JEE (Main) SOLVED PAPER

2022
29^{th} June Shift 1

ANSWERS WITH EXPLANATIONS

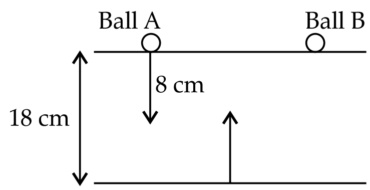
Physics

Section A

1. Option (D) is correct.

Explanation: As ball A is dropped from tower

$u = 0$ m/s



As the meeting point lies 100 m above ground, displacement of ball will be 80 m.

For ball A

$u = 0$, S = 80 m, $a = +g = +10$ m/s^2, time = t_1

$$S = ut + \frac{1}{2}at^2$$

$$80 = 0 + \frac{1}{2} \times 10 \times t_1^2$$

$$\frac{160}{10} = t_1^2$$

$$\Rightarrow \quad t_1 = 4\,s$$

As ball B is thrown after 2 seconds after release of A. Thus, time available for ball B is 2 seconds to cover a distance of 80 m.

Let speed be 'u' m/s, $t_2 = 4 - 2 = 2\,s$, S = 80 m, $a = +g = +10$ m/s^2

$$\therefore \quad 80 = u \times 2 + \frac{1}{2} \times 10 \times (2)^2$$

$$80 = u + 20$$

$$2u = 60$$

$$\Rightarrow \quad u = 30 \text{ m/s}$$

***Hint* :**

(i) As the motion is under gravity i.e., free fall it can be assumed to be uniformly accelerated motion.

(ii) Use $S = ut + \frac{1}{2}at^2$ two times, first for ball A and then for ball B.

(iii) Solve for initial velocity of ball B.

Shortcut:

As distance covered is same for both the balls, then

$$\frac{1}{2}gt^2 = u\,(t-2) + \frac{1}{2}g\,(t-2)^2 = 80 \quad ...(1)$$

$$\frac{1}{2}gt^2 = 80 \Rightarrow t = 4\,s$$

By substituting value of t in equation 1, we get:

$$u\,(t-2) + \frac{1}{2}g\,(t-2)^2 = 80$$

$$u\,(4-2) + \frac{1}{2}g\,(4-2)^2 = 80$$

$$u = 30 \text{ m/s}$$

2. Option (B) is correct.

Explanation: In the given problem a body of mass M explodes into three pieces of mass ratio 1:1:2

Thus, the mass of fragments will be x, x, $2x$

Hence, $M = x + x + 2x = 4x$ kg

As in the process of explosion no external forces are involved and explosion occurs due to internal forces. Thus, momentum of the system will be conserved.

Initially M is at rest

$p_{\text{initial}} = p_{\text{final}}$

By law of conservation of momentum x

$$M \times 0 = \frac{M}{4} \times 30\,\hat{i} + \frac{M}{4} \times 40\,\hat{j} + \frac{2M}{4}\vec{v}$$

Where $\vec{v}$ is the velocity of the third fragment.

$$\frac{M}{2}\vec{v} = -\frac{M}{4}(30\,\hat{i} + 40\,\hat{j})$$

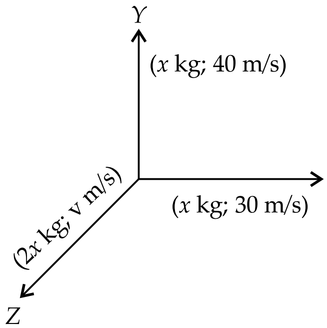
$$\vec{v} = -15\,\hat{i} - 20\,\hat{j}$$

Thus, magnitude of $\vec{v} = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{(-15)^2 + (-20)^2}$$

$$|\vec{v}| = \sqrt{625} = 25 \text{ m/s}$$

OSWAAL BOOKS LEARNING MADE SIMPLE



***Hint* :**

(i) As the explosion process does not include any external forces. Hence momentum of the system will be conserved.

(ii) Apply law of conservation of momentum

Shortcut:

Assume mass of the body to be 4 kg

$\Rightarrow$ Mass of fragments will be 1 kg, 1kg and 2 kg respectively.

$\Rightarrow$ Momentum of 3rd fragment = $-(\vec{p}_1 + \vec{p}_2)$

$$\vec{v}_3 = \frac{-(1\times 30\,\hat{i} + 1\times 40\,\hat{j})}{2} = -15\,\hat{i} - 20\,\hat{j}$$

$$(\vec{v}_3) = \sqrt{(-15)^2 + (-20)^2} = \sqrt{625} = 25 \text{ m/s}$$

3. Option (B) is correct.

Explanation: Given, at $t = 0 \Rightarrow$ Activity of Radioactive Sample = 2.56×10^{-3} Ci

Thus, $R_o = 2.56 \times 10^{-3}$ Ci

Half life of sample, $T_{½} = 5$ days

As, Radioactive decay is of first order reaction. Thus, Rate of decay or activity decreases exponentially.

By Radioactive Decay law,

$R = R_o e^{-\lambda t}$

$\Rightarrow 2\times 10^{-5} = 2.56 \times 10^{-3}\, e^{-\lambda t}$

Where, $R \Rightarrow$ Activity at time t

$\lambda \Rightarrow$ Activity constant of Radioactive sample

$$\lambda \Rightarrow \frac{ln 2}{T_{1/2}}$$

Taking logarithm on both sides

$ln\,(2 \times 10^{-5}) = ln\,(2.56 \times 10^{-3}) + ln\,(e^{-\lambda t})$

$ln\,(2 \times 10^{-5}) - ln\,(2.56 \times 10^{-3}) = -\lambda t$

$$ln\left(\frac{2\times 10^{-5}}{2.56\times 10^{-3}}\right) = -\lambda t$$

$$ln\left(\frac{1}{128}\right) = -\lambda t$$

$$-\,ln\,128 = -\lambda t$$

$$\Rightarrow \quad ln\,2^7 = \frac{ln 2}{T_{1/2}}t$$

$$\Rightarrow \quad 7 ln\,2 = \frac{ln 2}{T_{1/2}}t$$

$$\Rightarrow \quad t = 7\,T_{½}$$

$$\Rightarrow \quad t = 7 \times 5 = 35 \text{ days}$$

***Hint* :**

(i) Apply Law of Radioactive Decay

(ii) Use $R = R_o e^{-\lambda t}$

Shortcut:

We can calculate number of half lives taken for activity to change from 2.56×10^{-3} to 2×10^{-5} Ci By relation:

$$\frac{R}{R_o} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{2\times 10^{-5}}{2.56\times 10^{-3}}\right) = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{128}\right) = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^n$$

As, $\quad n = \dfrac{T}{T_{1/2}}$

Thus, $\quad n = 7$

Hence, $\quad T = n \times T_{1/2}$

$= 7 \times 5$

$= 35$ days

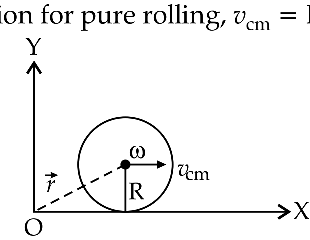
4. Option (C) is correct.

Explanation: Given: Mass of hollow sphere = 1 kg

Radius of sphere = R

As sphere is in pure rolling motion, then sphere will be rotating about its centre of mass with angular speed ω and its centre of mass will also perform translatory motion with velocity= v_{cm}

Condition for pure rolling, $v_{cm} = R\omega$



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$\Rightarrow$ Total Angular momentum of Rolling Sphere = Angular momentum due to rotation about it's centre of mass + Moment of linear momentum possessed by centre of mass about origin.

$\Rightarrow L_{net} = L_{cm} + \vec{r} \times (M\vec{v}_{cm})$

$\Rightarrow L_{net} = I\omega + Mv_{cm} \times r_{\perp ar}$

$\Rightarrow L_{net} = I\omega + Mv_{cm}R$

Where $r_{\perp ar}$ is the perpendicular distance between centre of mass and line passing from origin

$\therefore r_{\perp ar} = R$

Moment of inertia of hollow sphere about its centre of mass $= \frac{2}{3}MR^2$

$$L_o = \frac{2}{3}MR^2\omega^2 + 1 \times R\omega \times R$$

$$L_o = \frac{2}{3} \times 1 \times R^2\omega + R^2\omega$$

$$L_o = R^2\omega\left[\frac{2}{3}+1\right]$$

$$L_o = \frac{5}{3}R^2\omega$$

So, by comparing it with given value, we get, $a = 5$.

Hint: 1. Net Angular momentum of sphere about origin = Angular momentum due to Rotation about its centre of mass + Moment of linear momentum possessed by centre of mass

$$L_o = I\omega + \vec{r} \times (M\vec{v}_{cm})$$

2. Use condition of pure rolling i.e., $v_{cm} = R\omega$

Shortcut: $L_o = Mv_{cm}R + I\omega$

$$= 1 \times R\omega R + \frac{2}{3}MR^2\omega^2$$

$$= \omega R^2 + \frac{2}{3} \times 1\,\omega R^2$$

$$= \frac{5}{3}\omega R^2$$

Thus, $a = 5$

5. Option (C) is correct.

Explanation: As the cylinder is of fixed capacity it means its volume is fixed.

Hence, it is an Isochoric process.

Helium is at S.T.P. conditions (given).

By Avogadro Hypothesis, one mole of gas occupies 22.4 L volume at S.T.P. condition.

$\Rightarrow$ Thus, at S.T.P. 44.8 L volume will be occupied by 2 moles of Helium Gas.

$\Rightarrow$ As, Helium is monoatomic gas, its degree of freedom will be 3.

