UNIT – I: PHYSICAL WORLD AND MEASUREMENT

CHAPTER-1

UNITS AND MEASUREMENT

Topic-1 Units System and Measurement

Revision Notes

- **Units :** It is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.
- **Fundamental Units :** The physical quantities which are independent of each other and which can represent remaining physical quantities are called fundamental physical quantities and their units are called fundamental units.

  Seven Fundamental physical quantities in SI system of units are :

(a) Mass - kg (Kilogram)
(b) Length - m (Metre)
(c) Time - s (Second)
(d) Temperature - K (Kelvin)
(e) Electric current - A (Ampere)
(f) Luminous Intensity - cd (Candela)
(g) Amount of substance - mol (Mole)

- **Derived Units :** These are the units of measurement of all other physical quantities which can be obtained from fundamental units, e.g., Velocity - (m/s), Acceleration - (m/s^2), Pressure - (Pa), Force - (N) and so on.
- **Unit system**

(a) **F. P. S. system :** Foot, Pound, Second.
(b) **C. G. S. system :** Centimetre, Gram, Second.
(c) **M. K. S. system :** Meter, Kilogram, Second.

- **Significant figures :** The significant figures of a given number are those significant or important digits, which convey the meaning according to its accuracy. For example, 6.658 has four significant digits. These substantial figures provide precision to the numbers. They are also termed as significant digits.
- **Rules for significant figure :**

(a) All non-zero digits are significant. 198745 contains six significant digits.
(b) All zeros those occur between any two non-zero digits are significant. For example, 108.0097 contains seven significant digits.
(c) All zeros those are on the right of a decimal point and also to the left of a non-zero digit is never significant. For example, 0.00798 contained three significant digits.
(d) All zeros those are on the right of a decimal point are significant, only if, a non-zero digit does not follow them. For example, 20.00 contained four significant digits.
(e) All the zeros those are on right of the last non-zero digit, after the decimal point, are significant. For example, 0.0079800 contains five significant digits.
(f) All the zeros that are on the right of the last non-zero digit are significant if they come from a measurement. For example, 1090 m contains four significant digits.

Key Words

- **Fundamental Units :** Units of the physical quantities which are independent of each other and which can represent remaining physical quantities.
- **Derived Units :** Units of measurement of those physical quantities which can be obtained from fundamental units.

Key Formulae

- 1 AU = 1.496×10^{11} m.
- 1 ly = 9.46×10^{15} m.
- 1 parsec = 3.1×10^{16} m.
- 1 Å = 10^{-10} m; 1 nm = 10^{-9} m
  1 μm = 10^{-6} m, 1 mm = 10^{-3} m
- 60 seconds (of arc) = 1 min (arc)
- 60 min. (of arc) = 1 degree (of arc)
- 180 degree (of arc) = π radian
- Indirect methods for long and small distances :

  Angular diameter (θ) = $\dfrac{d}{D}$

  d = diameter, D = distance, radius = r
- **Magnification :**

(a) Linear Magnification = $\dfrac{\text{Size of image}}{\text{Size of object}}$

(b) Linear Magnification = $\sqrt{\text{Areal Magnification}}$

Mnemonics

Concept: The fundamental quantities of SI system:

Mnemonics: At the **l**ast **m**oment she **lu**ckily **c**aught the **t**rain.

Interpretation:

A - Amount of substance

t - Temperature (thermodynamic)

l - Length

m - Mass

lu - Luminous intensity

c - Current

t - Time

Topic-2 Dimensional Analysis

Revision Notes

- Dimensional analysis is the study of relationship between physical quantities and the fundamental quantities.
- Dimensional equation is the equation expressing of the relationship between physical quantities and the fundamental quantities.
- **Principle of homogeneity:** Dimensions of each term of a dimensional equation on both side should be same.
- Conversion of units from one system to another:

  n_1 and n_2 be the magnitudes of a physical quantity in two systems respectively. General dimensions of the physical quantity be $[M^aL^bT^C]$. If dimensions in two system be $u_1 = [M_1^aL_1^bT_1^c]$ and $u_2 = [M_2^aL_2^bT_2^c]$ respectively then $n_1 [M_1^aL_1^bT_1^c] = n_2 [M_2^aL_2^bT_2^c]$
- Dimensional analysis used to derive formula of a physical quantity.
- Correctness of formula of a physical quantity may be checked using principle of homogeneity.
- **Limitations of dimensional analysis:**
  - Cannot provide information about dimensional constant.
  - Formula containing trigonometric functions, logarithmic functions, and exponential function cannot be derived.
  - Cannot provide information whether the quantity is a vector or a scalar.
  - If the physical quantity depends on more than three quantities then it becomes difficult to derive the formula.

- It is not possible to find the formula correctly when the physical quantity is not related by multiplication of the other quantities.

Key Words

- **Dimensions of physical quantity** are the powers to which the symbols of fundamental quantities are raised to represent a derived unit of that quantity.
- **Dimensional formula of the given physical quantity** is the expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity.
- **Dimensional constants** are the quantities whose values are constant and they possess dimensions e.g., universal gravitational constant G etc.
- **Dimensional variables** are the quantities whose values are variable and they possess dimensions e.g., area, volume, etc.
- **Dimensionalless constants** are the quantities whose value are constant but they do not possess dimensions e.g., mathematical constants– π, e and numbers.
- **Dimensionalless variables** are the quantities whose values are variable and they do not have dimensions e.g., angle, strain, etc.

Key Formulae

- Conversion of one system of units into another

$$n_2 = \frac{n_1 u_1}{u_2} = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

UNIT – II: KINEMATICS

CHAPTER-2

MOTION IN A STRAIGHT LINE

Topic-1 Motion & Velocity

Revision Notes

- **Rest :** An object or a particle is said to be in the state of rest when it does not change its position with time w.r.t. same reference point.

  Depending upon the position of observer, the state of rest of a particle is of two types :

(a) Absolute state of rest,

(b) Relative state of rest.

- **Motion :** An object or a particle is said to be in the state of motion when it changes its position with time w.r.t. same reference point.

  The motion of an object can be either linear or curvilinear, circular or in a plane or in a space.

(a) Linear or Rectilinear or Translatory motion :

(i) It is the motion in which a particle moves along a straight line with respect to a point of reference.

(ii) A body is said to be in linear motion if every constituent particle of the body move along parallel straight line and covers same distance in the given time.

(b) Circular or Rotatory Motion :

(i) A motion in which a particle or a point mass body is moving in a circle.

(ii) In rotatory motion all its constituent particles move simultaneously along concentric circles.

(c) Oscillatory or Vibratory Motion :

(i) In oscillatory motion the body moves back & forth repeatedly in definite interval of time about a fixed point.

(ii) If the amplitude of oscillating body is very small, the motion is called vibratory motion.

➤ **Dimensional Motion**

(a) Motion in 1–D :

(i) It is that motion in which a particle moves in one particular direction with respect to a point of reference.

(ii) In 1–D, the particle or a body moves along a straight line or a well defined path. Therefore, one dimensional motion is sometimes known as linear motion.

(b) Motion in 2–D

(i) If two out of three coordinates specifying the position of the object change with respect to time, the motion is called 2–D motion.

(c) Motion in 3–D

(i) If all the three coordinates specifying the position of the object change with respect to time, the motion is called 3–D motion.

➤ **Path Length or Distance :**

(a) Path Length is defined as the actual path traversed by body during motion in a given interval of time.

(b) Distance is a scalar quantity.

(c) The S.I. unit of distance is metre and C.G.S. unit is centimeter.

(d) The value of distance traversed by a moving body can never be zero or negative.

➤ **Displacement :**

(a) Displacement of a body in a given time is defined as the change in the position of the body in a particular direction during that time. It may also defined as the shortest distance between initial and final position of the object.

(b) Displacement is a vector quantity as it possesses both magnitude and direction.

(c) The unit of displacement is same as that of path length.

(d) The value of displacement can be positive, zero or negative.

(e) The value of displacement can never be greater than the distance travelled.

(f) When a moving body returns to its starting point, then its effective displacement is zero.

➤ **Difference between Distance & Displacement :**

S. No.	Distance	Displacement
1.	Actual path traversed by object in given time.	Shortest distance between initial & final positions of object in given time.
2.	Scalar quantity.	Vector quantity.
3.	It cannot be zero or negative, it will be always positive.	It can be positive, negative or zero.
4.	It is either equal or greater than displacement but never less than displacement.	It is either equal or less than distance but never greater than distance.
5.	It can have many values depending upon path followed between two positions.	It has unique value.
6.	Between two positions of an object, it tells type of path followed.	It does not tell type of path followed.

➤ **Instantaneous velocity :** If a body is moving with a variable velocity, then the velocity of the body at a given instant of time is called its instantaneous velocity.

Key Words

➤ Uniform motion is said to be in an object when velocity is uniform i.e., it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.

➤ Non–uniform motion is said to be in an object when it undergoes equal displacements in unequal intervals of time, howsoever small these intervals may be.

Mnemonics

Concept: Motion In A Straight Line
Mnemonics: **D**elhi to **V**adodara via **T**undla **A**gra.

Interpretation: **D**isplacement/time = **V**elocity
Velocity / **t**ime = **A**cceleration

Key Formulae

- Path length or distance, D = Speed × Time
- Displacement = Velocity × Time

Topic-2 Uniformly Accelerated Motion

Revision Notes

- **Accelerated motion :** When an object is moving in non-uniform motion, the velocity is different at different instants i.e., the velocity keeps on changing with time. This motion is an accelerated motion.
- **Acceleration :** It is defined as the ratio of change in velocity & the corresponding time taken by the mirror object;

(a) It is vector quantity.

(b) It is either positive or negative.

(c) Negative acceleration is called retardation.

(d) Unit-m/s^2 in SI & cm/s^2 in CGS system.

(e) Dimensional formula– $[LT^{-2}]$.

(i) **Uniform acceleration :** An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

(ii) **Variable acceleration :**

(a) An object is said to be moving with a variable acceleration when its velocity changes by unequal amounts in equal intervals of time.

(b) The velocity time graph of a body having variable acceleration is represented by a curve.

(iii) **Average acceleration :** When an object is moving with a variable acceleration, then the average acceleration of the body is defined as the ratio of the total change in velocity during motion to the total time taken, i.e.,

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

(iv) **Instantaneous acceleration :**

(a) If a body is moving with a variable acceleration, then the acceleration of a body at the given instant of time is called instantaneous acceleration.

(b) If at an instant t, a body while moving with a variable acceleration shows a change in velocity $\Delta\vec{v}$ in a small interval of time Δt, so that $\Delta t \to 0$, then

$$\text{Instantaneous acceleration} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

where, $\frac{d\vec{v}}{dt}$ is the derivative of velocity $(\vec{v})$ w.r.t. time.

Key Terms

- Total displacement of the body in the given time is equal to the area which velocity time-graph encloses with time axis.
- Uniformly accelerated object in a given time-interval is represented by the slope on the velocity-time graph for a given time-interval.
- Acceleration of object is the slope of velocity-time graph of uniformly accelerated motion.

Key Formula

- Suppose

  u = initial velocity of body,

  a = uniform acceleration of the body,

  v = velocity of the body after time t,

  s = distance travelled by body in time t,

  s_n = distance travelled by body in n^{th} second.

(a) The equations of motion for accelerated body are :

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$ (iv) $s_n = u + \frac{a}{2}(2n-1)$

(b) The equations of motion for retarded body (here, a is negative) are :

(i) $v = u - at$ (ii) $s = ut - \frac{1}{2}at^2$

(iii) $v^2 = u^2 - 2as$ (iv) $s_n = u - \frac{a}{2}(2n-1)$

(c) The equations of motion for a body falling down under gravity (here, $a = +g$, $s = h$) are :

(i) $v = u + gt$ (ii) $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$ (iv) $h_n = u + \frac{g}{2}(2n-1)$

(d) The equations of motion for a body going up under gravity (here $a = -g$, $s = h$) are :

(i) $v = u - gt$ (ii) $h = ut - \frac{1}{2}gt^2$

(iii) $v^2 = u^2 - 2gh$ (iv) $h_n = u - \frac{g}{2}(2n-1)$

(e) The maximum height attained by a body thrown vertically upwards with initial velocity u is

$$h_{max} = \frac{u^2}{2g}$$

(f) Time taken to reach the maximum height is

$$t = \frac{u}{g}$$

(g) Total time taken by body in going up and coming down,

$$T = 2t = \frac{2u}{g}$$

(h) The initial velocity of body in order to attain height h is,

$$u = \sqrt{2gh}$$

CHAPTER-3
MOTION IN A PLANE

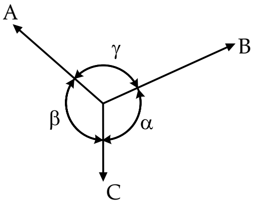
Topic-1 Scalar and Vector Quantities

Revision Notes

Scalar : A physical quantity which has only magnitude and no direction is called a scalar quantity or scalar.

Vector : A physical quantity which constitutes of magnitude as well as direction is called a vector quantity or vector. If it follows law of vector addition too.

