a) Constant: A constant is a symbol which retains the same value throughout a set of operations. So, a symbol which denotes a particular number is a constant. Constants are usually denoted by the symbols a, b, c, k, l, m, ... etc.

(b) Variable: It is a symbol which takes a number of values *i.e.,* it can take any arbitrary values over the interval on which it has been defined. *For example,* if x is a variable over R (set of real numbers) then we mean that x can denote any arbitrary real number. Variables are usually denoted by the symbols $x, y, z, u, v, \ldots$ etc.

(i) Independent variable: The variable which can take an arbitrary value from a given set is termed as an independent variable.

(ii) Dependent variable: The variable whose value depends on the independent variable is called a dependent variable.

9. Defining a Function

Consider A and B be two non-empty sets, then a rule f which associates **each element of** A **with a unique element of** B is called a function or the mapping from A to B or f maps A to B. If f is a mapping from A to B, then we write $f : A \rightarrow B$ which is read as 'f is mapping from A to B' or 'f is a function from A to B'.

If f associates $a \in A$ to $b \in B$, then we say that 'b **is the image of the element** a **under the function** f' or 'b **is the** f **- image of** a' or '**the value of** f **at** a' and denotes it by $f(a)$ and we write $b = f(a)$. The element a is called the **pre-image** or **inverse-image of** b.

Thus for a bijective function from A to B,

(a) A and B should be non-empty.

(b) Each element of A should have image in B.

(c) No element of A should have more than one image in B.

(d) If A and B have respectively m and n number of elements then the **number of functions defined from** A **to** B **is** n^m.

10. Domain, Co-domain and Range of A function

The **set** A **is called the domain** of the function f and the **set** B **is called the co- domain.** The set of the images of all the elements of A under the function f is called the **range of the function** f and is denoted as $f(A)$.

Thus range of the function f is $f(A) = \{f(x) : x \in A\}$.

Clearly $f(A) = B$ for a bijective function.

Note:
- It is necessary that every f-image is in B; but there may be some elements in B which are not the f-images of any element of A *i.e.,* whose pre-image under f is not in A.
- Two or more elements of A may have same image in B.
- $f : x \rightarrow y$ means that under the function f from A to B, an element x of A has image y in B.
- Usually we denote the function f by writing $y = f(x)$ and read it as 'y **is a function of** x'.

Example 1

Determine whether the function f : A → B defined by $f(x) = 4x + 7, x \in$ is one-one.

Show that no two elements in domain have same image in codomain.

Solution :

Given, f : A → B defined by $f(x) = 4x + 7, x \in$ A

Let, $x_1, x_2 \in A$, such that $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + 7 = 4x_2 + 7 \Rightarrow 4x_1 = 4x_2 \Rightarrow x_1 = x_2$

So, f is one-one function.

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. Let X = $\{x^2 : x \in N\}$ and the relation $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then, this function is

(A) injective only **(B)** not bijective
(C) surjective only **(D)** bijective

[CBSE TERM-I 2021-22]

Ans. Option (A) is correct.

Explanation: Let $x_1, x_2 \in N$

$$f(x_1) = f(x_2)$$
$$\Rightarrow x_1^2 = x_2^2$$
$$\Rightarrow x_1^2 - x_2^2 = 0$$
$$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$$
$$\Rightarrow x_1 = x_2$$
$$\{x_1 + x_1 \neq 0 \text{ as } x_1, x_2 \in N\}$$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in N. Thus, $f(x)$ is not surjective.

Q. 2. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is

(A) not one-one
(B) one-one
(C) not onto
(D) neither one-one nor onto

[CBSE TERM-I 2021-22]

Ans. Option (D) is correct.

Explanation: $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$$\Rightarrow 2 + x_1^2 = 2 + x_2^2$$
$$\Rightarrow x_1^2 = x_2^2$$
$$\Rightarrow x_1 = \pm x_2$$
$$\Rightarrow x_1 = x_2$$
$$\text{or} \quad x_1 = -x_2$$

Thus, $f(x)$ is not one-one.