For Isochoric Process

$Q = \Delta U$ where Q is the heat transfer $\Delta U \Rightarrow$ change in internal energy of gas $\Delta U = nC_V\Delta T$ wgere C_V is specific heat at constant volume

$$\Rightarrow \quad C_V = \frac{fR}{2} = \frac{3R}{2} \text{ for monoatomic gas}$$

(where, f = degree of freedom)

Given: $\Delta T = 20°C$

Thus, $Q = \Delta U = 2 \times \frac{3}{2} R \times 20 = 60\,R$

$\Rightarrow \quad Q = 60 \times 8.3 = 498$ J

Hint :

(i) By Avogadro Hypothesis, calculate number of moles of Helium gas.

(ii) Capacity of cylinder is fixed, so gas follows constant volume process.

(iii) Apply $Q = nC_V\Delta T$, because work done is zero in isochoric process.

Shortcut **:**

By first law of thermodynamics,

$$Q = \Delta U + w \quad ...(i)$$

As container is rigid so, change in volume = 0

Thus, work done = 0

$$Q = \Delta U = nC_V\Delta T$$

At S.T.P. condition 1 mole of gas occupies 22.4 L volume, so 2 moles of Helium will occupy 44.8 L volume.

As Helium is monoatomic gas, so its degree of freedom = 3

Thus, $C_V = \frac{f}{2} = \frac{3R}{2}$

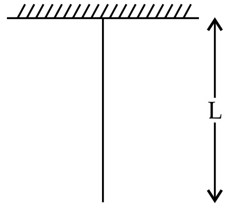
$$\Rightarrow \quad Q = 2 \times \frac{3R}{2} \times 20$$

$$= 60\,R = 60 \times 8.3 = 498 \text{ J}$$

6. Option (C) is correct.

Explanation: Given: When 1 kg mass is attached to wire the final length becomes L_1 and when 2 kg mass is attached to wire, the final length of wire becomes L_2.

Let the Young's modulus of wire material be Y, Then

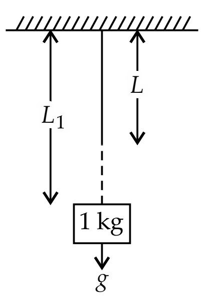


(a)

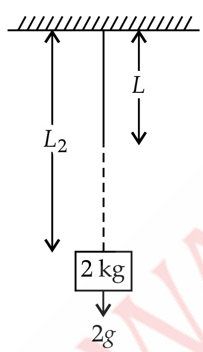
By Hooke's Law of elasticity $\Rightarrow \frac{F}{A} = Y\frac{\Delta l}{l}$

Let area of cross-section of wire be A
original length of wire = L
Case-I $\rightarrow$ When 1 kg load is attached to wire
change in length = $L_1 - L$
Using Hooke's Law

$$F_1 = mg = 1 \times g \Rightarrow \frac{1 \times g}{A} = Y\left(\frac{L_1 - L}{L}\right) \quad ...(1)$$



(b)



(c)

Length of wire

(a) in normal condition
(b) When 1 kg mass is suspended
(c) When 2 kg mass is suspended

Case-II $\rightarrow$ When 2 kg load is attached to wire
change in length = $L_2 - L$
Using Hooke's Law

$$F_2 = mg = 2 \times g$$

$$\Rightarrow \quad \frac{2 \times g}{A} = Y\left(\frac{L_2 - L}{L}\right) \quad ...(2)$$

Dividing equation (1) by (2)

$$\frac{1}{2} = \frac{L_1 - L}{L_2 - L}$$

$$L_2 - L = 2L_1 - 2L$$

$$\boxed{L = 2L_1 - L_2}$$

Hint :

(i) Apply Hooke's Law of elasticity.
(ii) Compare both the cases of loading and obtain relation.

Shortcut :

As loading is done on same wire
Thus, A, Y, L remains constant
Thus, Hooke's Law yield F α Δ *l*

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{\Delta l_1}{\Delta l_2}$$

$$\frac{1 \times g}{2 \times g} = \frac{L_1 - L}{L_2 - L}$$

$$\frac{1}{2} = \frac{L_1 - L}{L_2 - L}$$

$$L_2 - L = 2L_1 - 2L$$

Thus, $\boxed{L = 2L_1 - L_2}$

7. Option (B) is correct.

Explanation: In photoelectric effect ejection of electron from metal surface takes place only when :

Energy of incident photons is greater or equal to the Work Function of the Metallic Surface.

Where, work function is the minimum energy required to free an electron from metal surface.

∴ When energy of photon is less than work function, no electron will become free and photoelectric effect does not take place.

Thus, Assertion (A) is correct.

By equation of photoelectric effect

$$h\nu = \phi + K.E_{max}$$

As per Reason (R), if $K.E = 0$

Then $h\nu = \phi$

Thus, Reason (R) is also correct

But Reason (R) is not explaining Assertion (A) because it does not consider or explain why photoelectric effect does not take place if $h\nu < \phi$

***Hint* :** **(i)** Use equation of photoelectric effect and check the correctness of both statements.
(ii) If the combined statement makes sense then Reason (R) is correct explanation of Assertion (A) otherwise not.

Shortcut :

To free the electrons from metal surface, minimum energy required is equal to the work function of metal.

So, Assertion (A) is correct.

$$h\nu = \phi + K.E_{max}$$

if $\quad h\nu = \phi$

$\Rightarrow \quad K.E_{max} = 0$

Hence, Reason R is correct.

But, Reason (R) does not explains Assertion (A)

Thus, option B is correct.

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8. Option (D) is correct.

Explanation: Given: Velocity as a function of position as $v = bx^{5/2}$ ...(1)

By differentiating the above equation with respect to time

$$a = \frac{dv}{dt} = \frac{d}{dt}(bx^{5/2}) = \frac{5}{2}bx^{3/2}$$

$$\Rightarrow \quad a = \frac{5}{2}bx^{3/2} \times v \qquad \left[\text{since, } v = \frac{dx}{dt}\right]$$

$$\Rightarrow \quad a = \frac{5}{2}bx^{3/2} \times b \times x^{5/2} \Rightarrow a = \frac{5}{2}bx^4$$

Mass of body = 500 gm = 0.5 kg

Force acting on body = $m \times a$

(by Newton's II law of motion)

$$F = 0.5 \times \frac{5}{2}bx^2x^4 = \frac{5}{4}b^2x^4$$

By definition of work done $\Rightarrow$ W = $\int_{x_1}^{x_2} F dx$

$$\Rightarrow \quad W = \int_{x=0}^{x=4} \frac{5}{4}b^2x^4 dx$$

$$\Rightarrow \quad = \frac{5b^2}{4}\int_{x=0}^{x=4} x^4 dx$$

$$\Rightarrow \quad W = \frac{5}{4} \times \left(\frac{1}{4}\right)^2 \left[\frac{x^5}{5}\right]_{x=0}^{x=4}$$

$$= \frac{\cancel{5}}{4} \times \frac{1}{16} \times \frac{4^5 - 0}{\cancel{5}} = \frac{1024}{64}$$

$$\Rightarrow \quad W = 16 \text{ J}$$

Hint **:**

(i) By using definition of work, $W = \int_{x_1}^{x_2} F\, dx$.

(ii) By differentiating v, obtain acceleration (a) hence find force.

(iii) Integrate with proper limits and obtain work done.

Shortcut **:** As, velocity is given as a function of position Obtain velocity at x = 0 m and x = 4 m

Apply work-energy theorem

Net work done = Change in kinetic energy

$$W = \frac{1}{2}m[(v_{(4)})^2 - (v_{(0)})^2]$$

$$= \frac{1}{2} \times \frac{1}{2}[(8)^2 - (0)^2]$$

$$= \frac{1}{4} \times 64 = 16 \text{ J}$$

$$\left[\begin{aligned} \because v(4) &= \frac{1}{4} \times (4)^{5/2} \\ &= \frac{1}{4} \times (2)^5 \\ &= 8 \text{ m/s} \\ \because v(0) &= \frac{1}{4} \times (0)^{5/2} \\ &= 0 \text{ m/s} \end{aligned}\right]$$

9. Option (C) is correct.

Explanation: In a cyclotron, charge is rotated along circular path by magnetic field and it's kinetic energy is increased by electric field.

Radius of charged particle on circular path is given by

$$R = \frac{mv}{Bq}$$

Let the initial speed of charged particle be v

Due to application of electric field, it's kinetic energy increases to 4 times. Thus, new speed of charged particle

$$4K = \frac{1}{2}m(v)^2$$

$$\frac{v^2}{(v')^2} = \frac{1}{4} \Rightarrow \frac{v}{v'} = \frac{1}{2} \Rightarrow v = 2v$$

Thus, speed of charged particle becomes doubled.

Let initial radius of circular path be R_1 and final radius of circular path be R_2. Then,

$$R_1 = \frac{mv}{Bq} \quad ...(1)$$

$$R_2 = \frac{m \times 2v}{Bq} \quad ...(2)$$

from (1) and (2)

$$\boxed{R_2 = 2R_1}$$

Thus, Radius gets doubled.

Hint **:**

(i) As radius of charged particle in uniform magnetic field, $R = \frac{mv}{Bq}$

(ii) Mass, charge of charged particle and magnetic field remains constant.

(iii) Thus, $R \propto v \Rightarrow \frac{R_1}{R_2} = \frac{v_1}{v_2}$

(iv) As, kinetic energy becomes 4 times of initial, then speed gets doubled.

Shortcut **:**

As, $R = \frac{mv}{Bq} \Rightarrow R = \frac{p}{Bq} - \sqrt{\frac{2mK}{Bq}}$

where $p \Rightarrow$ momentum of particle
$K \rightarrow$ kinetic energy of particle

Thus, $\frac{R_1}{R_2} = \sqrt{\frac{K_1}{K_2}}$

$\frac{R_1}{R_2} = \sqrt{\frac{K}{4K}} = \frac{1}{2}$

$\boxed{R_2 = 2R_1}$

10. Option (C) is correct.

Explanation: At resonance $X_L = X_C$ and impedance of circuit is minimum and is equal to R and current is maximum.

At resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow$ If potential drop across capacitor is more than inductor, then current leads voltage and circuit becomes capacitive.

$\Rightarrow$ If potential drop across inductor is more than capacitor, then voltage leads current and circuit becomes inductive

Thus, $i_0X_L > i_0X_C$	$i_0X_C > i_0X_L$
Circuit is inductive	Circuit is capacitive
$X_L > X_C$	$X_C > X_L$
$\omega L > \frac{1}{\omega C}$	$\frac{1}{\omega C} > \omega L$
$\omega^2 > \frac{1}{\sqrt{LC}}$	$\omega < \frac{1}{\sqrt{LC}}$
$\omega > \sqrt{\frac{1}{\sqrt{LC}}}$	$\omega < \sqrt{\frac{1}{\sqrt{LC}}}$
$\omega > \omega_R$	$\omega < \omega_R$

Hint **:**

(i) At resonance, $\omega = \omega_R = \sqrt{\frac{1}{LC}}$, and $X_L = X_C$. Thus circuit is purely resistive and impedance is minimum.

(ii) $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

On increasing $\omega\uparrow$, $X_L > X_C$ circuit is inductive.

On decreasing $\omega\downarrow$, $X_C > X_L$ circuit is capacitive.

Shortcut **:**

$X_L \propto \omega$ and $X_C \propto \frac{1}{\omega}$

On increasing ω, X_L increases and circuit becomes inductive.

On decreasing ω, X_L decreases and X_C increases circuit becomes capacitive.

11. Option (C) is correct.

Explanation: As water ejected by jet strikes the block, it imparts momentum to the block

$\Rightarrow$ Change in momentum of water jet in 1 s = Momentum gained by block in 1 s.

$\Rightarrow$ Change in momentum of jet in second = Mass of jet ejected in 1 second $\times$ change in velocity

$= 1\ \frac{kg}{s} \times (10 - 0)$

$= 10$ kg m/s

$\Rightarrow$ Momentum gained by block = 10 kg m/s

By Newton's II law of motion $\Rightarrow$ F = Rate of change of momentum

$= \frac{\Delta p}{\Delta t}$

$F = \frac{10\ kg\ m/s}{1s} = 10\ \text{kg}\ \frac{m}{s^2} = 10\ N$

Thus, 10 N force acts on the block

By $F = ma$

$a = \frac{F}{m} = \frac{10}{2} = 5\text{m/s}^2$

Hint **:**

(i) Apply impulse-momentum equation on the block.

Shortcut **:**

As water is continuously ejecting out of jet.

Thus, it's a variable mass system

For variable mass system

$F = v\frac{dm}{dt}$

$F = 10\ \frac{\text{m}}{\text{s}} \times \frac{1kg}{\text{s}}$

$F = 10\ \text{kg}\ \frac{\text{m}}{\text{s}^2} = 10\ \text{N}$

By $F = ma \Rightarrow a = \frac{F}{m} = \frac{10}{2} = 5\text{m/s}^2$

12. Option (C) is correct.

Explanation: $\left[P + \frac{a}{V^2}\right]\left[V - b\right] = RT$

By principle of Dimensional Homogeneity, physical quantities with same dimensions only can be added or subtracted.

Thus, Dimensions of P and $\frac{a}{V^2}$ and dimensions of V and b must be same

$$\therefore \quad [P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] \quad ...(1)$$

$$[V] = [b]$$

Thus, Ratio of $\frac{a}{b} = \frac{[PV^2]}{[V]}$

$$= [PV]$$

Hint :

(i) Apply principle of Dimensional Homogeneity and obtain dimensions of a and b.

(ii) Calculate dimensions of $\frac{a}{b}$.

Shortcut :

As physical quantities of only same dimensions can be added or subtracted

$$[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] \quad ...(1)$$

$$[V] = [b] \quad ...(2)$$

On dividing (1) by (2), we get:

$$\left[\frac{a}{b}\right] = \frac{\left[PV^2\right]}{V} = [PV]$$

13. Option (C) is correct.

Explanation: Given $|\vec{A} + \vec{B}| = 2|\vec{A} - \vec{B}|$ ...(1)

Let angle between $\vec{A}$ and $\vec{B}$ be θ.

$$|A| = |B|$$

Magnitude of resultant of two vectors is given by

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(\pi - \theta)}$$

$$= \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Substituting values of $|\vec{A} + \vec{B}|$ and $|\vec{A} - \vec{B}|$ in equation (1), we get :

$$\sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= 2\sqrt{A^2 + B^2 - 2AB\cos\theta}$$

On squaring both the sides

$$(A^2 + B^2 + 2AB\cos\theta) = 4(A^2 + B^2 - 2AB\cos\theta)$$

As $\quad |A| = |8| \Rightarrow A = B$

$$2 + 2\cos\theta = 4(2 - 2\cos\theta)$$

$$6 = 10\cos\theta$$

$$\cos\theta = \frac{6}{10}$$

$$\therefore \quad \cos\theta = \frac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$$

Hint :

(i) Apply the formula of resultant of addition of two vectors and subtraction of two vectors.

(ii) As $|\vec{A} + \vec{B}| = 2|\vec{A} - \vec{B}|$

(iii) Put $A = B$ and solve for θ

Shortcut : $|\vec{A} + \vec{B}| = 2|\vec{A} - \vec{B}|$

Squaring both the sides

$$|\vec{A} + \vec{B}|^2 = 4|\vec{A} - \vec{B}|^2$$

Using identity $\vec{A}.\vec{A} = |\vec{A}|^2$

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 4(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$$

$$|A|^2 + |B|^2 + 2\vec{A} \cdot \vec{B} = 4|A|^2 + 4|B|^2 - 8\vec{A} \cdot \vec{B}$$

$$10\,\vec{A} \cdot \vec{B} = 3[|A|^2 + |B|^2]$$

$$10|A||B|\cos\theta = 3(|A|^2 |B|^2)$$

$$\text{As } |A| = |B|$$

$$10\cos\theta = 6$$

$$\cos\theta = \frac{6}{10} \Rightarrow \cos\theta = \frac{3}{5}$$

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

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14. Option (A) is correct.

Explanation: Escape velocity of a body on a planet is given by :

$$V_e = \sqrt{2gR} \text{ or } V_e = \sqrt{\frac{2GM}{R}}$$

where $g \Rightarrow$ acceleration due to gravity at the surface

$m \Rightarrow$ mass of planet and R = Radius of planet

Assuming plant to be solid sphere uniform, density ρ

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$M = \rho \times \frac{4}{3}\pi R^3$$

The expression for escape velocity can be written as

$$V_e = \sqrt{\frac{2 \quad \rho \times - \pi R}{R}} = \sqrt{\frac{}{3} G\rho R}$$

(V_e) for planet A = 12 km/s.

Let it's Radius R_A and density ρ

For planet $B \Rightarrow (V_e)_B \Rightarrow R_B = \frac{R_A}{2}, \rho_B = 4\,\rho_A$

$$\frac{(V_e)_A}{(V_e)_B} = \frac{\sqrt{\frac{8\pi G\rho_A R_A^2}{3}}}{\sqrt{\frac{8\pi G\rho_B R_B^2}{3}}}$$

$$= \sqrt{\frac{\rho_A}{\rho_B}\left(\frac{R_A}{R_B}\right)^2} = \sqrt{\frac{1}{4} \times 4} = 1$$

$\therefore \quad (V_e)_A = (V_e)_B = 12$ km/s

***Hint* :**

(i) Use the formula of escape velocity

$$V_e = \sqrt{\frac{2GM}{R}}$$

and convert mass in terms of density and volume.

(ii) Obtain relation of escape speed in terms of radius and density.

(iii) Compare escape velocity of both planets.

***Shortcut* :** $V_e \propto \sqrt{\frac{m}{R}}$

for a solid sphere

$$\Rightarrow \quad m = \frac{4}{3}\pi R^3 \times \delta$$

$\therefore \quad m \propto \rho$ and $m \propto R^3$

Thus, $\quad V_e \propto \sqrt{\frac{\rho R^3}{R}} \propto \sqrt{\rho R^2}$

$$\Rightarrow \quad V_e \propto R\sqrt{\rho}$$

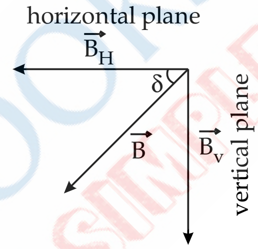
$$\Rightarrow \quad \frac{V_A}{V_B} = \frac{R_A}{R_B}\sqrt{\frac{\rho_A}{\rho_B}} \Rightarrow \sqrt{\frac{\rho_A}{4\rho_B}}$$

$$\Rightarrow \quad \frac{V_A}{V_B} = 2 \times \sqrt{\frac{1}{4}} = 2 \times \frac{1}{2} = 1$$

$\therefore \quad V_A = V_B = 12$ km/s

15. Option (A) is correct.

Explanation: Angle of dip at a place is defined as angle between earth's magnetic field and horizontal plane at that location.