- **Unit vector :**
  (i) A unit vector of $\vec{A}$ is written as $\hat{A}$ and is given by $\hat{A} = \vec{A}/|\vec{A}|$
  (ii) A unit vector is dimensionless quantity of magnitude equal to unity. Its magnitude is 1 and it can have any direction.
  (iii) In cartesian co-ordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x, y and z-axes, respectively.
- **Polar vectors :** These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.
- **Axial vectors or rotational vectors :** These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.
- **Some vector laws :**
  (1) General law for addition of vector : It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with tail of third vector and so on then resultant vector is represented in magnitude and direction by the line starting from tail of first vector to head of last vector.
  (2) Triangle Law : It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of triangle taken in opposite order.
  (3) Parallelogram Law : It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.
- **Lami's Theorem :** It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces, i.e.,



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Key Words

- **Modulus of vector** is the magnitude of vector.
- **Equal vectors** are those vectors which have equal magnitude and same direction.
- **Negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of given vector.
- **Co-initial vectors** are those vectors which have common initial point.
- **Collinear vectors** are those vectors which are having equal or unequal magnitudes and are acting along parallel straight lines.

- **Coplanar vectors** are those vectors which are acting in the same plane.
- **Localized vector** is that vector whose initial point is fixed.
- **Non-Localized vector** is that vector whose initial point is not fixed.
- **Zero or Null vector** is that vector which has zero magnitude and an arbitrary direction and represented by $\vec{0}$.
- **Displacement vector** is that vector which tells how much and in which direction an object has changed its position in a given interval of time.
- **Resultant vector** is defined as that single vector which produces the same effect as is produced by two or more individual vectors together.
- **Equilibrate vector** is a single vector which balances two or more vectors acting on a body at the same time.

Key Formulae

- $$\vec{R} = \vec{A} + \vec{B}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}; \beta = \text{angle of } \vec{R} \text{ with } \vec{A}.$$
- $$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin(180^\circ)}{A + B\cos(180^\circ)} = \frac{B\sin\theta}{A - B\cos\theta}$$
- $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $A_x = A\cos\theta$, $A_y = A\sin\theta$ (in 2D)
- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ (in 3D)
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}, \quad |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$
- $$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$
- **Unit Vector or $\hat{A}$ is**
$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$
- $$\vec{A}.\vec{B} = AB\cos\theta$$
$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$
- If two vectors are parallel to each other i.e., $\theta = 0°$
$$\vec{A}.\vec{B} = AB\cos 0° = AB$$
$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$
- If two vectors are perpendicular to each other i.e., $\theta = 90°$.
$$\vec{A}.\vec{B} = AB\cos 90° = 0$$
$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{j} = 0$$
- If two vectors are parallel to each other i.e., $\theta = 0°$

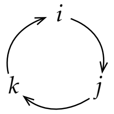
- $$\vec{A} \times \vec{B} = AB \sin 0^\circ \, \hat{n} = \vec{0}.$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- If two vectors are perpendicular to each other *i.e.*, $\theta = 90^\circ$

$$\therefore \quad \vec{A} \times \vec{B} = AB \sin 90^\circ = AB$$

- Trick to remember Cross product



$$\hat{i} \times \hat{j} = \hat{k}, \; \hat{j} \times \hat{k} = \hat{i}, \; \hat{k} \times \hat{i} = \hat{j}$$

$$\text{and} \quad \hat{i} \times \hat{k} = -\hat{j}, \; \hat{k} \times \hat{j} = -\hat{i}, \; \hat{j} \times \hat{i} = -\hat{k}$$

- **Area of triangle** $= \frac{1}{2} |\vec{A} \times \vec{B}|$
- **Area of parallelogram** $= |\vec{A} \times \vec{B}|$
- **Unit vector perpendicular** to $\vec{A}$ & $\vec{B}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\text{where,} \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$

then $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$

- $\sin \theta = \dfrac{|\vec{A} \times \vec{B}|}{|\vec{A}|\,|\vec{B}|}$
- $\cos \theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$
- $\tan \theta = \dfrac{\vec{A} \times \vec{B}}{\vec{A} \cdot \vec{B}}$

Mnemonics

Concept: Cross and dot product of two vectors.

Mnemonics: A and **B** **c**rossed **S**ikkim and **d**rove to **C**alcutta.

Interpretation:

A - $\vec{A}$

B - $\vec{B}$

c - Cross product

s - $\sin \theta$

d - dot product

c - $\cos \theta$

$\vec{A} \times \vec{B} = AB \sin \theta$

$\vec{A} . \vec{B} = AB \cos \theta$

Topic-2 Projectile Motion

Revision Notes

- **Projectile :**
  - **(a)** Projectile is defined as a body thrown with some initial velocity except vertically upward or downward and then allowed to move under the action of gravity alone, without being propelled by an engine or fuel or any source. The path followed by a projectile is known as its **trajectory**.
  - **(b)** The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.
  - **(c)** Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.
- The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.
- Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.
- **Centripetal force :**
  - **(a)** It is the force required to move the body in circular path with a constant angular velocity.
  - **(b)** The centripetal force acts on the particle along the radius which is directed towards the centre of circular path.
  - **(c)** The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path, therefore the work done by the centripetal force is zero.
  - **(d)** The centripetal force is provided in different ways, in different types of circular motions.

Key Words

- **Angular displacement** of the object moving around a circular path is defined as the angle traced out by radius vector at the centre of circular path in given time. It is vector quantity.
- **Angular velocity** of an object in circular motion is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Uniform circular motion** is the motion when a point object is moving on a circular path with a constant speed.

Key Formulae

- **For motion along *X*-axis,** $v_x = u_x + a_x t$ and $x = x_0 + u_x t + \frac{1}{2} a_x t^2$
- **For motion along *Y*-axis,** $v_y = u_y + a_y t$ and $y = y_0 + u_y t + \frac{1}{2} a_y t^2$
- **Velocity of projective at an instant of its flight is**

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

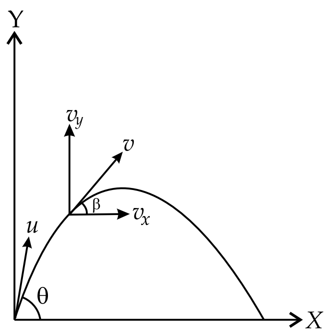
and $$\tan\beta = \frac{v_y}{v_x}$$

- **Angular projection of projectile :**

(i) Time of flight, $$T = \frac{2u\sin\theta}{g}$$

(ii) Maximum height, $$h = \frac{u^2\sin^2\theta}{2g}$$

(iii) Horizontal range, $$R = \frac{u^2\sin 2\theta}{g}$$



u = initial speed
θ = angle of projection

(iv) Maximum horizontal range $R_{max} = \frac{u^2}{g}$ for $\theta = 45°$

(v) Range is same for for angles θ and $(90° - \theta)$ if u & g remains unchanged

- **Circular Motion**
  - $\omega = \theta/t$
  - $\omega = 2\pi\nu = \frac{2\pi}{t}$
  - $a_c = \omega^2 r = \omega v = v^2/r$
  - $a_T = r\alpha$

where, a_c = centripetal acceleration
a_T = tangential acceleration
ω = angular velocity
ν = frequency
T = Time period

UNIT – III: LAWS OF MOTION

CHAPTER-4
LAWS OF MOTION

Topic-1 Newton's Laws of Motion

Revision Notes

- **Newton's I law :** A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some non-zero external force to change the state. This law defines forces and is also called law of inertia.
- **Inertia** of a body is of three types as follows :

(i) **Inertia of rest** of a body is inability to change by itself, its state on its own.

(ii) **Inertia of motion** of a body is the inability to change by itself its state of uniform motion i.e., body in uniform motion can neither accelerate nor retard on its own and comes to rest.

(iii) **Inertia of direction** of a body is inability to change by itself its direction of motion, i.e., body continues to move along the same straight line unless compelled by some external force to change it.

- **Linear momentum :** Linear momentum ($\vec{p}$) of a body is measured by the product of the mass (m) of the body and its velocity ($\vec{v}$) i.e.,

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity. Its direction is same as the direction of velocity of the body. The S.I. unit of linear momentum is kg ms^{-1}.

- **Newton's II law :** The rate of change of linear momentum ($\vec{p} = m\vec{v}$) of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force, i.e.,

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

or
$$\vec{F} = k\frac{d\vec{p}}{dt} = k\frac{d}{dt}(m\vec{v})$$

$$= km\left(\frac{d\vec{v}}{dt}\right) = km\vec{a}$$

where $\frac{d\vec{v}}{dt} = \vec{a}$, which is called acceleration of the body. Force can be defined in such a way that $k = 1$, then Newton's second law is written as

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

- **Newton's III law :** To every action, there is always an equal and opposite reaction. The action and reaction act on different bodies, so they never cancel each other.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$\vec{F}_{AB}$ = Force exerted on A by B

$\vec{F}_{BA}$ = Force exerted on B by A

Hence, $\vec{F}_{AB} + \vec{F}_{BA} = \vec{0}$

(i) Principle of conservation of linear momentum : From this principle, in an isolated system, the vector sum of the linear momentum of all the bodies of the system is conserved and is unaffected due to their mutual action and reaction. The total linear momentum of all the bodies in the system is given by

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = M\vec{v}_{c.m} = \text{constant}$$

where, M is the total mass of the system and $\vec{v}_{c.m}$ is the velocity of the centre of mass of the system.

(ii) Rocket propulsion : The propulsion of a rocket is based on the principle of conservation of linear momentum of Newton's third law of motion.

Suppose,

M_0 = Initial mass of rocket,

$\frac{\Delta M}{\Delta t}$ = Rate of ejection of fuel,

M = Mass of rocket at any instant,

$\vec{v}$ = Relative velocity of ejected gases w.r.t. rocket.

Then, thrust on the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \vec{v}$

Acceleration of the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M}$

Thrust on the rocket in the presence of gravity,

$$\vec{F} = \frac{\Delta M}{\Delta t} \times \vec{v} - M\vec{g}$$

Acceleration of the rocket in the presence of gravity,

$$\vec{a} = \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M} - \vec{g}$$

Mnemonics

Newton's Laws of Motion: Newton, Newton don't kick cow She may move ahead little bit now Newton hears her MAAA sound Cow gives Newton a kick rebound

Interpretation: 1st two lines of the rhyme depicts the 1st law of motion

Newton's 1st law: A body continues its state of rest or state of motion unless it is acted upon by an unbalanced force.

3rd line depicts the 2nd law of motion

Newton's 2nd law: F = ma

Last line depicts the 3rd law of motion

Newton's 3rd law: Every action has its equal and opposite reaction.

Key Words

- **Force** is an external effort in the form of push or pull which can try to produce motion in a body at rest, or stops or try to stop a moving body or can change or try to change the direction of motion of the body.
- **Inertia** is the inherent property of a body, by virtue of which, the body doesn't change its state of rest or of uniform motion along a straight line, on its own. It depends upon the mass of the body.
- **Impulse :** When a large force acts on a body for a short time, then the measure of the total effect of force is called impulse of force. It can be found out

$$\text{Impulse} = \text{Force} \times \text{Time} = \vec{F}_{av} \times \Delta t$$

Key Formulae

- **Linear Momentum :** $\vec{p} = m\vec{v}$
- **Force** $= \dfrac{d\vec{p}}{dt} = m\vec{a}$
- **Impulse** = Force × time

$$\vec{I} = \vec{F} \times t = m(\vec{v} - \vec{u})$$

- **Principle of Conservation of Linear Momentum**

$$m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = \text{Constant}$$

- **Recoil velocity of gun** $(\vec{v}_2) = \dfrac{-m_1\vec{v}_1}{m_2}$

m_2 = Mass of gun

m_1 = Mass of bullet

v_1 = Velocity of bullet

Topic-2 Friction & Dynamics of Circular Motion

Revision Notes

- **Equilibrium of Concurrent forces :** Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.

(i) **Conditions of equilibrium of concurrent forces :** If $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, ... are the concurrent forces acting at the same point, then the point will be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots = \vec{0}$$

- **Friction :** Friction is an opposing force which comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body. Frictional forces arise due to interlocking of irregularities present on the two surfaces which are in contact. From modern view, the frictional force arises due to strong atomic or molecular force of attraction between the two surfaces at the points of actual contact.

It is of two types :

(a) **Internal friction :** It arises on account of relative motion between every two layers of liquid. It is also known as viscosity of liquid.

(b) External friction : It arises when two bodies in contact with each other try to move or there is an actual relative motion between the two. It is also known as contact friction. Further it is of four types:

(i) Static friction is an opposing force which comes into play when one body tends to move over the surface of another body, but the actual motion has yet not started. It is a self-adjusting force.

(ii) Limiting friction is the maximum value of static friction. Limiting friction is the maximum opposing force that comes into play of one body is just at the average of moving over the surface of another body.

(iii) Dynamic or kinetic friction is the opposing force that acts between two surfaces in contact when one body is actually moving over the surface of another body.

(iv) Rolling friction is an opposing force that comes into play when one body is actually rolling over the surface of another body.

- **Laws of limiting friction :**

(a) The magnitude of the force of limiting friction (F) between two bodies in contact is directly proportional to the normal reaction (R) between them, i.e., $F \propto R$.

(b) The direction of the force of limiting friction is always opposite to the direction in which the body moves or tends to move.

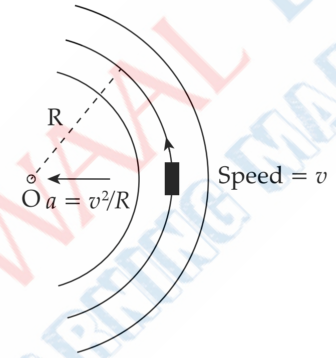
(c) The force of limiting friction is independent of the area of contact.

(d) The force of limiting friction between any two bodies in contact depends on the nature of the surfaces in contact.

