UNIT – I: ELECTROSTATICS

CHAPTER-1

ELECTRIC CHARGES AND FIELDS

Topic-1 Electric Field and Dipole

Concepts Covered • *Electric charge,* • *Electrostatic charge,* • *Properties of electric charge,* • *Coulomb's law,* • *Principle of superposition,* • *Electric field,* • *Electric field lines,* • *Electric dipole,* • *Torque on a dipole,* • *Electric dipole moment,* • *Electric field due to dipole*

Revision Notes

Properties of Electric Charge

Addition of charges

- If a system contains n point charges q_1, q_2, q_3, then the total charge of the system is

$$q = q_1 + q_2 + q_3 + \text{.........} + q_n$$

Conservation of charges

- The sum of positive and negative charges present in an isolated system, always remains constant.
- Charge can neither be created nor destroyed but only exists in positive-negative pairs.

Quantization of charges

- Electric charge is always quantized
- Net charge q_{net} of an object having N_e electrons, N_p protons and N_n neutrons is:

  $q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = \pm ne$, where n is an integer

Coulomb's Law

- The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of the distance between charges, written as:

$$F = k'\frac{|q_1||q_2|}{r^2}$$

where,

F = Force of attraction/repulsion between charges q_1 and q_2.

q_1, q_2= Magnitudes of charges

r = Distance between charges q_1, q_2

k' = Constant whose value depends on medium where charges are kept

When the charges are kept in vacuum, then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

ε_0 = Permittivity of vacuum or free space

$= 8.854 \times 10^{-12}$ F/m

$$F = \frac{1}{4\pi\varepsilon}\frac{|q_1||q_2|}{r^2}$$

ε = is the permittivity of that medium.

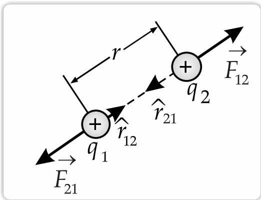
If k is the relative permittivity or dielectric constant of that medium, then $\varepsilon = k\varepsilon_0$

Then, $F = \dfrac{1}{4\pi k\varepsilon_0}\dfrac{|q_1||q_2|}{r^2}$

- The vector form of Coulomb force

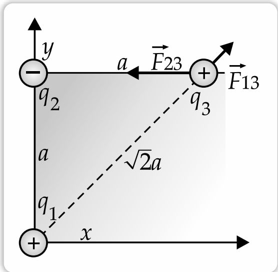
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \text{ and } \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$\Rightarrow \vec{F}_{21} = -\vec{F}_{12}$



Principle of Superposition

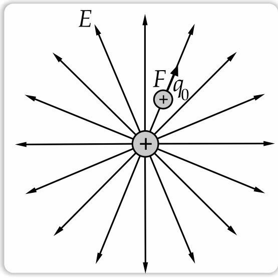
- The force on any charge due to other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.
- Force exerted by q_1 on q_3 = $\vec{F}_{13}$
- Force exerted by q_2 on q_3 = $\vec{F}_{23}$



- Net force exerted on q_3 is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$

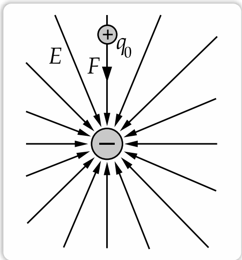
Electric field

- The space around a charge up to which its electric force can be experienced is called electric field.



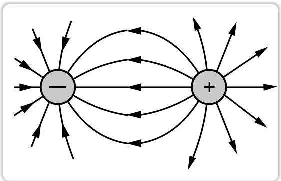
- If a test charge q_0 is placed at a point where electric field is E, then force on the test charge is $F = q_0 E$
- The electric field strength due to a point source charge 'q' at distance 'r' is given by:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3} \vec{r} \text{ or } E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



Electric field lines

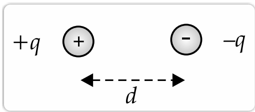
- Electric field lines are imaginary lines that originates from the positive charge and terminates at negative charge.



- The electric field lines never intersect each other.
- Strength of electric field is encoded in density of field lines.

Electric Dipole

- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.



- The force on a dipole in a uniform electric field is zero in both stable as well as unstable equilibrium.
- The potential energy of a dipole in an uniform electric field is minimum for a stable equilibrium and maximum for an unstable equilibrium.

Torque on a dipole

- In a dipole, when the net force on dipole due to electric field is zero and center of mass of dipole remains fixed, the forces on charged ends produce net torque τ about its center of mass.

$$\tau = F\, d\sin\theta = qE\, d\sin\theta = pE\sin\theta$$
$$\vec{\tau} = \vec{p}\times\vec{E}$$

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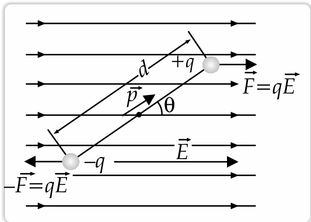


- If θ = 0° or 360°, dipole exists in stable equilibrium state.
- If θ = 180°, dipole exists in an unstable equilibrium state.

- In the uniform electric field, the dipole experiences a torque and a rotary motion. The net force on dipole is zero.
- In the non-uniform electric field, dipole experiences a torque, net force rotary and translatory motion..

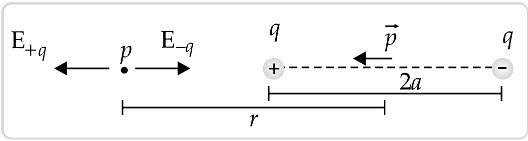
Electric Dipole Moment

- Dipole moment is a vector quantity
- $\vec{p} = \vec{q}\times 2a$ where q, $-q$, are separated by distance $2a$.

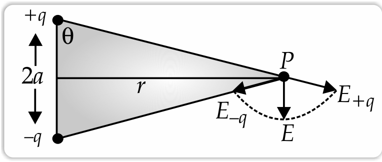
Electric field due to a dipole

- At point P at distance r from the centre of the dipole for $r>>a$, total field is

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \quad (\text{if } a << r)$$



- At point P on the equatorial plane due to a dipole at a large distance, $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$



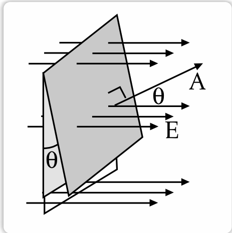
Topic-2 Gauss's Theorem and its Applications

Concepts Covered • *Electric flux,* • *Continuous charge distribution,* • *Gauss' theorem*

Revision Notes

Electric Flux

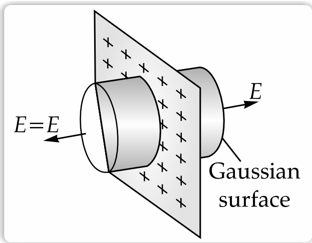
- Electric flux is proportional to algebraic number of electric field lines passing through the surface, outgoing lines with positive sign, incoming lines with negative sign.



- Due to arbitrary arrangement of electric field lines, electric flux can be quantified as $\phi_E = EA\cos\theta$
- In non-uniform electric field, the flux is $\phi_E = \int E.dA$

Gauss' theorem

- The net outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times of net charge enclosed by the surface.
- The electric field flux at all points on Gaussian surface is $\phi = E\oint dA = \frac{q}{\varepsilon_0}$.
- If there is a positive flux, net positive charge is enclosed.
- If there is a negative flux, net negative charge is enclosed.
- If there is zero flux, no net charge is enclosed.
- The expression for electric field due to a point charge on Gaussian surface is $E = \frac{q}{4\pi\varepsilon_0 r^2}$



- In an insulating sheet, the charge remains in the sheet, so electric field, $E = \frac{\sigma}{2\varepsilon_0}$
- Electric field due to uniformly charged shell:

  At a point outside the shell: $E = Q/4\pi\varepsilon_0 r^2$

  (r is the distance of the point from the centre of the shell.)

  At a point inside the shell: $E = 0$

  At a point on the surface of the shell: $E = Q/4\pi\varepsilon_0 R^2$

  (R is the radius of the sphere.)

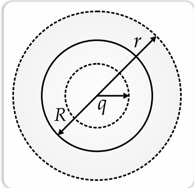
  Electric field due to a uniformly charged infinitely long straight wire: $E = 2\lambda/4\pi\varepsilon_0 r$

  (λ = linear charge density, R = distance of the point from the wire)

Key Formulae

- Coulomb's force: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$;

  where all alphabets have their usual meanings.
- Electric field due to point charge q: $E = \frac{k|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$
- Electric field due to a dipole at a point on the dipole axis: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$ $(r >>> a)$
- Electric field due to a dipole at a point on an equatorial plane: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$ $(r >>> a)$



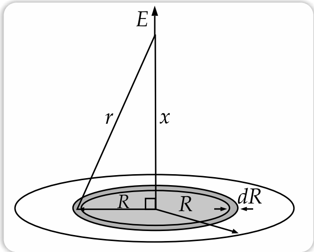
- Torque on an electric dipole placed in an electric field, $\tau = pE\sin\theta$
- Electric flux through an area A: $\phi = \hat{E}.\hat{A} = EA\cos\theta$
- Electric flux through a Gaussian surface: $\phi = \oint E.dS$
- Gauss's Law: $\phi = \frac{q_{enc}}{\varepsilon_0}$
- Electric Field due to an infinite line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$

  where, E = electric field [N/C],

  λ = charge per unit length [C/m]

  ε_0 = permittivity of free space = 8.85×10^{-12} [C²/N m²], r = distance (m), $k = 9 \times 10^9$ Nm²C⁻²
- Electric field due to a ring at a distance x is: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qx}{(r^2+x^2)^{3/2}}$
- When, $x >>> r$: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}$

  When $x <<< r$: $E = 0$
- Electric field due to a charged disc: $E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$



  where,

  E = electric field [N/C]

  σ = charge per unit area [C/m²]

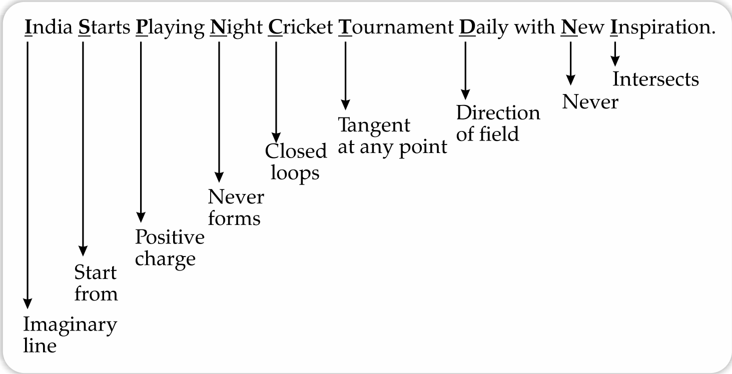
  $\varepsilon_0 = 8.85 \times 10^{-12}$ [C²/Nm²]

  x = distance from charge [m]

  R = radius of the disc [m]
- Electric field due to a thin infinite sheet: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

Mnemonics

Concept: Characteristics of Electric field lines
Mnemonic: India Starts Playing Night Cricket Tournament Daily with New Inspiration.
Interpretation:



CHAPTER-2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Topic-1 Electric Potential

Concepts Covered *Electric potential, potential difference, equipotential surfaces. Electrical potential energy of system of two point charges and of electric dipole.*

Revision Notes

Electric potential

- **Electric potential:** Amount of work done by an external force in moving a unit positive charge from infinity to a point in an electrostatic field without producing an acceleration.
- It is written as $V = \frac{W}{q}$

  where, W = work done in moving charge q through the field, q = charge being moved through the field.
- The SI units of electric potential are $\frac{\text{J}}{\text{C}}$, Volt, $\frac{\text{Nm}}{\text{C}}$.

Potential difference

- **Electric potential difference:** Amount of work done in moving a unit charge from one point to another in an electric field.

  Electric potential difference

$$= \frac{\text{Work}}{\text{Charge}} = \frac{\Delta PE}{\text{Charge}} = \frac{W}{q}$$

  Between two points A and B, $W_{AB} = -V_{AB} \times q$

  where, $V_{AB} = V_B - V_A$ is potential difference between A and B.
- In an electric field, the work done by electric field to move a test charge q by a distance dl is dW.

$$dW = q\vec{E}.\vec{dl}$$

$$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B q\vec{E}.\vec{dl}}{q} = -\int_A^B \vec{E}.\vec{dl}$$

Electric potential due to point charge

- The electric potential by point charge q, at a distance r from the charge is

$$V_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

where, ε_0 is absolute electrical permittivity of free space.

- **Electric potential** is a scalar quantity.
- Dimension of Electric potential is $[M L^2 T^{-3} A^{-1}]$.

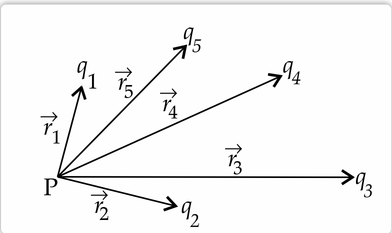
Dipole and system of charges

- Electric **dipole** consists of two equal but opposite electric charges which are separated by a certain distance.
- The net potential due to a dipole at any point on its equatorial line is always zero. So, work done in moving a charge on an equatorial line is zero.

- Electric potential due to dipole at a point at distance r and making an angle θ with the dipole moment p is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}.\hat{r}}{r^2} (r >> a)$$

- Potential at a point due to system of charges is the sum of potentials due to individual charges.
- In a system of charges $q_1, q_2, q_3, \ldots q_n$ having positive vectors $\hat{r}_1, \hat{r}_2, \hat{r}_3, \ldots.. \hat{r}_n$ relative to point P, the potential at point P due to total charge configuration is algebraic sum of potentials due to individual charges.



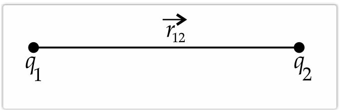
Key Words

Electric potential: The amount of work needed to move a unit charge from a reference point to a specific point against an electric field.
Dipole: A pair of equal and oppositely charged or magnetized poles separated by a certain distance.

Equipotential surfaces

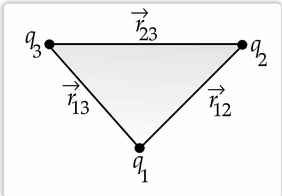
- Equipotential surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface.
- Electric field is always perpendicular to the equipotential surface.
- Spacing among equipotential surfaces allows to locate regions of strong and weak electric field.
- Equipotential surfaces never intersect each other. If they intersect then the intersecting point of two equipotential surfaces results in two values of electric potential at that point, which is impossible.
  - Potential energy of a system of two charges,

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$



  - Potential energy of a system of three charges,

$$U = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



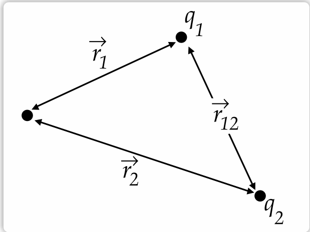
- Potential energy due to single charge in an external field:
  Potential energy of a charge q at a distance r in an external field,

$$U = qV(\vec{r})$$

Here, $V(\vec{r})$ is the external potential at a distance r.

- Potential energy due to two charges in an external field,

$$U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$



- Potential energy of a dipole in an external field:

  When a dipole of charge $q_1 = +q$ and $q_2 = -q$ having separation '$2a$' is placed in an external field ($\vec{E}$).

$$U(\theta) = -pE\cos\theta$$

  Here, $p = 2aq$ and θ is the angle between electric field and dipole.

Key Formulae

► Electric Potential, $V = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.

► Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$.

► Electric potential due to a point charge q at a distance r away: $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

► Finding V from E: $V_f - V_i = -\int_i^f \vec{E} . d\vec{r}$

► Potential energy of two point charges in absence of external electric field: $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right]$

► Potential energy of two point charges in presence of external electric field: $q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

► Maximum charge on a capacitor: $Q = VC$

► For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent and the sum of potential differences across the capacitors is equal to the emf of the charging battery. For capacitors connected in parallel, charges on different capacitors are different and potential drop across each capacitor is same,

► Electrical energy stored in a capacitor: [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

► Surface charge density or Charge per unit area: [C/m^2]

$$\sigma = \frac{q}{A}$$

- **Capacitors in series:**

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

- **Capacitors in parallel:**

$$C_{eff} = C_1 + C_2 \ldots$$

Mnemonics

Concept: Characteristics of equipotential surface
Mnemonics: Exclusive peace and No war; Noble India is super power.
Interpretations: Exclusive peace: Electric field is perpendicular to the surface
No war: No Work is done on moving a charge on the surface
Noble India: Never Intersects
Super Power: Same potential everywhere on the surface

Topic-2 Capacitance

Concepts Covered *Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics, electric polarization, capacitor and capacitance, combination of capacitors, Capacitance of a parallel plate capacitor, energy stored in capacitor.*

Revision Notes

Conductors and insulators

- **Conductors:** Materials through which charge can move freely. **Examples:** Metals, semi-metals as carbon, graphite, antimony and arsenic.
- **Insulators:** Materials in which the electrical current does not flow easily. **Examples:** Plastics and glass.

Dielectrics:

- Materials in which induced dipole moment is linearly proportional to the applied electric field.
- Electrical displacement or electrical flux density, $D = \varepsilon_r \varepsilon_0 E$.

  where, ε_r = Electrical relative permittivity, ε_0 = Electrical permittivity of free space and E is electric field.
- In dielectric, polarisation and production of induced charge takes place when dielectric is kept in an external electric field.

Electric polarization

- **Electric polarisation** P is the difference between electric fields D (induced) and E (imposed) in dielectric due to bound and free charges written as $P = \frac{D - E}{4\pi}$
- In term of electric susceptibility: $P = \chi_e E$
- In MKS: $P = \varepsilon_0 \chi_e E$,
- The dielectric constant $\kappa = 1 + \chi_e$ and is always greater than 1 as $\chi_e > 0$

Key Word

Electric polarisation: It is the separation of centre of positive charges and the centre of negative charges in a material. The separation can be caused by a sufficiently high electric field.

Capacitor & capacitance:

- A capacitor is a device which is used to store charge.
- Amount of charge 'Q' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate is connected to negative terminal of battery.
- Electric charge stored between plates of capacitor is directly proportional to potential difference between its plates, i.e.,

$$Q = CV$$

  where, C = Capacitance of capacitor, V = potential difference between the plates
- SI unit of capacitance is Farad (F)

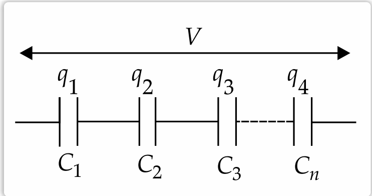
$$1\,F = \frac{1\,C}{1\,V} = 9 \times 10^{11} \text{ stat Farad}$$

- Units smaller than Farad

PREFIX	MULTIPLIER	
μ	10^{-6} (millionth)	$1\ \mu F = 10^{-6}$ F
n	10^{-9} (thousand-millionth)	$1\ nF = 10^{-9}$ F
p	10^{-12} (million-millionth)	$1\ pF = 10^{-12}$ F

Capacitors in series

(i)



Charge on each capacitor is same,

$$q_1 = q_2 = q_3 \ldots\ldots\ldots\ldots q_n = q$$

- Sum of potential differences across the capacitors is equal to V, the emf of the charging battery, i.e.,

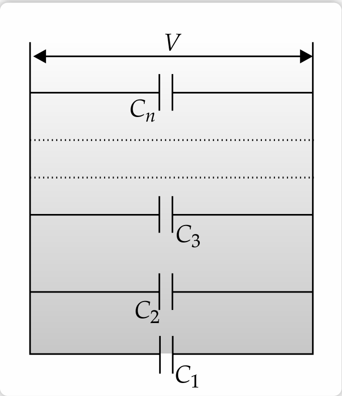
$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots + V_n$$

So, $q = V_1C_1 = V_2C_2 = V_3C_3 \ldots\ldots\ldots\ldots$

i.e., $V \propto \frac{1}{C}$

- Potential difference across largest capacitance is minimum.
- The equivalent capacitance is less than the smallest capacitance in combination.