$\delta \rightarrow$ Angle of Dip

On resolving $\vec{B}$ into two perpendicular components

$$B_H = B\cos\delta \quad \text{...(1)}$$

$$B_V = B\sin\delta \quad \text{...(2)}$$

Given horizontal component of earth's magnetic field at a location is 0.5 G

Using equation (1)

$$0.5 = B\cos 30°$$

$$0.5 = B \times \frac{\sqrt{3}}{2}$$

$$B = \frac{0.5 \times 2}{\sqrt{3}} = \frac{1}{\sqrt{3}} G$$

***Hint* :**

Use the relation $B_H = B\cos\delta$

***Shortcut* :** As, $B_H = B_H \cos\theta$

Given, $\quad B_H = 0.5\,G$

$\quad \delta = 30°$

Hence, $\quad 0.5\,G = B \times \cos 30°$

$$B = \frac{0.5}{\cos\delta} = \frac{0.5}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} G$$

16. Option (B) is correct.

Explanation: $y = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$

$y = 10\sin\left(2\pi nt - \frac{2\pi}{\lambda}x\right)$

On comparing it with standard equation of wave

$\Rightarrow \omega\ 2\pi n$ and $k = \frac{2\pi}{\lambda}$

OSWAAL BOOKS LEARNING MADE SIMPLE

Given : Maximum particle velocity = 4 × wave velocity

In wave propagation, medium particle oscillates and maximum velocity of oscillating particle is equal to $A\omega$

Where, $A \Rightarrow$ Amplitude

$\omega \Rightarrow$ Angular frequency

$\Rightarrow$ Wave velocity is given by $V = \frac{\omega}{k}$ where, k is angular wave number and $k = \frac{2\pi}{\lambda}$

$$V_{\text{wave}} = \frac{2\cancel{\pi}n}{2\cancel{\pi}\,\lambda} = n\lambda$$
$$V_{\text{particle}} = 4\,V_{\text{wave}}$$
$$A\omega = 4n\lambda$$
$$A \times 2\pi n = 4n\lambda$$
$$A \times 2\,\pi n = 4n\lambda$$
$$\lambda = \frac{2\pi A}{4} = \frac{\pi A}{2}$$

As, $A = 10 \Rightarrow \lambda = \frac{\pi \times 10}{2} = 5\pi$

Hint :

(i) Compare the given equation of wave with standard equation of wave $y = \sin(\omega t - kx)$

(ii) Obtion ω, k

(iii) Find V_{wave} and substitute in the given situation i.e., $V_{\text{particle}} = 4\,V_{\text{wave}}$

Shortcut :

$$y = 10\sin\left(2\pi nt - \frac{2\pi x}{\lambda}\right)$$

Standard equation of wave $y = A\sin(\omega t - kx)$

$\Rightarrow \omega = 2\pi$, $nk = \frac{2\pi x}{\lambda}$

Maximum velocity of oscillating particle = $A\omega$

$$A\omega = 4\,\frac{\omega}{k}$$
$$A = \frac{4}{\frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi A}{4}$$
$$= \frac{\pi A}{2} = \frac{\pi \times 10}{2} = 5\pi$$

Thus $\lambda = 5\pi$

17. Option (A) is correct.

Explanation: Potential energy stored in air capacitor of capacitance C, charged to potential difference V is given by:

$$U = \frac{1}{2}CV^2$$

If the above capacitor is filled completely with a dielectric of dielectric constant K, keeping the potential difference same, then potential energy of capacitor becomes

$$U' = \frac{1}{2}KCV^2 = KU$$

Given:

Initially K of dielectric = 10

$U_i \Rightarrow$ Initial potential energy = $\frac{1}{2} \times 10 \times CV^2$

$U_f \Rightarrow$ Final potential energy = $\frac{1}{2} \times 15 \times CV^2$

percentage increase in energy of capacitor

$$= \frac{U_f - U_i}{U_i} \times 100$$

$\Rightarrow$ percentage increase

$$= \frac{\frac{1}{2}CV^2\,[15-10]}{\frac{1}{2}CV^2 \times 10} \times 10$$
$$= \frac{5}{10} \times 100 = 50\%$$

Hint:

(i) Use the expression of potential energy stored in a capacitor filled with dielectric i.e.,

$$U = \frac{1}{2}KCV^2$$

(ii) Compare energy stored in both cases.

Shortcut:

As potential difference across capacitor remains same in both cases.

$$U = \frac{1}{2}KCV^2$$
$$\Rightarrow \quad U \propto K$$
$$\Rightarrow \quad \frac{U_2}{U_1} = \frac{K_2}{K_1}$$
$$\Rightarrow \quad \frac{U_2 - U_1}{U_1} = \frac{K_2 - K_1}{K_1}$$
$$\Rightarrow \quad \frac{U_2 - U_1}{U_1} \times 100 = \frac{K_2 - K_1}{K_1} \times 100$$
$$\Rightarrow \quad \text{percentage increase in } U = \frac{15-10}{10} \times 100$$
$$= 50\%$$

OSWAAL BOOKS LEARNING MADE SIMPLE SAMPLE

18. Option (D) is correct.

***Explanation:* Given:**

u = Initial velocity of charged particle = 200 m/s

m = mass of charged particle = 100 mg

$= 100 \times 10^{-6}$ kg $= 10^{-4}$ kg

$q \Rightarrow$ charge on particle = 40 μC = $40 \times 10^{-6}\, C = 4 \times 10^{-5}\, C$

$|E|$ = Electric field intensity = $1 \times 10^5\, N/C$

As particle is projected opposite to the direction of electric field, the electrostatic force will act on the particle opposite to velocity. Thus, its motion will be retarded.

As electric field is constant, $F = qE$ will be constants

Hence, $a = \frac{F}{m} = \frac{qE}{m}$ will also be constant.

Thus, equations of motion can be used

$$v^2 = u^2 + 2as \quad \ldots(1)$$

$$(0)^2 = (200)^2 + 2 \times \left(\frac{-qE}{m} \times S\right)$$

$$\frac{2 \times 4 \times 10^{-5} \times 1 \times 10^5}{10^{-4}} \times S = 4 \times 10^4$$

$$S = \frac{4 \times 10^4 \times 10^{-4}}{8} = \frac{1}{2}$$

$$m = 0.5 \text{ m}$$

Thus, $S = 0.5$ m

Hint:

(i) As electric field is uniform, if will apply a constant force, F = qE on charged particle.

Then, use $a = \frac{qE}{m}$

(ii) Use III equation of motion i.e., $v^2 = u^2 + 2as$ to calculate the required stopping distance.

Shortcut : By Work-Energy Theorem:

$\Rightarrow$ Work done = change in Kinetic energy

As force acting on charged particle is constant,

$$F = qE$$

Let the stopping distance by S.

$$K_{\text{inital}} = \frac{1}{2} \times m\,(200)^2$$

$$K_{\text{final}} = 0$$

$$\Rightarrow \quad W = K.E_f - K.E_i$$

$$qE \times S = +\,K.E_i$$

$$S = \frac{10^{-4} \times (200)^2}{2 \times 4 \times 10^{-5} \times 1 \times 10^5}$$

$$\Rightarrow \quad S = 0.5 \text{ m}$$

19. Option (D) is correct.

Explanation: As, $\beta = \frac{\lambda D}{d}$

Case 1: $$0.5 \text{ mm} = \frac{5000\text{Å} \times D}{d_1} \quad \ldots(1)$$

Case 2: $$\beta_2 = \frac{6000\text{Å} \times D}{2d_1} \quad \ldots(2)$$

Dividing Eq. (1) by (2), we get

$$\frac{0.5}{\beta_2} = \frac{5000}{6000} \times \frac{2d_1}{d_1}$$

$$\beta_2 = \frac{3}{10} = 0.3 \text{ mm}$$

Hint:

(i) Use the expression of fringe width in double slit $\beta \propto \frac{\lambda D}{d}$

(ii) Compare β for both the cases

Shortcut: Fringe width in double slit experiment = $\frac{\lambda D}{d}$

As D is kept constant

$$\beta \propto \frac{\lambda}{d}$$

Thus, $$\frac{\beta_1}{\beta_2} = \left(\frac{\lambda_1}{\lambda_2}\right)\left(\frac{d_2}{d_1}\right)$$

$$\Rightarrow \quad \frac{0.5 \text{ mm}}{\beta_2} = \frac{5000}{6000} \times \frac{2}{1}$$

$$\Rightarrow \quad \beta_2 = \frac{3}{10} \text{ mm} = 0.3 \text{ mm}$$

20. Option (B) is correct.

***Explanation:* Given:**

Wavelength of signal = 1000 mm = $10^3 \times 10^{-9}$

$= 10^{-6}$ m

c = Speed of light = 3×10^8 m/s

As $$c = n\lambda$$

$$n = \frac{c}{\lambda} = \frac{3 \times 10^8}{10^{-6}} = 3 \times 10^{14} \text{ Hz}$$

Thus, frequency of optical signal is 3×10^{14} Hz

Given only 2% of optical source frequency is available for communication system.

$$= 0.75 \times 10^9 = 75 \times 10^7$$

$\therefore$ Available frequency $= \dfrac{2}{100} \times 3 \times 10^{14}$

$$= 6 \times 10^{12} \text{ Hz}$$

Bandwidth of communication system

$$= 6 \times 10^{12} \text{ Hz}$$

Bandwidth of each channel to be accommodated

$$= 8 \text{ kHz} = 8 \times 10^3 \text{ Hz}$$

Thus, number system of channel

$$= \frac{\text{Bandwidth of system}}{\text{Bandwidth of each channel}}$$

$$n = \frac{6\times10^{12}}{8\times10^3}$$

$$= 0.75 \times 10^9 = 75 \times 10^7$$

Hint:

(i) Calculate the frequency of communicating system by $c = n\lambda$

(ii) By given percentage, calculate the available bandwidth of frequency.