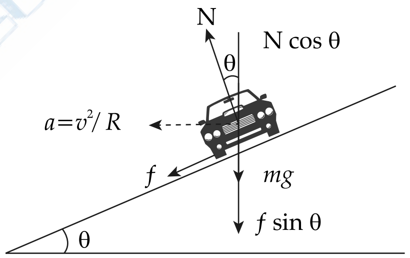
- **Coefficient of limiting friction** between two surfaces in contact (μ) is defined as the ratio of the force of limiting friction (F) and normal reaction (R) between them, i.e.,

$$\mu = F/R$$

- **Motion of car on banked road:**



$$V_{max} = \sqrt{\mu_s Rg}$$



$$V_{max} = \sqrt{\left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}\right]}$$

For $\mu_s = 0$ i.e., for a frictionless banked road

$$V_{max} = \sqrt{rg\tan\theta}$$

where, μ_s is the coefficient of friction and R is the radius of the circle.

Key Terms

- **Centripetal force** is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.

- **Angle of friction** is the angle in which the direction of resultant of the force of friction and normal reaction makes with the direction of normal reaction. It is represented by θ.

$$\tan\theta = \mu$$

- **Angle of repose** is the maximum angle of inclination of a plane with the horizontal, at which the body placed on the plane is just in limiting equilibrium.
- **Centrifugal force** is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Key Formulae

- **Angle of friction,** $\tan\theta = \mu.$
- **Angle of repose,** $\mu = \frac{F}{R} = \tan\theta.$
- **Centripetal force** $= \frac{mv^2}{r} = mr\omega^2.$

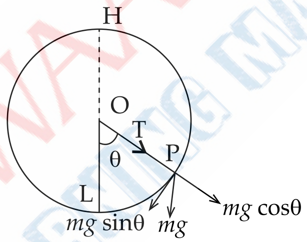
$$= mr(2\pi\nu)^2.$$

- $\tan\theta = v^2/rg.$
- $\tan\theta = \frac{h}{\sqrt{b^2-h^2}} = v^2/rg.$

h = height between outer edge and inner edge

b = breadth of road

- Maximum speed of car on circular banked road $= v_{max} = \left[\frac{rg(\mu_s+\tan\theta)}{(1-\mu_s\tan\theta)}\right]^{1/2}$
- **At any position of angular displacement** θ **along a vertical circle**



$$T = \frac{mv^2}{r} + mg\cos\theta.$$

- **At lowest point of vertical circle,** $\theta = 0°;\ T_L = \frac{mv_L^2}{r} + mg.$
- **At highest point of vertical circle,** $\theta = 180°$

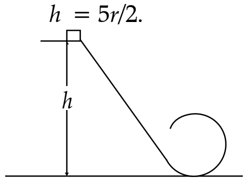
$$T_H = \frac{mv_H^2}{r} - mg.$$

- **Minimum velocity at highest point, at H** $v_H = \sqrt{gr}.$
- **Minimum velocity at lowest point, at L** $v_L = \sqrt{5gr}.$
- **When,** $\theta = 90°$

$$v = \sqrt{3gr}.$$

- **Height through which a body should fall for looping the vertical loop**

$$h = 5r/2.$$



UNIT – IV: WORK, ENERGY AND POWER

CHAPTER-5

WORK, ENERGY AND POWER

Topic-1 Work and Power

Revision Notes

➤ **Work :** Work is done when the body is displaced actually through some distance in the direction of applied force.

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs \cos \theta$$

- Work done is a scalar quantity. However, work done is positive when θ lies between 0 (zero) and π/2. Work done is negative when θ lies between π/2 and 3π/2.
- S. I. unit of work is joule (J) and the C.G.S unit of work is erg, where 1 joule = 10^7 erg.
- Work done by a body does not depend on the time taken to complete the work.

➤ **Internal work or zero work:** The work in which muscles are strained, but work done is not useful, is called internal work. For example, when a person carrying load keeps on standing at the same place, work done is zero, but he gets tired on account of internal work.

Dimensions : $[ML^2 T^{-2}]$

Power : Power of a body is defined as the time rate of doing work by the body. Thus, in power, time taken by the body to complete the work is significant.

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

- Here, θ is the angle between force $\vec{F}$ and velocity $\vec{v}$ of the body.
- **Unit** : 1 W = 1 Js^{-1}
- **Dimensions :** $[ML^2T^{-3}]$
- Power is a scalar quantity.

Key Words

➤ **Conservative force** is a force if work done by or against the force in moving a body depends only on the initial & final positions of the body and not on the nature of path followed between initial and final positions, e.g., gravitational force, electrostatic force between two electric charges, all central forces, etc.

➤ **Non-conservative force** is a force if work done by or against the force in moving a body from one position to another depends on the path followed between these two positions. e.g., frictional forces, elastic forces, etc.

Key Formulae

➤ Work= Force × Displacement in the direction of force

➤
$$W = \vec{F} . \vec{s}$$
$$= Fs \cos \theta$$

- Maximum work
  When $\cos\theta = 0°$
  $W = Fs$
- Minimum Work when $\theta° = 90°$
  Then $W = Fs \times \cos 90° = 0$
- Work done by variable force :

$$W = \int_{x_A}^{x_B} \vec{F}.d\vec{x}$$

- $$\text{Power} = \frac{\text{Work done}}{\text{Time Taken}}$$
- $$P = \vec{F}\cdot\vec{v}$$
- $$P = Fv\cos\theta.$$

Mnemonics

Concept: Positive work, negative work
Fernandez **d**'souza **O**rdered **d**onut **W**ith **n**oodles
Fernandez **d**'souza was **S**erved **d**onut **W**ith **P**izza.
Interpretation:
F - Force
d - Displacement
o - opposite
d - direction
w - Work done
n - Negative
If Force and displacement are in opposite directions work done is negative.
F - Force
d - Displacement
s - Same
d - direction
w - Work done
p - positive
If Force and displacement are in same directions work done is positive

Topic-2 Energy & Collision

Revision Notes

- **Energy :** Energy of a body is defined as the capacity of the body to do the work. Energy is a scalar quantity. It has the same units and dimensions as those of work. Some practical units of energy and their relation with S.I. unit of energy (joule) are :
  (i) 1 calorie = 4.2 J
  (ii) 1 kiloWatt hour (kWh) = 3.6×10^6 J
  (iii) 1 electron volt (1 eV) = 1.6×10^{-19} J
- **Work-Energy Theorem :** According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body and i.e.,

$$W = K_f - K_i$$
$$K_f = \text{final } K.E.$$
$$K_i = \text{initial } K.E.$$

- **Collisions :** When a body strikes against another body such that there is exchange of energy and linear momentum then the two are said to collide. Collisions are of two types :
  (i) Perfectly elastic collision is that in which there is no change in kinetic energy of the system, i.e.,
  Total *K.E.* before collision = Total *K.E.* after collision.
  e.g., collisions between atomic and subatomic particles are perfectly elastic collisions.
  (ii) Perfectly inelastic collision is that in which *K.E.* is not conserved. Here, the bodies stick together after impact. Linear momentum is conserved in every collision elastic as well as inelastic, further total energy is also conserved in all such collisions. Kinetic energy alone is not conserved in inelastic collisions.

Key Words

- **Kinetic Energy** is the energy possessed by the body by virtue of its motion. It is always positive.
- **Potential Energy** is the energy possessed by the body by virtue of its position. It can be both negative as well as positive.
- **Gravitational Potential Energy** is the energy possessed by the body by virtue of its position with respect to center of Earth or other body.
- **Potential Energy of spring** is the energy associated with the state of compression or expansion of an elastic spring.
- **Internal Energy** is the energy possessed by the body by virtue of particular configuration of its molecules.
- **Coefficient of Restitution or Coefficient of Resilience** is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by '*e*'.

Key Formulae

- $$\text{Kinetic Energy } (K.E.) = \frac{1}{2}mv^2,$$

  where, m = mass, v = velocity of particles
- $$\text{Potential Energy } (P.E.) = mgh$$
- $$\text{Velocity } (v) = \sqrt{2gh}$$
- $$\text{Force in Spring } (F) = -kx,$$

  where, k = spring constant, x = compression.
- **Mass Energy Equivalence :**

  $$E = mc^2$$

  where, m = mass that disappears

  E = energy that appears

  c = velocity of light
- **Coefficient of Resilience** $$(e) = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$$

  $$e = \frac{v_2 - v_1}{u_2 - u_1}$$

  for perfectly elastic collision, $e = 1$

  for perfectly inelastic collision, $e = 0$
- **Elastic collision in 1- Dimension :**

  $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

  $$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$
- **Inelastic Collision in 1–Dimension :**

  $$v = \frac{m_1u_1}{m_1 + m_2} \text{ if } u_2 = 0$$

  $$\text{Loss in } K.E. = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

UNIT – V: MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

CHAPTER-6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Topic-1 Centre of Mass & Motion of Rotational Particles

Revision Notes

- **Kinds of Motion of Rigid Body**

  (i)Pure Translational Motion : All the particles of body are moving together with same velocity at particular instant of time. eg., A car moving is a straight line.

  (ii)Pure Rotational Motion : A rigid body rotates about a fixed axis. Every particle of the body moves in a circle which lies in a plane perpendicular to axis and has its centre on the axis. eg., A potter's wheel.

  (iii)Combination of Translational and Rotational Motion : The motion of rigid body, which is not pivoted or fixed in some way is either pure translation motion or a combination of translation and rotation. eg., A vehicle's wheel.

- **Center of Mass of a two particle system :** Position vector of centre of mass of a two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to sum of the products of masses of two particles and their respective position vectors.

- **Momentum Conservation :** Total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

  $\vec{p} = M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 + \text{..........} + m_n\vec{v}_n$

  Differentiating it,

  $$\frac{d\vec{p}}{dt} = M\frac{d\vec{v}}{dt} = M\vec{a} = \vec{F}_{ext}.$$

  This is Newton's II law.

  For isolated system, $\vec{F}_{\text{ext}} = \vec{0}$.

  $$\therefore \quad \frac{d\vec{p}}{dt} = F_{\text{ext}} = 0 \text{ or } \vec{p} = \text{constant}$$

  $$\therefore \quad M\vec{v} = \text{Constant.}$$

- **Moment of Force or Torque :** Torque due to a force is moment of force and measures the turning effect to the force about the axis of rotation. The general expression for torque is

  $$\vec{\tau} = \vec{r} \times \vec{F}$$

- **Angular Momentum and its Conservation :** Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about the axis. It is equal to the product of linear momentum of the particle and the perpendicular distance of the line of action of linear momentum from the axis of rotation. It is the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

  $$\vec{L} = \vec{r} \times \vec{p} = rp \sin \phi$$
  $$= \vec{d} \times \vec{p}$$

  where, $d = r\sin \phi$ = perpendicular distance of line of action of $\vec{p}$ from the axis. Angular momentum is a vector quantity, whose direction is given by **right handed screw rule.**

- $\vec{L} \perp \vec{d}$ and $\vec{L} \perp \vec{p}$
- **Rate of change of angular momentum is torque**, i.e., $\vec{\tau} = d\vec{L}/dt$.

As $$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\tau}_{ext}.$$

for isolated system $\vec{\tau}_{ext} = \vec{0}$.

$\therefore$ $$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} = \vec{0}.$$

So, $$\vec{L} = \text{constant}.$$

- **Equilibrium of Rigid Bodies :**

  1st Condition : A rigid body is said to be in translational equilibrium, if it remains at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all external forces acting on the body must be zero i.e., $\Sigma \vec{\tau}_i = \vec{0}$.

  Translational Static Equilibrium is of 3 types :

  (i) Stable Equilibrium
  (ii) Unstable Equilibrium
  (iii) Neutral Equilibrium

  2nd Condition : A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body must be zero i.e., $\Sigma \vec{\tau}_i = \vec{0}$.

- **Principle of Moments**

  According to principle of moments, body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

Key Words

- **A rigid body** is defined as a system of particles, in which distance between any two particles does not change under the influence of external forces, where, size and shape of the body will remain unaffected under the effect of external forces.
- **The centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated. If all the forces acting on the body were applied on the centre of mass, the nature of motion of the body shall remain unaffected.
- **Centre of gravity** of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.

Key Formulae

- **Position vector of centre of mass of *n* particles system**

$$\vec{r} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$$, where, M is the total mass to the system i.e., $M = \sum_{i=1}^{n} m_i$

- **For two particle system**

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

- **Coordinates of centre of mass**

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$z = \frac{m_1z_1 + m_2z_2}{m_1 + m_2}$$

- **Velocity of C.M. of a system of two particles is**

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

- **Angular Momentum**

$$\vec{L} = \vec{r} \times m\vec{v}$$

- **Equations of Rotational Motion :**

(a) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

(b) $v = r\omega$

$\omega = 2\pi\nu = \frac{2\pi}{T}$

(c) $a = r\alpha$

(d) Centripetal acceleration $= \frac{v^2}{r} = r\omega^2$

- **Torque :** $\vec{\tau} = \vec{r} \times \vec{F}$

(i) Work done by torque $dW = \tau(d\theta)$

(ii) Power of torque $P = \frac{dW}{dt} = \tau\left(\frac{d\theta}{dt}\right)$

$P = \tau\omega$

Mnemonics

Concept: Position of centre of mass of different objects.

Mnemonics: R S Das in a **C**inema hall met **C**hiranjeet **m**all **a**rea and **R**am **C**handran behind **C**entral **d**oor.

Interpretation:

R - Ring
S - Sphere
D - Disc
C - Centre

Ring, sphere, disc have centres of mass at their respective centre.

C - Cylinder
m - mid-point
a - Axis

Cylinders have centres of mass at the mid-point on the respective axis.

R - Rectangular lamina
c - cube
c - cross-point
d - diagonal

Rectangular lamina, cube have centres of mass at the cross point of respective diagonals.