Capacitors in parallel

- 

Charges on different capacitors are different.

$$q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n = VC_p$$

- Potential drop across each capacitor is same, i.e.,

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \ldots\ldots = \frac{q_n}{C_n}$$

Capacitance of parallel plate capacitor with and without dielectric medium between the plates

- Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium.
- Capacitance of parallel-plate capacitor with area A separated by a distance d is

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

- Capacitance of parallel plate capacitor depends on plate area A, distance d between the plates, medium between the plates (κ) and not on charge on the plates or potential difference between the plates.
- If number of dielectric slabs of same area as the plates of the capacitor and thicknesses t_1, t_2, t_3,.... and dielectric constant κ_1, κ_2, κ_3...... inserted between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \frac{t_3}{\kappa_3} + ...}$$

Where, $d = t_1 + t_2 + t_3 + ...$

- If slab of conductor of thickness t is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{\frac{t}{\kappa} + \frac{(d-t)}{1}} = \frac{\varepsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}}$$

$$C = \frac{\varepsilon_0 A}{d-t} \qquad (\because \kappa = \infty \text{ for a conductor})$$

- When the medium between the plates consists of dielectric slabs of same thickness but areas A_1, A_2, A_3,... and dielectric constants κ_1, κ_2, κ_3..., then

$$C = \frac{\varepsilon_0(\kappa_1 A_1 + \kappa_2 A_2 + \kappa_3 A_3 ....)}{d}$$

- When space between the plates is partly filled with dielectric of thickness t and dielectric constant κ, then capacitance:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}$$

When there is no medium between the plates, then $\kappa = 1$, so $C_{\text{vacuum}} = \frac{\varepsilon_0 A}{d}$

Energy stored in capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Key Formulae

- Electric Potential, $V = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.
- Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$.
- Electric potential due to a point charge q at a distance r away: $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$
- Finding V from E: $V_f - V_i = -\int_i^f \vec{E}.d\vec{S}$
- Potential energy of two point charges in absence of external electric field: $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right]$
- Potential energy of two point charges in presence of external electric field: $q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$
- Capacitance, $C = \frac{Q}{V}$, measured in Farad; 1 F = 1 coulomb/volt
- **Parallel plate capacitor:** $C = \kappa\varepsilon_0 \frac{A}{d}$
- **Cylindrical capacitor:** $C = 2\pi\kappa\varepsilon_0 \frac{L}{\ln(b/a)}$

  where, L = length, b = radius of the outer conductor, a = radius of the inner conductor.

- **Spherical capacitor:** $C = 4\pi\kappa\,\varepsilon_0\left(\frac{ab}{b-a}\right)$

  where, b = radius of the outer conductor, a = radius of the inner conductor
- Maximum charge on a capacitor: $Q = VC$
- For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the **dielectric constant** κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ and Q is unchanged. In vacuum, $\kappa = 1$ and when dielectrics are used, replace ε_0 with $\kappa\,\varepsilon_0$.
- Electrical energy stored in a capacitor: [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

- Surface charge density or Charge per unit area: [C/m^2]

$$\sigma = \frac{q}{A}$$

- **Energy density:**
  - Electric energy density is also called Electrostatic pressure.
  - Electric force between plates of capacitor, $F = \frac{1}{2}\varepsilon_0 E^2 \cdot A$
  - Energy stored in terms of Energy density, $\frac{E}{A\times d} = \frac{1}{2}\varepsilon_0 E^2$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

  where, U = energy per unit volume [J/m^3], ε_0= permittivity of free space, = 8.854×10^{-12} C^2/Nm^2, E = energy [J]
  - **Capacitors in series:**

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

  - **Capacitors in parallel:**

$$C_{eff} = C_1 + C_2 \ldots$$

UNIT – II: CURRENT ELECTRICITY

CHAPTER-3

CURRENT ELECTRICITY

Topic-1 Electric Current & Cells

Concepts Covered • *Electric current,* • *drift velocity,* • *Ohm's law,* • *V-I characteristics, Resistivity and conductivity,* • *Temperature dependence of resistance,* • *Cells and their combinations.*

Revision Notes

Electric current

- Electric current is defined as the rate of flow of charge, across the cross section of conductor, i.e., $I = \frac{dq}{dt}$

Flow of electric charges in metallic conductor

- When an electric field is applied to a metal at certain points, free electrons experience force and start moving.
- Drift velocity is an average velocity which is obtained by certain particles like electrons due to the presence of electric field.
- Drift velocity = $\vec{v}_d = -\frac{e\vec{E}}{m}\tau$

where, relaxation time, $\tau = \frac{\lambda}{v}$,

here, e = charge, m = mass,

λ = mean free path

- When electric current is set up in a conductor, electrons drift through the conductor with velocity v_d, is given as

$$v_d = \frac{I}{neA} \text{ or } I = neAv_d$$

where, I = Electric current through conductor,
n = Number of free electrons per unit volume,
A = Area of cross-section, e = Charge of electron

- Mobility is the drift velocity of an electron when applied electric field is unity.

Mobility, $\mu = \frac{v_d}{E}$

or, $\mu = \frac{e\tau E / m}{E} = \frac{e\tau}{m}$

Electrical resistivity and conductivity

- Resistivity is the resistance of a conductor having unit length and unit area of cross-section.

$$\rho = \frac{m}{ne^2\tau}$$

- Conductivity is the reciprocal of resistivity shown as : $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

Ohm's law

- The flow of current through conductor is directly proportional to the potential difference established across the conductor, provided other physical conditions remain constant.

or, $I \propto V$

or, $I = GV$

Here, $G = \frac{1}{R}$

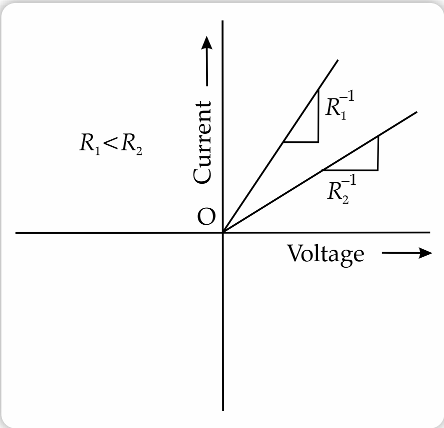
or, $I = \frac{1}{R}V$

or, $V = IR$

where, R = resistance of conductor

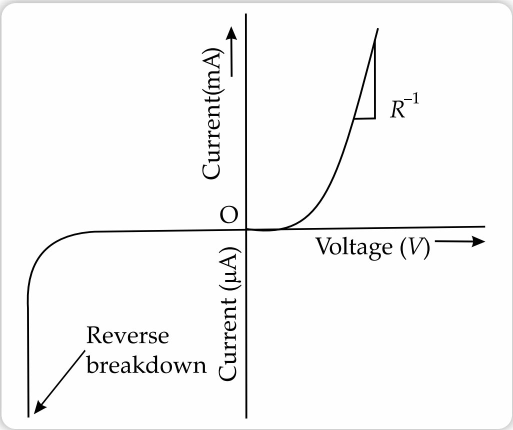
Linear *V-I* Characteristics

- Ohm's law and their *V-I* characteristic curve are straight lines passing through the origin.



Non-linear *V-I* Characteristics

- Semiconductors, p-n junction diodes do not obey Ohm's law and their V-I characteristic curves are non-linear.



Electrical energy and power

- Electrical energy due to conduction of charged particles in a conductor causing electric current (i) is

$$E = V \times I \times t = I^2 \times R \times t = \frac{V^2}{R} \times t$$

where, E = Electrical energy, V = Potential difference, t = Time taken, I = Current, R = Resistance

The SI unit of energy is Joule (J).

- Power is the work done per unit time.

$$P = \frac{W}{t}$$

$$P = I^2R = \frac{V^2}{R}$$

The unit of power is J/s or W (Watt).

Temperature dependence of resistance and resistivity

- With small change in temperature, resistivity varies with temperature as :

$$R = R_0(1 + \alpha\Delta t)$$
$$\rho = \rho_0(1 + \alpha\,\Delta T)$$

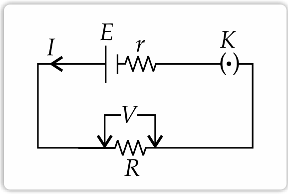
where, α = temperature coefficient of resistance and resistivity.

Internal resistance of cell

- Cell is a device that maintains the potential difference between the two electrodes as a result of chemical reaction.
- Internal resistance is the resistance of electrolyte which resists the flow of current when connected to a circuit.
- Emf E is the potential difference between the electrodes of cell, when no current flows through it.

Potential difference and emf of a cell

- **The emf and terminal potential difference of a cell :** Let emf of a cell be E and its internal resistance, r. If an external resistance R be connected across the cell through a key, then



$$E = V + Ir$$

$\Rightarrow$ $I = \frac{E - V}{r}$

So $V = E - Ir$

$\therefore$ $V < E$. (if there is flow of current)

Combination of cells in series and parallel

- **(i) Series combination of cells :** Let n cells, each of emf E and internal resistance r be connected in series across an external resistance R, then the current in the circuit will be

$$I_S = \frac{nE}{R + nr}$$

(ii) Parallel combination of cells: When m cells are connected in parallel across a resistance R, then current through the resistance is given by

$$I_P = \frac{E}{R + r/m} = \frac{mE}{mR + r}$$

If m cells of emfs $E_1, E_2, E_3, ...E_m$ and of internal resistances $r_1, r_2, r_3, ...r_m$ are connected in parallel across an external resistance R, then the current through the external resistance is given by

$$I_P = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} ..... + \frac{E_m}{r_m}}{R + \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_m}\right)}$$

- **Conductance :** The reciprocal of resistance. Its SI unit is Siemen, S.

Key Formulae

- Electric current $I = neAv_d$
- Mobility $\mu = v\hat{d}/E$
- Resistivity $\rho = m/ne^2T$
- Conductivity $\sigma = ne^2T/m$
- Ohm's Law
  $V = IR$
  $I = GV$
- Power $= I^2R = V^2/R$
- Temperature dependence of resistance and resistivity
  $R = R_0(1 + \alpha\Delta t)$
  $\rho = \rho_0(1 + \alpha\Delta t)$
- Potential difference and emf of a cell
  $V = E - Ir$
- For series cell combination
  $I_s = nE/(R + nr)$
- For parallel cell combination
  $I_P = mE/(mR + r)$

Topic-2 Kirchhoff's Rules & Wheatstone Bridge

Concepts Covered • Kirchhoff's rules, • Wheatstone bridge.

Revision Notes

Kirchhoff's rules

First rule

- **Kirchhoff's first rule (junction rule):**

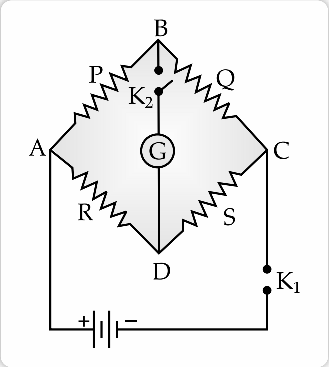
  The algebraic sum of all currents meeting at a junction in a closed circuit is zero. i.e., $\Sigma I = 0$
- This follows the law of conservation of charge.

Second rule

- **Kirchhoff's second rule (loop rule):** Around any closed loop in a circuit, sum of the emfs and the potential differences across all elements is zero.
  i.e., $\Sigma V = 0$ or $\Sigma V = \Sigma IR$
- This follows the law of conservation of energy.

Wheatstone Bridge

- It is a circuit having four resistances P, Q, R and S, a galvanometer (G) and a battery connected as shown.
- **Balanced condition:** $P/Q = R/S$



Key Formulae

- **Kirchhoff's 1st law:**
  $\Sigma I = 0$
- Kirchhoff's 2nd law:
  $\Sigma V = \Sigma IR$
- Balance condition of Wheatstone Bridge
  $P/Q = R/S$

UNIT – III: MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

CHAPTER-4

MOVING CHARGES AND MAGNETISM

Topic-1

Magnetic Field and Biot-Savart law

Concepts Covered • *Magnetic field,* • *Oersted's experiment,* • *Biot-Savart law and its application to current carrying circular loop.*

Revision Notes

Concept of Magnetic field

- **Magnetic field** is a region around a magnet where force of magnetism acts which affects other magnets and magnetic materials.
- Magnetic field also known as *B*-field can be pictorially represented by magnetic field lines.
- Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.
- **Lorentz Force:** When a charge *q* moving with velocity *v* enters a region where both magnetic fields and electric fields exist, both fields exert a force on it.

  Lorentz Force, $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

  where, $\vec{F}$ = magnetic force, $\vec{q}$ = charge, $\vec{v}$ = velocity, $\vec{B}$ = magnetic field, $\vec{E}$ = electric field, $q\vec{E}$ = electric force on the charge, $q(\vec{v} \times \vec{B})$ = magnetic force on the charge
- SI unit of magnetic field is Tesla.

  1 Tesla = 10^4 Gauss

- When a test charge q_0 enters a magnetic field $\vec{B}$ directed along negative z-axis with a velocity $\vec{v}$ making an angle θ with the z-axis, then,

$$\vec{F}_m = q_0(\vec{v}\times\vec{B}) = q_0 vB\sin\theta\,\hat{n}$$

Characteristics of motion of particle in magnetic field

- Velocity and kinetic energy of particle do not change, as force is always perpendicular to velocity.
- Direction of velocity will continuously change, if θ = 90°.
  When θ = 0°, no force will act on the particle, hence there will be no change in velocity.
- When θ = 90°, test charge describes a circle of radius

$$\frac{mv}{q_0B}$$

  where, m is mass of the particle; larger the momentum, bigger the circle described.
- In case of θ being any angle other than 0° and 90°, test charge will show circular path of radius $\frac{mv\sin\theta}{q_0B}$, which moves along the direction of magnetic field with speed of $v\cos\theta$.
- Momentum along the direction of magnetic field will remain same.
- Angular speed of test charge $\frac{q_0B}{m}$ is independent of initial speed of particle.
- Centripetal force on test charge q_0v B sin θ is independent of the mass of particle.
- When the particle enters the magnetic field with the same momentum, then radius of path will be,

$$r = \frac{mv}{q_0B}$$

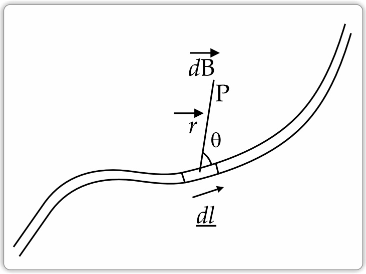
where, $$r \propto \frac{1}{q_0}$$

Oersted's experiment

Oersted observed that:

- When there is no current, compass needle below a wire shows no deflection.
- When the flow of current is in single direction, then the compass needle deflects in a particular direction.
- When the flow of current is reversed, deflection in compass needle occurs in the opposite direction.
- From the experiment, it is concluded that an electrical current produces a magnetic field which surrounds the wire.
- The magnetic field due to a current element at a nearby point is given by:

where, $$\vec{dB} = \left[\frac{\mu_0}{4\pi}\right] I\frac{\vec{dl}\times\vec{r}}{r^3}$$



$\vec{dB}$ = Magnetic field produced by current element

$\vec{dl}$ = Vector length of small section of wire in direction of current

$\vec{r}$ = Positional vector from section of wire to the point where magnetic field is measured

I = Current in the wire

θ = Angle between $\vec{dl}$ and $\vec{r}$

μ_0 = Permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am

The magnitude of magnetic field,

$$|\vec{dB}| = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl\sin\theta}{r^2}$$

Key Words

Permeability: It is also called magnetic permeability, is a constant of proportionality that exists between magnetic induction and magnetic field intensity.

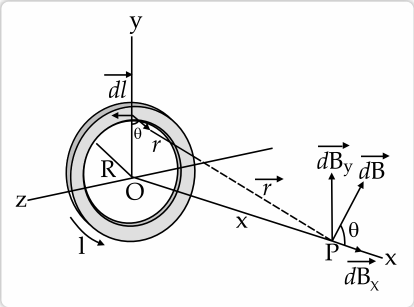
Applications of Biot-Savart's Law

- Magnetic field at a point due to a current carrying circular loop will be:

$$\vec{B} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \vec{r} \qquad [\text{Here, } r^2 = R^2 + x^2]$$

- Magnetic field at the centre of the coil

$$\vec{B} = \frac{\mu_0 N I}{2R} \vec{r} \qquad [x = 0]$$



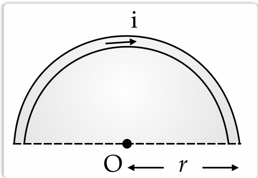
- Magnetic field at very large distance from the centre: $B = \frac{2\mu_0 N i A}{4\pi x^3}$ [Here, $R^2 << r^2$ or, $R^2 + x^2 \approx x^2$]

  where, A = Area of circular loop

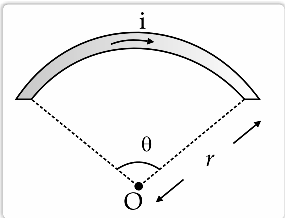
  $= \pi R^2$

  Magnetic field due to current carrying circular arc with centre O will be:

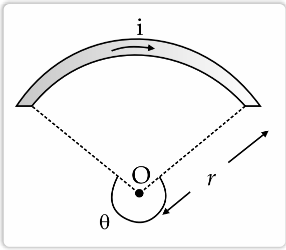
  (i) $B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$



  (ii) $B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$

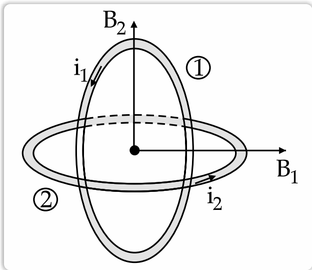


  (iii) $B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta) i}{r}$



- Magnetic field at common centre of non-coplanar and concentric coils, where both coils are perpendicular to each other will be:

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



Key Formulae

- Lorentz force, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- In uniform magnetic field B, frequency of circular motion of charged particle,

$$f = \frac{qB}{2\pi m}$$

and $$KE_m = \frac{q^2r^2B^2}{2m}$$

- Biot-Savart's law, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(\vec{dl} \times \vec{r})}{r^3}$
- Magnetic field at a point due to circular loop,

$$\vec{B} = \frac{\mu_0}{2} \cdot I \frac{R^2}{(R^2 + x^2)^{3/2}}$$

Mnemonics

Concept: Path of a charged particle in a uniform magnetic field depending on the angle between the magnetic field and the velocity of the particle:

Mnemonics: There is space below the page. Can you please write the line in the following manner?

Circle -circle ninety (90°) angle,

go straight if it is zero (0°),

Go for helical, all other angle

magnetic field is hero.

Circle circle ninety (90°) angle ⟹	Path is a circle if angle between magnetic field and velocity of charged particle is 90°.
Go **straight** if it zero **(0°)** ⟹	Path is a straight line if angle between magnetic field and velocity of charged particles is 90°
Go for **helical,** all **other angle** ⟹	Path is helix if any other angle between magnetic field and velocity of charged particle.