(iii) Use expression n

$$= \frac{\text{Available Bandwidth}}{\text{Bandwidth of one channel}}$$

Shortcut:

Frequency of signal having wavelength 1000 nm is

$$\frac{c}{\lambda} = \frac{3\times10^8}{1000\times10^{-9}} = 3 \times 10^{14} \text{ Hz}$$

$\Rightarrow$ Frequency available for communication

$$= \frac{2}{100} \times 3 \times 10^4 = 6 \times 10^{12} \text{ Hz}$$

Bandwidth for one channel = 8000 Hz

$\therefore$ Number of channel $= \dfrac{6\times10^{12}}{8\times10^3} = 75 \times 10^7$

21. Correct answer is [15].

Explanation: Let there be two coils A and B.

Let the potential difference across source be V

Using $\quad H = \dfrac{V^2}{R}t$

For Coil (1) $\quad H = \dfrac{V^2}{R_1} \times 60 \quad \ldots(1),$

For Coil (2) $\quad H = \dfrac{V^2}{R_2} \times 20 \quad \ldots(2)$

When coils (1) and (2) connected in parallel combination across voltage source.

Total Heat generated $= H_1 + H_2$

Let the total time be 't' when two coils together in parallel are operated.

As $\quad H_{\text{total}} = H$

$$\Rightarrow \quad H = \frac{V^2}{R_1}t + \frac{V^2}{R_2}t$$

$$H = \frac{H}{60}t + \frac{H}{20}t$$

From equation (1) and (2)

$$1 = \left(\frac{1}{60} + \frac{1}{20}\right)t$$

$$\Rightarrow \quad t = \frac{1200}{80} = 15 \text{ minutes}$$

Hint:

(i) Use expression $H = \dfrac{V^2}{R}t$ to calculate heat generated, as potential drop across each coil is same.

(ii) As coils are combined in parallel.

Apply $H_{\text{total}} = H_{\text{coil (1)}} + H_{\text{coil (2)}}$

Shortcut: As heat generated in both coil is same

i.e., $\quad \dfrac{V^2}{R_1} \times 60 = \dfrac{V^2}{R_2} \times 20$

$$\frac{3}{1} = \frac{R_1}{R_2}$$

Assume $R_1 = 3\Omega$, $R_2 = 1\Omega$

Then, $\quad \dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{3} + \dfrac{1}{1} = \dfrac{4}{3}$

And, $\quad \dfrac{V^2}{R_{eq}} \times t = \dfrac{V^2}{R_1} \times 60$

$$t = \frac{R_{eq}}{R_1} \times 60 = \frac{3}{4\times3} \times 60$$

$$= 15 \text{ minutes}$$

22. Correct answer is [43].

Explanation: As $I = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)$

On substituting value of I, ε_0 and c, we get

$$0.22 = \frac{1}{2}(8.85 \times 10^{-12}) E_0^2 \times 3 \times 10^8$$

$$E_0 = \sqrt{\frac{2\times0.22}{8.85\times10^{-12}\times3\times10^8}}$$

$$= 12.9 \text{ N/C}$$

As $c = \frac{E_0}{B_0}$

$$\Rightarrow \quad B_0 = \frac{12.9}{3\times10^8} = 4.3\times10^{-8} = 43\times10^{-9}\ T$$

Hint:

(i) Apply relation of intensity i.e., $\frac{1}{2}\varepsilon_0 E_0^2 \times c$ and calculate E_0.

(ii) As, speed of light $c = \frac{E_0}{B_0}$, B_0 can be calculated.

Shortcut: $I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)\times c = \frac{1}{2}\frac{B_0^2}{\mu_0}c$

$$0.22 = \frac{1}{2}\frac{B_0^2\times3\times10^8}{4\pi\times10^{-7}}$$

$$B_0 = \sqrt{\frac{0.22\times2\times4\times3.14\times10^{-7}}{3\times10^8}}$$

$$= 4.3\times10^{-8} = 43\times10^{-9}\ T$$

23. Correct answer is [21].

Explanation: Since, $\frac{Q}{t} = \frac{KA\Delta T}{d}$

Here, all alphabets are is their usual meanings

where $\frac{Q}{t}$ = Rate of heat transfer

$$\Rightarrow \quad \frac{Q_1}{t} = \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{100-T_i}{4/KA} = \frac{T_i - 0}{2.5/2\,KA}$$

$$\Rightarrow \quad \frac{100-T_i}{4} = \frac{2T_i}{2.5}$$

$$\Rightarrow \quad 250 - 2.5\,T_i = 8\,T_i$$

$$T_i = \frac{250}{10.5}$$

As the two plates are in series, same thermal current will flow in composite rod as well

DT for composite plate = 100°C

$$\Rightarrow \quad \frac{100-0}{\frac{6.5}{K_{eq}A}} = \frac{\frac{250}{10.5}-0}{\frac{2.5}{2\,KA}}$$

$$\Rightarrow \quad \frac{K_{eq}\times100}{6.5} = \frac{250\times2K}{10.5\times2.5}$$

$$K_{eq} = \frac{130}{105}K \quad \frac{26}{21}K = \left(\frac{1+5}{21}\right)K$$

on comparing with $K_{eq} = \left(1+\frac{5}{21}\right)K$, we get

$$\alpha = 21$$

Hint:

(i) The two plates of same cross-section area are connected in series.

(ii) As thermal conduction obeys Ohm's Law, the two plates act as thermal resistors which are connected in series.

(iii) Use $R_{eq} = R_1 + R_2$

Hence, $\frac{L_{eq}}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$

$$\Rightarrow \quad K_{eq} = \frac{(K_1K_1)\,L_{eq}}{L_1K_1 + L_2K_1}$$

Shortcut: As two plates are connected in series

Thus, same thermal current (heat transfer $\mathring{Q}$)

$\Rightarrow$ Thermal Resistance also follow same combination laws as that of electrical resistors.

In above diagram, compound plate is made by joining two plates in series

$$\therefore \quad R_{th} = R_{th_1} + R_{th_2}$$

$$\frac{L_1+L_2}{K_{eq}A} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$

Thus, $\frac{1}{K_{eq}} = \frac{L_1K_2 + L_2K_1}{K_1K_2\,(L_1+L_2)}$

$$\Rightarrow \quad \frac{1}{K_{eq}} = \frac{4\times2K + 2.5\times K}{2K^2\times6.5} = \frac{10.5K}{13K^2}$$

$$\Rightarrow \quad K_{eq} = \frac{13K}{10.5} = \frac{130}{105}K$$

$$= \frac{26}{21}K = \left(1+\frac{5}{21}\right)K$$

24. Correct answer is [700].

Explanation: Let the equation of particle performing SHM be:

$$x = A\sin(\omega t) \qquad \ldots(i)$$

On differentiating equation (1) we get,

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$\Rightarrow \quad v = a\omega\sqrt{1-\sin^2 \omega t}$$

$$\Rightarrow \quad v = A\omega\sqrt{1-\left(\frac{x}{A}\right)^2} \quad \text{[From eq (1)]}$$

$$v = \omega\sqrt{A^2 - x^2}$$

Initially, the amplitude of SHM is 10 cm.

When the particle is at a location of 5 cm i.e., $\frac{A}{2}$, it's speed is increased by air jet to 3V.

$\Rightarrow$ This will increase the total energy of SHM and and hence amplitude will also increase.

$\Rightarrow$ Angular frequency i.e., $\omega = \sqrt{\frac{\text{Elastic Factor}}{\text{Inertial factor}}}$ remains same because neither mass has changed nor any force.

$$v = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} \qquad \left[\text{When } x = \frac{A}{2}\right] \ldots(1)$$

$$3v = \left[\omega\sqrt{(A')^2 - \left(\frac{A}{2}\right)^2}\right]\left[\text{at } x = \frac{A}{2}\right] \qquad \ldots(2)$$

Divide (1) by (2)

$$\frac{1}{3} = \sqrt{\frac{A^2 - \frac{A^2}{4}}{A'^2 - \frac{A^2}{4}}}$$

On Sauaring both sides, we get;

$$\frac{1}{9} = \frac{\frac{3A^2}{4}}{A'^2 - \frac{A^2}{4}}$$

$$\Rightarrow \quad A'^2 - \frac{A^2}{4} = \frac{27A^2}{4}$$

$$\Rightarrow \quad A'^2 = \left(\frac{27}{4} + \frac{1}{4}\right)\frac{A^2}{4} = 7A^2$$

$$\Rightarrow \quad A' = \sqrt{7}A$$

On comparison with $\sqrt{x}$

$$x = 700 \text{ [As, } A = 10 \text{ cm]}$$

Hint:

(i) Use the relation $v = \omega\sqrt{A^2 - x^2}$

(ii) Write velocity in both cases in terms of amplitude and solve them to find new amplitude.

Shortcut: Kinetic energy of an oscillating particle is gives as $\Rightarrow K = \frac{1}{2} m\omega^2 (A^2 - x^2)$

$\Rightarrow$ As velocity is tripled, kinetic energy becomes g times

$$\frac{K}{9K} = \frac{A^2 - \frac{A^2}{4}}{A'^2 - \frac{A^2}{4}}$$

$$\Rightarrow \quad A'^2 - \frac{A^2}{4} = 9A^2 - \frac{9A^2}{4}$$

$$\Rightarrow \quad A'^2 = \frac{28\,A^2}{4}$$

$$= 7A^2$$

$$\Rightarrow \quad A' = \sqrt{7}A$$

$$\Rightarrow \quad A' = \sqrt{7} \times 10 \text{ cm}$$

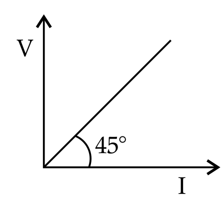
$$\Rightarrow \quad A' = \sqrt{700} \text{ cm}$$

By comparing it with given value, we get;

$$\therefore \quad x = 700$$

25. Correct answer is [144].

Explanation: We know that slope of *V-I* graph gives Resistance.