Topic-2 Moment of Inertia & Radius of Gyration

Revision Notes

- **Principle of Conservation of Angular Momentum :** According to this principle, when no external torque acts on a system of particles, then the total angular momentum of system always remains a constant. i.e.,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots\ldots + \vec{L}_n = \text{constant}.$$

- **Laws of Rotational Motion**

I Law : A body continues to be in a state of rest or in a state of uniform rotation about a given axis unless an external torque is applied on the body.

II Law : The rate of change of angular momentum of a body about a given axis is directly proportional to external torque applied on the body.

III Law : When a rigid body A exerts a torque on another rigid body B in contact with it, then the body B would exert an equal and opposite torque on the body A.

- **Moment of inertia** of a body about a given axis is the property by virtue of which, the body opposes any change in its state of rest or state of uniform rotation about that axis. For a single particle, moment of inertia (I) is equal to product of mass (m) of the particle and square of perpendicular distance (r) of the particle from the axis of rotation., i.e., $I = mr^2$.

Moment of inertia is a scalar quantity, whose unit is kgm^2. It plays the same role in rotational motion as is played by the mass in linear motion.

- **Radius of gyration** of a body about a given axis is the distance (K) of a point from the given axis, where if whole mass of the body is concentrated, the body would have the same moment of inertia, as it has with the actual distribution of mass.
- **Kinetic energy of rotation** of a body is the energy possessed by body on account of its rotation about a given axis.

Key Formulae

- **Moment of Inertia—**

(i) Circular ring : (perpendicular to plane, at centre) $I = MR^2$

(ii) Circular disc : (perpendicular to plane, at centre) $I = \frac{1}{2}MR^2$

(iii) Angular disc (or ring) —

(R = outer radius, r = inner radius) $I = \frac{1}{2}(R^2 + r^2)$

(iv) Thin rod : (axis perpendicular to its length at mid point) $I = \frac{Ml^2}{12}$, Where l is the length of the rod.

(v) Soild cylinder : (along axis of cylinder) $I = \frac{1}{2}MR^2$

(vi) Hollow cylinder : $I = MR^2$

about its long axis of symmetry.

(vii) Solid sphere : (about its diameter.) $I = \frac{2}{5}MR^2$

(viii) Hollow sphere (or thin spherical shell) : (about its diameter) $I = \frac{2}{3}MR^2$

(ix) Solid cylinder (or ring) : (about central axis) $I = \frac{1}{2}MR^2$

(x) (a) Solid cylinder (about axis through its CM) : $I = M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$

(b) Hollow cylinder : $I = M\left(\frac{l^2}{12} + \frac{R^2}{2}\right)$

about an axis passing through C.G. & perpendicular to its own axis

(xi) Uniform rectangular lamina (or thin slab)— about an axis passing through C.G. & perpendicular to its plane.

$$I = M\left(\frac{l^2 + b^2}{12}\right)$$

Where, l = length, b = breadth

(xii) Elliptical disc :

$$I = \frac{M}{4}\left(a^2 + b^2\right)$$

about an axis passing through its C.G. & perpendicular to its plane.

(a = semi-major axis, b = semi-minor axis.)

(xiii) Uniform cone : about an axis joining the vertex to centre of its base.

$$I = \frac{3}{10}MR^2$$

(xiv) Triangular lamina—

$$I_1 = M \times h^2/6 \text{ (about the base as axis)}$$

$$I_2 = \frac{b^2}{6} . M \text{ (about the height as axis)}$$

$$I_3 = \frac{Mb^2h^2}{6\left(b^2 + h^2\right)} \text{ (about the hypotenuse as axis)}$$

- **Kinetic energy of rotation :**

$$K.E. = \frac{1}{2} I\omega^2,$$

- **Angular Momentum—** $L = I\omega$
- **Relation between Angular Momentum & Torque :**

$$\tau = I\alpha, \tau = dL/dt$$

- **From Principle of Conservation of Angular momentum :**

$$\frac{I_1}{I_2} = \frac{\tau_1}{\tau_2}$$

- **Radius of Gyration—**

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ...r_n^2}{n}}$$

Here, $r_1, r_2 ....r_n$ = perpendicular distance of particles from axis of rotation

n = total no. of particles.

UNIT – VI: GRAVITATION

CHAPTER-7

GRAVITATION

Topic-1 Kepler's Laws, Universal Law of Gravitation, Acceleration Due to Gravity

Revision Notes

- **Kepler's Laws of Planetary Motion :**

(a) Kepler's I Law (Law of Orbits) : Each planet revolves around the Sun in an elliptical orbit. The Sun is situated at one foci of the ellipse.

(b) Kepler's II Law (Law of Areas) : The position vector of the planet from the Sun sweeps out equal area in equal interval of time. That is the areal velocity of the planet around the Sun is constant.

(c) Kepler's III Law (Law of Periods) : The square of the time period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Universal Law of Gravitation : It states that every body in universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them and its direction is along line joining the two masses.

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = \frac{G m_1 m_2}{r^2}$$

- **Gravitational constant (G) :** It is defined as the force of attraction acting between two bodies each of unit mass, whose centres are placed at unit distance apart. The value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula of $G = [M^{-1}L^3T^{-2}]$. The value of $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}.

  The value of G being too small, we do not experience gravitational force in our daily life.
- **Gravity :** It is the force of attraction exerted by Earth towards its centre on a body lying on or near the surface of the Earth.

  Gravity is the measure of weight of the body.

  The weight of the body = mass (m) × acceleration due to gravity (g) = mg.

  The unit of weight of the body will be the same as those of force.

  Gravity is a vector quantity. It is always directed towards the centre of Earth. Gravity holds the atmosphere around the Earth.
- **Acceleration due to gravity (g) :** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone.

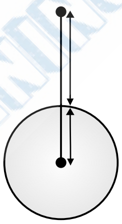
  Acceleration due to gravity is a vector quantity. It is directed towards the centre of Earth.

  The unit of g is ms^{-2} and its dimensional formula is $[M^0LT^{-2}]$.

  (a) The acceleration due to gravity does not depend upon (a) the mass of body, (b) shape or size of the body.
- **Variation of acceleration due to gravity.**

  (a) Effect of altitude : $g' = g\left(1 - \frac{2h}{R}\right)$

  (b) Effect of depth : $g' = g\left(1 - \frac{d}{R}\right)$, d = depth below Earth surface



  (c) Rotation of Earth : $g = g' + R\omega^2 \cos^2 \lambda$

  or $g' = g - R\omega^2 \cos^2 \lambda$

  where, ω is the angular velocity of rotation of Earth about its polar axis and λ is the latitude of a place.

  (i) At the equator, $\lambda = 0°$,

  so $g' = g - R\omega^2 \cos^2 0° = g - R\omega^2$

  (ii) At the poles, $\lambda = 90°$,

  so $g' = g - R\omega^2 \cos^2 90° = g$

  Hence, the value of acceleration due to gravity increases from equator to pole due to rotation of Earth. It means the value of g increases with latitude.

  (iii) When the Earth stops rotating about its own axis, there will be no change in the value of g on the poles, but there will be increase in the value of g by about 0.35% at the equator.

  (d) Shape of the Earth. Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. The polar radius of Earth is smaller than its equatorial radius by 21 km. As $g = GM/R^2$, so $g \propto 1/R^2$.

  It means the value of acceleration due to gravity increases as we go from equator to pole.

Key Words

- **Areal velocity** may be defined as the area swept by the radius vector in unit time.
- **Cavendish method** determines the value of G.
- **Geodesic** is the shortest distance between two points on Earth.

- **Aphelion** is the nearest point of planet from Sun.
- **Perihelion** is the farthest point of planet from Sun.

Key Formulae

- **Kepler's Law :**

$$T^2 = kr^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

Here, T = Time period and r = Semi major axis.

- **Newton's Gravitational Law :**

$$F = \frac{Gm_1m_2}{r^2}$$

- **Relation between g & G** $$g = \frac{GM}{R^2}$$

Mnemonics

Concept: Kepler's laws of planetary motion:
Mnemonics: Take **e**ssential **f**oods **e**veryday **2/3** **t**imes.
Interpretation:
e - elliptical
f - focus
1st law: Planet moves is **elliptical** orbit with sun at one of the **foci.**
e equal - 2nd law: A planet covers the **equal** area of space in **equal** interval of time.
2/3 - 2/3
t - time period
3rd law: Semi-major axis of the orbit is proportional to **(time period)$^{2/3}$**

Topic-2 Gravitational Potential Energy & Satellites

Revision Notes

- **Gravitational potential energy (U) :** The amount of work done in bringing a body from infinity to a point in gravitational field.

$$U = \frac{GMm}{r}$$

- **Escape Velocity :** The minimum velocity with which a body must be projected up in the space, so as to enable it to just overcome the gravitational pull.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}. \qquad \text{as } \left[g = \frac{GM}{R^2}\right]$$

- **Satellite :** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.
  - **(i) Natural satellites :** All those satellites were made by nature. e.g., Jupiter–16 Moons. Saturn–18 Moons.
  - **(ii) Artificial satellites :** All man-made satellites e.g., Aryabhatta, etc.
- Gravitational forces are **central forces** as they act along the line joining the centres of the two bodies. The gravitational forces are **conservative forces.**

Key Words

- **Gravitational field** is the space around a material body in which its gravitational pull can be experienced.
- **Gravitational field intensity** of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

- **Gravitational potential** at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.
- **Mass** of a body is the quantity of matter possessed by body.
- **Inertial mass of a body** is equal to the magnitude of external force required to produce unit acceleration in the body.
- **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.
- **Centre of Gravity (C.G.)** of a body placed in the gravitational field is that point where the net gravitational force of the field acts.

Key Formulae

- **Gravitational Intensity :** $I = \frac{F}{m_0} = \frac{GM}{R^2} = g$
- **Gravitational Potential :** $V = \frac{W}{m_0}$

$$V = -\frac{GM}{r}$$

- **Gravitational Potential Energy :** $U = \frac{-GMm}{r}$
- **Satellite :**
  - **(a)** Orbital speed : $v = R\sqrt{\frac{g}{R+h}} = \sqrt{\frac{GM}{R+h}}$
  - **(b)** Time period of revolution : $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$
  - **(c)** Height of satellite : $h = \left(\frac{T^2R^2g}{4\pi^2}\right) - R$
- **Escape speed**

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \left[g = \frac{GM}{R^2}\right]$$

UNIT – VII: PROPERTIES OF BULK MATTER

CHAPTER-8

MECHANICAL PROPERTIES OF SOLIDS

Topic-1 Elastic Behaviour of Solids

Revision Notes

- **Stress** is defined as the restoring force acting per unit area of a deformed body, i.e.,

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

The S.I. unit of stress is N/m^2 and its dimensional formula = $[ML^{-1}T^{-2}]$. Stress is a tensor quantity. Normal Stress have following three types :

(a) Longitudinal stress : If a body changes its length under a deforming force and the stress is normal to the surface of the body then the stress is called longitudinal stress. The longitudinal stress can be a tensile stress or compression stress. The longitudinal stress produced because of increase in length of body under deforming force is known as **tensile stress.** The longitudinal stress produced due to decrease in length of body under a deforming force is known as **compression stress.**

(b) Volumetric stress : If a body changes its volume under a normal deforming force acting on every surface of the body, the stress set up in the body is volumetric stress.

(c) Tangential stress : It is also called shearing stress. When a deforming force applied tangentially to the surface of the body changes the shape of the body without changing its volume, the stress set up is known as tangential stress. The shape of the body changes or the body gets twisted due to tangential stress.

- **Strain** is defined as the ratio of change in configuration of the body because of a deforming force on it, to the original configuration of the body it means

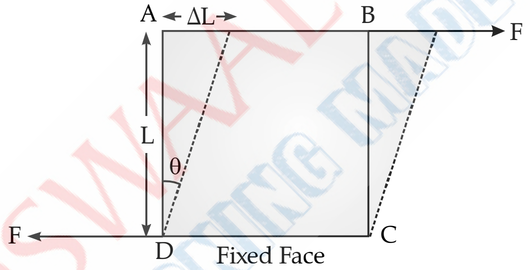
$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain can be of following three types, depending upon the direction of force applied :

(a) $$\textbf{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l}$$

(b) $$\textbf{Volumetric strain} = \frac{\text{change of volume}}{\text{original volume}} \quad \frac{\Delta V}{V}$$

(c) Shearing strain is produced when the deforming force is applied parallel to the surface of a body and body changes its shape without changing its volume. Shearing strain is defined as the angle through which a vertical line perpendicular to the fixed surface gets rotated under the effect of a tangential deforming force.



Shearing strain is also defined as the ratio of displacement of a surface (ΔL) under the tangential deforming force to the perpendicular distance (L) of the displaced surface from the fixed surface, i.e.,

$$\text{Shearing strain,} \quad \theta = \frac{\Delta L}{L}$$

Strain has no units and dimensions.

(a) If a beam is bent, both compression strain as well as extension strain are produced.

- Hooke's law states that within elastic limit, stress is directly proportional to strain, i.e.,
Stress ∝ Strain.

Key Words

- **Deforming force** is that force which when applied changes the configuration of the body.
- **Elasticity** is the property of the body by virtue of which the body regains its original configuration (length, volume or shape) when the deforming forces are removed.
- **Perfectly elastic body** is that body which perfectly regains its original form on removing the external deforming force from it, e.g., quartz.
- **Plastic body** is that body which does not regain its original form at all on the removal of deforming force, howsoever small the deforming force may be, e.g., putty and paraffin wax.
- **Elastic limit** is the upper limit of deforming force up to which if deforming force is removed, the body regains its original form completely and beyond which if the force is increased, the body loses its property of elasticity and it gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of material of a body.