Topic-2 Ampere's Circuital Law and its Applications

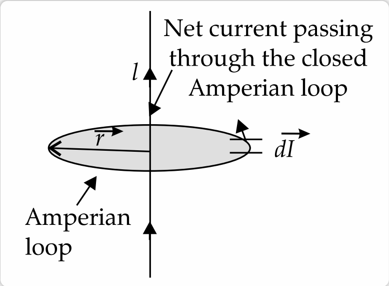
Concepts Covered • *Ampere's law and it applications to infinitely long straight wire,* • *straight solenoid*

Revision Notes

- Ampere's circuital law states that the line integral of magnetic field around a closed path is μ_0 times of total current enclosed by the path, $\oint B.dI = \mu_0 I$

where,

B = Magnetic field
dl = Infinitesimal segment of the path
μ_0 = Magnetic permeability of free space
I = Enclosed electric current by the path



- Magnetic field at a point will not depend on the shape of Amperian loop and will remain same at every point on the loop.

Forces between two parallel currents

- Two parallel wires separated by distance r having currents I_1 and I_2 where magnetic field strength at second wire due to current flowing in first wire is given as:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

  - In this, the field is orientated at right-angles to second wire where force per unit length on the second wire will be:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

  - Magnetic field-strength at first wire due to the current flowing in second wire will be:

$$B = \frac{\mu_0 I_2}{2\pi r}$$

- One ampere is the magnitude of current which, when flowing in each parallel wire one metre apart, results in a force between the wires as 2×10^{-7} N per metre of length.

Applications of Ampere's law to infinitely long straight wire, straight solenoids

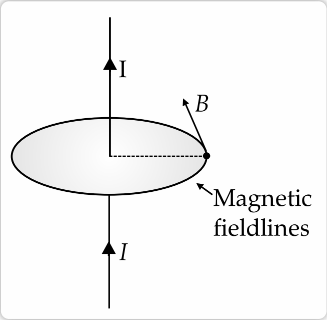
(i) Magnetic Field due to long straight wire

Ampere's law describes the magnitude of magnetic field of a straight wire as:

$$B = \frac{\mu_0 I}{2\pi r}$$

where,

- Field **B** is tangential to a circle of radius r centred on the wire.
  - Magnetic field **B and path length L** will remain parallel where magnetic field travels.



(ii) Magnetic Field due to Solenoid

Solenoid:

Solenoid is a tightly wound helical coil of wire whose diameter is small compared to its length.

Magnetic field generated in the centre, or core of a current carrying solenoid is uniform and is directed along the axis of solenoid.

Magnetic field due to a straight solenoid:

- at any point in the solenoid, $B = \mu_0 nI$
- at the ends of solenoid, $B_{\text{end}} = \frac{\mu_0 nI}{2}$

where, n = number of turns per unit length, I = current in the coil.

Key Formulae

- Ampere's circuital law: $\oint \vec{B}.\vec{dl} = \mu_0 I$
- Magnetic field at the surface of a solid cylinder:

$$B = \frac{\mu_0 I}{2\pi R}$$

- Magnetic field inside the solenoid:

$$B = \mu_0 nI$$

- Magnetic field in a toroid with mean radius r:

$$r = \frac{\mu_0 NI}{2\pi r}$$

- Force between two parallel wires,

$$F = \frac{\mu_0}{4\pi} \times \frac{2I_1 I_2}{a} \times l$$

- Force between two moving charge particle,

$$F_m = \frac{\mu_0}{4\pi} \times \frac{q_1 q_2 v_1 v_2}{r^2}$$

Topic-3 Torque and Galvanometer

Concepts Covered • *Torque experienced by a current loop in uniform magnetic field,* • *moving coil galvanometer and its current sensitivity* • *conversion to ammeter and voltmeter.*

Revision Notes

- **Galvanometer:** A galvanometer is an electromechanical measuring instrument for electric current.

Torque experienced by a current loop in uniform magnetic field

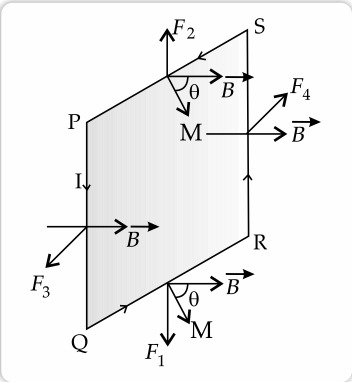
- If a rectangular loop of length l, breadth b with current I flowing through it is in a uniform magnetic field of induction, B where angle θ is between the normal and in direction of magnetic field, then the torque experienced will be:

$$\tau = nBIA \sin\theta$$

where, n = number of turns in the coil

∴ $nIA = M$

Further, $\tau = MB \sin\theta$

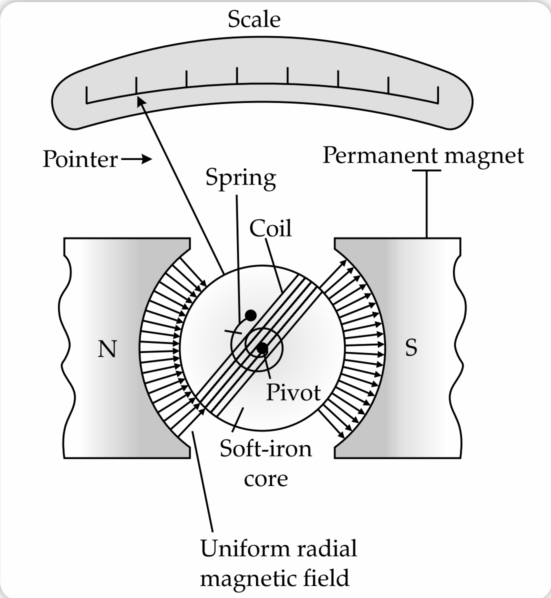


- Torque is maximum when the coil is parallel to magnetic field and is zero when coil is perpendicular to magnetic field.

- In vector notation, torque $\vec{\tau}$ experienced will be $\vec{\tau} = \vec{M} \times \vec{B}$

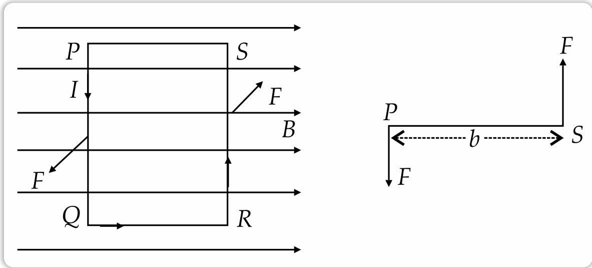
Moving coil galvanometer

- It is an instrument used for detection and measurement of small electric currents.
- In this, when a current carrying coil is suspended in uniform magnetic field, it experiences a torque which rotates the coil.
- The force experienced by each side of the galvanometer is $F = BIl$ which are opposite in direction.



- Opposite and equal forces form the couple which generates deflecting torque on the coil having number of turns n is given as:

$$\tau = F \times b$$
$$= nBIl \times b$$
$$= nBIA$$



- In moving coil galvanometer, current in the coil is directly proportional to the angle of the deflection of the coil, $I \propto \theta$ i.e., where, θ is the angle of deflection.

Current sensitivity of galvanometer

- Current sensitivity: The deflection produced per unit current

  Current Sensitivity, $$\frac{\theta}{I} = \frac{nBA}{C}$$

- Voltage sensitivity: The deflection per unit voltage.

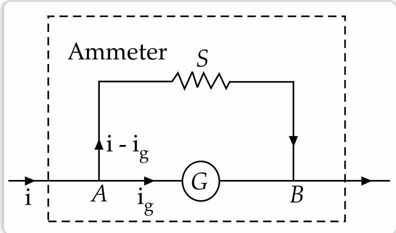
  Voltage Sensitivity, $$\frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$$

  where, G = galvanometer resistance, C = torsional constant.
- Increase in sensitivity of moving coil galvanometer depends on:
  (i) number of turns n
  (ii) magnetic field B
  (iii) area of coil A and
  (iv) torsional constant.

Conversion of galvanometer into ammeter

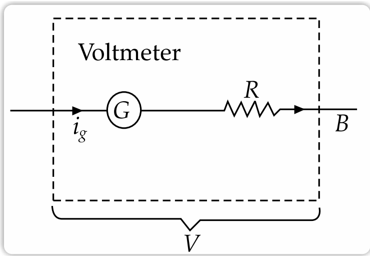
- Galvanometer can be converted into ammeter by connecting a low resistance known as shunt in parallel with the galvanometer coil.

  $$S = \frac{Gi_G}{(i-i_G)}$$



Conversion of galvanometer into voltmeter

- Galvanometer can be converted to voltmeter by connecting high resistance in series with galvanometer coil.
  i_G = current for full scale deflection of galvanometer.



Key Formulae

- $\tau_{max} = NBiA$
- Current Sensitivity = $\frac{\theta}{I} = \frac{nAB}{C}$
- Voltage Sensitivity = $\frac{\theta}{V} = \frac{nAB}{CG}$

CHAPTER-5
MAGNETISM AND MATTER

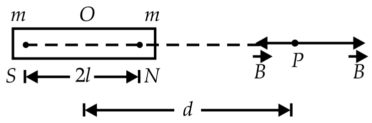
Revision Notes

Magnetic field intensity due to magnetic dipole (bar magnet) along its axis and perpendicular to its axis

- **Magnetic dipole moment** of bar magnet:
  $M = m \times 2l = 2ml$ (where m = pole strength, $2l$ = magnetic length)
- Direction of magnetic dipole moment is from South pole to North pole.

Point lies on axial line of bar magnet:

- Magnetic field at point P due to:



North pole of magnet (N)

$$B = \frac{\mu_0(+m)}{4\pi(d-l)^2}$$

Key Words

- **Magnetic length:** It is the distance between the two poles of a magnetic dipole.
- **Magnetic dipole moment:** It is the product of pole strength and separation between two poles. It is denoted by M.
- **Magnetic intensity:** It is the magnetic moment per unit volume.
- **Magnetic dipole:** A system of two equal and opposite magnetic poles separated by small distance.

South pole of magnet (S)

$$B = \frac{\mu_0(-m)}{4\pi(d+l)^2}$$

Hence, resultant at point P when $2l << d$:

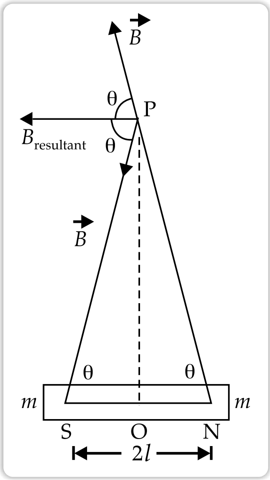
$$B = \frac{\mu_0 2M}{4\pi d^3}$$

where, $M = m \times 2l$.

Point lies on equatorial line of bar magnet

- Magnetic field at point P due to:
  North pole of magnet (direction N–P)

$$B = \frac{+\mu_0(m)}{4\pi(d^2+l^2)}$$



South pole of magnet (direction P–S)

$$B = \frac{-\mu_0 m}{4\pi(d^2 + l^2)}$$

Hence, resultant at point P when $2l << d$:

$$B = \frac{\mu_0 M}{4\pi d^3}$$

where, $M = m \times 2l$

Torque on a magnetic dipole (bar magnet) in uniform magnetic field

- A bar magnet with length $2l$ and pole strength m in uniform magnetic field induction B at angle θ with force mB acting on North and South pole along the direction opposite to magnetic field results as a couple where torque τ due to couple is

τ= force × perpendicular distance

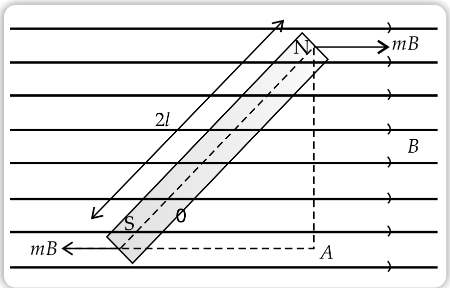
$\tau = F \times NA$

$= mB \times NA$

$= mB \times 2l\sin\theta$

$= MB\sin\theta$

where, $M = m \times 2l$



In vector form:

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of τ is perpendicular to plane containing, so when $B = 1$ and $\theta = 90^0$,

$$\tau = M$$

- Magnetic moment M of the magnet will be equal to the torque required to keep the magnet at right angles to a magnetic field of unit magnetic induction.

Magnetic field lines

- Imaginary closed loops which continuously represent the direction of magnetic field at any point. Tangent at any point of these loops gives the direction of magnetic field at that point.
- Concentration of field lines gives strength of magnetic field.
- Two field lines can never intersect each other. If they would, there would be two tangents at the point of intersection, which means two directions of magnetic lines, which is impossible.
- In a 'uniform' magnetic field, the field lines are parallel and equidistant.

Magnetic needle placed in magnetic field:

- If a magnetic needle is placed in a magnetic field and left, it will experience a torque and start oscillating in simple harmonic motion.

  Time period of oscillations of magnetic needle,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

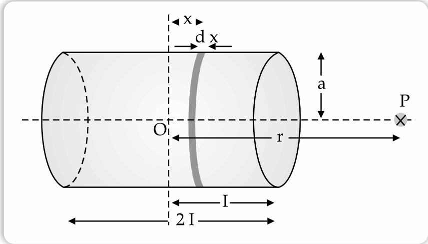
  where, I is moment of inertia of magnetic needle.
- Potential energy of a magnet in a magnetic field,

$$U = -\vec{M}.\vec{B}$$

Bar magnet as an equivalent solenoid

- If a solenoid of length $2l$, radius a with current I having n number of turns per unit length, then the magnetic moment of solenoid,

$$M = NIA, \; B = \frac{\mu_0 2M}{4\pi d^3}$$



- Magnetic moment of a bar magnet is equal to magnetic moment of an equivalent solenoid that produces same magnetic field.
- **Properties of Dia, para and Ferromagnetic Substances:**

S.No.	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
1.	These substances acquire feeble magnetism opposite to the direction of magnetic field when placed in magnetic field. $\longrightarrow$ H, M $\longleftarrow$	These substances acquire feeble magnetism in the direction of magnetic field when placed in magnetic field. $\longrightarrow$ H, $\longrightarrow$ M	These substances acquire strong magnetism in the direction of magnetic field when placed in magnetic field. $\longrightarrow$ H, $\longrightarrow$ M
2.	Permeability is less than one ($\mu<1$).	Permeability is slightly greater than one ($\mu>1$).	Permeability is much greater than one ($\mu>>1$).
3.	Susceptibility is small, negative, and independent of temperature.	Susceptibility is small and positive and inversely proportional to the absolute temperature, $\chi \propto \frac{1}{T}$ (Curie's Law)	Susceptibility is large and positive and follow Curie's Law, $\chi \propto \frac{1}{T}$ (Curie's Law) At Curie temperature, ferromagnetic substance changes to paramagnetic substance.

Key Formulae

- Magnetic field due to short dipole at distance *'d'* on axial line:

$$B_{axial} = \frac{\mu_0 2M}{4\pi d^3}$$

- Magnetic field due to short dipole at distance *'d'* on equatorial line:

$$B_{equi} = \frac{\mu_0 M}{4\pi d^3}$$

- Torque on a magnetic dipole in uniform magnetic field: $\tau = MB \sin\theta$

UNIT – IV: ELECROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

CHAPTER-6 ELECTROMAGNETIC INDUCTION

Topic-1 Magnetic Flux and Faraday's Laws

Concepts Covered • *Electromagnetic induction, Faraday's laws & Lenz's Law.*

Revision Notes

Electromagnetic induction

- Electromagnetic induction is the process of generating the electric current with a changing magnetic field.

- If magnetic field is changing, the changing magnetic flux will be $\phi_B = NBA\cos\theta$, where, θ is the angle between magnetic field and normal to the plane.

Magnetic flux density

- The change in **magnetic flux** per unit change in area is called magnetic flux density.

Key Word

Magnetic flux: Magnetic flux is a measurement of the total magnetic field which passes through a given area. It is the number of magnetic field lines passing through a given closed surface.
It is the product of the average magnetic field and the perpendicular area that it penetrates.

$$d\phi = \vec{B}\cdot d\vec{A}$$

- For $\vec{B}$ parallel to $d\vec{A}$,

$$d\phi = B(dA)\cos 0° = B(dA)$$

Therefore, $$B = \frac{d\phi}{dA} \quad \text{....(i)}$$

i.e., **magnetic induction** is equal to the magnetic flux density.

Unit of $$B = \frac{\text{Unit of } d\phi}{\text{Unit of } dA}$$

Or, $$T = \frac{\text{Wb}}{\text{m}^2}$$

i.e., Tesla = Weber per square metre.

Faraday's Laws of Electromagnetic Induction

- **Faraday's First Law:** Whenever a conductor is placed in varying magnetic field, an emf is induced which is known as induced emf and if the conductor circuit is closed, current is also induced which is called induced current.
- **Faraday's Second Law:** The induced emf is equal to the rate of change of magnetic flux linkage where flux linkage is the product of number of turns in the coil and flux associated with the coil.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

- Where ϕ_B is magnetic flux through the circuit and is represented as $\phi_B = \int \vec{B}.d\vec{A}$
- With N loops of similar area in a circuit and ϕ_B being the flux through each loop, emf is induced is

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

Key Words

- **Induced emf:** A short-lived voltage generated in a conductor or coil, while moving in a magnetic field.
- **Induced electric field:** Field generated due to changing magnetic flux with time.

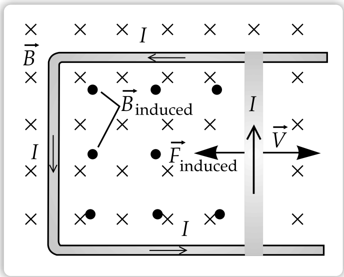
Induced current

- When a magnet is moved towards a loop of wire connected to an ammeter, ammeter shows current induced in the loop.
- When a magnet is held stationary, there is no induced current in the loop.
- When a magnet is moved away from the loop, the opposite current induced in the loop.

Motional emf

- The relationship between an induced emf ε in a wire or a conductor of length l moving at a constant speed v through a magnetic field B is given by:

$$|\varepsilon| = Blv$$



- An induced emf according to Faraday's law is generated which opposes the change in flux.
- Magnetic and electric forces on charges in a rod moving perpendicular to magnetic field is given as:

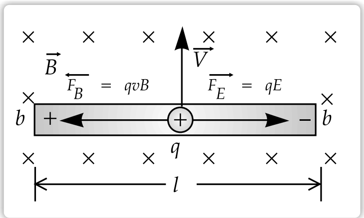
At equilibrium, $F_E = F_B$

$$qE = qvB$$

$$E = vB$$

$$\frac{\varepsilon}{l} = vB \qquad \left[\text{Here, } E = \frac{\varepsilon}{l}\right]$$

$$\varepsilon = Bvl$$



Lenz's law

- The direction of an induced emf always opposes the change in magnetic flux which causes the emf.
- It explains the negative sign in Faraday's rule, $\varepsilon = -\frac{d\phi_B}{dt}$ showing that the polarity of induced emf tends to produce a current that opposes the cause i.e., change in magnetic flux.
- As per conservation of energy, induced emf opposes its cause, making mechanical work to continue with the process which gets converted into electrical energy.

Electric Generators and Back Emf

- Electric generator rotates a coil in a magnetic field inducing an emf which is given as a function of time

$$\varepsilon = NBA\omega \sin(\omega t).$$

where, A = Area of N-turn coil rotating at constant angular velocity ω in uniform magnetic field $\vec{B}$.
- The **peak emf** of a generator is,

$$\varepsilon_0 = NBA\omega$$

- Any rotating coil produces an induced emf. In motors, it is known as **back emf** as it opposes the emf input to the motor.