Thus, Slope $= R = \tan\theta = \tan 45° = 1$

Thus, $R = 1\Omega$

By using $R = \frac{\rho l}{A}$

$$\Rightarrow \quad 1 = \frac{x \times 10^{-5} \times 31.4 \times 10^{-2}}{\frac{\pi \times (2.4 \times 10^{-2})^2}{4}}$$

$$\Rightarrow \quad x = \frac{2.4 \times 2.4 \times 10^{-4}}{4 \times 10^{-6}} = \frac{2.4 \times 2.4 \times 10^{2}}{4}$$

$$x = \frac{24 \times 24}{4}$$

$$= 144$$

Hint:

(i) By using slope of *V-I* graph, find resistance.

(ii) Use Relation $R = \frac{\rho l}{A}$ to evaluate x

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Shortcut:

By slope of V-I graph, $R = 1\Omega$ and $R = \frac{\rho l}{A}$

$$\Rightarrow \quad x \times 10^{-5} = \frac{3.14 \times (2.4 \times 10^{-2})^2}{31.4 \times 10^{-2} \times 4}$$

$$\Rightarrow \quad x = \frac{2.4 \times 2.4}{4} \times 10^2 \Rightarrow x = 144$$

26. Correct answer is [102].

Explanation: Assuming the engine to be Carnot engine

$$\therefore \quad \frac{Q_{\text{absorbed}}}{Q_{\text{lost}}} = \frac{T_{\text{source}}}{T_{\text{sink}}} \quad \ldots(1)$$

On substituting values in equation (1)

$$\frac{300 \text{ cal}}{225 \text{ cal}} = \frac{500K}{T_{\text{sink}}}$$

$$T_{\text{sink}} = \frac{225}{300} \times 500 = 375\ K$$

$$T_{\text{sink}} (\text{in } ^\circ\text{C}) = 375 - 273 = 102\ ^\circ\text{C}$$

Hint:

(i) Use the relation, $\frac{T_H}{T_L} = \frac{Q_{\text{absorbed}}}{Q_{\text{Lost}}}$

(ii) Substitute the required values in above expression.

Shortcut: As engine operates on cyclic process

$$Q_{\text{net}} = W_{\text{net}}$$

$$\therefore \quad W_{\text{net}} = +300 - 225 = 75 \text{ cal}$$

$$\eta \Rightarrow \text{efficiency of Carnot engine} = 1 - \frac{T_L}{T_H}$$

$$\eta \Rightarrow \quad \frac{W}{Q_{\text{absorbed}}} = 1 - \frac{T_L}{T_H}$$

$$\frac{75}{300} = 1 - \frac{T_L}{T_H}$$

$$\frac{T_L}{500} = \frac{3}{4}$$

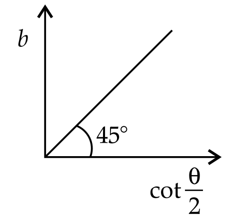
$$\Rightarrow \quad T_L = 375\ K$$

$$T_L = 375 - 273 = 102^\circ\text{C}$$

27. Correct answer is [3].

Explanation: $\cot\frac{\theta}{2} \propto b$

i.e., more the impact parameter lesser is the angle of scattering.



Given: $\sqrt{d_1}$ = impact parameter in 1st case

$\theta_1 = 60^\circ$

$\sqrt{d_2}$ = impact parameter in 2nd case

$\theta_2 = 90^\circ$

$$\text{Hence, } \frac{\cot\frac{\theta_1}{2}}{\cot\frac{\theta_2}{2}} = \sqrt{\frac{d_1}{d_2}}$$

On squaring both sides, we get;

$$\Rightarrow \quad \frac{\cot^2\frac{\theta_1}{2}}{\cot^2\frac{\theta_2}{2}} = \frac{d_1}{d_2}$$

$$\Rightarrow \quad \frac{(\sqrt{3})^2}{1} = \frac{d_1}{d_2}$$

$$\Rightarrow \quad d_1 = 3d_2$$

Hint:

(i) As $\cot\frac{\theta}{2} \propto b$

$$\frac{\cot\frac{\theta_1}{2}}{\cot\frac{\theta_2}{2}} = \frac{b_1}{b_2}$$

(ii) Put data in avobe relation and solve.

Shortcut: $\sqrt{d} \propto \cot\frac{\theta}{2}$

$$d_1 = xd_2$$

$$\cot^2\left(\frac{60^\circ}{2}\right) = x\cot^2\left(\frac{90^\circ}{2}\right)$$

$$\cot^2 30^\circ = x\cot^2 45^\circ$$

$$3 = x \times 1$$

Thus, $x = 3$

28. Correct answer is [2].

Explanation: Power gain is defined as the ratio of output power to the input power.

$$\text{Power gain} = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{I^2_{\text{output}} \times R_{\text{output}}}{I^2_{\text{input}} \times R_{\text{input}}}$$

$$\frac{\Delta i_c}{\Delta i_b} = \frac{I_{\text{output}}}{I_{\text{input}}} = \beta = \text{current gain}$$

in common emitter mode.

$$\beta = \frac{10^{-2} A}{10^{-4} A} = 100$$

$$\therefore \quad \text{Power gain} = (\beta)^2 \times \frac{R_{\text{out}}}{R_{\text{in}}}$$

$$= 10^4 \times \frac{2\times10^3}{1\times10^3} = 2 \times 10^4$$

Thus, value of $x = 2$

Hint:

(i) In Common-emitter mode:

$$\text{Current gain, } \beta = \frac{\Delta I_c}{\Delta I_b} = \frac{10\ mA}{100\ \mu A}$$

$$= 10^2 = 100$$

(ii) $$P_{\text{gain}} = \beta^2 \times \frac{R_{\text{out}}}{R_{\text{in}}}$$

Shortcut: $\Delta I_b = 100\ \mu A = 10^{-4} A$

$\Delta I_c = 10\ mA = 10^{-2}\ A$

$$\beta = \frac{\Delta I_c}{\Delta I_b} = 10^2$$

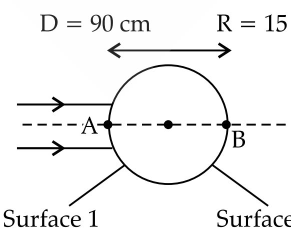
$$P_{\text{gain}} = \beta^2 = \frac{R_{\text{out}}}{R_{\text{in}}} = (10^2)^2 \times \frac{2}{1}$$

$$= 2 \times 10^4$$

Thus, $x = 2$

29. Correct answer is [225].

Explanation: Parallel Beam of light strikes at surface 1



Thus, 1st refraction takes place at surface 1 and, 2nd refraction takes place at surface 2

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at surface 1

All distances are measured from A

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{-\infty} = \frac{1.5-1}{+15}$$

[Given: $u = -\infty$, $v = ??$

$\mu_2 = 1.5, \mu_1 = 1$

$R = +15$ cm]

$$\frac{1.5}{v} = \frac{0.5}{15}$$

$$V = +45 \text{ cm}$$

for refraction at surface 2

All distance will be measured from B

Image after refraction at surface 1 will play role of object for surface 2

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{+15} = \frac{1-1.5}{-15}$$

$$\frac{1}{v} - \frac{1}{10} = \frac{0.5}{-15}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{10}$$

$$v = \frac{300}{40} = 7.5 \text{ cm}$$

Thus, distance of final image from B is 7.5 cm

Thus, beam of light converges at point 15 + 7.5= 22.5 cm = 225 mm from centre of sphere.

Hint:

(i) Use the relation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{\text{R}}$

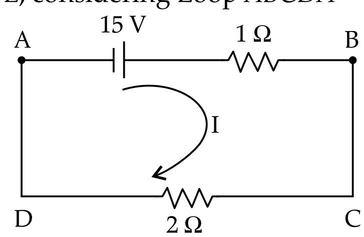
(ii) Apply the above equation two times first for surface 1 and then for surface 2. Obtain the final position image.

30. Correct answer is [10].

Explanation: By Kirchhoff's Loop Rule,

Potential difference while traversing a closed loop = 0

By *KVL*, considering Loop *ABCDA*



$$+ 15 - I \times 1 - 2 \times I = 0$$

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$$15 = 3I$$
$$\Rightarrow \quad I = 5A$$
Now, $V_A + 15 - 1 \times 5 = V_B$
$$V_B - V_A = 15 - 5 = 10\ V$$

Hint:

(i) Apply Kirchhoff's Voltage Law in the given loop and find current.

(ii) Find potential difference across the branch AB.

Shortcut:

$V_B - V_A$ = Potential Difference across 2 Ω resistor

As 2 Ω and 1 Ω are connected in series.

$R_{aq} = 2 + 1 = 3\Omega$

The $\quad I_{\text{circuit}} = \dfrac{V}{R_{eq}} = \dfrac{15}{3} = 5A$

Thus, by using Ohm's law, the potential difference across 2 Ω is $V_B - V_A = 2 \times 5 = 10\ V$

Chemistry

Section A

31. Option (C) is correct.

Explanation : Moles of $Fe_3O_4 = 4.640 \times \dfrac{10^3}{232}$

$$= 20 \text{ mol}$$

Moles of CO $= 2.52 \times \dfrac{10^3}{28} = 90$ mol

$Fe_3O_4\,(s) + 4\,CO\,(g) \rightarrow 3\,Fe\,(l) + 4\,CO_2\,(g)$

Since, 1 mole of Fe_3O_4 produces 3 moles of Fe.

So, 20 moles of Fe_3O_4 produces 20×3 moles of Fe.

Moles of Fe produced $= 20 \times 3 = 60$ moles.

So, weight of Fe = No. of moles produced × molar mass of Fe.

Weight of Fe $= 60 \times 56 = 3360$ g.

32. Option (D) is correct.

Explanation: (a) The electronic configuration of Cr is [Ar] $3d^5\,4s^1$ is correct.