For onto

Let $f(x) = y$ such that $y \in R$

$$\therefore x^2 = y - 2$$
$$\Rightarrow x = \pm\sqrt{y-2}$$

Put $y = -3$, we get

$$\Rightarrow x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$$

Q. 3. A function $f : R \to R$ defined by $f(x) = 2 + x^3$ is :

(A) One-one but not onto
(B) Not one-one but onto
(C) Neither one-one nor onto
(D) One-one and onto

[CBSE TERM-I SQP 2021-22]

Ans. Option (D) is correct.

Explanation: $f(x) = x^3$ is a bijective function.

Q. 4. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Based on the given information, f is best defined as :

(A) Surjective function **(B)** Injective function
(C) Bijective function **(D)** None of these

[CBSE TERM-I SQP 2021-22]

Ans. Option (B) is correct.

Explanation: F is injective since every element in set B has atmost one pre-image in set A.

Q. 5. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

(A) 720 **(B)** 120
(C) 0 **(D)** None of these

Ans. Option (C) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is

$n!$ if $m = n$

0, if $m \neq n$

Given that, $m = 5$ and $n = 6$

$\therefore m \neq n$

Number of one-one and onto mapping = 0

Q. 6. Let $A = \{1, 2, 3, ...n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is

(A) ${}^nP_2$ **(B)** $2^n - 2$
(C) $2^n - 1$ **(D)** None of these

Ans. Option (B) is correct.

Explanation: Total number of functions from A to $B = 2^n$

Number of into functions = 2

Number of surjections from A to $B = 2^n - 2$

Q. 7. Let $f : R \to R$ be defined by $f(x) = \dfrac{1}{x}$, $\forall x \in R$. Then f is

(A) one-one **(B)** onto
(C) bijective **(D)** f is not defined

Q. 8. Which of the following functions from Z into Z are bijections?

(A) $f(x) = x^3$ **(B)** $f(x) = x + 2$
(C) $f(x) = 2x + 1$ **(D)** $f(x) = x^2 + 1$

Ans. Option (B) is correct.

Explanation: For bijection on Z, $f(x)$ must be one-one and onto.

Function $f(x) = x^2 + 1$ is many-one as $f(1) = f(-1)$

Range of $f(x) = x^3$ is not Z for $x \in Z$.

Also $f(x) = 2x + 1$ takes only values of type $= 2k + 1$ for $x \in k \in Z$

But $f(x) = x + 2$ takes all integral values for $x \in Z$

Hence $f(x) = x + 2$ is bijection of Z.

Q. 9. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

Ans. Option (D) is correct.

Explanation: We know that $f : R \to R$ is defined as $f(x) = x^4$.

Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow \quad x^4 = y^4$

$\Rightarrow \quad x = \pm y$

$\therefore \quad f(x) = f(y)$

does not imply that $x = y$.

For example, $f(1) = f(-1) = 1$

$\therefore$ f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that $f(x) = 2$.

$\therefore f$ is not onto.

Hence, function f is neither one-one nor onto.

Q. 10. Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Check whether the function $f : \mathrm{R} \to \mathrm{R}$ defined as $f(x) = x^3$ is one-one or not.

Q. 2. A relation R in the set of real numbers R defined as $R-\{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify

R&U **[CBSE SQP - 2021]**

Sol. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ **1**

$\therefore \quad \sqrt{a} = b$ is not a function

[CBSE SQP Marking Scheme 2021]

Q. 3. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not. R **[CBSE SQP 2020-21]**

Short Answer Type Questions-I (2 marks each)

Q. 1. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one. A

Q. 2. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is onto. A

Q. 3. Show that the function $f : R \to R$ defined as $f(x) = x^2$ is neither one-one nor onto.

Sol. Given, a function $f : \mathrm{R} \to \mathrm{R}$ defined as $f(x) = x^2$

For one-one Here, at $x = 1, f(1) = 1$

and at $x = -1, f(-1) = (-1)^2 = 1$

Thus, $f(1) = f(-1) = 1$, but $1 \neq -1$

So, f is not one-one.

For onto Let $y \in \mathrm{R}$ (codomain) be any arbitrary element.

Then, $\quad y = f(x)$

$\Rightarrow \quad y = x^2$

$\Rightarrow \quad x = x^2$

$\Rightarrow \quad x = \pm\sqrt{y}$

Now, for $y = -2 \in \mathrm{R}, x = \notin \mathrm{R}$

So, f is not onto.

Hence, given function is neither one-one nor onto.