Key Words

- **Electric generator:** Device for converting mechanical work into electrical energy that induces an emf by rotating a coil in magnetic field.
- **Motional emf:** Voltage produced by the movement of conducting wire or a conductor in a magnetic field.
- **Back emf:** The emf generated by a running motor due to coil that turns in a magnetic field which opposes the voltage that powers the motor.

Mnemonics

Concept: Induced emf in a conductor moving in a magnetic field:
Mnemonics: I eat **P**lain **L**oaf and **B**oiled **V**egetables.
Interpretations:
I: Induce
eat: emf
Plain: product of
Loaf: Length of Conductor
Boiled: B (magnetic field)
Vegetables: V (Velocity)
Induced emf = *Blv*

Topic-2 Self and Mutual Induction

Concepts Covered • *Self Inductance* • *Mutual Inductance.*

Revision Notes

Mutual Induction

- The production of induced emf in a circuit, when the current in the neighbouring circuit changes.
- The mutual induction between two coils depends on:
  - The number of turns of primary and secondary coils.
  - The shape, size or geometry of the two coils. i.e., the area of cross-section and the length of the coils.

Coefficient of mutual induction:

- For instantaneous current I in the primary if the magnetic flux linked with the secondary coil be ϕ then

$$\phi \propto I \quad \text{or} \quad \phi = MI \qquad \text{...(i)}$$

where, M is the constant of proportionality. It is called coefficient of mutual induction.

The induced emf ε in the secondary coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \qquad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law i.e., the induced emf in the secondary coil opposes the variation of current in the primary coil.

Taking magnitude of induced emf from the equation (ii),

$$M = \frac{\varepsilon}{(dI/dt)}$$

$$\text{Unit of } M = \frac{\text{V}}{\text{As}^{-1}} = \text{VA}^{-1}\text{s}$$

If n_1, n_2 be the number of turns per unit length in primary and secondary coils per unit length and r be their radi, then coefficient of mutual induction is given as

$$M = \mu_0 n_1 n_2 \pi r^2 l$$

Key Word

Inductance: Inductance is the tendency of an electrical conductor to oppose a change in electric current flowing through it.
Inductance is defined as the ratio of the induced voltage to the rate of change of current causing it. .
The unit of inductance is the henry (H).

Self-Induction:

- The production of induced emf in a circuit, when the current in the same circuit changes is known as **self-induction**.

For instantaneous current I in the coil if the magnetic flux linked is ϕ, then:

$$\phi \propto I \text{ or } \phi = LI \qquad \text{...(i)}$$

where, L is the constant of proportionality. It is called **coefficient of self-induction.**

The induced emf ε in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \qquad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law, i.e., the induced emf opposes the variation of current in the coil.

Taking the magnitude of the induced emf from the equation (ii),

$$L = \varepsilon / (dI / dt) \qquad \text{...(iii)}$$

Then, the coefficient of self-induction is the ratio of induced emf in the circuit to the rate of change of the current in the circuit.

- Inductance in the electrical circuit is equivalent to the inertia (mass) in mechanics.
- When a bar magnet is dropped into a coil, the electromagnetic induction in the coil opposes its motion, so the magnet falls with acceleration less than that due to gravity.
- Unit of inductance, $H = \frac{\text{Wb}}{\text{A}} = \frac{\text{Vs}}{\text{A}} = \Omega\text{s}$
- The self inductance of a circular coil is given by:

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu_0}{4\pi}\cdot\frac{(2\pi NI)}{rI} \times AN \qquad \left[\because B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi NI}{r}\right]$$

$$L = \frac{\mu_0 N^2}{2r} A = \frac{\mu_0 N^2}{2r} \times \pi r^2$$

or $$L = \frac{\mu_0 N^2 \pi r}{2}$$

- The self inductance of a solenoid of length l is given by

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left(\frac{\mu_0 NI}{l}\right)\frac{AN}{I} \qquad \left[\because B = \frac{\mu_0 NI}{l}\right]$$

or $$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al = \mu_0 n^2 V \qquad \left[\because n = \frac{N}{l}\right]$$

Here, $n = N / l$ = Number of turns per unit length and $V = Al$ = Volume of the coil.

- If two coils of inductance L_1 and L_2 are coupled together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

where, k is called the coupling constant.

- The value of k lies between 0 and 1.

For perfectly coupled coils, $k = 1$, it means that the magnetic flux of primary coil is completely linked with the secondary coil.

- If a current I is set up in a coil of inductance L, then the magnetic field energy stored in it is given by

$$U_m = \frac{1}{2}LI^2$$

Key Formulae

- Magnetic flux: $\phi_m = \int \vec{B}.\vec{dA}$
- Faraday's law: $\varepsilon = -N\dfrac{d\phi_m}{dt}$
- Motional induced emf: $\varepsilon = Blv$
- Motional emf around a circuit:

$$\varepsilon = \oint E.dl = -\frac{d\phi_m}{dt}$$

- emf produced by an electric generator

$$\varepsilon = NBA \sin \omega t$$

- For Self Induction

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

- For Mutual Induction

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$$

- The inductance in series:

$$L_s = L_1 + L_2 + L_3 + ...$$

- The inductance in parallel:

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ...$$

- Mutual Inductance of two coils

$$M = \frac{\mu_0 \mu_r N_P N_S A}{l}$$

where, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Hm^{-1}).
μ_r is the relative permeability of the soft iron core.
N_S is number of turns in secondary coil.
N_P is number of turns in primary coil.
A = area of the coils
l = length of the coils
- For a.c. Generator $\varepsilon = \varepsilon_0 \sin \omega t$ or $\varepsilon = \varepsilon_0 \sin 2\pi\nu t$

CHAPTER-7

ALTERNATING CURRENT

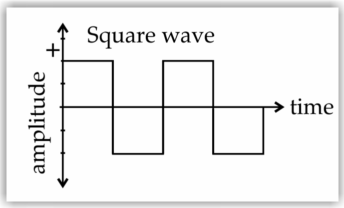
Topic-1 Alternating Current

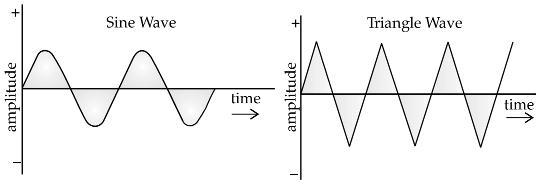
Concepts Covered • *Alternating current, its peak and rms value, reactance and impedance,*

Revision Notes

Alternating current

- Alternating current is represented by sine curve or cosine curve as $I = I_0\sin \omega t$ or $I = I_0\cos \omega t$ where, I_0 is peak value of current and I is instantaneous value of current.
- Frequency f, is defined as the number of cycles completed per second. Unit: Hertz (Hz). In India, the frequency of ac is 50 Hz.
- The time period T, is time taken to complete one cycle.
- AC waveforms are:





Peak and rms value of alternating current/voltage:

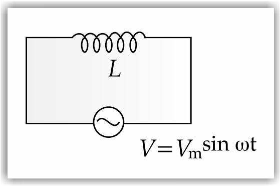
- rms value or effective value or virtual value of ac is represented as I_{rms}, I_{eff} or I_v :

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707I_0$$

- rms voltage value is represented as:

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\,V_0$$

- AC voltage applied to pure **inductive circuits**:



$$V = V_m \sin \omega t$$

$$I = I_m\left(\sin \omega t - \frac{\pi}{2}\right)$$ [which shows current lags the voltage by $\frac{\pi}{2}$]

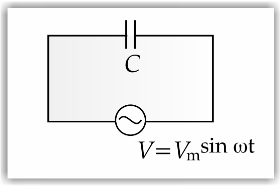
Average power $= P_L = \frac{I_m V_m}{2}[\sin (2\omega t)] = 0$

[Since average of sin $2\omega t$ over a complete cycle is zero]

Key Words

Inductive circuits: A pure inductive circuit that contains a pure inductor with inductance L henry.
Capacitive circuit: A pure capacitor circuit is a circuit that contains a pure capacitor with capacitance C farads.
Impedance: For A.C. circuits, impedance is the measure of the total opposition that a circuit presents to electric current. Impedance includes both resistance and reactance.

- **AC applied to pure capacitive circuit:**

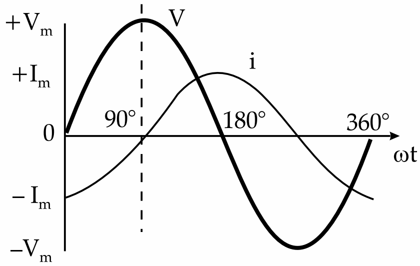


$$V = V_m \sin \omega t$$

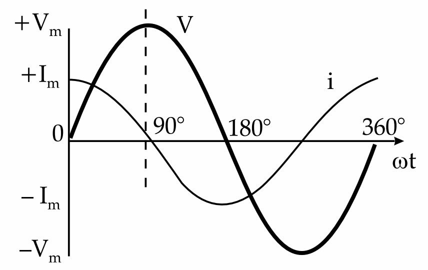
$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$ [which shows current leads the voltage by $\frac{\pi}{2}$]

Average power $= P_C = \frac{I_m V_m}{2}\sin(2\omega t) = 0$ [Since average of sin $2\omega t$ over a complete cycle is zero]

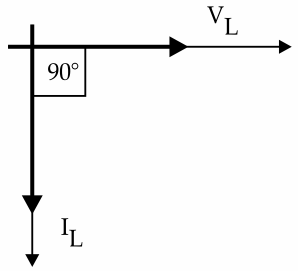
- **Phasor-diagram:** A phasor diagram represents sinusoidal ac current and sinusoidal voltage in a circuit along with the phase difference between current and voltage.



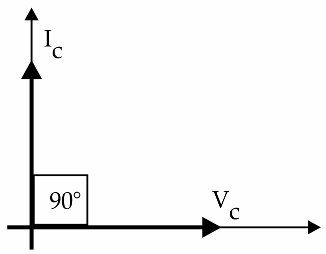
Waveform of pure inductive circuit



Waveform of pure capacitive circuit



Phasor diagram of pure inductive circuit



Phasor diagram of pure capacitive circuit

Reactance and Impedance

- In a pure resistive ac circuit, voltage drop is in phase with the current. Resistance is measured in ohms (Ω).
- Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is 90° out of phase with the current.
- Reactance is represented by "X" and its unit is ohms (Ω).
- Inductive reactance (X_L) is the resistance offered by an inductor. $X_L = \omega L = 2\pi f L$
- Through a pure inductor, alternating current lags behind the alternating emf by phase angle of 90^{o}.
- Capacitive reactance (X_C) is the resistance offered by a capacitor.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

- Through a pure capacitor, alternating current leads the alternating emf by a phase angle of 90^{o}.

  Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between 0^{o} and 90^{o} which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X^2}$$

where, Z = Impedance of circuit, X = Reactance

- **Impedance**: In an ac, the impedance is analogous to resistance in a dc circuit that measures the combined effect of resistance, capacitive reactance and inductive reactance.

Key Formulae

- rms value for current

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

- rms value for voltage

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

- Power $P = V_{rms} I_{rms}$
- In a purely inductive circuit if,

$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right),$$

where $I_m = \frac{V_m}{X_L}$ and $X_L = \omega L$

$$(P_{avg})_L = 0$$

- In a purely capacitive circuit if,

$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $I_m = \frac{V_m}{X_C}$ and $X_C = \frac{1}{\omega C}$

- Average Power $= \frac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$ (where, $\cos \phi = \frac{R}{Z}$ is power factor)

- $Z = \sqrt{R^2 + (X_L - X_C)^2}$

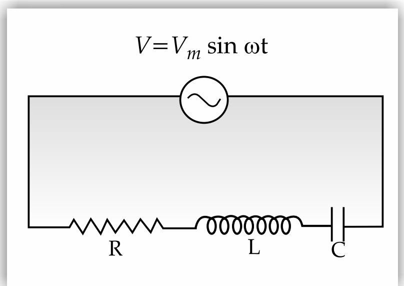
Topic-2 LCR Series Circuit

Concepts Covered • *LCR series circuit (phasors only), resonance, power in AC circuits, power factor, wattless current.*

Revision Notes

LCR series circuit

- In an LCR series circuit



the applied potential difference is equal to the sum of the magnitudes of potential differences across R, L and C elements as

$$V = V_m \sin \omega t = RI + L\frac{dI}{dt} + \frac{1}{C}q$$

where, q is the charge on capacitor.

- The **steady state** situation will be

$$I = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \text{ and}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}}$$

where, $\phi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$

- From the equation, steady-state current varies sinusoidal with time, so steady-state current can be written as

$$I = I_m \sin (\omega t - \phi)$$

- In an LCR circuit:

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here, Z = Impedance of the circuit, X = Reactance of the circuit, X_L and X_C = Inductive and Capacitive reactance.

- For steady-state currents, maximum current I_m is related to maximum potential difference V_m by

$$I_m = \frac{V_m}{Z}$$

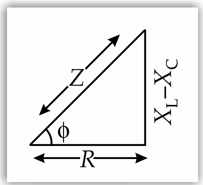
- Total effective resistance of LCR circuit is called Impedance (Z) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle by which alternating voltage leads the alternating current in LCR circuit is given by

$$\tan \phi = \frac{X_L - X_C}{R}$$

- Impedance triangle in a series LCR circuit:



- Sign for phase difference (ϕ) between I and V for a series LCR circuit:

ϕ is positive, when $X_L > X_C$.

ϕ is negative, when $X_L < X_C$.

ϕ is zero, when $X_L = X_C$.

Resonance

- When current in a series LCR becomes maximum then the circuit is called as series resonant circuit.

- The necessary condition for resonance in LCR series circuit is: $V_C = V_L$

$X_L = X_C$ and Z = R which gives

$$\omega^2 = \frac{1}{LC} \text{ or } f = \frac{1}{2\pi\sqrt{LC}}$$

$$I_m = \frac{V_m}{Z} = \frac{V_m}{R}$$

This is the maximum current at resonance.

- The sharpness of tuning at resonance is measured by Q factor or quality factor of the circuit given as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Power in AC circuits

- If the instantaneous values of the voltage and current in an ac circuit are given by

$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t - \phi)$$

where ϕ is the phase difference between voltage and the current. Then, the **instantaneous power**

$$P_{in} = V \times I = V_m I_m \sin \omega t . \sin(\omega t - \phi)$$

or average power $P_{avg} = \frac{1}{2} V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= V_{\text{rms}} \times I_{\text{rms}} \times \cos\phi$$

where, $\cos\phi$ is known as power factor.

$$\cos\phi = \frac{\text{Effective power}}{\text{Apparent power}}$$

- The value of power factor varies from 0 to 1.
- The current in an AC circuit is said to be Wattless Current when the average power consumed in such circuit corresponds to zero. This happens in case of pure inductive and capacitive circuits.

Key Formulae

- Impedance for a series *LCR* circuit,

$$Z = \sqrt{R^2 + X^2} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}.$$

- Average power, $P = \frac{V_m I_m}{2} \cos\phi = V_{rms} I_{rms} \cos\phi$
- Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Mnemonics

Concept: Current leads in pure capacitive circuit, voltage leads in pure inductive circui.

Mnemonics: Chocolate Cookies are Very Interesting!

Interpretations:

C: **C**urrent leads

C: **C**apacitive circuit

V: **V**oltage leads

I: **I**nductive circuit

Topic-3 AC Generator and Transformer

Concepts Covered • *AC generator and transformer.*

Revision Notes

AC generator

- An AC generator is an electrical machine which converts mechanical energy into alternating electrical energy.
- It works on the principle of electromagnetic induction where a coil rotates in uniform magnetic field and sets an induced emf given as:

$$e = e_0 \sin \omega t = NBA\omega \sin \omega t$$

Transformer

- Transformer is an electrical device used for changing the amplitude of alternating voltages.
  It is based on the phenomenon of mutual induction.
- One main use of transformer is in transmission of ac over long distances at extremely high voltages which reduces the energy losses in transmission.
- It comprises of two sets of coils which are insulated on one another on **soft-iron** core.
- One coil is called primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- Transformation Ratio:

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$ is defined as the transformation ratio.

- In step-up transformer $N_s > N_p$, $V_s > V_p$ and $I_s < I_p$.
  The transformation ratio $k > 1$
- **Step-down transformer:** $N_s < N_p$, this, $V_s < V_p$ and $I_s > I_p$
  The transformation ratio $k < 1$

Key Words

Transformation Ratio: The transformation ratio is defined as the ratio of output voltage to the input voltage of a transformer. This gives the information about the change in voltage level by the transformer.

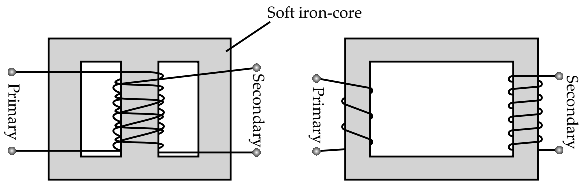
$$\text{Transformation ratio} = k = \frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

If $k > 1$, then it is a step-up transformer.
If $k < 1$, then it is a step-down transformer.

- **Efficiency of transformer:**

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$



- The efficiency in a transformer is usually above 90%.
- An ideal transformer is 100% efficient as it delivers all energy it receives.
- Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- Energy losses in transformers are due to:
1. Flux Leakage
2. Resistance of windings
3. Eddy currents
4. Hysteresis

Key Formulae

- **For transformer:**

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- $$V_s = \left(\frac{N_s}{N_p}\right)V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

- The value of transformation ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.

- $$\%\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$
$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100\%$$

For generator:

- $e = e_0 \sin \omega t = NBA\,\omega \sin \omega t$
- $I = \frac{e}{r} = \frac{NBA\omega \sin \omega t}{R}$

UNIT – V : ELECTROMAGNETIC WAVES

CHAPTER-8

ELECTROMAGNETIC WAVES

Topic-1 Electromagnetic Waves and Maxwell's Equations

Concepts Covered • *Basic idea of displacement current,Electromagnetic waves and their characteristics, their transverse nature.*

Revision Notes

Basic idea of displacement current:

- Displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of ***electric flux.***
- Displacement current has the units of electric current and has associated magnetic field similar as actual currents.
- Displacement current is defined as:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}, \left(I_d = \varepsilon_0 A \frac{dE}{dt}\right)$$

Key Word

Electric flux: It is the rate of flow of electric field of lines passing through a given area.

where, $\phi_E = \int \vec{E}.\vec{dA}$ is the electric flux and ε_0 is the permittivity of free space.

- As per Ampere Maxwell law, line integral of magnetic field around any closed path is equal to $\mu_0 \times$ (sum of conduction current and displacement current through that path.)

i.e.,
$$\oint \vec{B}.\vec{dl} = \mu_0[I_c + I_d]$$
$$\oint \vec{B}.\vec{dl} = \mu_0 I_C + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$$
$$= \mu_0 I_C + \mu_0\varepsilon_0 \frac{d}{dt}(E.A)$$

This proves that change in electric field E, is responsible for the induction of magnetic field.

Electromagnetic waves

- Waves that can travel through vacuum of outer space and do not need the presence of material medium for transporting energy from one location to another.
- EM waves are produced by accelerated charged particles.
- The electric and magnetic fields produced by accelerated charge change with time, which radiate electromagnetic waves.