(b) The magnetic quantum number may have a negative value because it depends upon azimuthal quantum number so, $m = -l$ to $+l$ is correct.

(c) According to Aufbau principle, the ground state of an atom, the orbitals are filled in order of their increasing order of their degenerate energy levels so, this statement is also correct.

(d) The total number of nodes are given by $n - 1$ so, this statement is incorrect.

33. Option (C) is correct.

Explanation: The covalent nature depends upon the polarizing power of the ion. As the size of the cation increases, its polarizing power decreases. Hence, the covalent character of the compound decreases. So, chloride (Cl) compounds of alkali metals, the decreasing order of covalent character is

LiCl > NaCl > KCl > RbCl > CsCl.

Thus, option (C) is correct.

34. Option (D) is correct.

Explanation: Adding a common ion decreases solubility because a dissociation reaction causes the equilibrium to shift left toward the reactants to relieve the stress of the excess product, causing precipitation. This effect is know as Common Ion Effect. Hence, solubility of AgCl will be maximum in deionised water because there is no common ion effect.

35. Option (D) is correct.

Explanation: When the colloidal particle have similar charges, they will repel each other which prevents them from aggregating when they come close to each other. Hence, the presence of equal and similar charges on colloidal particles provides stability to the colloidal solutions.

36. Option (A) is correct.

Explanation: The electronic configuration of Pt (Z = 78) = [Xe] $4f^{14}\,5d^9\,6s^1$.

37. Option (D) is correct.

Explanation: For ZnS, KCN is used as depressant to separate ZnS and PbS ores in froth flotation process. For Gold and silver, CaCN is used in cyanide process of extracting silver and gold from their ores by dissolving them in a dilute solution of sodium cyanide or potassium cyanide.

Cyanide salts are not involved in the extraction of copper metal. Hence, option (D) is correct.

38. Option (C) is correct.

Explanation:

$$\underbrace{2\overset{+7}{Mn}O_4^- + 2H_2O_2 \longrightarrow 2\overset{+4}{Mn}O_2}_{\text{Reduction}} + 3O_2 + 2H_2O + 2OH^-$$

Since, the oxidation state of Mn changes from + 7 to + 4. So, Mn gets reduced.

Therefore, H_2O_2 is acting as a reducing agent in the above reaction.

39. Option (A) is correct.

Explanation: Range of visible region : 390 nm to 760 nm

LiC—Crimson Red – 670.8 nm

NaCl—Golden yellow – 589.2 nm

RbCl—Violet – 780.0 nm

CsCl—Blue – 455.5 nm

40. Option (A) is correct.

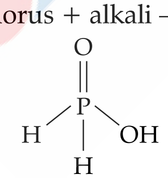
Explanation: Caesium is used devising photoelectric cells. Boron fibres are used in making bullet-proof vest. Silicones being surrounded by non-polar alkyl groups are water repelling in nature. Gallium is less toxic and has a very high boiling point, so it is used in high-temperature thermometers.

41. Option (B) is correct.

Explanation:

$$P_4 + NaOH + H_2O \rightarrow PH_3 + NaH_2PO_2$$
$$\downarrow HCl$$
$$H_3PO_2$$

White phosphorus + alkali → Phosphinic acid



Phosphinic acid

42. Option (A) is correct.

Explanation : Air pollutants like SO_2, NO_x are released in the atmosphere through various human activities. When these pollutants combines and reacts with water, vapours present in atmosphere forms more acidic pollutants.

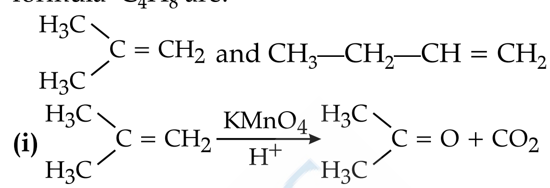
$$2SO_2 + O_2 \longrightarrow 2SO_3$$
$$SO_3 + H_2O \longrightarrow H_2SO_4$$

This H_2SO_4 formed reacts with $CaCO_3$ of Taj Mahal and is responsible for its damage.

$$CaCO_3 + H_2SO_4 \longrightarrow CaSO_4 + H_2O + CO_2$$

43. Option (D) is correct.

Explanation: Two isomers with molecular formula C_4H_8 are:

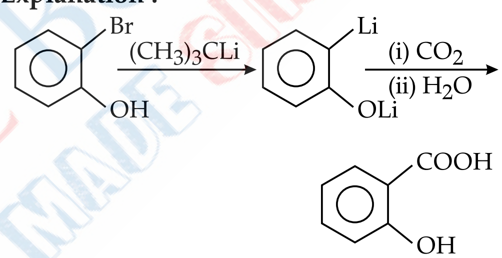


(ii) $CH_3—CH_2—CH{=}CH_2 \xrightarrow[H^+]{KMnO_4} CH_3—CH_2COOH + CO_2$

So, Isomer (A) is $(H_3C)_2\ C = CH_2$ as it gives ketone and CO_2.

44. Option (B) is correct.

Explanation :



45. Option (A) is correct.

Explanation :

$$R-\overset{O}{\overset{\|}{C}}-\boxed{OH}+\boxed{H}-O-R' \xrightleftharpoons{H^+} R-\overset{O}{\overset{\|}{C}}-O-R'+\boxed{H-O-H}$$

In nucleophilic acyl substitution, attack of the carbonyl carbon atom of an acyl derivative by (a) nucleophile yields a tetrahedral intermediate. The tetrahedral intermediate then eject a leaving group. Also, electron withdrawing group on carboxylic acid will increase the rate of esterification.

$$R-OH-R + \overset{O}{\overset{\|}{C}}-OH \longrightarrow R-O-\overset{O}{\overset{\|}{C}}-R$$

Nucleophilic acyl substitution

46. Option (C) is correct.

Explanation :

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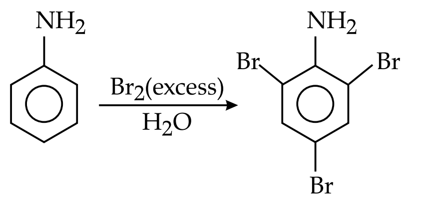
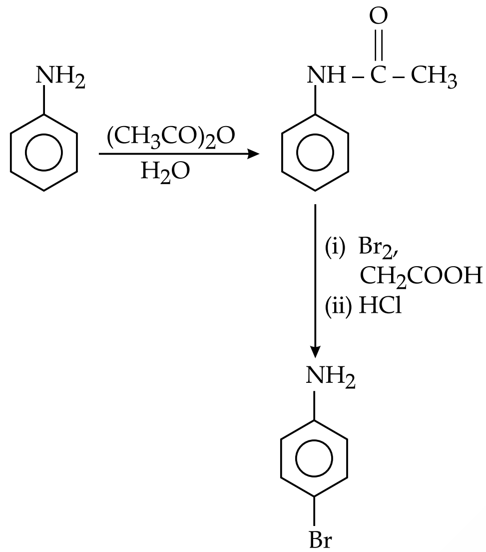


2,4,6-tribromoaniline
(Major Product)



4-bromoaniline

47. Option (C) is correct.

Explanation : Bakelite can be moulded very quickly, decreasing production time.

Nylon 6, 6 has high mechanical strength, high toughness, stiffness and hardness.

Buna – N is synthetic rubber which can be stretched and retains its original status on releasing the force.

Terylene is a very strong fibre and will suffer very little loss in strength when wet. It is elastic in nature and possess the property of resist creasing.

48. Option (B) is correct.

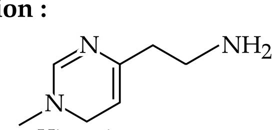
Explanation:

DNA contains = β-D – 2 deoxyribose

RNA contains = β-D – ribose

49. Option (C) is correct.

Explanation :



Histamine

Histamine is nitrogenous compound so, it does not contain sulphur atom.

50. Option (C) is correct.

Explanation : Phenol is a weak acid.

Since, the phenoxide ion formed after the removal of H^+ ion is resonance stabilised due to which it's stability increases, so phenol is more acidic than alcohol and water.

Section B

51. Correct answer is [152].

Explanation : Assuming ideal behaviour

$P = dRT/M$

Given : $P = \frac{100}{760}$ atm

$T = 257 + 273 = 530$ K

$d = 0.46$ g/L

$M = dRT/P$

$= \frac{0.46 \times 0.082 \times 530 \times 760}{100} = 152$

The molar mass of geraniol is 152 g mol^{-1}.

52. Correct answer is [117].

Explanation :

17 g NH_3 contains 1 mol NH_3.

So, 85 g NH_3 will have $\frac{85}{17} = 5$ mol of NH_3

Enthalpy change for 1 mol of vaporisation of $NH_3 = 23.4$ kJ mol^{-1}

So, enthalpy change required for 5 mol of vaporistion, $\Delta H = 5 \text{ mol} \times 23.4$ kJ

$= 117$ kJ

Therefore, the enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is 117 kJ.