Key Formulae

- Normal stress (S) = F/A
- Breaking force = Breaking stress × area of cross-section
- Longitudinal strain = $\dfrac{\Delta l}{l}$
- Volumetric strain = $\dfrac{\Delta V}{V}$
- Shearing strain, $\theta = \dfrac{\Delta L}{L}$

Topic-2 Modulus of Elasticity

Revision Notes

- **Modulus of elasticity or coefficient of elasticity (E)** of a body is defined as the ratio of stress to the corresponding strain produced, within the elastic limit, i.e.,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of elasticity is of three types :

(a) **Young's modulus of elasticity (Y)** is defined as the ratio of longitudinal stress to the longitudinal strain, within the elastic limit, i.e.,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/A}{\Delta l / l} = \frac{F}{A} \times \frac{l}{\Delta l}$$

Y is the property of solid material only. **Y** increases on mixing the impurity in the solid and decreases on increasing the temperature of the solid body.

(b) **Bulk modulus of elasticity (K)** is first defined by Maxwell. It is defined as the ratio of volume stress to the volumetric strain, within the elastic limit, i.e.,

$$K = -\frac{\text{Volume stress}}{\text{Volumetric strain}}$$

$$K = \frac{P}{\frac{\Delta V}{V}} = -\frac{PV}{\Delta V}$$

K is the property for solids, liquids and gases.

- **Modulus of Rigidity** (η) is defined as the ratio of tangential stress to the shearing strain, within the elastic limit, i.e.,

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

η is the characteristic of solid material only as the liquids and gases do not have fixed shape. η for liquid is zero.

- **Poisson's ratio** (σ). It is defined as the ratio of lateral strain to the longitudinal strain, i.e.,

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\Delta D / D}{\Delta l / l} = -\frac{\Delta D . l}{D . \Delta l}$$

Numerically value of σ lies between -1 and $+\frac{1}{2}$ but practical value of σ lies between 0 and $+\frac{1}{2}$.

Key Words

➢ **Compressibility** is defined as the reciprocal of bulk modulus of elasticity.

$$\text{Compressibility (c)} = \frac{1}{K} = -\frac{\Delta V}{PV}$$

➢ **Elastic fatigue** is the loss in strength of a material caused due to repeated alternating strains to which the material is subjected.

➢ Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

Key Formulae

➢ Young's modulus $\quad Y = \dfrac{Fl}{A\Delta l}$

➢ Bulk modulus $\quad K = -\dfrac{FV}{A\Delta V} = -P\dfrac{V}{\Delta V}$

➢ Modulus of Rigidity $\quad \eta = \dfrac{FL}{A\Delta L}$

➢ Poisson's Ratio $\quad \sigma = \dfrac{-\Delta D.l}{D.\Delta l}$

➢ **Relation Among Various Elastic Constants :**

(i) Relation between Y, K and σ, $\quad Y = 3K(1-2\sigma)$

(ii) Relation between Y, η and σ, $\quad Y = 2\eta(1+\sigma)$

(iii) Relation between K, η and σ, $\quad \sigma = \dfrac{3K-2\eta}{2\eta+6K}$

(iv) Relation between Y, K and η, $\quad \dfrac{9}{Y} = \dfrac{1}{K} + \dfrac{3}{\eta}$

➢ Elastic potential energy in stretched wire, $U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of wire}$

➢ Elastic potential energy per unit volume of wire,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} Y \times (\text{Strain})^2$$

➢ Work done in a stretching wire $\quad W = \dfrac{1}{2} \times \text{Load} \times \text{Extension.}$

Mnemonics

Concept: Value of Poisson's ratio.

Mnemonics: Priyanka **R**oy started from **zero** mark and ran for **half** an hour.

Interpretation:

P - Poisson's

R - Ratio

zero - 0

half - ½

Value of Poisson's ratio lies between 0 and ½.

CHAPTER-9

MECHANICAL PROPERTIES OF FLUIDS

Topic-1 Fluids at Rest

Revision Notes

- **Pressure :**

(i) Pressure is defined as the thrust acting per unit area of the surface in contact with liquid, i.e.,

$$P = \frac{\text{Thrust}(F)}{\text{Area}(A)} = \frac{F}{A} = h\rho g$$

(ii) Liquid pressure is independent of shape of the liquid surface as well as area of the liquid surface, but depends upon height of liquid column.

(iii) Total pressure at a depth h below liquid surface is $P = h\rho g + P_0$, where P_0 is the atmospheric pressure.

(iv) S.I. unit of pressure is Nm^{-2} or pascal (denoted by Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

(v) Pressure is a scalar quantity because a liquid at rest exerts equal pressure in all directions at all points in the same horizontal plane.

- **Pascal's Law :** It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same. Pascal's law also states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of liquid provided the gravity effect is neglected.

- **Atmospheric pressure :**

(i) It is defined as the pressure exerted by atmosphere.

(ii) At S.T.P., the value of atmospheric pressure is 1.01×10^5 Nm^{-2} or 1.01×10^6 dyne/cm^2.

- **Archimedes' principle :** It states that when a body is immersed partly or wholly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Observed weight of body = True weight – Weight of liquid displaced.

If w is the observed weight of body of density ρ when immersed in a liquid of density σ, then

$$\begin{aligned} w &= Mg - mg \\ &= Ah\rho g - Ah\sigma g \\ &= Ahg(\rho - \sigma) \\ &= Ahg\rho\left(1 - \frac{\sigma}{\rho}\right) = W\left(1 - \frac{\sigma}{\rho}\right) \end{aligned}$$

$\therefore$ True weight,
$$W = \frac{\text{apparent weight}}{(1 - \sigma/\rho)}$$

- **Laws of floatation :** It states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.

(a) When true weight of the body $W > w$ (weight of the liquid displaced), the body will sink to the bottom of the liquid. It will be so when the density of solid body (ρ) is greater than the density of liquid (σ), i.e., $\rho > \sigma$.

(b) When $W < w$, the body will rise above the surface of liquid to such an extent that the weight of the liquid displaced by immersed part of the body (i.e., upward thrust) becomes greater than the weight of the body. The body then will float. In this case the density of solid body is less than the density of liquid, i.e., $\rho < \sigma$.

(c) When $W = w$, the body is at rest anywhere in the liquid. The body will float with its whole volume just immersed in the liquid. In this case the density of body is equal to density of liquid, i.e., $\rho = \sigma$.

There will be equilibrium of floating body when

(i) Weight of liquid displaced by the immersed part of body is equal to the weight of the body.

(ii) The centre of gravity of the body and the centre of buoyancy lie along the same vertical line.

(iii) If the centre of gravity of the body lies vertically below the meta centre then body is in stable equilibrium. The body will be in unstable equilibrium if centre of gravity lies vertically above the meta centre.

Key Words

- **Fluid** is the name given to a substance which begins to flow when external force is applied on it. It includes liquid and gas.
- **Thrust :** The total normal force exerted by liquid at rest on a given surface in contact with it is known as thrust of liquid on that surface. It is due to collision of molecules of liquid while moving at random, with the walls of the container and rebounding from them.
- **Buoyancy** is the upward force acting on the body immersed in a fluid.
- **Metacentre** is a point where the vertical line passing through the centre of buoyancy intersects the central line.

Key Formulae

- Pressure $= \frac{F}{A} = h\rho g$ (due to h height of liquid)

  h = height, ρ = Density of liquid, g = Acceleration due to gravity.
- Gauge pressure = Total pressure – Atmospheric pressure
- For Hydraulic lift; $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

  F_1, F_2 = Forces on pistons of area of cross - sections A_1, A_2
- Density $= \frac{\text{Mass}}{\text{Volume}}$, Relative density $= \frac{\text{Density of substance}}{\text{Density of water at 4°C}}$
- **Archimedes' Principle :**

  Loss of weight of body in liquid = Weight of liquid displaced = Volume × Density of liquid × g
- **Law of floatation :**

  A body will float if, weight of body = Weight of liquid displaced.

Topic-2 Surface Energy & Surface Tension

Revision Notes

- **Surface Tension :** It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.
  - **(a)** Quantitatively, surface tension of a liquid is measured as the force acting per unit length of a line imagined to be drawn tangentially any where on the free surface of the liquid at rest. It acts at right angles to this line on both the sides and along the tangent to the liquid surface, i.e., $S = F/l$.
  - **(b)** Surface tension of a liquid is also defined as the amount of work done in increasing the free surface of liquid at rest by unity at constant temperature, i.e., $S = W/A$.

    or $\quad W = S \times A$ = Surface tension × Area of liquid surface formed.
  - **(c)** Surface tension is a molecular phenomenon and it arises due to electromagnetic forces. The explanation of surface tension was first given by Laplace.
  - **(d)** S.I. unit of surface tension is Nm^{-1} or Jm^{-2} and C.G.S. unit is dyne-cm^{-1} or erg-cm^{-2}.
  - **(e)** Dimensional formula of surface tension = $[ML^0T^{-2}]$
  - **(f)** Surface tension is a scalar quantity as it has no specific direction for a given liquid.
  - **(g)** Surface tension does not depend upon the area of the free surface of liquid at rest.
- **Surface Energy :** It is defined as the amount of work done against the force of surface tension in forming the liquid surface of a given area at a constant temperature, i.e.,

  Surface energy = Work done = Surface tension × Surface area of liquid.

  The S.I. unit of surface energy is Joule and C.G.S. unit is erg.
  - **(a)** If small drops combine together to form a big drop, the surface area decreases, so surface energy decreases. Therefore the energy is released. If this energy is taken by drop, the temperature of drop increases.
  - **(b)** If a big drop is splitted into number of smaller drops, the surface area of drops increases. Hence, surface energy increases. So energy is spent. If this energy is pronated by drop, the temperature of drop decreases e.g., spray.

Key Words

- **Surface film** is the top most layer of liquid at rest with thickness equal to the molecular range.
- **Angle of contact** between a liquid and solid in contact is defined as the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid.
- **Capillary tube** is a tube with a fine and uniform bore throughout its length.
- **Capillarity** is the phenomenon of rise or fall of liquid in a capillary tube.

Key Formulae

- Surface tension, $S = F/l$
- Surface energy, E = Work done
- Work done, $W = S \times$ Increase in area
- Excess of pressure inside the liquid drop is

$$P = P_i - P_o = \frac{2S}{r};$$

P_i = Pressure inside bubble

- Excess of pressure inside the soap bubble is

$$P = P_i - P_o = \frac{4S}{r};$$

P_0 = Pressure outside bubble

- Total pressure in the air bubble at a depth h below the surface of liquid of density ρ is

$$P = P_o + h\rho g + \frac{2S}{r}$$

- Ascent/Decent Formula : $h = \dfrac{2S\cos\theta}{r\rho g}$

where,
r = radius of capillary tube
ρ = density
S = Surface tension
θ = angle of contact

Mnemonics

Concept: Excess pressure inside liquid drop, air bubble and soap bubble.
Mnemonics: **E**mily **p**urchased **t**wo **S**wiss **r**olls for **L**ata **d**idi and **A**mit **b**haiya and **f**our **s**wiss **r**olls for **S**hanti **b**ahin.
Interpretation:
E - Excess
P - pressure
T - Two
s - Surface tension
r - Radius
L - Liquid
d - drop
A - Air
b - bubble
Excess pressure in liquid drop and in air bubble are 2S/R.
f - four
s - Surface tension
r - Radius
S - soap
b - bubble
Excess pressure in soap bubble is 4S/R.

Topic-3 Viscosity & Bernoulli's Theorem

Revision Notes

- **Bernoulli's theorem :** Bernoulli's theorem state that the total energy per unit volume (pressure energy, P.E. and K.E.) per unit volume of an incompressible non-viscous liquid in steady flow remain constant throughout the flow of the liquid $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$.
- **Torricelli's theorem :** According to this theorem, velocity of efflux i.e., the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

Key Words

- **Viscosity** is the property of liquid due to which a backward dragging force called viscous force act tangentially on the layer of the liquid in motion.
- **Terminal velocity** is the maximum constant velocity acquired by the body while falling freely in a viscous medium.
- **Streamline flow of a liquid** is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.
- **Streamline** is the actual path followed by the procession of particles in a steady flow which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point.
- **Tube of flow** is the bundle of streamlines having the same velocity of the liquid particle over any cross-section perpendicular to the direction of flow of liquid.
- **Laminar flow** is a flow in which the liquid moves in layers.
- **Turbulent flow** is a flow when a liquid moves with a velocity greater than its critical velocity, the motion of particles of liquid becomes disorderly or irregular.
- **Critical velocity** is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.
- **Reynold number** is a pure number which determines the nature of flow of liquid through a pipe.