Example:

- Electron jumping from its outer to inner orbits radiates EM waves.
- Electrical oscillations in **LC circuit** produce EM waves.
- Electric sparking generates EM waves. **Characteristics of EM waves:**
- EM waves are propagated as electric and magnetic fields oscillating in mutually perpendicular directions.
- EM waves travel in vacuum along a straight line with the velocity 2.997924591×10^8 m/s which is often assumed as 3×10^8 m/s.
- EM waves are not affected by electric and magnetic fields.
- Relation between electric and magnetic field components is:

$$B_0 = E_0/c$$

where, $c \cong 3 \times 10^8$ m/s. and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

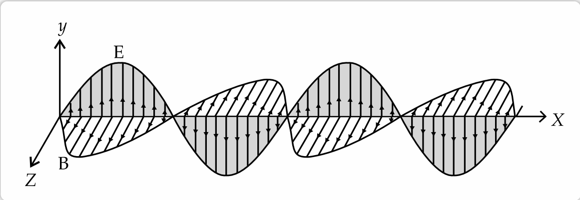
The λ and f are related as

$$c = f\lambda.$$

where λ is the wavelength and f is the frequency.

Transverse nature of electromagnetic waves

- In electromagnetic wave, electric and magnetic field vectors are perpendicular to each other in the direction of propagation of wave which shows its transverse nature.



- A plane EM wave travelling in the x-direction is of the form:

$E(x, t) = E_{max}\cos(kx - \omega t + \phi)$

$B(x, t) = B_{max}\cos(kx - \omega t + \phi)$

where, E = electric field vector, B = magnetic field vector

- In this, wave propagates along z-axis, the electric and magnetic field propagation will be:

$E = E_0\sin(kz - \omega t)$

$B = B_0\sin(kz - \omega t)$

Key Formulae

- Displacement current between the plates of a capacitor

$$I_D = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$I_D = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

Here, E = Electric field between the plates of the capacitor, V = Potential difference, d = Separation between the plates, C = Capacitance of the capacitor, A = Area of plates.

- For the EM waves, the energy density is given by

$$U_E = \frac{1}{2}\varepsilon_0 E^2 \text{ (Due to electric field)}$$

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0} \text{ (Due to magnetic field)}$$

- The energy transported by EM waves per unit area per second is called Poynting vector ($\vec{S}$).

It is given by $\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0}$.

Since, $\vec{E} \perp \vec{B}$, hence $S = \frac{EB}{\mu_0}$.

- In EM waves, the total energy density of EM waves is

$$U = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$U = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \quad \left[\text{As, } E = \frac{B}{\sqrt{\mu_0 \varepsilon_0}}\right]$$

- In the EM waves: $E = E_0 \sin(\omega t - kx)$,
$B = B_0 \sin(\omega t - kx)$.

Maxwell's Equations:

1. $\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$ (Gauss's law for electricity).
2. $\oint \vec{B}.\vec{dA} = 0$ (Gauss's law for magnetism).
3. $\oint \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$ (Faraday's law).
4. $\oint \vec{B}.\vec{dl} = \mu I_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$ (Ampere-Maxwell law).

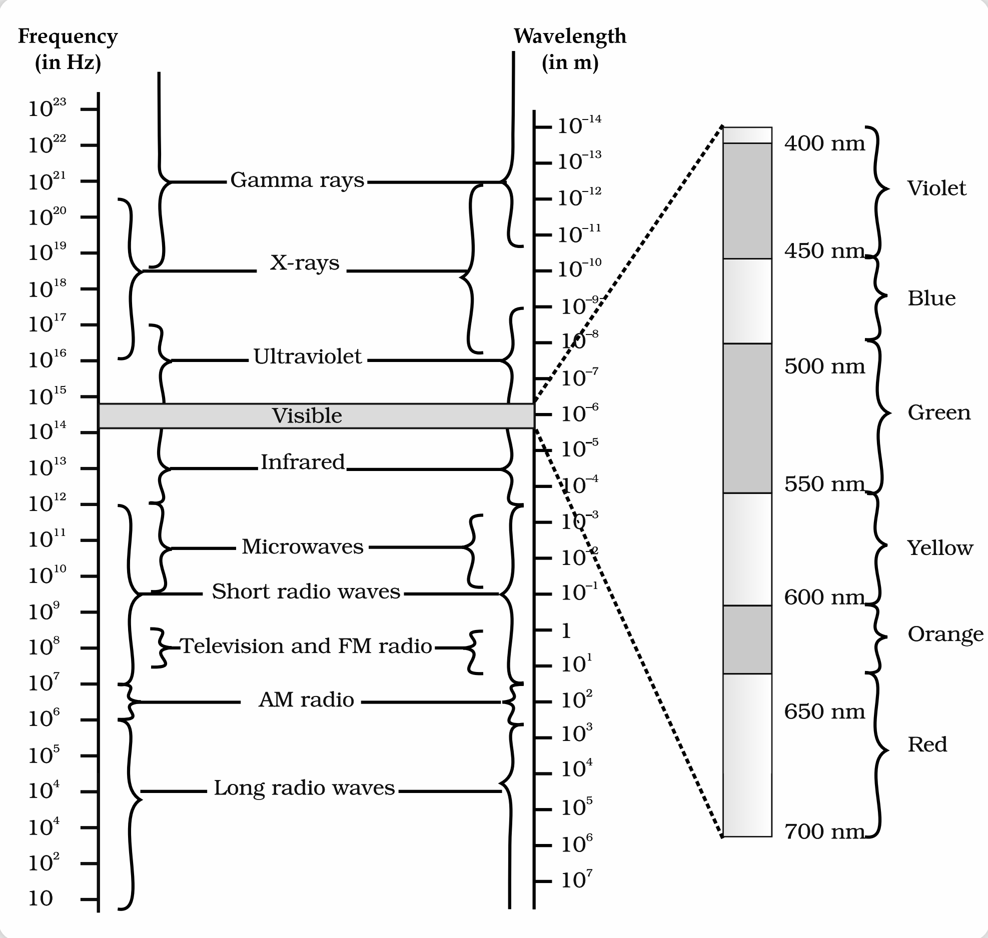
Topic-2 Electromagnetic Spectrum

Concepts Covered • *Electromagnetic spectrum (radio waves,microwaves, infrared, visible, UV, X-rays, gamma rays) including elementary facts about their uses*

Revision Notes

Electromagnetic spectrum

- Classification of EM-waves is based on their frequency or wavelength range.
- EM radiations are classified as per the frequency and wavelength of wave such as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.



General properties of electromagnetic waves (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays)

- Electromagnetic waves require no medium to travel or propagate.
- Varying electric and magnetic fields are the sources of electromagnetic waves.
- Electromagnetic waves are transverse waves.

Frequency and wavelength of various electromagnetic radiations:

Type of radiation	Frequency range	Wavelength range
Gamma rays	$> 3 \times 10^{20}$	<1 fm
X-rays	$3 \times 10^{17} - 3 \times 10^{20}$	1 fm – 1 nm
Ultraviolet	$7.5 \times 10^{14} - 3 \times 10^{17}$	1 nm – 400 nm
Visible	$4 \times 10^{14} - 7.5 \times 10^{14}$	0.4 μm – 0.75 μm
Near-infrared	$10^{14} - 7.5 \times 10^{14}$	0.75 μm – 3.0 μm
Midwave infrared	$5 \times 10^{13} - 10^{14}$	3.0 μm – 6 μm
Long wave infrared	$2 \times 10^{3} - 5 \times 10^{13}$	6.0 μm – 15 μm
Extreme infrared	$3 \times 10^{13} - 2 \times 10^{13}$	15 μm – 15 μm
Micro and radio waves	$< 3 \times 10^{11}$	> 1 mm

Uses of Electromagnetic waves:

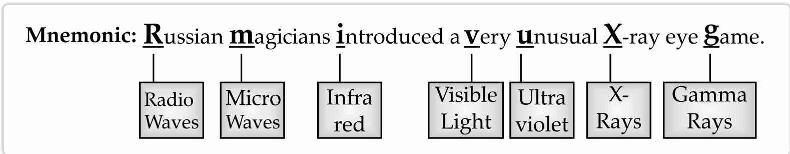
Band designation	Applications
Audible	Acoustics
Extremely Low Frequency (ELF) Radio	Electronics, Submarine Communications
Infra Low Frequency (ILF)	Not applicable
Very Low Frequency (VLF) Radio	Navigation, Weather System Forecasting
Low Frequency (LF) Radio	Navigation, Maritime Communications, Information and Weather Systems, Time Systems
Medium Frequency (MF) Radio	Navigation, AM Radio, Mobile Radio
High Frequency (HF) Radio	Citizens Band Radio, Mobile Radio, Maritime Radio
Very High Frequency (VHF) Radio	Amateur (Ham) Radio, VHF TV, FM Radio, Mobile Satellite, Mobile Radio, Fixed Radio
Ultra High Frequency (UHF) Radio	Microwave, Satellite, UHF TV, Paging, Cordless Telephone, Cellular and PCS Telephony, Wireless LAN (Wi-Fi)
Super High Frequency (SHF) Radio	Microwave, Satellite, Wireless LAN (Wi-Fi)
Extremely High Frequency (EHF) Radio	Microwave, Satellite, Radiolocation
Infrared Light (IR)	Wireless LAN Bridges, Wireless LANs, Fiber Optics Remote control
Visible Light	Photographic plate, photocells.
Ultraviolet (UV)	Photocells, kill bacteria and germs.
X-Rays	In medical, Geiger tubes, ionisation chamber.
Gamma and Cosmic Rays	In medical (cancer cell killing)

Types of Electromagnetic waves, Production and Detection:

Type of radiation	Production	Detection
Radio	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	Klystron valve or magnetron valve	Point contact diodes
Infra-red	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eyes, Photocells Photographic film
Ultraviolet	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	*X*-ray tubes or inner shell electrons	Photographic film, Geiger tubes, Ionisation chamber
Gamma rays	Radioactive decay of the nucleus	Photographic film, Geiger tubes, Ionization chamber

Mnemonics

Concept: Electromagnetic spectrum: Arrangement of em waves with increasing frequency (decreasing wavelength).
Mnemonics: Russian magicians introduced and very unusual X-ray eye Game.



UNIT – VI : OPTICS

CHAPTER-9

RAY OPTICS AND OPTICAL INSTRUMENTS

Topic-1 Reflection by Spherical Mirrors

Concepts Covered *Reflection of light, spherical mirrors, image formation by concave and convex mirror, mirror formula, magnification by mirror*

Revision Notes

- Light is a form of energy. Ray of light represents the direction of propagation of light energy.
- The speed of light in vacuum is the highest speed attainable in nature. Its approximate value is 3.0×10^8 m/s.
- When light falls on any object/surface, there are three possible optical phenomenon i.e., reflection, refraction and absorption of light by the object/surface.

Spherical Mirror

- Curved shaped mirrors are known as spherical mirrors. On the basis of curve of reflecting surface, spherical mirrors are categorised as:
  - **Concave mirror:** A spherical mirror, whose reflecting surface is curved inwards.
  - **Convex mirror:** A spherical mirror whose reflecting surface is curved outwards.

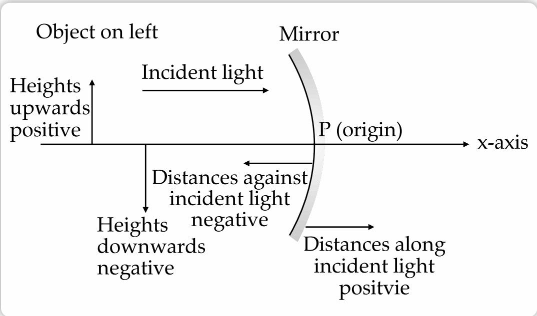
Important terms related to spherical mirrors

- The mid point or the centre of the reflecting surface of the mirror is known as **pole** and represented by P.
- The centre of the hollow sphere from which the mirror is made, is known as **centre of curvature** and represented by C.
- An imaginary straight line which joins the pole and centre of curvature of the mirror is known as **principal axis** and the distance between the centre of curvature and pole of the mirror is called the **radius of curvature.** It is represented by R.
- The point on the principal axis at which the incident rays parallel to the principal axis, meet or appear to diverge from, after reflection from the mirror, is called focus.
  Distance of focus from pole is the focal length (f) of the mirror.
- The relation between R and f is

$$f = \frac{R}{2}$$

- If rays emanating from a point actually meet at another point, then the point is real image of the object. The image is virtual if the rays do not actually meet but appear to meet at the point when produced backward.

The Cartesian sign conventions



- **Image formation by concave mirror for different positions of object**

Position of the object	Position of the image	Size of the image	Nature of the image
At infinity	At the focus F	Highly diminished, point sized	Real and inverted
Beyond C	Between F and C	Diminished	Real and inverted
At C	At C	Same size	Real and inverted

Between C and F	Beyond C	Enlarged	Real and inverted
At F	At infinity	Highly enlarged	Real and inverted
Between P and F	Behind the mirror	Enlarged	Virtual and erect

- **Image formation by convex mirror for different positions of object**

Position of the object	Position of the image	Size of the image	Nature of the image
At infinity	At the focus F, behind the mirror	Highly diminished, point sized	Virtual and erect
Between infinity and the pole P of the mirror	Between P and F, behind the mirror	Diminished	Virtual and erect

- **Mirror formula:** In a spherical mirror, there is a relation between object distance u, image distance v and focal length f.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- **Magnification by Mirror:** The extent by which mirror extends or reduces the size of image with respect to object is called the magnification factor of mirror and represented by m. If size of an object is h_o and its image by spherical mirror is h_i. Then magnification factor of mirror is

$$m=-\frac{v}{u}=\frac{h_i}{h_o}$$

Key Formulae

- Relation between focal length and radius of curvature:
  $f = R/2$
  Mirror formula: $1/v + 1/u = 1/f$
  Magnification by spherical mirror:
  $m = -v/u = hi/ho$

Topic-2 Refraction through Glass Slab, Prism, Lenses and Total Internal Reflection

Concepts Covered *Total internal reflection, refraction at spherical surfaces, lenses, thin lens formula, lens maker's formula, magnification, power of lenses, refraction of light through prism.*

Revision Notes

- **Refraction of light:** Refraction is deviation of light path when it obliquely travels from one medium to another.

Laws of Refraction of Light

- The incident ray, the refracted ray and the normal to the interface of two transparent media, at the point of incidence, all lie in the same plane.
- The ratio of sine of angle of incidence to the sine of angle of refraction is a constant, for the light of a given colour and for the given pair of media. This constant value is called the refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r}=\text{constant } (n_{21})$$

  This is known as Snell's law.
- From Snell's law

$$\sin i = \sin r \times n_{21}.$$

  It shows that if $\angle i = 0$, then $\angle r$ is also zero. This shows that the light rays do not deviate when they travel normally from one medium to another.
- If the first medium is air, then the refractive index is known as the absolute refractive index of the second medium. The absolute refractive index of a medium is expressed by

$$n_2 = \frac{\text{Velocity of light in free space}}{\text{Velocity of light in mediun}} = \frac{c}{v}$$

  since, $c > v \Rightarrow n_2 > 1$

- If light ray bends away from normal after refraction, then second medium is optically rarer. If the light ray bends towards normal, then second medium is optically denser.

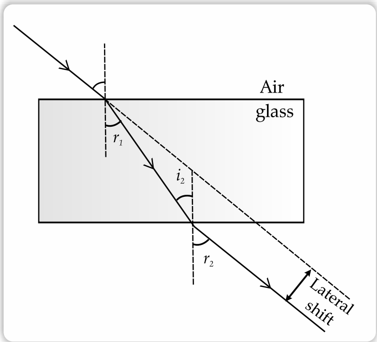
Principle of Reversibility

- According to the principle of reversibility, if light travels through several media say medium 1 to medium 2 and then to medium 3, then to medium 1 then.

$$n_{21} \times n_{32} \times n_{13} = 1$$

Refraction through Glass Slab

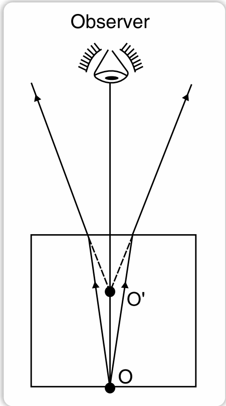
- Emergent ray is parallel to the incident ray but it suffers lateral displacement.



- The apparent depth of the object is always less than actual depth.

$$\text{Rise of image} = \text{Real depth}\left(1 - \frac{1}{n_{21}}\right)$$

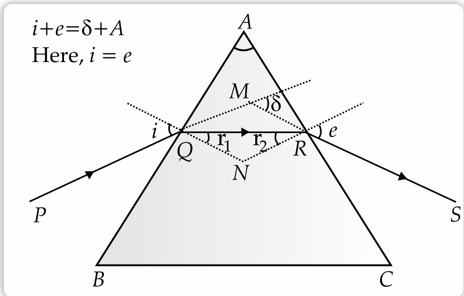
Here, $n_{21} = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

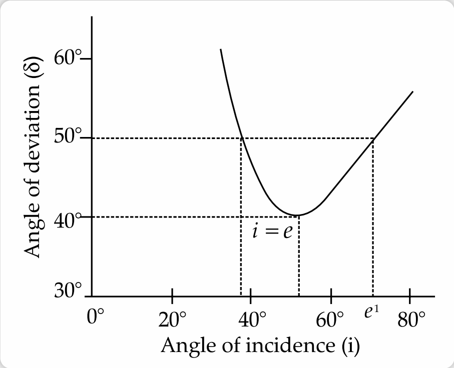


- The following phenomena occur due to the refraction of light:-
  - Bottom surface of water pool seems to be raised.
  - The letter appears to be raised when we observe it through a glass slab.
  - Object looks bigger than its actual size and raised when we dip it into liquid.
  - Twinkling of stars.
  - Delayed sunset and early sunrise.

Refraction through Prism

- Refracted ray always bends towards the base.
- Angle of deviation , $\delta = (i - r_1) + (e - r_2)$





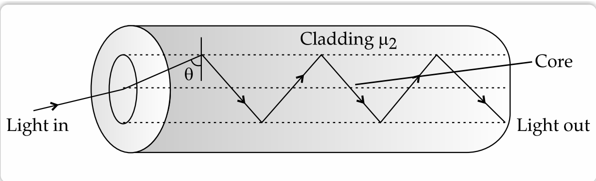
- **Angle of minimum deviation:** When incident angle is gradually increased, the angle of deviation initially decreases and after obtaining a minimum value, it starts increasing again. This angle obtained at the lowermost point is called angle of minimum deviation δ_m.
  - At minimum deviation, $i = e\ (= i)$ and $r_1 = r_2\ (= r)$, then

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin[(A+\delta_m)/2]}{\sin[A/2]}$$

- For thin prism, $\delta_m = (n_{21} - 1)A$.
- **Total internal reflection:** When light travels from an optically denser medium to a rarer medium at the interface, it is reflected back into the same medium when angle of incidence is greater than critical angle. This reflection is called total internal reflection.
  - Critical angle is that value of incident angle for which angle of refraction is 90°. The critical angle for water-air, glass-air and diamond-air are 45°, 42° and 24° respectively.

$$n_{12} = \frac{1}{\sin C} \qquad \text{(where, } C \text{ is critical angle)}$$

- Conditions for total internal reflection are:
  - The light should travel from denser medium to the rarer medium.
  - Angle of incidence should be larger than the critical angle.
- **Optical fibre:** Optical fibre is the technology associated with data transmission using light pulses travelling along with a long fibre which is usually made of plastic or glass.
  - Optical fibre works on the principle of total internal reflection.