Q. 4. Show that the function $f : N \to N$, given by $f(x) = 2x$ is one-one but not onto.

Sol. Given, a function $f : N \to N$, defined as $f(x) = 2x$

For one-one Let, $x_1, x_2 \in N$, such that $f(x_1) = f(x_2)$

$\Rightarrow \quad 2x_1 = 2x_2$

$\Rightarrow \quad x_1 = x_2$

So, f is one-one

For onto Let $y \in \mathrm{N}$ (codomain) be any arbitrary element.

Then, $\quad y = f(x)$

$\Rightarrow \quad y = 2x$

$\Rightarrow \quad x = \frac{y}{2}$

Now, for $y = 1, x = \frac{1}{2} \notin N$.

Thus, $y = 1 \in N$ (codomain) does not have a pre-image in domain (N). So, f is not onto.

Short Answer Type Questions-II (3 marks each)

Q. 1. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$ is neither one-one nor onto.

Sol. Checking for one-one:

here $f(x) = f\left(\frac{1}{x}\right)$. For example $f(2) = f\left(\frac{1}{2}\right)$

These questions are for practice and their solutions are available at the end of the chapter

$\therefore f$ is not one-one 1½

Checking for onto:

Let $y = 1 \in R$ (co-domain) . Then

$y = f(x) \Rightarrow \dfrac{x}{x^2+1} = 1$

$\Rightarrow x^2 - x + 1 = 0$, which has no real roots.

$\therefore R_f \neq$ co-domain $\Rightarrow f$ is not onto. 1½

[CBSE SQP Marking Scheme 2020 (Modified)]

Q. 2. Show that the function $f : N \to N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$ for every $x > 2$, is onto but not one-one.

Sol. We have a function $f : N \to N$, defined as

$f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$.

For one-one Since $f(1) = f(2) = 1$, therefore 1 and have same image, namely 1. So, f is not one-one.

For onto Note that $y = 1$ has two pre-images, namely 1 and 2. Now, let $y \in N, y \neq 1$ be any arbitrary element.

Then, $y = f(x) \Rightarrow y = x - 1$

$\Rightarrow x = y + 1 > 2$ for every $y \in N, y \neq 1$.

Thus, for every $y \in N$, $y \neq 1$, there exists $x = y + 1$ such that

$$f(x) = f(y + 1) = y + 1 - 1 = y$$

Hence, f is onto.

Q. 3. Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

R [CBSE Delhi Set III-2019]

Sol. For one-one. Let $x_1, x_2 \in N$.

$$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$
$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$
$$\Rightarrow x_1 = x_2 \text{ as } x_1 + x_2 + 1 \neq 0$$
$$(\because x_1, x_2 \in N) \quad 1½$$
$$\Rightarrow f \text{ is one-one.}$$

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ 1½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $f(x) = x^2 + x + 1$

for $x_1, x_2 \in N$

$$f(x_1) = f(x_2)$$
$$x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$
$$x_1^2 - x_2^2 + x_1 - x_2 = 0$$
$$(x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$$
$$(x_1 - x_2)(x_1 + x_2 + 1) = 0$$

Therefore, the given function is one-one.

Also, f is not onto as for $1 \in N$, there does exist any 'x' in $f(x) = 1$.

Long Answer Type Questions (5 marks each)

Q. 1. Check which of the following function is onto or into.

(i) $f : A \to B$, given by $f(x) = 3x$, where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$.

(ii) $f : Z \to Z$, given by $f(x) = 3x + 2$, where Z = set of integers.

Sol. (i) We have a function $f : A \to B$, given by $f(x) = 3x$, where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$

Let $y \in B$ be any arbitrary element.

Then, $y = f(x) \Rightarrow y = 3x \Rightarrow x - \dfrac{y}{3}$

Now, at $y = 0$, $x = \dfrac{0}{3} = 0 \in A$

At $y = 3$, $x = \dfrac{3}{3} = 1 \in A$

At $y = 6$, $x = \dfrac{6}{3} = 2 \in A$

Thus, for each element y of B, there is a pre-image in A.

(ii) We have a function $f : Z \to Z$, given by $f(x) = 3x + 2$.

Let $y \in Z$, (codomain of f) be any arbitrary element.