Key Formulae

- **Newton's viscous drag force :** $F = \pm \eta A \frac{dv}{dx}$

  η = coeff. of viscosity, A = area of layer of liquid, dv/dx = velocity gradient.
- **Poiseuille's Formula :** $V = \frac{\pi p r^4}{8\eta l}$

  p = Pressure difference across length l of horizontal tube of radius r & V = volume.
- **Stoke's Law :** $F = 6\pi\eta r v$
- **Terminal velocity :**

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where,

ρ = density of spherical body

σ = density of medium

r = radius of spherical body

η = coefficient of viscosity

➢ **Reynold's Number :**

$$R_N = \frac{\rho D v}{\eta}$$

➢ **Bernoulli's Theorem :**

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant} \qquad \text{or} \quad \frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

$\frac{P}{\rho}$ = pressure energy per unit mass $\qquad$ $\frac{P}{\rho g}$ = Pressure head, h = gravitational head

$gh = P.\,E.$ per unit mass $\qquad$ $\frac{v^2}{2g}$ = Velocity head

$\frac{1}{2}v^2 = K.E.$ per unit mass

Velocity of efflux : $\qquad v = \sqrt{2gh}$

CHAPTER-10

THERMAL PROPERTIES OF MATTER

Topic-1 Thermal Expansion & Heat Capacities

Revision Notes

➢ **Four Scales of Temperature :**

S. No.	Scale	Ice point	Steam point	No. of divisions	Smallest division
1.	Centigrade scale	0°C	100°C	100	1°C
2.	Fahrenheit scale	32°F	212°F	180	1°F
3.	Thermodynamical scale or Absolute Kelvin scale	273 K	373 K	100	1 K

➢ **Thermal Expansion of solids. :** It is the phenomenon of expansion of solids on heating. It is of three types :

(a) Linear Expansion : It is the increase in length of a solid on heating. α is called coefficient of linear expansion.

(b) Area Expansion : It is the increase in surface area of a solid on heating. β is called coefficient of area expansion.

(c) Volume Expansion : It is the increase in volume of a solid on heating. γ is called coefficient of volume expansion.

➢ **Expansion of liquids :**

(a) Coefficient of real expansion of a liquid is defined as the real increase in volume of the liquid per unit original volume per °C rise in temperature. If γ_r is the coefficient of real expansion of a liquid, then

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

(b) Coefficient of apparent expansion of a liquid is defined as the apparent increase in volume per unit original volume per °C rise in temperature. If γ_a is the coefficient of apparent expansion of a liquid, then

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Key Words

- **Heat** is a form of energy, which produces the sensation of warmth. The thermal energy in matter is present in the form of translational, rotational and vibrational energy of its atoms/molecules.
- **Temperature** of a body is a measure of degree of hotness/coldness of the body. This macroscopic property determines the direction of flow of heat, when the given body is placed in contact with some other body.
- Anomalous expansion of water is the decrease of volume of water with increase in temperature from 0°C to 4°C.
- **Specific heat of a substance** is the amount of heat required to raise the temperature of unit mass of substance through unit degree.
- **Molar specific heat of a substance** is the amount of heat required to raise the temperature of 1 g mole of substance through 1°C.
- **Heat capacity or thermal capacity** of a body is the amount of heat required to raise the temperature of whole body through 1°C or 1 K.
- **Water equivalent** is the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature & represented by W.
- **Change of state** is the conversion of one of the states of matter to another.
- **Latent heat** of a substance is the amount of heat required to change the state of unit mass of the substance at constant temperature ($Q = ML$). Its units are cal/g or joule/kg and its dimensions are [L^2T^{-2}].

Key Formulae

- **Temperature**

  (a) Relation between °C and °F is

$$\frac{C}{5} = \frac{F-32}{9}$$

  (b) T K = (t°C + 273) or t°C = (T K – 273)

  (c) Temperature. diff. of 1°C = Temp. diff. of 1 K.

  (d) Normal body temperature of a person is 98.6°F or 37°C.

  (e) The temp. of – 40° is same in °C and °F.

- **Thermal expansion**

  (a) Coefficient of linear expansion $\alpha = \dfrac{\Delta L}{L\Delta T}$

  (b) Coefficient of Area expansion $\beta = \dfrac{\Delta S}{S(\Delta T)}$

  (c) Coefficient of volume expansion $\gamma = \dfrac{\Delta V}{V(\Delta T)}$

  (d) $\beta = 2\alpha$; $\gamma = 3\alpha$

  (e) In liquids. $\gamma_r = \gamma_a + \gamma_g$

where,

γ_r = Coefficient of real expansion of liquid

γ_a = Coefficient of apparent expansion of liquid

γ_g = Coefficient of expansion of vessel

- **Specific heat.** $\Delta Q = ms\Delta T$
- **Molar specific heat** $C = M \times s$
- **Latent heat** $\Delta Q = ML$

  L = Latent heat
- **Specific heat of gases** $C_p - C_V = R$, Here $R = \dfrac{PV}{T}$
- Calorimetric principle: The heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings.

Mnemonics

Concept: Relation between coefficients of thermal expansion.

Mnemonics: **A**tal and **V**ikie stood **2nd** and **3rd** respectively in **last** exam.

Interpretation:

A - Areal thermal expansion

V - Volume thermal expansion

2nd - 2

3rd - 3

last - Linear thermal expansion

Coefficient of Areal Expansion = 2 × Coefficient linear expansion Coefficient of Volume Expansion = 3 × Coefficient linear expansion

Topic-2 Heat Transfer

Revision Notes

- **Thermal Conductivity :**

  (i) Coefficient of Thermal Conductivity : It is equal to rate of flow of heat per unit area per unit temperature gradient across the solid at steady state. It is represented by K and its value depends on nature of material of solid.

$$K = \frac{\Delta Q \Delta x}{\Delta T A}$$

$$K = \Delta Q, \text{ when } \left(\frac{\Delta x}{\Delta T}\right) = 1, A = 1$$

  (ii)Thermal resistance corresponds to electrical resistance (V/i) and is given by the ratio of temperature difference and rate of flow of heat i.e.,

$$R_{\text{Th}} = \frac{T_1 - T_2}{dQ/dt}$$

$$= \frac{x}{KA}$$

- **Total emittance or emissive power** of a body at a certain temperature is the total amount of thermal energy emitted per unit time per unit area of the body for all possible wavelengths. It is represented by e'.

$$e = \int_0^\infty e_\lambda d\lambda$$

- **Emissivity** (ε) of a body at a given temperature is the ratio of emissive power of the body (e) to the emissive power of perfectly black body (E) at that temperature,

  i.e., $$\varepsilon = \frac{e}{E}$$

  Similarly, we can define monochromatic absorptance or spectral absorptive power. Total absorptance or absorbing power

$$a = \int_0^\infty a_\lambda d\lambda$$

➢ **Kirchhoff's Law.** From this law, at a given temperature and for a given wavelength, the ratio of spectral emissive power (e_λ) to spectral absorptive power (a_λ) for all bodies is constant which is equal to spectral emissive power of a perfectly black body (E_λ) at the same temperature and for the same wavelength, i.e., $\frac{e_\lambda}{a_\lambda} = E_\lambda$ clearly, $e_\lambda \propto a_\lambda$ it means good emitters are good absorbers. The law implies that at a particular temperature, a body can absorb only those wavelengths, which it is capable of emitting.

➢ **Wien's law :** From this law, the wavelength (λ_m) corresponding to which energy emitted/sec/ unit area by a perfectly black body is maximum, is inversely proportional to the absolute temperature (T) of the black body.

$$\lambda_m \propto \frac{1}{T}$$

or,
$$\lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is known as Wien's constant $b = 2.898 \times 10^{-3}$ mK.

➢ **Newton's law of cooling.** According to this law, when difference in temperature of a liquid and its surroundings is small (~ 30°C), then the rate of loss of heat of the liquid is directly proportional to difference in temperatures of the liquid and the surroundings, i.e.,

$$-\frac{dQ}{dt} \propto (\theta - \theta_0)$$

or,
$$-\frac{dQ}{dt} = K(\theta - \theta_0)$$

where K is constant of proportionality.

➢ **Stefan's law :** From this law, the total energy (E) emitted/sec/ unit area by a perfectly black body corresponding to all wavelengths is directly proportional to fourth power of the absolute temperature (T) of the body *i.e.*

$$E \propto T^4$$

or,
$$E = \sigma T^4$$

There σ is a constant of proportionality and is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-8} \text{ watt m}^{-2}\text{K}^{-4}$$

If Q is the total amount of heat energy emitted by the black body, then by definition,

$$E = \frac{Q}{At}$$

$$\therefore \quad Q = At \times E = At(\sigma T^4)$$

If the body is not perfectly black and has an emissivity e, then $Q = eAt(\sigma T^4)$

➢ **Stefan Boltzman law :** From this law, the net amount of radiation emitted per second per unit area of a perfectly black body at temperature T is equal to difference in the amounts of radiation emitted/sec/unit area by the body and by the black body enclosure at T_0.

$\Rightarrow$
$$E' = E - E_0$$
As
$$E = \sigma T^4$$
and
$$E_0 = \sigma T_0^4$$
$\therefore$
$$E' = \sigma T^4 - \sigma T_0^4 = \sigma(T^4 - T_0^4)$$

Proceeding as above, total energy lost

$$Q' = E'At = At\sigma(T^4 - T_0^4)$$

When the body and enclosure are not perfectly black and have emissivity ε, then

$$Q' = \varepsilon At\sigma(T^4 - T_0^4)$$

Key Words

➢ **Conduction** is the mode of transfer of heat from one part of the body to another, from particle to particle in the direction of fall of temperature without any actual movement of heated particles.

➢ **Thermal convection** is the phenomenon of transfer of heat by actual mass motion of the medium. All liquids and gases are heated by convection.

- **Radiation** is the phenomenon of transfer of heat from source to the receiver without any actual movement of source or receiver and without heating the intervening medium. For example, heat comes to us from the Sun through radiation.
- **Perfectly black body** is that which absorbs all the radiations incident upon it. Thus absorptive power of a perfectly black body is unity (*i.e.*, 100%). When such a body is heated to high temperature, it would emit radiations of all wavelengths.

Key Formulae

- **Rate of conduction of Heat,** $\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$
- **Thermal resistance,** $R_{Th} = \frac{T_1 - T_2}{dQ/dt}$
- **Emissive power,** $e' = \int_0^{\infty} e_{\lambda} d\lambda$
- **Emissivity,** $\varepsilon = \frac{e}{E}$

UNIT – VIII: THERMODYNAMICS

CHAPTER-11

THERMODYNAMICS

Revision Notes

- **Thermodynamics:** Thermodynamics is the branch of physics that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy. Thermodynamics is a macroscopic science. It deals with bulk systems and does not go into the molecular constitution of matter.
- **Thermal equilibrium:** A system is in equilibrium if the macroscopic variables (pressure, volume, temperature, mass and composition) those characterise the system do not change with time.
- **Adiabatic wall:** Adiabatic wall is an insulating wall that does not allow flow of energy (heat) from one to another.
- **Diathermic wall:** Diathermic wall is a conducting wall that allows flow of energy (heat) from one to another.
- **Zeroth Law of Thermodynamics:** Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.
- **Internal energy:** Every bulk system consists of a large number of molecules. Internal energy is the sum of the kinetic energies and potential energies of these molecules.
- **Equivalence of work and heat:** Work is a form of heat energy. 4.18×10^3 Joule of work is equivalent to 1 kilocalorie of heat.

$$W = JQ$$

J is the mechanical equivalent of heat.

- **Sign convention of heat:**

(i) Negative when heat is given from a system to its surroundings.

(ii) Positive when heat is taken from the surroundings by the system.

- **First law of thermodynamics:** If an amount of heat ΔQ is given to a system, a part of it is increases the internal energy ΔU of the system and the rest is utilized in doing work ΔW by the system.

$$\Delta Q = \Delta U + \Delta W$$

  - **In cyclic process:** In cyclic process, a system is taken from one initial state to other different states and finally brought back to its initial state.

    So, there is no change in internal energy i.e.

$$\Delta U = 0$$

So, $\Delta Q = \Delta W$

- **In isobaric process:** In isobaric process the pressure remains constant.

$$\text{Work done } = \Delta W = P\Delta V$$

So, $\Delta Q = \Delta U + P\Delta V$

- **In isochoric process:** In isochoric process the volume remains constant.

So, work done $= \Delta W = P\Delta V = 0$

So, $\Delta Q = \Delta U$

- In isothermal process: In isothermal process the temperature remains constant.

$$\Delta Q = \Delta U + \Delta W$$

(For ideal gas, the internal energy depends on temperature only. As temperature is constant, then $\Delta U = 0$.

So, $\Delta Q = \Delta W$)

- In adiabatic process: In adiabatic process heat neither enters nor leaves the system.

Hence, $\Delta Q = 0$

So, $\Delta U = -\Delta W$

➢ **Thermodynamic processes :**

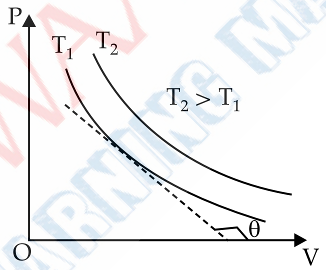
(a) **Isothermal process,** where the temperature remains constant. The pressure and volume of a given mass of gas changes.

Essential conditions :

(i) Walls of container must be perfectly conducting.

(ii) Changes must be slow.

Isothermal process obeys Boyle's law i.e., PV = constant.



Variation of P with V at constant temperature is represented by Isothermal curves.

Slope of an isothermal graph is negative.

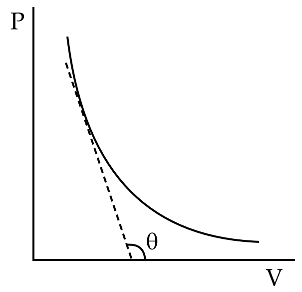
(b) **Adiabatic process,** where the heat content of a gaseous system remains constant. The pressure and volume of given mass of gas change with consequent change in temperature.

Essential conditions are :

(i) Walls of container must be perfectly insulating.

(ii) Changes must be sudden.

Adiabatic process obeys Poission's law i.e., PV^{γ} = constant.