- **Refraction at spherical surface:** If the rays are incident from a medium of refractive index n_1, to another medium of refractive index n_2, then

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Where, R = Radius of curvature of spherical surface and object is placed at rarer medium.
u = Object distance from spherical surface
v = Image distance from spherical surface

- **Lens:** A lens is a piece of transparent glass bounded by two surfaces out of which at least one surface is spherical. There are two types of lenses:
  - **Convex lens:** A convex lens is one which is thinner at edges and thick at centre.
  - **Concave lens:** A concave lens is one which is thick at edges and thin at centre.
- **Relation between object distance, image distance with focal length of lens:**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

- **Magnification by lens:**

$$m = \frac{\text{Height of the image } (h')}{\text{Height of the object } (h)} = \frac{v}{u}$$

- **Power of a lens:**

The power of a lens is defined as the reciprocal of its focal length, represented by the letter P and expressed as

$$P = \frac{1}{f}$$

The SI unit of power is dioptre when focal length is in metre.

- When two or more lenses are combined, then the power of combined lens is sum of individual power of lenses.

$$P = P_1 + P_2 + ...$$

- **Lens maker's Formula:**

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left(\text{where, } n_{21} = \frac{n_2}{n_1}\right)$$

- **Image formation in convex lens for different positions of object**

Position of the object	Position of the image	Relative size of the image	Nature of the image
At infinity	at focus F_2	Highly diminished, point sized	Real and inverted
Beyond $2F_1$	Between F_2 and $2F_2$	Diminished	Real and inverted
At $2F_1$	at $2F_2$	Same sized	Real and inverted
Between $2F_1$ and $2F_2$	Beyond $2F_2$	Enlarged	Real and inverted
At Focus F_1	At infinity	Infinitely enlarged	Real and inverted
Between focus F_1 and optical centre	On the same side of the lens as object	Enlarged	Virtual and erect

- **Image formation in concave lens for different positions of object**

Position of the object	Position of the image	Relative size of the image	Nature of the image
At infinity	At focus F_1	Highly diminished point sized	Virtual and erect
Between infinity and the optical centre O of the lens	Between focus F_1 and optical centre O	Diminished	Virtual and erect

Key Formulae

- Snell's law of refraction, $\frac{\sin i}{\sin r}$ = constant (n_{21})
- Absolute refractive index = $n_{21} = \frac{c}{v}$
- $n_{21} \times n_{32} \times n_{13} = 1$
- Rise of image = Real depth $\left(1 - \frac{1}{n_{21}}\right)$
- Deviation through prism, $\delta = (i - r_1) + (e - r_2)$
- For thin prism, $\delta_m = (n_{21} - 1)A$
- Relation between refractive index, angle of prism and minimum deviation

$$n_{21} = \frac{\sin\frac{(\delta_m + A)}{2}}{\sin\left(\frac{A}{2}\right)}$$

- $n_{21} = \frac{1}{\sin C}$ (for Total internal reflection)
- Lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ and Magnification = $m = \frac{h'}{h} = \frac{v}{u}$
- Power of lens, $P = \frac{1}{f}$

- When two or more lenses are combined, then

  $P_{eff} = P_1 + P_2 + ...$
- Lens maker's Formula,

  $$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \left(\text{where, } n_{21} = \frac{n_2}{n_1}\right)$$

Mnemonics

Concept name: Mirror Formula and lens formula:

Go **M** means **M**ore → **M**irror $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Go **L** means **L**ess → **L**ens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Magnification will have opposite sign:

$m = -\frac{v}{u}$ (For mirror)

$m = \frac{v}{u}$ (For lens)

Topic-3 Optical Instruments

Concepts Covered *Simple and compound microscope, telescope, astronomical telescope and their magnifying powers.*

Revision Notes

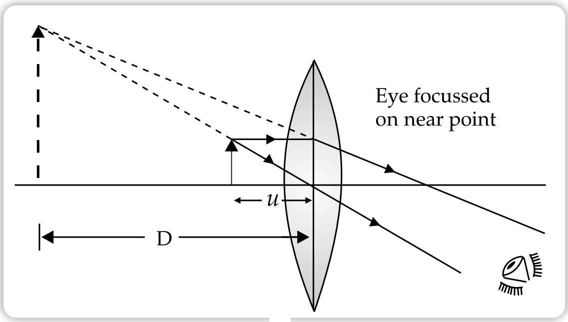
- **Microscope** is an optical instrument which helps us to see and study micro objects or organisms.
- **Telescope** is an optical instrument which helps us to see and study far off objects magnified and resolved (with clarity).
- We generally set these instruments at two different image vision positions:
  - Image at at least distance of distinct vision: When image is formed at the least distance of human eye (25cm).
  - Image at relaxed vision: When image is formed at infinity.

  Simple Microscope: Convex lens behaves as simple microscope.

  The magnifying power of the simple microscope

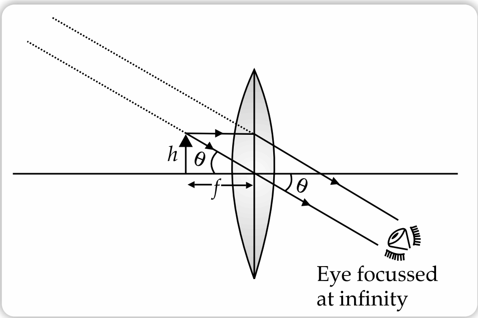
  (i) For least distance of distinct vision, $m = 1 + \frac{D}{f}$

  where, D is the least distance of distinct vision of the eye and f is focal length of the lens.



(ii) For relaxed vision, $m = \frac{D}{f}$

For larger magnifying power, the focal length of the convex lens should be small.



The angular magnification by optical instruments is the linear magnification by lenses which is written as

$$m = \frac{h'}{h}$$

where, h is size of object (in one dimension) and h' is the size of image.

Compound Microscope: It is a combination of two convex lenses placed co-axially and the distance between them is adjustable.

- The lens towards the object is called objective and that towards the eye is called eyepiece.
- The final image is magnified and inverted.
- Total magnification,

$$m = m_o \times m_e$$

where, m_o is magnification by objective lens and m_e is magnification by eyepiece.

- For least distance of distinct vision, magnification by objective lens is

$$m_o = \frac{v_o}{u_o} \approx \frac{L}{f_o}$$

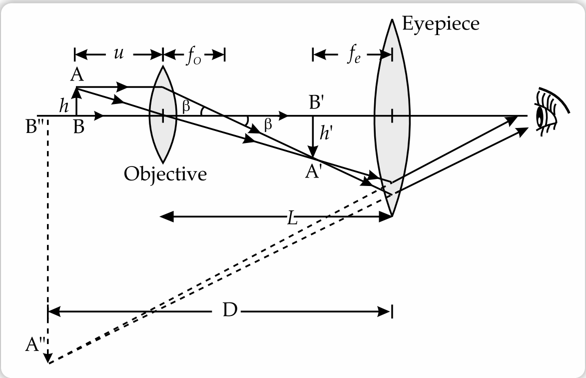
where, L is the distance between the second focal point of the objective and the first focal point of the eyepiece. It is called the tube length of the compound microscope.

Magnification by eyepiece lens is

$$m_e = 1+\frac{D}{f_e}$$

Hence, Magnification by compound microscope

$$= \frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$



For Relaxed Eye (normal adjustment)

For relaxed eye, the magnification by objective lens remain same, the magnification by eyepiece is $+\frac{D}{f_e}$

Hence, the total magnification of compound microscope in relaxed vision condition is

$$m = \frac{L}{f_o} \times \frac{D}{f_e}$$

Properties of Compound Microscope

- For large magnification of a compound microscope, both f_o and f_e should be small.
- Generally f_o is much smaller. So, the object is placed very near to principal focus.
- The aperture of the eyepiece is generally small so that whole of the light may enter the eye.
- The aperture of the objective is also small, so the field of view may be restricted.

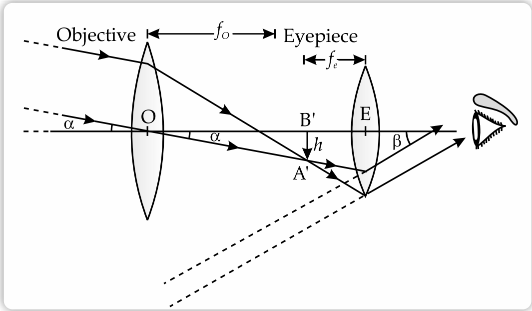
Telescope

- Far off objects make much smaller angle at our eye. Telescope makes that angle larger without much intensity loss.
- To maximise the intensity, aperture size of objective lens is quite large.
- Magnification for least distance of distinct vision,

$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

Magnification for Relaxed Eye (normal adjustment)

$$m = \frac{\alpha}{\beta} = -\frac{f_o}{f_e}$$



Properties of astronomical telescope

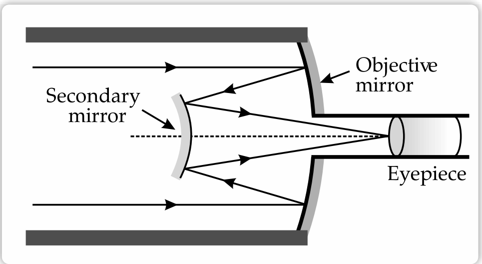
- For larger magnifying power, f_o should be large and f_e should be small.
- The length of the tube of an astronomical telescope is $L = f_o + f_e$ for relaxed vision adjustment.

Limitations of refractive telescope

- Large objective lens makes the telescope very heavy.
- It has spherical and chromatic aberrations.

Reflecting Telescope

- Reflecting telescope consists of a concave mirror of large radius of curvature in place of objective lens
- A secondary convex mirror is used to focus the incident light, which passes through a hole in the objective primary mirror .
- The magnifying power of the reflecting telescope is $m = \frac{f_o}{f_e}$



Advantages of reflecting telescope

- Very sharp image by objective mirror removes spherical aberrations.
- As it is very light, large aperture of parabolic mirror can be used for desired magnification.
- This is based on the principle of reflection and there is no chromatic aberrations.

Key Formulae

- Magnification by simple microscope

$$m = 1+\frac{D}{f} \text{ (for distinct vision)}$$

$$m = \frac{D}{f} \text{ (For relaxed eye)}$$

- Magnification by compound microscope

$$\frac{L}{f_o}\left(1+\frac{D}{f_e}\right) \text{ or } \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) \text{ (for distinct vision)}$$

$$\frac{L}{f_o}\times\frac{D}{f_e} \text{ or } \frac{v_o}{u_o}\times\frac{D}{f_e} \text{ (for relaxed vision)}$$

- Magnification by telescope

$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) \text{ (for distinct vision)}$$

$$m = -\frac{f_o}{f_e} \text{ (for relaxed vision)}$$

CHAPTER-10
WAVE OPTICS

Topic-1 Wave Theory and Huygens' Principle

Concepts Covered • *Wave front, Huygens' principle, reflection and refraction of plane wave at a plane surface using wave fronts, proof of laws of reflection and refraction using Huygens' principle.*

Revision Notes

- According to the corpuscular theory, “sources of light emit large number of tiny massless particles known as corpuscles in a medium surrounding the source. They are perfectly elastic, rigid and have high speed".

  This theory could explain reflection and refraction of light but could not explain many other optical phenomenon like interference and diffraction of light.
- **Huygens proposed wave theory of light**; According to the theory, light travels in the form of longitudinal waves with uniform speed in a homogenous medium.
- As longitudinal and mechanical waves need medium to travel, he assumed a hypothetical medium known as ‘ether’.
- Michelson Morley experiment discarded the existence of ether.

  Maxwell's theory of electromagnetic waves and Young’s famous double slit experiment firmly established the wave theory. Maxwell explained that light is an electromagnetic wave which does not need medium and its speed in vacuum is 3×10^8 m/s. Phenomenon of optical interference, diffraction and polarisation can be explained with wave nature of light.
- Wave theory could not explain Photoelectric effect and Compton effect.

  Polarisation established that light is not a longitudinal wave but a transverse wave.
- Huygens' principle brings concept of formation of new wave fronts and its propagation in forward direction.
- Wavefront is locus of all points in which light waves are in same phase. Propagation of wave energy is perpendicular to the wavefront.

Huygens' Principle:

- Every point of a wavefront becomes secondary source of light.
- These secondary sources give their own light waves. Within small time, they produce their own wave called secondary wavelets. These secondary wavelets have same speed and wavelengths as waves by primary sources.
- At any instant, a common tangential surface on all these wavelets gives new wavefront in forward direction.

Shapes of wavefronts:

Source	Wavefronts
Point source	Spherical wavefront
Line source	Cylindrical wavefront
Plane source	Plane wavefront
Point source very far away	Plane wavefront

Concave lens converts plane wavefront to convex wavefront and convex lens convert plane wavefront to concave wavefront.

- Laws of reflection and refraction of light can be proved by Huygen's principle.

Topic-2 Superposition of Light Waves (Interference and Diffraction)

Concepts Covered • *Interference, Young's double slit experiment, Coherent sources, Diffraction due to a single slit.*

Revision Notes

- According to superposition principle, "At a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each wave".

  Two coherent waves are given by

  $y_1 = a\cos\omega t$ and $y_2 = a\cos\omega t$.

  Then due to superposition, resultant displacement at that point will be

  $y = y_1 + y_2 = 2a\cos\omega t$.

  Hence, the total intensity at that point is:

  $$I = 4I_0$$

  where, $I_0 \mu a^2$; maximum intensity due to one wave.

Interference

- **Constructive Interference:** If the crest and trough of both waves reach at a point in the same instant, constructive interference occurs. The resultant amplitude of the wave is the sum of individual amplitudes.

  $$a = a_1 + a_2$$

- **Destructive Interference:** If the crest of one wave and trough of other wave reach at a point in same instant, then destructive interference occurs. The resultant amplitude of the wave is the difference of individual amplitudes.
- Two independent sources can never be coherent. We may create two coherent sources by deriving them from one source.

Condition for constructive Interference:

- Waves should be coherent in nature. Coherent wave means that they should have equal frequency and constant phase difference (0, 2π, ... 2nπ).

  Path difference between waves at this phase difference = 0 , λ, ... nλ, Here, n = 0, 1, 2, 3 ......

  $$a_r = a_1 + a_2$$

  if $$a_1 = a_2 = a$$

  then $$a_r = 2a$$

  ∵ $$I \propto a^2$$

  $$I_r = 4a^2$$

Condition for destructive interference:

- Waves would be coherent in nature. The phase between the waves should be odd multiples of π, i.e., 0, π, ... 2nπ

  Path difference between waves at this phase difference

  $$= \frac{\lambda}{2}, \frac{3\lambda}{2}, ...(2n+1)\frac{\lambda}{2}, \text{ Here, } n = 1, 2, 3, 4....$$

$a_r = a_1 - a_2$

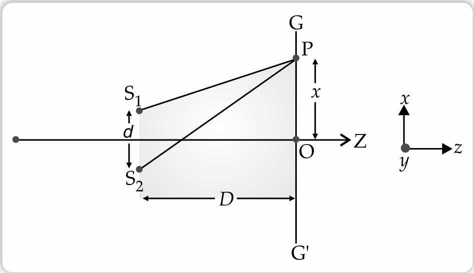
if $a_1 = a_2$

then $a_r = 0$

$\because$ $I \propto a^2$

$I_r = 0$

Young's double slit Experiment:



- Central maxima forms at O. Here, path difference $(S_2P - S_1P) = 0$
- At "P", which is at x distance from O path difference

$$(S_2P - S_1P) = \frac{xd}{D}$$

- **Condition for P to be a bright spot:**

$$\frac{xd}{D} = 0, \lambda, 2\lambda........n\lambda$$

$$x_{n\text{th bright}} = \frac{nD}{d}\lambda$$

where, n is number of bright fringes after central fringe.

- **Condition for P to be a dark spot:**

$$\frac{xd}{D} = 0, \frac{3\lambda}{2}..........(2n+1)\frac{\lambda}{2}$$

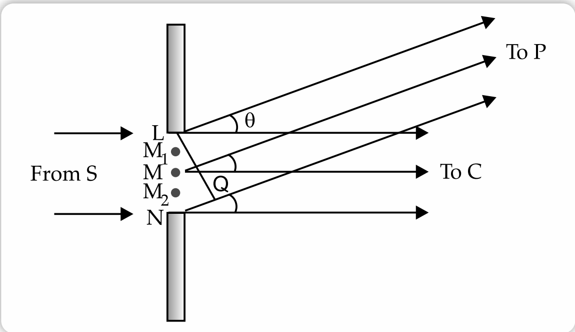
$$x_{n\text{th dark}} = \frac{(2n+1)D}{2d}\lambda$$

Here, n is the number of dark fringes after central fringe.

- Width of the bright fringe $(\beta_B) = x_{nB} - x_{(n-1)B} = \dfrac{D\lambda}{d}$
- Width of the dark fringe $(\beta_D) = x_{nD} - x_{(n-1)D} = \dfrac{D\lambda}{d}$
- Width of the central fringe $(\beta_C) = \dfrac{D\lambda}{d}$
- Hence, $\beta_B = \beta_D = \beta_C$

Diffraction

It is the bending of light around the corners of an obstacle or aperture into the region where we expect shadow of the obstacle.



If width of the opening = a

θ is the angle of elevation of point P from principal axis.

Path difference between ray from L and ray from $N = NQ = a\sin\theta$

$$a\sin\theta = \lambda$$

$$\theta = \frac{\lambda}{a} \qquad (\because \sin\theta \cong \theta \text{ for small value of } \theta)$$

For first maxima

It is observed that when path difference = $\lambda, 2\lambda \ldots\ldots n\lambda$, P is a dark point.

When Path difference $\theta = \frac{3\lambda}{2}, \ldots\ldots.(2n+1)\frac{\lambda}{2}$, P is a bright point.

Elevation angle for first bright fringe, $\theta_{1D} = \frac{3\lambda}{2a}$

- Height of first dark fringe, $x_{1D} = \frac{3\lambda D}{2a}$
- Elevation angle for first dark fringe, $\theta_{1D} = \frac{\lambda}{a}$
- Width of the bright fringe = $\frac{D\lambda}{a}$
- Width of the dark fringe = $\frac{D\lambda}{a}$
- Width of the central fringe = $\frac{2D\lambda}{a}$

There is no gain or loss of energy in interference or diffraction, which is consistent with the principle of conservation of energy. Energy only redistributes in these phenomena.

Key Formulae

- **Condition for constructive interference for coherent waves:**
  - Phase difference$(0, 2\pi, \ldots\ldots 2n\pi)$
  - Path difference = $0, \lambda \ldots\ldots n\lambda$
- **Condition for destructive interference for coherent waves:**
  - Phase difference$(0, \pi, \ldots\ldots (2n+1)\pi)$ with each other at any time interval t.
  - Path difference = $\frac{\lambda}{2}, \ldots\ldots (2n+1)\frac{\lambda}{2}$
- **In Interference Pattern**
  - Width of the bright fringe = $\frac{D\lambda}{d}$
  - Width of the dark fringe = $\frac{D\lambda}{d}$
  - Width of the central fringe = $\frac{D\lambda}{d}$
- **In Diffraction Pattern:**
  - Angle of elevation of any point P on screen = $\frac{\lambda}{a}$
  - Condition that P would be dark point when path difference = $\lambda, 2\lambda \ldots. n\lambda$
  - Condition that P would be bright point when path difference = $\frac{3\lambda}{2}, \ldots.(2n+1)\frac{\lambda}{2}$
  - Width of the bright fringe = $\frac{D\lambda}{a}$
  - Width of the dark fringe = $\frac{D\lambda}{a}$
  - Width of the central fringe = $\frac{2D\lambda}{a}$
  - Height of first bright fringe $x_{1B} = \frac{3\lambda D}{2a}$

Mnemonics

Concept: Phase difference for constructive and destructive interference:

Mnemonics: **P**ranab **D**hawan **d**eparted **O**tty by **C**hennai **E**xpress.