Q. 2. Let R be the set of all non-zero real number. Then, show that $f : R \to R$, given by $f(x) = \dfrac{1}{x}$ is one-one and onto.

Sol. Given, $f(x) = \dfrac{1}{x}$

For one-one Let $x_1, x_2 \in R$, such that $f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{1}{x_1} = \dfrac{1}{x_2} \quad \left[\text{put } x_1 \text{ and } x_2 \text{ in } f(x) = \dfrac{1}{x}\right]$

$\Rightarrow \quad x_1 = x_2$

So, f is one-one

For onto Let $y \in R$ be any arbitrary element.

Then, $y = f(x)$

$\Rightarrow \quad y = \dfrac{1}{x}$

$\Rightarrow \quad x = \dfrac{1}{y}$ [expressing x in terms of y]

It is clear that for every $y \in R$(codomain), $x \in R$(domain)

Thus, for each $y \in R$(codomain), there exist

$x = \dfrac{1}{y} \in R$ (domain), such that $f(x) = f\left(\dfrac{1}{y}\right) = \dfrac{1}{\left(\dfrac{1}{y}\right)} = y$

[*i.e.*, every element of codomain has pre-image in domain]

So, f is onto.

COMPETENCY BASED QUESTIONS

Case based MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

ONE – NATION
ONE – ELECTION
FESTIVAL OF DEMOCRACY
GENERAL ELECTION - 2019

$R = \{(V_1, V_2) : V_1, V_2 \in$ I and both use their voting right in general election – 2019$\}$

[CBSE QB 2021]

Q. 1. Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election - 2019. Which of the following is true?

(A) $(X, Y) \in R$
(B) $(Y, X) \in R$
(C) $(X, X) \notin R$
(D) $(X, Y) \notin R$

Ans. Option (D) is correct.

Explanation: $(X, Y) \notin$ R.

$\because$ X exercised his voting right while, Y did not cast her vote in general election-2019

And $R = \{(V_1, V_2) : V_1\ V_2 \in$ I and both use their voting right in general election-2019$\}$

Q. 2. Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

(A) both (X, W) and $(W, X) \in R$
(B) $(X, W) \in R$ but $(W, X) \notin R$
(C) both (X, W) and $(W, X) \notin R$
(D) $(W, X) \in R$ but $(X, W) \notin R$

Ans. Option (A) is correct.

Q. 3. Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then which of the following is true?

(A) $(F_1, F_2) \in R, (F_2, F_3) \in R$ and $(F_1, F_3) \in R$
(B) $(F_1, F_2) \in R, (F_2, F_3) \in R$ and $(F_1, F_3) \notin R$
(C) $(F_1, F_2) \in R, (F_2, F_2) \in R$ but $(F_3, F_3) \notin R$
(D) $(F_1, F_2) \notin R, (F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

Ans. Option (A) is correct.

Q. 4. The above defined relation R is __________

(A) Symmetric and transitive but not reflexive
(B) Universal relation
(C) Equivalence relation
(D) Reflexive but not symmetric and transitive

Ans. Option (C) is correct.

Explanation: R is reflexive, since every person is friend or itself.

i.e., $(F_1, F_2) \in R$

Further, $(F_1, F_2) \in R$

$\Rightarrow F_1$ is friend of F_2

$\Rightarrow F_2$ is friend of F_1

$\Rightarrow (F_2, F_1) \in R$

$\Rightarrow R$ is symmetric

Moreover, $(F_1, F_2), (F_2, F_3) \in R$

$\Rightarrow F_1$ is friend of F_2 and F_2 is friend of F_3.

$\Rightarrow F_1$ is a friend of F_3.

$\Rightarrow (F_1, F_3) \in R$

Therefore, R is an equivalence relation.

Q. 5. Mr. Shyam exercised his voting right in General Election - 2019, then Mr. Shyam is related to which of the following?

(A) All those eligible voters who cast their votes
(B) Family members of Mr. Shyam
(C) All citizens of India
(D) Eligible voters of India

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}, B = \{1, 2, 3, 4, 5, 6\}$ **[CBSE QB 2021]**

Q. 1. Let R : B → B be defined by R = {(*x*, *y*) : *y* is divisible by *x*} is

(A) Reflexive and transitive but not symmetric

(B) Reflexive and symmetric but not transitive

(C) Not reflexive but symmetric and transitive

(D) Equivalence

Ans. Option (A) is correct.