The variation of P with V at constant heat content is represented by an Adiabatic curve.

$$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \gamma$$

Since, γ is always greater than 1, adiabatic curve is steeper than isothermal curve.

(c) **Isobaric process,** where pressure is kept constant.
Since, temperature changes, so does internal energy. The heat absorbed goes partly to increase internal energy and partly to do work. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant pressure.

(d) **Isochoric process,** where volume is kept constant.
In an isochoric process, V is constant. No work is done on or by the gas. The heat absorbed by the gas goes entirely to change its internal energy and its temperature. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant volume.

- **Work done in isothermal and adiabatic process:**
  Area under the curve is the measurement of work done.
  - For same expansion,
    Work done by isothermal process > work done in adiabatic process.
  - For same compression:
    Work done by adiabatic process > work done in isothermal process.
- **2nd law of thermodynamics:**
  There are two statements:
  - Kelvin-Planck statement: No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.
  - Clausius statement: No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.
- **Reversible process:** The process which can be reversed in such a way that all changes taking place in the direct process are exactly repeated in reverse order and opposite sense is called reversible process.
- **Irreversible process:** The process which cannot be reversed is called irreversible process.

Key Words

- **Thermodynamical system & thermodynamical parameters.**
  A gaseous system is called a thermodynamical system. The state of the system is represented in terms of pressure (P), volume (V), temperature (T) and heat content (Q) of the gas. These four quantities are called thermodynamical parameters of the system.
- **Open system :** System which exchanges both energy & matter with surroundings.
- **Closed system :** System which exchanges only energy with surroundings.
- **Isolated system :** System which Exchanges neither energy nor matter with surroundings.
- **Equation of state** is the equation connecting pressure, volume and temperature of the gas.
- **Isothermal process,** where the temperature remains constant.
- **Adiabatic process,** where the heat content of a gaseous system remains constant.
- **Isobaric process,** where pressure is kept constant.
- **Isochoric process,** where volume is kept constant.
- **Reversible process** is a process which can be reversed back to initial state.
- **Irreversible process** is a process which cannot be traced back in opposite direction.

Key Formulae

- **Equation of state for:**

(a) Isothermal process: $PV = \text{Constant.}$

(b) Isobaric process: $\frac{V}{T} = \text{Constant.}$

(c) Isochoric process: $\frac{P}{T} = \text{Constant.}$

(d) Adiabatic process: PV^{γ} = constant;

$TV^{\gamma-1}$ = constant and

$$\frac{P^{\gamma-1}}{T^{\gamma}} = \text{constant.}$$

➢ **Work done during expansion of gas:** $dW = PdV$ (for constant pressure)

or $W = \int_{V_1}^{V_2} PdV$ (for variable pressure)

(a) In an isothermal process :

$$W = 2.3026\, RT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$W = 2.3026\, RT \log_{10}\left(\frac{P_1}{P_2}\right)$$

(b) In an adiabatic process :

$$W = \frac{R}{1-\gamma}(T_2 - T_1)$$

$$= \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

Mnemonics

➢ **Concept:** Four thermodynamic processes.

➢ **Mnemonics:** **T**oday **I**ncome **T**ax was **p**aid in **I**ndian **B**ank by **V**ijoy in **I**ndian **C**urrency, **ad**jacent to **N**ational **h**ighway.

Interpretation:

T → temperature is constant
I → Iso
T → Thermal

} Temperature remains constant in isothermal process.

P → pressure
I → Iso
B → baric

} Pressure remains constant in isobaric process.

V → volume
I → Iso
C → Choric

} Volume remain constant in isochoric process.

Ad → adiabatic
N → No
h → heat transfer

} No heat transfer between system and surrounding takes place in adiabatic process.

UNIT – IX: BEHAVIOUR OF PERFECT GASES AND KINETIC THEORY OF GASES

CHAPTER-12

KINETIC THEORY

Revision Notes

➢ **Ideal Gases :**

(a) Strictly obeys gas laws, like Boyle's law, Charles's law etc.

(b) The size of the gas molecules is almost zero and the volume of the gas molecule is also almost zero.

(c) There is no force of attraction or repulsion amongst the molecules.
(d) Collisions between molecules are perfectly elastic.

- **Equation of state of Ideal gas:**

(a) **From Boyel's law:** $P_1V_1 = P_2V_2$

(b) **From Charles' law:** $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

(c) **Combining Charles' and Boyel's law:** $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

or $PV = KT$

K is a proportionality constant.

Again, $K = nk_B$

Where n = number of molecules

K_B = Boltzmann constant

$\therefore$ $PV = nk_B T$

For 1 gram mole, $PV = RT$

For μ mole, $PV = \mu RT$

Where $\mu = \frac{m}{M}$

m = mass of gas sample

M = molecular weight

$\therefore$ $PV = \frac{m}{M}RT$

Number of molecules in 1 mole (i.e., M gram) gas = Avogadro number (N)

So, number of moles in m gram gas $= \frac{Nm}{M}$

So, $\frac{m}{M} = \frac{n}{M}$

$\therefore$ $PV = \frac{n}{N}RT$

Value of R = 8.31 J mol$^{-1}K^{-1}$

Value of k = 1.38×10^{-23} J K^{-1}

- **Assumptions of Kinetic Theory of Gases :**

(a) A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for different gases.
(b) The molecules of a gas are in a state of continuous, rapid and random motion.
(c) The volume occupied by the molecules is negligible in comparison to the volume of the gas.
(d) The molecules do not exert any force of attraction or repulsion on each other, except during collision.
(e) The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic.
(f) Molecular density is uniform throughout the gas.
(g) A molecule moves along a straight line between two successive collisions.
(h) The collisions are almost instantaneous.

- **Brownian Motion :** It is defined as continuous zig-zag motion of particles of macroscopic size (10^{-5} to 10^{-6} m) suspended in fluid.

Brownian motion increases :

(a) When size of suspended object increases.
(b) When density of fluid is decreases.
(c) When temperature of medium increases.
(d) When viscosity of medium decreases.

- **Pressure of an ideal gas:**

Pressure exerted by an ideal gas is equal to two third of its translational kinetic energy per unit volume.

$$P = \frac{2}{3}E = \frac{1}{3}\frac{mn}{V}\overline{v^2} = \frac{1}{3}\rho\overline{v^2}$$

Where, $\overline{v^2}$ = mean square velocity of the molecules

mn = Mass of the whole gas

V = Volume of the gas

ρ = Density of gas

- **Root mean square speed**

Root mean square speed of the molecules is defined as the square root of the mean of the squares of speed of all the gas molecules.

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2}{n}}$$

Since, pressure of a gas $= P = \frac{1}{3}\rho\overline{v^2}$

$$\therefore \quad v_{rms} = \sqrt{\frac{3P}{\rho}}$$

- **Interpretation of temperature:**

The root mean square speed of the molecules of a gas is directly proportional to the square root of absolute temperature of gas.

$$\therefore \quad v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} \propto \sqrt{T}$$

When $T = 0, v_{rms} = 0$

So, absolute temperature is the temperature at which the motion of the molecules of the gas becomes zero and no temperature below absolute zero is possible.

- **Degrees of freedom:**

Degrees of freedom: The degree of freedom for a dynamic particle is the number of directions in which that can move freely or the total number of coordinates required to describe completely the position and configuration of the particle.

A gaseous molecule has three types degrees of freedom: translational, rotational and vibrational.

- Translational degrees of freedom arise from the ability of gas molecules to move freely in space. So, the translational motion of the molecule of gas has three degrees of freedom associated with it. This is applicable for all gas molecules, whether they are monatomic, diatomic or polyatomic,
- Rotational degrees of freedom represent the number of unique ways the molecule may rotate in space about its center of mass. A monatomic gaseous molecule has no rotational degrees of freedom. Linear molecule has two rotational degrees of freedom.

  However, non-linear molecules have three rotational degrees of freedom.
- Vibrational degrees of freedom arises when the bonds of the molecules behave like a spring and the molecule execute simple harmonic motion.

Atomicity		Degrees of freedom			
1		3	0	0	3
2		3	2	1	6
3	linear	3	2	4	9
	Non-linear	3	3	3	9
N	Linear	3	2	3N - 5	3N
	Non-linear	3	3	3N - 6	3N

- **Law of Equipartition of energy :** It states that the energy for each degree of freedom in thermal equilibrium is 12 k_BT.
- **Molar specific heat:**
  - **For monoatomic gas:** The molecule of a monatomic gas has only three translational degrees of freedom.

The molar specific heat at constant volume, $C_V = \frac{3}{2}R$

$$C_P = C_V = R$$

$$\therefore \quad C_P = \frac{3}{2}R$$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

- **For diatomic gas:** A diatomic molecule has 3 translational and 2 rotational degrees of freedom.

$$C_V = \frac{5}{2}R$$

$$C_P - C_V = R$$

$$\therefore \quad C_P = \frac{7}{2}R$$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{9}{7}$$

Key Words

- **Most probable speed** of the molecules of a gas is that speed which is possessed by maximum fraction of total number of molecules of the gas.
- **Mean speed or average speed** is the average speed with which molecules of a gas move.
- **Root mean square speed** is defined as the square root of the mean of the squares of random velocities of individual molecules of a gas.
- **Absolute zero of temperature** may be defined as that temperature at which the root mean square velocity of gas molecules reduces to zero.
- **Pressure** exerted by gas is due to continuous bombardment of gas molecules against the walls of container.
- **Degrees of freedom** of a dynamic system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system.
- **Mean free path** is the average distance covered between two successive collisions by the gas molecule moving along the straight line.

Key Formulae

- **Most probable speed :** $c_{mp} = \sqrt{\frac{2k_BT}{m}}$
- **Average speed:** $c_{av} = \sqrt{\frac{8k_BT}{\pi m}}$
- **Root mean square speed:** $c_{rms} = \sqrt{\frac{3k_BT}{m}}$

  k_B = Boltzman constant, T = Temperature, m = mass
- **Ratio among speeds:** $c_{mp} : c_{av} : c_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$
- **Pressure exerted by ideal gas:** $P = \frac{2}{3}E = \frac{1}{3}\frac{mn}{V}\overline{v^2} = \frac{1}{3}\rho\overline{v^2}$
- **Root mean square velocity:** $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2}{n}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$
- **Law of equipartition of energy:** $E_t = \frac{1}{2}k_BT$
- **Specific Heat Capacity of :**

  (a) **Monoatomic gas:** $\gamma = \frac{C_P}{C_V}$ or $\gamma = \frac{5}{3} = 1.67$

  (b) **Diatomic gas:** $\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$

  (c) **Triatomic gas:**

  Linear gas molecules $\gamma = \frac{9}{7} = 1.28$

Non-linear gas molecules $\gamma = \frac{4}{3} = 1.33.$

(d) **Polyatomic gas:** $\gamma = \left(1+\frac{2}{n}\right)$, where n is the degree of freedom

➢ **Mean free path:** $\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$

Mnemonics

Concept: Degrees of freedom.

Mnemonics: Baa Baa Black Sheep

Have you any wool?

Yes, sir, **M**om has **3** bags full.

Dadi needs **5** bags **n**ormally rule.

Papa keeps **6** bags normal rule.

Papa **D**adi each needs **2** bags more

High **c**old whenever, be very sure.

Interpretation: Mom has **3** bags full → Degrees of freedom of Monoatomic gas is 3.

Dadi needs **5** bags **n**ormally cool

Degrees of freedom of diatomic gas at normat → (room) temperature is 5.

Papa keeps **6** bags **n**ormal rule → Degrees of freedom of Polyatomic gas at normal (room) temperature is 6.

Papa, **D**adi each needs **2** bags more → Degrees of freedom of Polyatomic gas at high temperature is 6 + 2 = 8.

High **c**old whenever, be very sure → Degrees of freedom of Diatomic gas at high temperature is 5 + 2 = 7.

UNIT – X: OSCILLATIONS AND WAVES

CHAPTER-13

OSCILLATIONS

Revision Notes

➢ **Harmonic Oscillations :** Those oscillations which can be expressed in terms of single harmonic function. i.e., (sine function or cosine function).

$$y = a\sin \omega t \text{ or } y = a\cos \omega t.$$

➢ **Non-Harmonic Oscillations :** Those oscillations which cannot be expressed in terms of single harmonic function i.e.,

$$y = a\sin \omega t + b\sin 2\,\omega t.$$

➢ **Periodic Functions :** Those functions which are used to represent periodic motion i.e.,

$$f(t) = f(t + T) = f(t + 2T).$$

sine & cosine functions are periodic functions.

➢ **Phase :** Phase of vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of particle at that instant with respect to its mean position.

➢ **Some Facts :**

(a) In oscillatory motion, the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalised equation of motion of the vibrating particle.

(b) When the displacement of the particle executing a vibratory motion is represented by $y = a\sin(\omega t + \phi)$, then $(\omega t + \phi)$ is called phase of the vibrating particle.

(c) ϕ is called the initial phase of the vibrating particle.

➢ **Simple Harmonic Motion :** It is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean (*i.e.*, equilibrium) position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean (i.e., equilibrium) position at that instant, i.e.,

$$F = -ky$$

where k is known as force constant. Here negative sign represent that the restoring force (F) is always directed towards the mean position.

- **Geometrical interpretation of S.H.M. :** S.H.M. is defined as the projection of a uniform circular motion on any diameter of a circle of reference.

(a) S.H.M. may be linear and angular S.H.M.

(b) The linear S.H.M. is always along a straight line about a fixed point on a line, whereas the angular S.H.M. is always along an arc of a circle about a fixed point on the arc.

(c) The linear S.H.M. is controlled by force law, where $F = -ky$, where k is the restoring force constant, i.e., force per unit displacement.