Interpretations:

Even [i.e. $2n\pi$]
For **C**onstructive interference
Odd [i.e. $(2n+1)\pi$]
for **D**estructive interference
Difference
Phase

UNIT – VII : DUAL NATURE OF RADIATION AND MATTER

CHAPTER-11

DUAL NATURE OF RADIATION AND MATTER

Topic-1 Photoelectric Effect

Concepts Covered • *Photoelectric effect,* • *Hertz and Lenard's observations,* • *Einstein's photoelectric equation,* • *Particle nature of light,* • *Experimental study of photoelectric effect*

Revision Notes

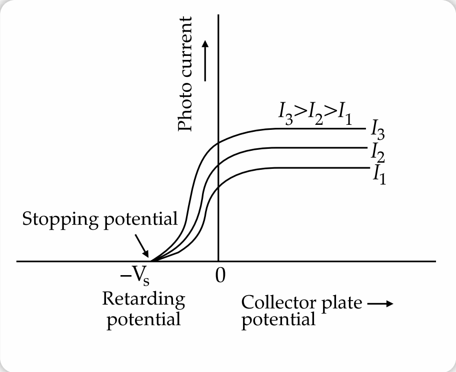
- **Electron Emission:**
  - Free electrons in metals cannot come out from the surface due to force by positive ions present in metals. Electrons can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull.
  - **Work Function of a metal:** Work function of a metal is the minimum amount of work done (energy given) to its electron so that it can escape from the metal surface. It is measured in electron volt (eV).
  - **Thermionic emission:** When electron emission occurs by heating the metal, it is known as thermionic emission. Emitted electrons are called thermionic electrons.
  - **Field emission:** When electron emission occurs by applying a strong electric field, it is known as field emission and emitted electrons are called field electrons.
  - **Photoelectric effect:** When electron emission is occurred by illumination of metal by the light of suitable frequency, it is known as photoelectric emission. Here, emitted electrons are called photo electrons.

    When light falls on the metal surface, free electrons absorb energy from light and if this energy is more than the work function of metal, the electron escapes from the surface.
- **Hallwachs' and Lenard's detailed study of Photoelectric effect:**
  - In 1888, Lenard observed that when ultraviolet light falls on zinc metal, metal becomes positively charged. With the discovery of electrons, it was established that this is due to emission of electrons. The current produced by these photoelectrons is called photoelectric current.
  - The frequency of the incident light below which emission of electrons does not take place, is called its threshold frequency.
- **Hertz and Lenard's experiment:** This experiment led the formation of quantum theory of light as wave theory could not explain photoelectric effect.

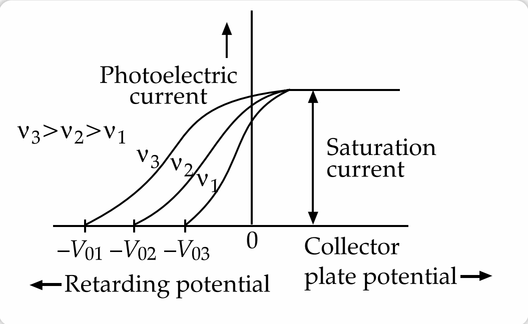
- **Experimental outcome:**

(i) intensity of light has linear relationship with photoelectric current at a potential higher than the stopping potential.

(ii) For a given frequency of the incident radiation, the stopping potential is independent of its intensity.



- **(iii)** Photoemission current starts only at certain minimum frequency of light known as **threshold frequency**. Below this frequency, photoemission does not take place inspite of increase of intensity of light.



- Wave theory failed to explain the photoelectric effect due to the following reasons:
  - According to wave theory, higher amplitude means higher energy but experiments show that even larger amplitude (higher intensity) of light below threshold frequency does not show photoelectric effect.
  - According to wave theory, same intensity of different colour should have same energy but experiment shows that energy depends upon frequency, not on amplitude.
  - According to wave theory, wavefront should take some time to give energy to electron but experimentally, it was found that ejection of electron is instantaneous.
- In 1900, Max Planck stated that electromagnetic energy can be emitted only in quantised form.

$$E = h\nu$$

where, h is Planck's constant.
- Based upon this postulate, Einstein established quantum theory of radiation and was able to explain photoelectric phenomenon. It states that light energy packets are known as photons (Particle nature of light).
- In photoelectric effect, an electron absorbs a quantum of energy ($E = h\nu$) of radiation. If this absorbed quantum of energy exceeds, the minimum energy needed for the electron to escape from the metal surface (work function ϕ_0), the electron is emitted with maximum kinetic energy.

$$K.E. = h\nu - \phi_0$$

where, ϕ_0 is the work function of the metal.
  - At stopping potential, kinetic energy of the ejected electron is zero. Below this potential, the electrons can not be ejected. Hence, maximum kinetic energy of an electron is calculated by
    $K.E._{max} = eV_0$

where, V_0 is stopping potential.
Work function of metal, $\phi_0 = h\nu_0$

where, ν_0 is the cut-off frequency or threshold frequency.
  - Maximum speed of emitted photoelectrons can be calculated as

$$\nu_{max} = \sqrt{\frac{2K.E._{max}}{m}}$$

- **Characteristics of photons:**
  - All photons of specific light frequency have equal energy. Intensity of light only increases the number of photons per unit area and not the energy of photons.
  - Photons are electrically neutral and are not deflected by electric and magnetic field.
  - Photon has energy to propagate, hence it has momentum.

    Momentum of photon $p = \frac{h\nu}{c}$
  - In photon- electron collision, the number of electrons or photons are not conserved but energy and momentum are conserved.
- As interference, diffraction and polarisation cannot be explained by quantum theory of light, hence it was said that light has dual nature. When it travels in a medium, it travels as wave and while interacting with other medium, it acts like particles (photons).

Key Formulae

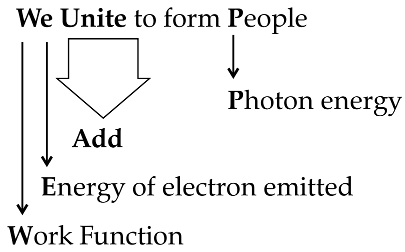
- **Einstein equation of photoelectric effect**
  $K.E. = h\nu - \phi_0$
- Work function $\phi_0 = h\nu_0$
  where, ν_0 is the cut-off frequency.
- Maximum speed of emitted photoelectrons can be calculated as

$$v_{max} = \sqrt{\frac{2KE_{max}}{m}}$$

- Momentum of photon, $p = \frac{h\nu}{c} = \frac{h}{\lambda}$ as $c = \lambda\nu$
- 1eV (one electron volt) = 1.6×10^{-19} J
- Energy of light $E = \frac{hc}{\lambda} = \frac{12375}{\lambda(\text{in Å})}(\text{eV})$
- If applied potential is V, the emitted energy $E = eV$

Mnemonics

Concept: Einstein's equation of photoelectric effect:
Mnemonics:



Interpretations: So, Work function + Energy of electron emitted = Energy of incident photon.

Topic-2 Dual Nature of Matter

Concepts Covered • *Matter waves,* • *de-Broglie wavelength.*

Revision Notes

- de-Broglie's postulate is based upon the symmetry of nature. If radiation has dual nature, then matter should also have dual nature.
- According to his hypothesis, moving particles of matter should display wave nature under suitable conditions.

He named the wave as **matter wave**. It is the third type of wave. It is different from mechanical wave and electromagnetic wave.

➤ **Properties of matter wave:** Whenever a particle moves, the matter wave envelops it and controls its motion.

- de-Broglie proposed that the wavelength λ known as de-Broglie wavelength; associated with momentum of particle p is given as

$$\lambda = \frac{h}{p}$$

  Hence, de-Broglie's wavelength of particle,

$$\lambda = \frac{h}{mv}$$

- **Calculation of wavelength of electron-wave:**

  In photoelectric equation, kinetic energy of electron at potential V is $K = eV$. Putting this value of kinetic energy in de-Broglie wavelength equation,

$$\lambda_e = \frac{h}{\sqrt{2meV}}$$

  By putting the value of mass of electron, its charge and Planck's constant, it becomes $\lambda_e = \frac{1.227}{\sqrt{V}}$ nm. This is theoretical calculation of de-Broglie wavelength of electron, where, V is the magnitude of accelerating potential in volts.

Key Word

Matter waves:

According to de-Broglie, a wave is associated with each moving particle which is called matter waves.

(i) Lighter the particle, larger the de-Broglie wavelength.

(ii) Faster the particle moves, the smaller is its de-Broglie wavelength.

(iii) de-Broglie's wavelength is independent of the charge.

- From this formula, wavelength of particle is inversely proportional to the mass of particle and its velocity. Hence, heavier particles have shorter wavelengths.

Key Formulae

➤ de Broglie wavelength associated with momentum of particle p as

$$\lambda = \frac{h}{p} \text{ or } \lambda = \frac{h}{mv}$$

➤ $\lambda_e = \frac{h}{\sqrt{2meV}}$ or $\lambda_e = \frac{1.227}{\sqrt{V}}$ nm, where V is the magnitude of accelerating potential in Volts.

UNIT – VIII : ATOMS & NUCLEI

CHAPTER-12

ATOMS

Revision Notes

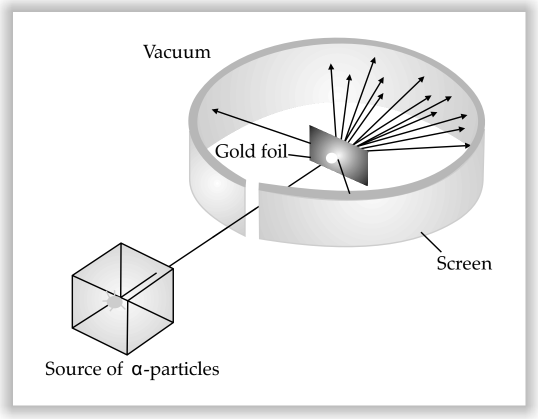
Rutherford's atomic model:

- According to this model "The entire positive charge and most of the mass of the atom is concentrated in a small volume called the nucleus, with electrons revolving around the nucleus just as planets revolve around the Sun."

- **Geiger** and **Marsden** experimentally proved Rutherford's atomic model.

Geiger and Marsden scattering experiment or Alpha particle scattering experiment:

Experimental setup:



- Radioactive element $^{214}_{83}Bi$ was taken as collimated source of α-particles.
- Thin foil of highly malleable heavy metal gold was used as target.
- The detector was made from ZnS.

Experimental observations:

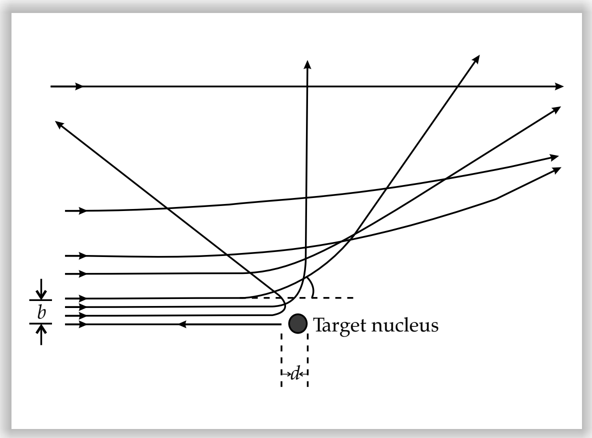
- Most of the α-particles passed roughly in a straight (within 1°) without deviation.
- A very small number of α-particles were deflected. (1 out of 8000)

Conclusions:

- Most of the space in the atom is mostly empty (only 0.14% scatters more than 1°).
- Experiment suggests that all positively charged particles are together at one location at centre. It was called nucleus.
- Size of the nucleus is calculated to be about 10^{-15} m to 10^{-14} m. According to kinetic theory, size of one atom is of the order of 10^{-10} m.

Alpha-particle trajectory:

- **Impact parameter:** It is the perpendicular distance between the direction of the given α-particle and centre of the nucleus. It is represented by '*b*'.
- **Distance of closest approach:** It is the distance between centre of nucleus and the α-particle where it stops and reflects back. It is represented by '*d*'. This distance gives an approximation of nucleus size.



Electron Orbits

- The electrostatic force of attraction, F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits.

$$F_e = F_c$$

For hydrogen atom, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$

or, $$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

Electron has kinetic energy, $K = \frac{1}{2}mv^2$. Putting the value of mv^2 in the above equation

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

And $$v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

P.E. of an electron, $U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$ (negative sign shows that it is due to attractive force)

Total energy, $$E = K + U$$

$$E = \frac{e^2}{\pi 8\varepsilon_0 r} + \left(-\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}\right)$$

$$= -\frac{e^2}{8\pi\varepsilon_0 r}$$

- Due to this negative energy, the electron is bound to the nucleus and revolves around it.
- Providing this amount of energy to this electron, it gets free.

Atomic Spectra:

- Each element has two types of **atomic spectra**: Emission atomic spectra and absorption atomic spectra.
- **Emission atomic spectra:** Due to excitation of atom light of particular wavelength is emitted. This atomic spectra is known as emission spectra.
- **Absorption atomic Spectra:** If atoms are excited in presence of white light, it absorbs its emission spectral colours and black lines appear in the same places of that atoms' emission spectra. This type of spectra is known as absorption spectra.

Spectral series:

- Hydrogen is the simplest atom and has the simplest spectrum.
- Balmer Series:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, ...$$

  Longest wavelength = 6566.4 Å
  Shortest wavelength = 3648 Å
  This is in visible range.

- Lyman Series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4, 5, ...$ This is in UV range.
  Longest wavelength = 1216 Å
  Shortest wavelength = 912 Å

- Paschen Series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$; $n = 4, 5, 6, ...$
  Longest wavelength = 18761.14 Å
  Shortest wavelength = 8208 Å
  This is in far infrared region.

- Brackett Series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$; $n = 5, 6, ...$
  Longest wavelength = 40533.33 Å
  Shortest wavelength = 14592 Å
  This is in far infrared region.

- Pfund Series: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$; $n = 6, 7, 8 ....$
  Longest wavelength = 74618.1 Å
  Shortest wavelength = 22800 Å
  This is in far infrared region.

Limitation of Rutherford model:

- It could not explain the stability of the atom.
- It could not explain the nature of energy spectrum.

Bohr's Model and Postulates:

- An electron can revolve in certain stable orbits without emission of radiant energy. These orbits are called stationary states of the atom.
- Electron revolves around nucleus only in those orbits for which the angular momentum is the integral multiple of $\frac{h}{2\pi}$, where, h is Planck's constant.
- Hence, angular momentum, $L = \frac{nh}{2\pi}$
- An electron may make a transition from one of its specified non-radiating orbit to another of lower energy. When it does so, a photon is radiated having energy equal to energy difference between initial and final state.

$$h\nu = E_i - E_f \quad \text{(where, } \nu \text{ is frequency)}$$

Angular momentum, $L = mv_n r_n$

According to Bohr's postulate, $L = \frac{nh}{2\pi}$

Hence, $$mv_n r_n = \frac{nh}{2\pi}$$

$$mr_n = \frac{nh}{2\pi v_n}$$

For hydrogen atom, $$v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

Combining these two equations, we get

$$v_n = \frac{1}{n} \cdot \frac{e^2}{4\pi\varepsilon_0} \frac{1}{(h/2\pi)}$$

This equation depicts that electron speed in n^{th} orbit falls by a n factor.

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$$

For innermost orbit $n = 1$; the value of r_1 is known as Bohr's radius a_0.

$$a_0 = \frac{h^2\varepsilon_0}{\pi m e^2}$$

If we put values of all constants, we get

$a_0 = 5.29 \times 10^{-11}\, m \approx 0.53$ Å

It can also be observed that radii of n^{th} orbit increases by n^2 times.

By putting this value in total energy of an electron and convert the unit in eV, we get

$$E_n = \frac{-13.6}{n^2} \text{eV}$$

Negative value shows that electron is bound to nucleus.

Limitation of Bohr's atomic model:

- Bohr's model is for hydrogenic atoms. It does not hold true for a multi-electron model.

Key Formula

- **Radius of orbit,**

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

- **Kinetic energy of electron in its orbit,**

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

- **PE of an electron,**

$$U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

- **Velocity of electron in its orbit,**

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$$

- **Total energy of an electron in an orbit,**

$$E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Spectral series

- **Balmer Series:** $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $n = 3, 4, 5$..... This is in visible range.
- **Lyman Series:** $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4, 5$, ...This is in UV range.

- **Paschen Series:**
  $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$; $n = 4, 5, 6$ .....
- **Brackett Series:**
  $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$; $n = 5, 6, 7$...
- **Pfund Series:**
  $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$; $n = 6, 7, 8$....

These series are in infrared region.

- **Relation between speed, total energy of an electron and its radius with respect to orbital number *n*:**

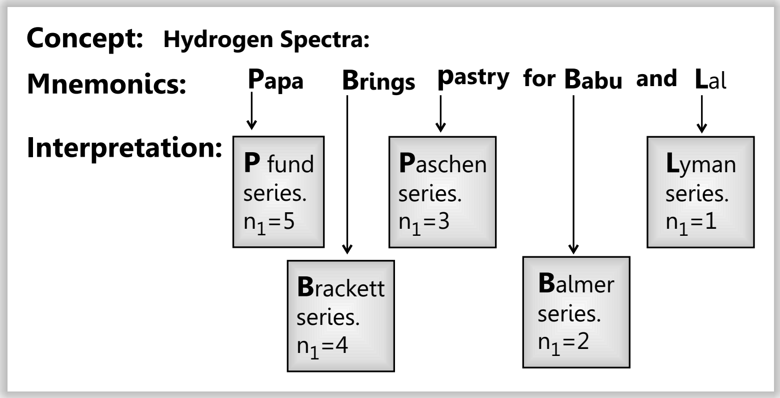
$$v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{(h/2\pi)}$$

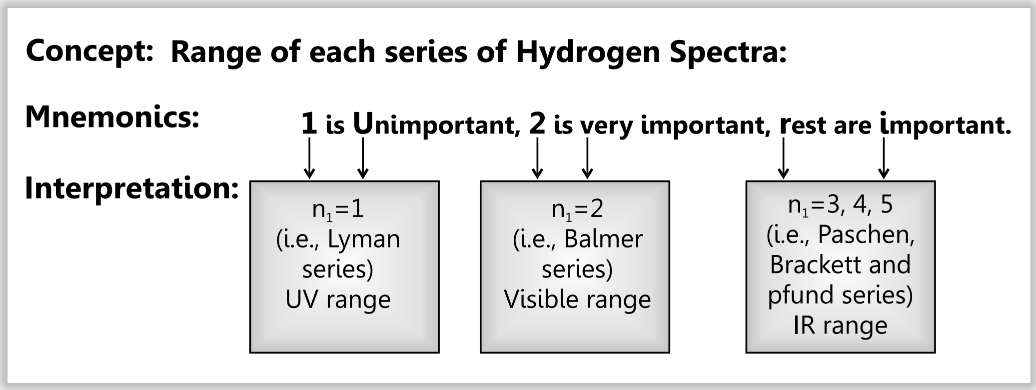
$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)\frac{4\pi\varepsilon_0}{e^2}$$

Bohr radius, $a_0 = \frac{h^2\varepsilon_0}{\pi me^2} = 0.53$ Å

Energy for n^{th} orbiting electron, $E_n = \frac{-13.6}{n^2}$ eV

Mnemonics





CHAPTER-13
NUCLEI

Revision Notes

- All mass and positive charge of an atom is concentrated in its centre known as nucleus.
- The composition of a nucleus can be described using the following terms and symbols :
  - Z = Atomic number = Number of protons (equal to the number of electrons)
  - N = Neutron number = Number of neutrons
  - A = Atomic mass number = $(Z + N)$ = Total number of protons and neutrons
- An atom is represented by ${}^{A}_{Z}X$, where

  X = Symbol of element

  A = Mass number

  Z = Atomic number
- **Isotopes :** Two atoms of an element having the same atomic number (Z is same) but different atomic mass number (due to the different number of neutrons) are said to be isotopes.
- **Isobars :** Two atoms of different elements having the same mass number but different atomic numbers are said to be isobars.
- **Isotones:** Two atoms of different elements having different mass numbers and atomic numbers such that their difference is same are said to be isotones. It means they have same number of neutrons.