Explanation: R is reflexive, since every element of B i.e.,

$B = \{1, 2, 3, 4, 5, 6\}$ is divisible by itself.

i.e., $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$

further, $(1, 2) \in R$

but $(2, 1) \in R$

Moreover,

$(1, 2), (2, 4) \in R$

$\Rightarrow$ $(1, 4) \in R$

$\Rightarrow R$ is transitive.

Therefore, R is reflexive and transitive but not symmetric.

Q. 2. Raji wants to know the number of functions from A to B. How many number of functions are possible?

(A) 6^2 **(B)** 2^6

(C) $6!$ **(D)** 2^{12}

Ans. Option (A) is correct.

Q. 3. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is

(A) Symmetric

(B) Reflexive

(C) Transitive

(D) None of these

Ans. Option (D) is correct.

Explanation: $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

R is not reflexive. $(3, 3) \notin 4$

Since, $(1, 1), (3, 3), (4, 4), (6, 6) \in R$

R is not symmetric.

Because, for $(1, 2) \in R$ there

$(2, 1) \notin R$.

R is not transitive.

Because for all element of B there does not exist,

$(a, b)\ (b, c) \in R$ and $(a, c) \in R$.

Q. 4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

(A) 6^2 **(B)** 2^6

(C) $6!$ **(D)** 2^{12}

Ans. Option (D) is correct.

Q. 5. Let $R : B \to B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is

(A) Symmetric

(B) Reflexive and Transitive

(C) Transitive and symmetric

(D) Equivalence

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

An organization conducted bike race under 2 different categories–boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. **[CBSE QB 2021]**

Ravi decides to explore these sets for various types of relations and functions

Q. 1. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

(A) 2^6 **(B)** 2^5

(C) 0 **(D)** 2^3

Ans. Option (A) is correct.

Q. 2. Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$, Then this relation R is_______

(A) Equivalence

(B) Reflexive only

(C) Reflexive and symmetric but not transitive

(D) Reflexive and transitive but not symmetric

Ans. Option (A) is correct.

Explanation:

$R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$

R is reflexive, since, $(x, x) \in R$

R is symmetric, since, $(x, y) \in R$ and $(y, x) \in R$

R is transitive. For $a, b, c \in B$

$\exists\ (a, b)\ (b, c) \in R$

and $(a, c) \in R$.

Therefore R is equivalence relation.

Q. 3. Ravi wants to know among those relations, how many functions can be formed from B to G?

(A) 2^2 **(B)** 2^{12}

(C) 3^2 **(D)** 2^3

Ans. Option (D) is correct.

Q. 4. Let $R : B \to G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then R is__________

(A) Injective

(B) Surjective

(C) Neither Surjective nor Injective

(D) Surjective and Injective

Ans. Option (B) is correct.

Explanation:

$R : B \to G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

R is surjective, since, every element of G is the image of some element of B under R, i.e., For $g_1, g_2 \in G$, there exists an elements $b_1, b_2, b_3 \in B$,

$(b_1\, g_1)\, (b_2, g_2), (b_3, g_1) \in R$.

Q. 5. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

(A) 0 **(B)** 2!

(C) 3! **(D)** 0!

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

[CBSE QB 2021]

Q. 1. Let relation R be defined by $R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$ then R is______ relation

(A) Equivalence

(B) Only reflexive

(C) Not reflexive

(D) Symmetric but not transitive

Ans. Option (A) is correct.

Explanation: Let relation R be defined by

$R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$.

R is reflexive, since every line is parallel to itself.

Further, $(L_2, L_1) \in R$

$\Rightarrow L_1$ is parallel to L_2

$\Rightarrow L_2$ is parallel to L_1

$\Rightarrow (L_2, L_1) \in R$

Hence, R is symmetric.

Moreover, $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow L_1$ is parallel to L_3

$\Rightarrow (L_1, L_3) \in R$

Therefore, R is an equivalence relation

Q. 2. Let $R = \{(L_1, L_2) : L_1 \perp L_2$ where $L_1, L_2 \in L\}$ which of the following is true?

(A) R is Symmetric but neither reflexive nor transitive

(B) R is Reflexive and transitive but not symmetric

(C) R is Reflexive but neither symmetric nor transitive

(D) R is an Equivalence relation

Ans. Option (A) is correct.