(d) The angular S.H.M. is controlled by torque law, where $\tau = -C\theta$, where C is the restoring torque constant, i.e., restoring torque per unit twist.

- **Characteristics of S.H.M. :**

(a) **Displacement :** The displacement of a particle executing linear S.H.M. at an instant is defined as the distance of the particle from the mean position at that instant.

(b) **Velocity :** is defined as the time rate of change of the displacement of the particle at the given instant.

(c) **Amplitude :** The maximum displacement on either side of mean position.

(d) **Acceleration :** It is defined as the time rate of change of the velocity of the particle at the given instant.

(e) **Time Period :** It is defined as the time taken by the particle executing S.H.M. to complete one vibration.

- **Restoring Force & Force Constant :** Force constant is the force required to give unit displacement to the body.

$$F = -ky$$ Here, k is force constant.

- **Simple Pendulum :** It is most common example of S.H.M. An ideal simple pendulum consists of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

$$\text{Time period, } = 2\pi\sqrt{\frac{l}{g}}$$

- **Energy of S.H.M :** A particle executing S.H.M. possesses two types of energy :

  (a) Potential Energy : This energy is on account of the displacement of the particle from its mean position.

  (b) Kinetic Energy : This energy is on account of the velocity of the particle.

Key Words

- **Periodic motion :** The motion which is identically repeated after a fixed interval of time.
- **Oscillatory motion or vibratory motion :**

  The motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.

  Some terms related to periodic motion :

  Time period : It is the least interval of the time after which the periodic motion of a body repeats itself.

  Frequency : It is defined as the no. of periodic motions executed by the body per second.

  Angular frequency : It is equal to the product of frequency of the body with factor 2π. i.e., $\omega = 2\pi\nu$.

  Displacement : It is the change in position under consideration with time in a periodic motion.

Key Formulae

- **Periodic Motion.**

(a) **Frequency,** $\nu = \frac{1}{T}$

(b) **Angular Frequency,** $\omega = \nu \times 2\pi$

or $\omega = \frac{2\pi}{T}$

➢ Phase $= (\omega t + \phi) = \left(\frac{2\pi t}{T} + \phi\right) = (2\pi \nu t + \phi)$

➢ **Simple Harmonic Motion :**

(a) Differential Equation,

(i) Linear S.H.M. $= \frac{d^2y}{dt^2} + \omega^2 y = 0$, where $\omega^2 = k/m$, here, m is the mass of the body

(ii) Angular S.H.M. $= \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$, where $\omega^2 = C/I$, here, I = moment of inertia

(b) General *equation*—

(i) Linear S.H.M. $y = y_0 \sin(\omega t + \phi)$

(ii) Angular S.H.M. $\theta = \theta_0 \sin(\omega t + \phi_0)$

(c) Displacement, $y = A \sin \omega t$

or $y = A \cos \omega t$

(d) Velocity, $v = \omega\sqrt{A^2 - y^2}$

(e) Acceleration, $a = \frac{dv}{dt} = -\omega^2 A \sin \omega t$

(f) Time Period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ or $2\pi\sqrt{\frac{I}{C}}$

➢ **Time period :**

For simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

Potential energy of SHM: $U = \frac{1}{2} m\omega^2 y^2$

Kinetic energy of SHM: $K = \frac{1}{2} m\omega^2(a^2 - y^2)$

Total energy: $E = \frac{1}{2} m\omega^2 a^2$

Mnemonics

Concept: Time period of a simple pendulum.

Mnemonics: **T**anu **P**aul **s**peaks **l**ot **o**f **A**sian languages.

Interpretation:

T - Time period

P - proportional to

S - square root of

L - length

O - on

A - acceleration due to gravity

Time period

$\propto \sqrt{\frac{\text{Length}}{\text{Acceleration due to gravity}}}$

CHAPTER-14

WAVES

Topic-1 Waves & Wave Motion

Revision Notes

➢ **Wave motion** is a kind of disturbance which travels through a medium on account of repeated periodic vibrations of the particles of the medium about their mean position.

- **The medium** for wave propagation should have three properties :
  - **(a)** elasticity
  - **(b)** inertia
  - **(c)** minimum frictional resistance.

➢ **Kinds of waves :**

(i) Mechanical waves or elastic waves: The waves which require a material medium for propagation. Example: sound waves, waves on the surface of water, waves on strings.

(ii) Electromagnetic waves: The waves which require no medium for propagation. Example: light waves, radio waves.

(iii) Longitudinal waves : In which particles vibrate in the direction of propagation of waves. Longitudinal waves travel through a medium in the form of compressions and rarefactions involving changes in pressure and volume and can travel in all modes and cannot be polarised. The medium required must possess elasticity of volume. Example: Sound waves in air are longitudinal.

(iv) Transverse waves : In which particles vibrate in a direction perpendicular to the direction of propagation of waves. Transverse waves travel through a medium in the form of crests and troughs involving changes in shape can travel in solid and liquid and can be polarised. The medium required must possess elasticity of shape. Example: Vibrations in strings

(v) Matter waves: These waves are associated with constituents of matter - electrons, protons, neutrons, atoms and molecules. Those arise in quantum mechanical description of nature.

(vi) Capillary waves and gravity waves: The waves on the surface of water are of two kinds: capillary waves and gravity waves.

Ripple wave: These are ripples of fairly short wavelength—not more than a few centimetre—and the restoring force that produces them is the surface tension of water.

Gravity waves: These are have wavelengths typically ranging from several metres to several hundred meters. The restoring force that produces these waves is the pull of gravity, which tends to keep the water surface at its lowest level.

➢ **Laplace correction :** According to Laplace, the changes in pressure & volume of a gas, when sound waves propagate through it are not isothermal but it is adiabatic. since,

(a) Velocity of sound in gas is quite large.

(b) A gas is bad conductor of heat.

$\therefore$ Velocity of sound, $v = \sqrt{\dfrac{B_a}{\rho}}$

B_a = Bulk modulus = γP, ρ = density of gas

Key Words

➢ **Some parameters related to wave motion**

(a) Displacement of a particle is the distance covered by the particle from the mean position.

(b) Amplitude is the maximum displacement of the particle from the equilibrium position.

(c) One wavelength is the distance travelled by the wave, during the time the particle completes one vibration about its mean position. We may also define, one wavelength = smallest distance between two particles vibrating in the same phase = distance between the centres of two consecutive crests/troughs/compressions/rarefactions.

(d) Angular wave number or propagation constant : It is 2π times the no. of waves that can be accommodated per unit length. It is denoted by K.

(e) Frequency : It is the no. of complete wavelengths traversed by the wave in one second.

(f) Time period : It is equal to time taken by wave to travel a distance equal to wavelength.

(g) Particle velocity = velocity of particle executing SHM = $\dfrac{dx}{dt}$. Its value changes with time.

(h) Wave velocity is the velocity with which disturbance travels in the medium.

$$v = n\lambda = \frac{\lambda}{T} = \text{constant for a wave motion.}$$

- **Phase or phase angle** is the physical quantity which tells us by what amount the two waves or the two particles lag or lead in terms of angle or time or distance.

Key Formulae

- **A longitudinal wave** can be represented by

$$x = a\sin(\omega t \pm kx)$$

A transverse wave can be represented by

$$y = a\sin(\omega t \pm kz) \; ; \; y = a\sin(\omega t \pm kx)$$
$$z = a\sin(\omega t \pm kx) \; ; \; z = a\sin(\omega t \pm ky)$$
$$x = a\sin(\omega t \pm ky) \; ; \; x = a\sin(\omega t \pm kz)$$

- **Relation between particle velocity & wave velocity.**

$$u(x, t) = -v \frac{d}{dx}[y(x, t)]$$

Where u = particle velocity, v = wave velocity.

- **Particle Acceleration :** $a(x, t) = -(2\pi\nu)^2 y = -\omega^2 y.$
- **Plane progressive wave :**

(a) **Standard Equations :**

$$y = r\sin\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$$

or

$$y = r\cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$$

where y = displacement, r = amplitude, T= time period, λ = wavelength, x = starting distance of wave from origin.

(b) **Angular frequency :** $\omega = \dfrac{2\pi}{T}$

(c) **Propagation constant :** $k = \dfrac{2\pi}{\lambda}$

(d) **Velocity of wave :** $v = \nu\lambda = \dfrac{\lambda}{T}$

(e) **Velocity of particle :** $u = \dfrac{dy}{dt}$, $u_{max} = r\omega$

(f) **Acceleration of particle :** $a = \dfrac{d^2y}{dt^2}$, $a_{max} = -\omega^2 r$

(g) **Phase:**

For wave $y(x, t) = a\sin(kx - \omega t + \phi)$

$$\text{Phase} = (kx - \omega t + \phi)$$

Mnemonics

Concept: Transverse and longitudinal wave.

Mnemonics: **T**eacher **P**unished lazy **d**og.

Interpretation:

T - Transverse wave

P -perpendicular to direction of propagation

L - Longitudinal wave

D - direction of propagation

If constituents of the medium oscillate perpendicular to the direction of wave propagation, we call the wave a transverse wave. If constituents of the medium oscillate along the direction of wave propagation, we call the wave a longitudinal wave.

Topic-2 Superposition Principle

Revision Notes

➢ **Principle of Superposition of Waves :**

(a) According to this principle, overlapping waves add algebraically to produce a resultant wave or a net wave.

or

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of displacements due to each wave at that time.

i.e., $$y = y_1 + y_2 + \text{........} + y_n$$

Important phenomenon related to Superposition of waves:

(i) Stationary waves : When two identical transverse or longitudinal progressive waves travel in a bounded medium with same speed but in opposite directions, then by their superposition a new type of wave is produced which appears stationary in the medium. This is known as stationary or standing wave.

(ii) Beats: When two sound waves of nearly equal frequencies superimpose, the intensity of the resultant wave increases and decreases alternately with time. Then phenomenon is known as beats.

(iii) Interference of waves: When two waves of same frequency travel in a medium simultaneously in the same direction, then due to their superposition, the resultant intensity at any point of the medium becomes different from the sum of the intensities of the two waves. At some pints the intensity of resultant wave is very large while at some other points it is very low or zero. This phenomenon is known as interference.

➢ **Laws of Vibrations of Stretched Strings.**

Fundamental Frequency of vibration of stretched string

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

(a) Law of Length : $$\nu \propto \frac{1}{L}$$

Fundamental frequency is inversely proportional to length.

(b) Law of Tension : Fundamental frequency is directly proportional to square root of tension. i.e.,

$$\nu \propto \sqrt{T}$$

(c) Law of Mass : Fundamental frequency is inversely proportional to the square root of mass :

$$\nu \propto \frac{1}{\sqrt{m}}$$

(d) Law of Diameter : $$\nu \propto \frac{1}{D}$$

(e) Law of Density : $$\nu \propto \frac{1}{\sqrt{\rho}}$$

➢ **Vibration of air column in closed organ pipe:**

The tube which is closed at one end and open at other end is called closed organ pipe.

When air is blown at open end, a longitudinal wave travels towards the closed end and reflects back towards the open end. These two waves superimpose and form stationary longitudinal wave.

$$\lambda = \frac{4L}{2m-1}$$

$$\nu = \frac{v}{4L} \times (2m-1)$$

where $m = 1, 2, 3, 4.......$

➢ **Vibration of air column in closed organ pipe:**

The tube which is open at both ends is called closed organ pipe.

When air is blown at one open end, a longitudinal wave travels towards the other end and reflects back towards the first open end. These two waves superimpose and form stationary longitudinal wave.

$$\lambda = \frac{2l}{m}$$

$$\nu = \frac{v}{2L} \times m$$

where $m = 1, 2, 3, 4.......$

Key Words

- **Stationary Waves :** If two waves of same type having same amplitude, same frequency and same wavelength, travelling with same speed in opposite directions along a straight line superimpose each other, they give rise to a new kind of waves known as stationary wave.
- **Beats** is the phenomenon of regular variation in the intensity of sound when two sources of nearly equal frequencies are sounded together.
  - **(i)** **Beat period** is the time interval between two successive beats.
  - **(ii)** **Beat frequency** is the number of beats produced per second.

Key Formulae

- **Equation of stationary wave :**

When incident wave: $y_1 = A \sin \frac{2\pi}{\lambda} (vt - x)$ and

Reflected wave: $y_2 = A \sin \frac{2\pi}{\lambda} (vt - x)$

Then the standing wave: $y = 2A \sin \frac{2\pi}{\lambda} vt \cos \frac{2\pi}{\lambda} x$

- **Normal modes of vibration of strings :**

(a) **Frequency of fundamental tone:** $\nu_0 = \frac{1}{2l}\sqrt{\frac{T}{m}}$

(b) **1st overtone or 2nd harmonic :** $\nu_1 = 2\nu_0$

(c) **2nd overtone or 3rd harmonic :** $\nu_2 = 3\nu_0$

- **Organ Pipes :**

(a) **Closed organ pipe :**

(i) Fundamental note $\nu_0 = \frac{v}{4L}$

(ii) 1st overtone or 3rd harmonic $\nu_1 = 3\nu_0$

(iii) 2nd overtone or 5th harmonic $\nu_2 = 5\nu_0$

(b) **Open organ pipe :**

(i) Fundamental note $\nu_1 = \frac{v}{2L}$

(ii) 1st overtone or 2nd harmonic $\nu_1 = 2\nu_0$

(iii) 2nd overtone or 3rd harmonic $\nu_2 = 3\nu_0$

- **Beats :** Beat frequency $n_1 - n_2$ or $n_2 - n_1$