- **Size of the nucleus :** A nucleus of mass number A has a radius

$$R = R_0 A^{1/3} \quad \text{where, } R_0 = 1.2 \times 10^{-15} \text{ m}$$

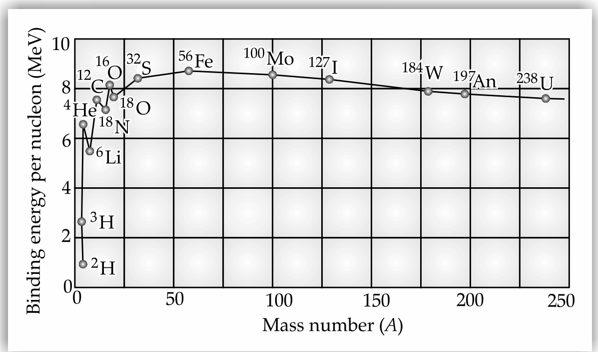
- Nuclear matter density = 2.3×10^{17} kgm^{-3}
- **Mass defect :** The difference in mass of a nucleus and its constituents, ΔM, is called the mass defect.

$\Delta M = [Zm_p + (A - Z)m_n] - M.$

- **Binding energy: Nuclear binding energy** of a nucleus is that quantity of energy which when given to nucleus, its nucleons will become free and will leave the nucleus.

$E_b = \Delta Mc^2$ where, E_b is binding energy.

- **Binding energy per nucleon (E_b/A):** The average energy required to remove an individual nucleon from nucleus is the binding energy per nucleon, E_{bn} or E_b/A, which is the ratio of the binding energy E_b of a nucleus to the number of the nucleons.
- **Relation between E_b/A and Stability of elements**



•Higher the Binding Energy per Nucleons, more stable is the element.

Key Word

Nuclear Binding Energy:

Nuclear binding energy is the minimum amount of energy required to separate the nucleons of a nucleus, or, equivalently, the energy that would be liberated by combining individual nucleons into a single nucleus.

Nucleus is made up of protons and neutrons. But the mass of a nucleus is always less than the sum of the individual masses of the protons and neutrons which constitute it. The difference is a measure of the nuclear binding energy which holds the nucleus together. This binding energy can be calculated from the Einstein relationship, $E = \Delta mc^2$

- Most of the atoms where atomic mass number are in the range $30 < A < 200$, the binding energy per nucleon is fairly constant and quite high. It is maximum for $A = 56$ about 8.75 MeV.
- For $A < 30$ and $A > 170$; Binding energy per nucleon is quite low and remain constant.

- If a nucleus of lower binding energy is converted into higher binding energy, then energy is released.
- There are two methods of converting lower binding energy nucleons into higher binding energy nucleons.
  - **Fission :** A heavy nucleus (low binding energy per nucleon) is broken into two lighter nuclei (higher binding energy per nucleon) with the release of energy. This process is known as fission.

Example:

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^1_0n + 200 \text{ MeV}$$

  - **Fusion :** Two light nucleus (low binding energy per nucleon) are joined to form one nucleus of higher binding energy per nucleon and energy is released. This process is known as fusion.

Example : $^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \nu + 0.42 \text{ MeV}$

- **Nuclear force :**
  - This high binding energy per nucleon counters the repulsive force between protons and bind both protons and neutrons into the tiny nuclear volume.
  - The nuclear force is much stronger than the Coulomb force acting between charges or the Gravitational forces between masses. But it is a short range force.

Key Formula

- Size of the nucleus:
  $R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m
- Mass defect : $\Delta M = [Zm_p + (A - Z)m_n] - M$
- Binding energy : $E_b = \Delta M c^2$
- Binding energy per nucleon (BE/N): $E_{bn} = \frac{E_b}{A}$.

Mnemonics

Concept: Isotope, Isotone and Isobar

Mnemonics: **Zakir with a <u>topee</u>, found Ansari in a <u>bar</u> singing in nasal <u>tone</u>.**

	Zakir	topee	Ansari	bar	nasal	tone
Interpretations:	**Same Z** (i.e., same number of protons)	**ISOTOPE**	**Same A** (i.e., same number of nucleons)	**ISOBAR**	**Same** number of neutrons	**ISOTONE**

[Isotopes have the same number of protons. Isobars have the same number of nucleons. Isotones have the same number of neutrons.]

UNIT – IX : ELECTRONIC DEVICES

CHAPTER-14

SEMICONDUCTOR ELECTRONICS: MATERIALS, DEVICES AND SIMPLE CIRCUITS

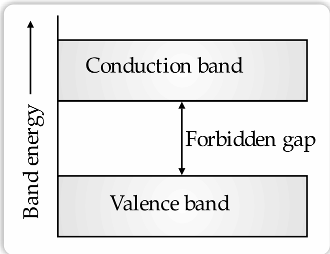
Topic-1 Energy Bands

Concepts Covered *Energy bands in conductors, semiconductors and insulators, intrinsic and extrinsic semiconductors.*

Revision Notes

Energy bands:

- **Energy bands** consist of large number of closely spaced energy levels that exist in crystalline materials.
- In solids, there are three important energy bands such as Valence band, Conduction band, forbidden band or forbidden gap.

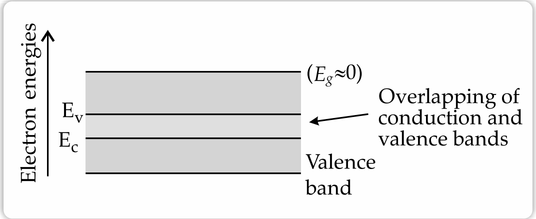


- The collection of energy levels of free electrons which move freely around the material are called conduction band.

- There is an extra energy required for valence electrons to move to conduction band which is known as forbidden energy.
- The collection of energy levels which are partially or wholly filled by electrons are known as valence band.

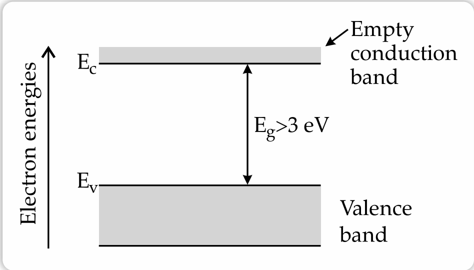
Energy bands in Conductors:

- In conductors, the conduction and valence bands are overlapped.



Energy bands in Insulators:

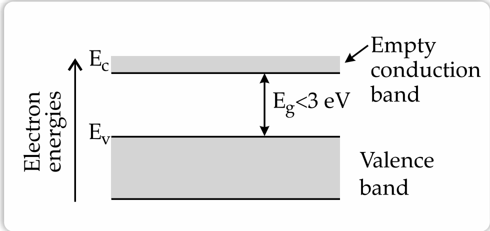
- In insulators, conduction band and valence band are separated by very wide forbidden energy gap.



- The gap between conduction band and valence band exceeds by 3 eV as electrons that transfer from valence band to conduction band need more energy.
- Due to requirement of more energy, insulators do not conduct any electric current.

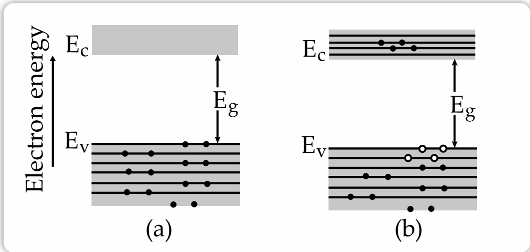
Energy bands in Semiconductors:

- Semiconductors, are materials in which, conduction band and valence band are neither overlapped nor have wide gap.
- In such materials, the energy provided by the heat at room temperature is sufficient to lift the electrons from the valence band to the conduction band.



- Semiconductors behave as insulators at 0 K as no electron exist in conduction band.
- Silicon and Germanium have energy gaps 1.12 eV and 0.75 eV respectively.
- Intrinsic semiconductors: Pure semiconductors are called intrinsic semiconductors. in electronic circuits as their conductivity is low.
- For intrinsic semiconductors, the number of free electrons is equal to the number of holes.

$$n_e = n_h = n_i$$



- An intrinsic semiconductor (a) at $T = 0$ K behaves like insulators. (b) At $T > 0$ K forms thermally generated electron hole pairs.

Key Word

Energy band gap is the distance between the valence band and the conduction band. Essentially, the band gap represents the minimum energy required to excite an electron to shift it to conduction band where it can participate in conduction.

This band gap is also known as forbidden band gap.

- Extrinsic semiconductors: When a small fixed amount of charged impurity is mixed with intrinsic semiconductors, they become extrinsic.
- In extrinsic semiconductors:
  - **(i)** Conductivity increases
  - **(ii)** Conductivity is controlled by dopant
- In extrinsic semiconductors, the number of free electrons is not equal to number of holes.

$$n_e \neq n_h$$

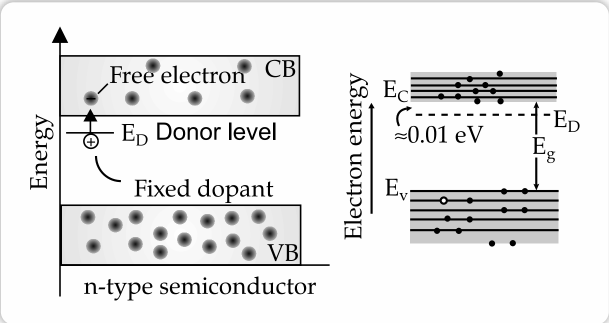
- For raising electrical conductivity, semiconductors are mixed with either pentavalent impurity such as Antimony (Sb), Arsenic (As) and Phosphorus (P) or trivalent impurity such as Indium (In), Gallium (Ga) or Boron (B).

***n*-Type Semiconductor:**

- On doping an intrinsic semiconductor with pentavalent impurity like Antimony (Sb) or Arsenic (As), extrinsic semiconductor so obtained is known as *n*-type.
- *n*-type semiconductor has large number of free electrons known as majority (charge) carriers and small number of holes known as minority (charge) carriers.

$$n_e >> n_h$$

- Impurity atom in *n*-type semiconductor is called donor as it generates new energy level below the conduction band, known as E_D.



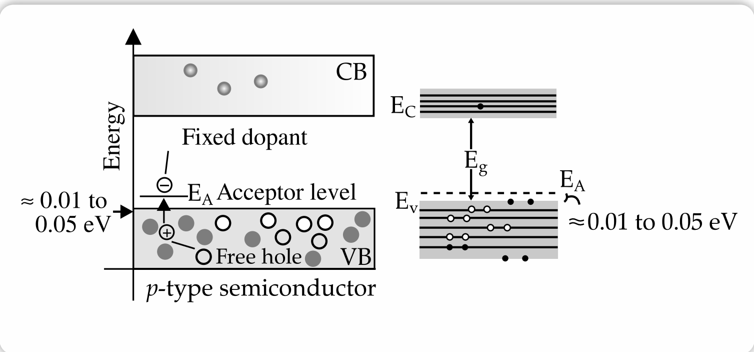
Energy bands for *n*-type semiconductor at T > 0 K

***p*-Type Semiconductor**

- On doping an intrinsic semiconductor with trivalent impurity like Indium (In) or Gallium (Ga), extrinsic semiconductor so obtained is known as *p*-type.
- *p*-type semiconductor has large number of holes known as majority (charge) carriers where number of free electrons is less known as minority (charge) carriers.

$$n_h >> n_e$$

- Impurity atom in *p*-type semiconductor is known as acceptor as it generates new energy level above the valance band, known as E_A. .



Energy bands for *p*-type semiconductor at *T* > 0 K

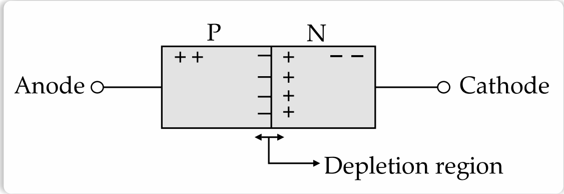
- In extrinsic semiconductors $n_e \neq n_h$ but $n_e.n_h = n_i^2$

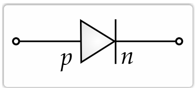
Topic-2 Semiconductor Diodes and Applications

Concepts Covered *Semiconductor diode and its I-V characteristics in forward and reverse bias, Diode as a rectifier.*

Revision Notes

Semiconductor diode



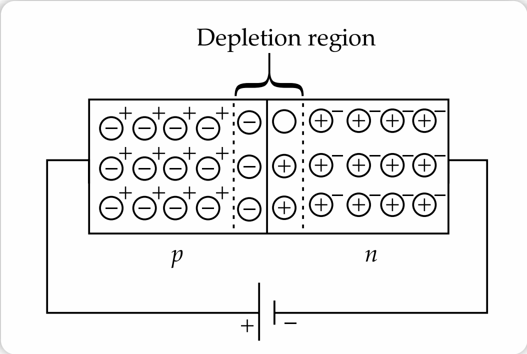


- When a *p*-type semiconductor material is suitably joined to *n*-type semiconductor, the contact surface is called a *p-n* junction.
- It is an electrical device that allows current only in one direction.
- At junction, electrons and holes diffuse and recombine to form the depletion region.

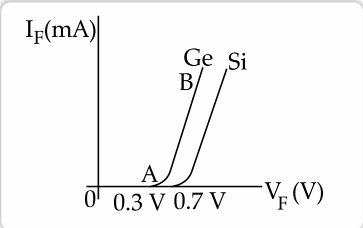
  It acts as a barrier for majority charge carriers.

Forward Bias

- When the negative terminal of battery is connected to *n*-side and positive terminal to *p*-side, the barrier potential and the width of the depletion layer get reduced and only majority carriers can across the junction. Thus the resistance is reduced.

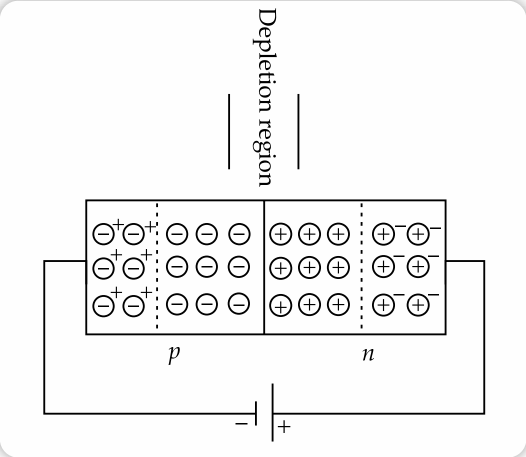


- The positive terminal of battery repels majority carriers, holes in *p*-region while negative terminal repels electrons in *n*-region which pushes them towards the junction.

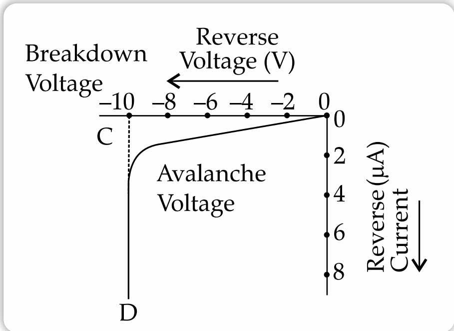


Reverse Bias

- When the positive terminal of a battery is connected to *n*-side and negative terminal to *p*-side, then barrier potential and width of depletion region increases and only minority charge carriers can across junction.



- Current is quite small and is independent of external voltage.
- Beyond certain voltage, diode breaks down with avalanche breakdown mechanism or zener breakdown mechanism.
- Here, negative terminal of battery attracts majority charge carriers, holes in *p*-region and positive terminal attracts majority charge carriers electrons in *n*-region which pulls them away from the junction.
- As a result of this, the width of depletion region increases.

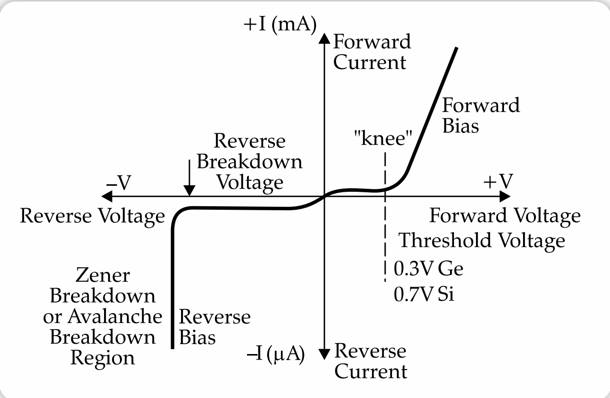


Key Word

Diode:

Diode is an electrical component which allows current to flow only in one direction.

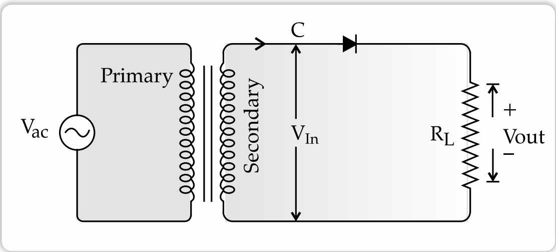
V-I Characteristics of Diode



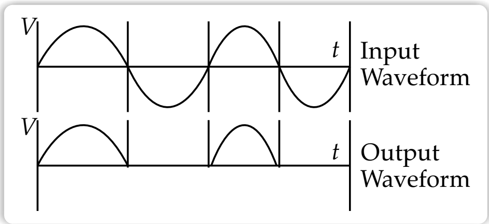
Diode as rectifier

- Rectifier is a circuit which converts AC supply into unidirectional DC supply.
- There are two types of rectifier: half-wave and full-wave.

Half-wave rectifier



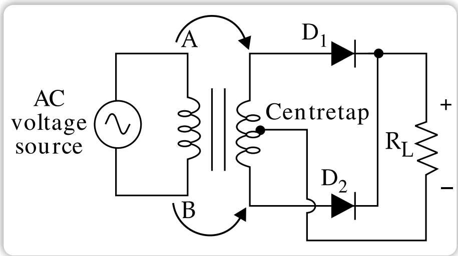
- The half-wave rectifier with single diode, allows current to flow in one direction.
- If V_{ac} is in positive cycle, the diode is forward biased and conducts. As a result voltage becomes available across the load.
- If V_{ac} is in negative half cycle, the diode is reversed biased and does not conduct. No voltage becomes available across the load.
- Voltage waveform across load resistor is shown below.

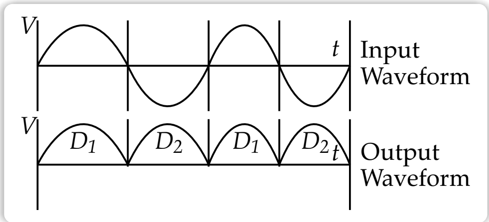


- The output voltage V_{dc} is not exactly similar to the output of a battery.
- The output of half-wave rectifier is bumpy as only one half of input AC is rectified.

Full-wave rectifier

- For rectifying AC input for both half cycles full wave rectifier is used.





- A simple kind of full-wave rectifier uses a centre tap transformer with two diodes.
- In full wave rectifier, for positive half cycle, A is +ve, B is –ve, then only D1 diode conducts, while D2 diode blocks the current. For negative half cycle, A is –ve and B is +ve, only D2 diode conducts while the D1 diode blocks the current.

❐❐