Explanation: R is not reflexive, as a line L_1 can not be perpendicular to itself, i.e., $(L_1, L_1) \notin R$.

R is symmetric as $(L_1, L_2) \in R$

As, L_1 is perpendicular to L_2

and L_2 is perpendicular to L_1

$$(L_2, L_1) \in R$$

R is not transitive. Indeed, it L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3, then L_1 can never be perpendicular to L_3.

In fact L_1 is parallel to L_3,

i.e., $(L_1, L_2) \in R$, $(L_2, L_3) \in R$ but $(L_1, L_3) \notin R$

i.e., symmetric but neither reflexive nor transitive.

Q. 3. The function $f : R \to R$ defined by $f(x) = x - 4$ is___________

(A) Bijective

(B) Surjective but not injective

(C) Injective but not Surjective

(D) Neither Surjective nor Injective

Ans. Option (A) is correct.

Explanation:

The function f is one-one,

for $f(x_1) = f(x_2)$

$\Rightarrow x_1 - 4 = x_2 - 4$

$\Rightarrow x_1 = x_2$

Also, given any real number y in R, there exists $y + 4$ in R

Such that $f(y + 4) = y + 4 - 4 = y$

Hence, f is onto

Hence, function is both one-one and onto, *i.e.*, bijective.

Q. 4. Let $f : R \to R$ be defined by $f(x) = x - 4$. Then the range of $f(x)$ is ________

(A) R **(B)** Z

(C) W **(D)** Q

Ans. Option (A) is correct.

Explanation: Range of $f(x)$ is R

Q. 5. Let $R = \{(L_1, L_2) : L_1 \parallel L_2$ and $L_1 : y = x - 4\}$ then which of the following can be taken as L_2?

(A) $2x - 2y + 5 = 0$ **(B)** $2x + y = 5$

(C) $2x + 2y + 7 = 0$ **(D)** $x + y = 7$

Ans. Option (A) is correct.

Explanation: Since, $L_1 \parallel L_2$

then slope of both the lines should be same.

Slope of $L_1 = 1$

$\Rightarrow$ Slope of $L_2 = 1$

And $2x - 2y + 5 = 0$

$$-2y = -2x - 5$$

$$y = x + \frac{5}{2}$$

Slope of $2x - 2y + 5 = 0$ is 1

So, $2x - 2y + 5 = 0$ can be taken as L_2.

Case based Subjective Questions (4 mark each)

(Each Sub-part carries 2 marks)

I. Read the following text and answer the following questions on the basis of the same:

Rohan is confused in the Mathematics topic 'Relation and equivalence relation'. To clear his concepts on the topic, he took help his elder brother. He has following notes on this topic.

Relation : A relation R from a set A to a set B is a subset of the cartesian product A × B obtained by describing a relationship between first element x and the second element 'y' of the ordered pairs in A × B. A relation R in a set A is called. :

Reflexive : If $(a, a) \in R\ \forall\ a \in A$.

Symmetric : If $(a_1, a_1) \in R \Rightarrow (a_2, a_1) \in R\ \forall\ a_1, a_2 \in R$.

Transitive : If $(a_1, a_1) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R\ \forall\ a_1, a_2, a_3 \in A$

Equivalence Relation : A relation R in a set A is an equivalence relation if R is reflexive, symmetric and transitive.

Q. 1. Show that relation defined by $R_1 = \{(x, y) \mid x^2 = y^2\}$ $x, y \in R$ is an equivalence relation.

Sol. Given relation $R_1 = \{(x, y) \mid x^2 = y^2\}$

Reflexive : For all $x \in R$, $x^2 = x^2$, so, $(x, x) \in R^1$

Hence, R_1 is reflexive relation.

Symmetric : For all $x, y \in R$

If $x^2 = y^2$ then $y^2 = x^2$

Hence, R_1 is symmetric relation. **1**

Transitive : For all $x, y \in R$, $x^2 = y^2$ and for all $y, z \in R$ $y^2 = z^2$

$\therefore x^2 = y^2 = z^2$, for all $x, y, z \in R$

Hence, R_1 is transitive.

Thus, R_1 is an equivalence relation. **1**

Q. 2. Check whether the relation (R) 'x greater than y' for all $x, y \in N$ is reflexive, symmetric or transitive.

Sol. Given, x greater than y, $\forall\ x, y \in N$

$\Rightarrow x > y\ \forall\ x, y \in N$

Reflexive : Now, for $(x, x) \in R$

Therefore, $x > x$ is not true for any $x \in N$

Thus, R is not reflexive.

Symmetric : Now, let $(x, y) \in R$, then $x > y$

If $x > y$, then $y \not> x$ for any $x, y \in N$

Thus, R is not symmetric.

Transitive : Now, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x > y$ and $y > z$

Therefore, $x => (x, z) \in R$ for all $x, y, z \in N$ **1**

Thus, R is transitive.

Solutions for Practice Questions (Topic-1)

Multiple Choice Questions

6. Option (C) is correct.

Explanation: Consider that aRb, if a is congruent to b, $\forall\ a, b \in T$.

Then, $aRa \Rightarrow a \cong a$,

Which is true for all $a \in T$

So, R is reflexive, ...(i)

Let $aRb \Rightarrow a \cong b$

$\Rightarrow b \cong a$

$\Rightarrow bRa$

So, R is symmetric. ...(ii)

Let aRb and bRc

$\Rightarrow b \cong b$ and $b \cong a$

$\Rightarrow a \cong c \Rightarrow aRc$

So, R is transitive ...(iii)

Hence, R is equivalence relation.

7. Option (B) is correct.

Explanation: $aRb \Rightarrow a$ is brother of b.

This does not mean b is also a brother of a as b can be a sister of a.

Hence, R is not symmetric.

$aRb \Rightarrow a$ is brother of b

and $bRc \Rightarrow b$ is a brother of c.

So, a is brother of c.

Hence, R is transitive.

Very Short Answer Type Questions

3. $[(1, 3)] = \{(x, y) \in A \times A : x + 3 = y + 1\}$

$= \{(x, y) \in A \times A : y - x = 2\}$

$= \{(1, 3), (2, 4)\}$ **1**

[CBSE Marking Scheme 2017-18]

Short Answer Type Questions-I

2. **(i)** $1, 2 \in \mathbb{R}$ such that $1 < 2 \Rightarrow (1, 2) \in R$, but since 2 is not less than $1 \Rightarrow (2, 1) \notin R$. Hence R is not symmetric. **1**

(ii) Let $(a, b) \in R$ and $(b, c) \in R$, $\therefore a < b$ and $b < c$

$\Rightarrow a < c \Rightarrow (a, c)\ R \therefore R$ is transitive. 1

[CBSE SQP Marking Scheme 2020]

Detailed Answer:

(i) It is not symmetric because if $a < b$ then $b < a$ is not true.

(ii) Here, if $a < b$ and $b < c$ then $a < c$ is also true for all $a, b, c \in$ Real numbers. Therefore R is transitive.

Commonly Made Error

- Students use examples to show that the relation is transitive which is wrong.

Answering Tip

- Use only arbitrary elements to prove transitivity.

5. Let $(a, b), (b, c) \in R, f(a) = f(b), f(b) = f(c) \Rightarrow f(a) = f(c)$, $(a, c) \in R$. Thus, Relation is transitive.

Short Answer Type Questions-II

5. **Reflexive:**

R is reflexive, as $1 + a.a = 1 + a^2 > 0 \Rightarrow (a, a) \in R$ $\forall\, a \in R$ **1**

Symmetric:

If $(a, b) \in R$

then, $1 + ab > 0$

$\Rightarrow$ $1 + ba > 0$

$\Rightarrow$ $(b, a) \in R$

Hence, R is symmetric. **1**

Transitive:

Let $a = -8, b = -1, c = \dfrac{1}{2}$

Since, $1 + ab = 1 + (-8)(-1) = 9 > 0$

$\therefore (a, b) \in R$

also, $1 + bc = 1+(-1)\left(\dfrac{1}{2}\right)=\dfrac{1}{2}>0$

$\therefore (b, c) \in R$

But, $1 + ac = 1+(-8)\left(\dfrac{1}{2}\right)=-3<0$

Hence, R is not transitive. **1**

[CBSE Marking Scheme 2018 (modified)]

Commonly Made Error

- Students use counter example to prove reflexive and symmetric.

Answering Tip

- Counter examples can be used only to show exceptions.

Long Answer Type Questions

2. Given $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$

and $A = \{1, 2, 3, 4, 5\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)$
$(1, 3), (1, 5), (2, 4), (3, 5), (3, 1),$
$(5, 1), (4, 2), (5, 3)\}$ **2**

(i) $\forall\, a \in A, (a, a) \in R,$

$\therefore$ R is reflexive. **1**

[As $\{(1, 1), (2, 2), (3, 3), (4, 4)\ (5, 5)\} \in R$]

(ii) $\forall\, (a, b) \in A, (b, a) \in R,$

$\therefore$ R is symmetric. **1**

[As $\{(1,3), (1,5), (2,4), (3,5)\ (3,1), (5,1), (4,2), (5,3)\} \in R$]

(iii) $\forall\, (a, b), (b, c) \in R, (a, c) \in R$

$\therefore$ R is transitive. **1**

[As $\{(1, 3), (3, 1) \in R \Rightarrow (1, 1) \in R$ and similarly others]

$\therefore$ R is an equivalence relation.

[CBSE Marking Scheme 2015 (Modified)]

Solutions for Practice Questions (Topic-2)

Multiple Choice Questions

7. Option (D) is correct.

Explanation: We have, $f(x) = \dfrac{1}{x}, \ \forall\, x \in R$

For $x = 0$, $f(x)$ is not defined.

Hence, $f(x)$ is a not defined function.

10. Option (A) is correct.

Explanation: $f : R \to R$ is defined as $f(x) = 3x$.

Let $x, y \in R$ such that $f(x) = f(y)$

$\Rightarrow$ $3x = 3y$

$\Rightarrow$ $x = y$

$\therefore$ f is one-one.

Also, for any real number (y) in co-domain R, there exists $\dfrac{y}{3}$ in R such that $f\left(\dfrac{y}{3}\right)=3\left(\dfrac{y}{3}\right)=y$.

$\therefore$ f is onto.

Hence, function f is one-one and onto.

Very Short Answer Type Questions

1. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in$ R **1**

$\Rightarrow$ $(x_1)^3 = (x_2)^3$

$\Rightarrow$ $x_1 = x_2$

Hence $f(x)$ is one-one.

[CBSE SQP Marking Scheme 2021]

Commonly Made Error

- Students get confused between one-one and many-one functions.

Answering Tip

- Injectivity should be determined considering the domain and co-domain. A function which is one-one in a domain may not be one-one in another domain.

3. Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \rightarrow B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ *i.e.* $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one. **1**

Short Answer Type Questions-I

1. Given $f(x) = \dfrac{4x+3}{6x-4}$

Let, $f(x_1) = f(x_2)$,

then $\dfrac{4x_1+3}{6x_1-4} = \dfrac{4x_2+3}{6x_2-4}$ ½

or $(4x_1 + 3)(6x_2 - 4) = (6x_1 - 4)(4x_2 + 3)$ ½

or $24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$

or $-16x_1 + 18x_2 = 18x_1 - 16x_2$ ½

or $-16x_1 - 18x_1 = -18x_2 - 16x_2$

or $-34x_1 = -34x_2$

or $x_1 = x_2$

or f is one-one. ½

2. Let, $y \in B = R - \left\{\dfrac{2}{3}\right\}$

$\therefore$ $y = f(x)$

or $y = \dfrac{4x+3}{6x-4}$

or $y(6x - 4) = 4x + 3$ ½

or $6xy - 4y = 4x+3$

or $6xy - 4x = 4y + 3$

or $x(6y - 4) = 4y + 3$

or $x = \dfrac{4y+3}{6y-4} \in B = R - \left\{\dfrac{2}{3}\right\}$ **1**

or For every value of y except $y = \left\{\dfrac{2}{3}\right\}$, there is a pre-image $x = \dfrac{4y+3}{6y-4} = g(y)$.

or $x \in A$

$\therefore$ f is onto. ½

REFLECTIONS

- In this chapter we have covered the different types of relations and functions. Look around you and pick some real life relations say 'is the father of', 'is the friend of' etc and check whether they are reflexive, symmetric and transitive.