53. Correct answer is [5].

Explanation : Given, volume of acetic acid (v) = 1.2 mL

Density of acetic acid $(d) = 1.02$ g mL^{-1}

So, mass of acetic acid (m) = Density × volume

$= 1.02 \times 1.2$

$= 1.224$ g

Now, molar mass of acetic acid = 60 g mol^{-1}

So, No. of moles of acetic acid $= \frac{\text{Mass}}{\text{Molar mass}}$

$= \frac{1.224}{60} = 0.0204$

So, Moles = 0.0204 mole in 2L

So, Molality = 0.0102 mol/kg

$\Delta T_f = i \times K_f \times m$

$0.0198 = i \times 1.85 \times 0.0102$

$$i = \frac{0.0198}{1.85 \times 0.0102} = 1.049$$

$$i \approx 1.05$$

Since, $\alpha = \frac{i-1}{n-1} = \frac{1.05-1}{1} = 0.05$

$$\% \alpha = 0.05 \times 100\%$$
$$= 5\%$$

The percentage of dissociation of the acid is = 5%

54. Correct answer is [127].

Explanation : Given: 2 F produces = 3/2 mole of gas

$$i = 0.10 \text{ A}$$
$$T = 2 \text{ h}$$

Reaction at anode

$2H_2O \rightarrow O_2(g) + 7H^+ + 4e^-$

At cathode: $2H^+ + 2e^- \rightarrow H_2(g)$

Number of equivalent = $\frac{i \times t}{96500}$

= 0.10 × 2 × 3600 coulomb produce

$$= \frac{3 \times 0.1 \times 2 \times 3600}{2 \times 2 \times 96500}$$

Volume of gas produced = 0.0056 × 22.7 L

= 0.127 L

= 127 mL

55. Correct answer is [1].

Explanation : $\ln k_2/k_1 = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$$\ln 310/300 = \frac{532611}{8.3}\left[\frac{1}{300} - \frac{1}{310}\right]$$

$$\ln \frac{k_{310}}{k_{200}} = \frac{532611}{8.3}\left(\frac{10}{93000}\right)$$

$$\ln \frac{k_{310}}{k_{300}} = 6.9$$

$$\ln \frac{k_{310}}{k_{300}} = 3 \times \ln 10$$

$$\frac{k_{310}}{k_{300}} = 10^3$$

$$k_{300} = k_{310} \times 1 \times 10^{-3}$$

i.e., $k_{300} = 1 \times 10^{-3}\, k_{310}$

So, $x = 1$

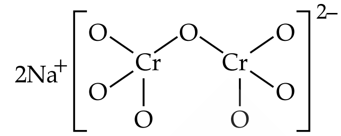
56. Correct answer is [6].

Explanation :

$FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + CO_2$

$Na_2CrO_4 + H^+ \rightarrow Na_2Cr_2O_7 + Na^+ + H_2O$

(A) (B)



The number of terminal oxygen atoms present in the product B is 6.

57. Correct answer is [0]

Explanation :

$$\overset{+6}{Mn}O_4^{2-} \xrightarrow{H^+} \overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + H_2O$$

In Mn^{7+}, there is no unpaired electron.

So, spin only magnetic moment value of MnO_4^- is 0.

58. Correct answer is [64].

Explanation :

$$\% N = \frac{1.4 \times N \times V}{\text{Mass of organic compound}}$$

$$\% N = \frac{1.4 \times 2 \times 12.5}{0.55} = 63.63\%$$

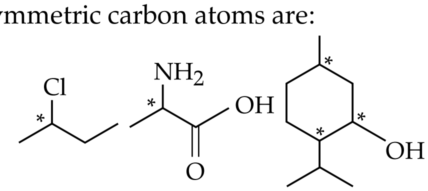
[∵ Normality = Molarity × n factor

N = 1 × 2 = 2]

= 64%

59. Correct answer is [3].

Explanation : Compound which possesses asymmetric carbon atoms are:



60. Correct answer is [1]

Explanation :

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} CH_3CH_2OH \xrightarrow{\text{Zymase}} H-\overset{\overset{\displaystyle O}{\|}}{C}-\bar{O}\overset{+}{N}a + CHI_3$$

So, in HCOONa (B), numbers of carbon atom is 1.

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Mathematics

Section A

61. Option (C) is correct.

Explanation: Let M be a 2×2 matrix such that

$$M = \begin{bmatrix} m & n \\ o & p \end{bmatrix} \text{ and}$$

For M to be a singular matrix, $|M| = 0$

$\Rightarrow \quad mp - on = 0$

Case 1: All four elements are equal

$m = n = o = p$

$\Rightarrow \quad mp - on = 0$

So, number of matrices possible = 10

Case 2: When two prime numbers are used

$\Rightarrow$ Either $m = n$ and $o = p$ or $m = o$ and $n = p$

So, number of matrices possible = ${}^{10}C_2 \times 2! \times 2!$

$$= \frac{10 \times 9}{2} \times 2 \times 2$$

$$= 180$$

So, number of matrices possible = 10 + 180 = 190

And total number of matrices that can be formed $= 10 \times 10 \times 10 \times 10 = 10^4$

So, required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

Hint: Take two cases : when all elements are equal and when two prime numbers are used to form the matrix.

Shortcut: Number of matrices possible = when all elements are equal + when two prime number are used

$$= 10 + {}^{10}C_2 + 2! \times 2!$$

$$= 190$$

Total number of matrices $= 10^4$

$$\text{probability} = \frac{190}{10^4} = \frac{19}{10^3}$$

62. Option (A) is correct.

Explanation: Given differential equation is

$$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$$

$$\Rightarrow \quad x\frac{dy}{dx} = y + \sqrt{y^2 + 16x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y + \sqrt{y^2 + 16x^2}}{x} \quad \ldots(1)$$

Let $\quad y = tx$

$$\Rightarrow \quad \frac{dy}{dx} = t + x\frac{dt}{dx} \quad \ldots(2)$$

From equation (1) and (2), we get

$$t + x\frac{dt}{dx} = \frac{y + \sqrt{y^2 + 16x^2}}{x}$$

$$\Rightarrow \quad t + x\frac{dt}{dx} = \frac{tx + \sqrt{t^2x^2 + 16x^2}}{x}$$

$$\Rightarrow \quad t + x\frac{dt}{dx} = t + \sqrt{t^2 + 16}$$

$$\Rightarrow \quad x\frac{dt}{dx} = \sqrt{t^2 + 16}$$

$$\Rightarrow \quad \frac{dt}{\sqrt{t^2 + 16}} = \frac{dx}{x}$$

Integrating both the sides, we get

$$\int \frac{dt}{\sqrt{t^2 + 16}} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \text{In}\,[t + \sqrt{t^2 + 16}\,] = \ln x + \ln c$$

$$\Rightarrow \quad t + \sqrt{t^2 + 16} = xc$$

$$\Rightarrow \quad \frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 16} = xc$$

$$\Rightarrow \quad \frac{y}{x} + \sqrt{\frac{y^2 + 16x^2}{x}} = xc$$

$$\Rightarrow \quad y + \sqrt{y^2 + 16x^2} = x^2c$$

Now, $\quad y(1) = 3$

$\Rightarrow \quad 3 + \sqrt{9 + 16} = c$

$\Rightarrow \quad c = 8$

$\Rightarrow \quad y + \sqrt{y^2 + 16x^2} = 8x^2$

So, at $x = 2$,

$y + \sqrt{y^2 + 64} = 32$

$\Rightarrow \quad \sqrt{y^2 + 64} = (32 - y)$

$\Rightarrow \quad y^2 + 64 = y^2 + (32)^2 - 64y$

$\Rightarrow \quad 64 = 960$

$\Rightarrow \quad y = 15$

$\therefore \quad y\,(2) = 15$

63. Option (C) is correct.

Explanation: Given: The mirror image of (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c)

As we know, the mirror image of the point (x, y, z) in the plane $ax + by + cz + d = 0$ is given by,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

$$\Rightarrow \frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-2(6+(-4)+28-2)}{(3)^2+(-1)^2+(4)^2}$$

$$\Rightarrow \frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-2\,(28)}{26}$$

$$\Rightarrow \frac{a-2}{3}=\frac{-28}{13},\ \frac{b-4}{1}=\frac{28}{13},\ \frac{c-7}{4}=\frac{-28}{13}$$

$$\Rightarrow a=2-\frac{84}{13},\ b=\frac{28}{13}+4,\ c=7-\frac{112}{13}$$

$$\Rightarrow a=\frac{-58}{13},\ b=\frac{80}{13},\ c=\frac{-21}{13}$$

$$\Rightarrow 2a+b+2c=2\left(\frac{-58}{13}\right)+\frac{80}{13}+2\left(\frac{-21}{13}\right)$$

$$\Rightarrow 2a+b+2c=-6$$

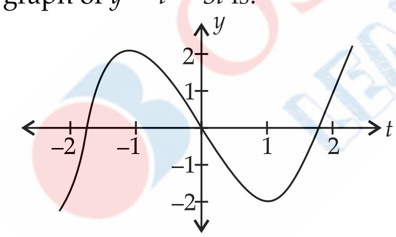
Hint: The mirror image of the point (x, y, z) in the plane $ax+by+cz+d=0$ is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

64. Option (C) is correct.

Explanation: Given: $f: R \to R$

$$f(x)=\begin{cases}\max\limits_{t\le x}\{t^3-3t\} & x\le 2\\ x^2+2x-6 & 2<x\le 3\\ [x-3]+9 & 3<x\le 5\\ 2x+1 & x>5\end{cases}$$

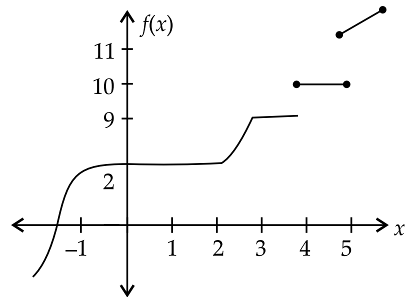
The graph of $y=t^3-3t$ is:



For $x \le -1$, $\max\limits_{t\le x}\{t^3-3t\}=x^3-3x$

For $-1<x\le 2$, $\max\limits_{t\le x}\{t^3-3t\}=2$

$$\therefore\quad f(x)=\begin{cases}x^3-3x & x\le -1\\ 2 & -1<x\le 2\\ x^2+2x-6 & 2<x\le 3\\ 9 & 3<x\le 4\\ 10 & 4<x<5\\ 11 & x=5\\ 2x+1 & x>5\end{cases}$$



As we know, a function is not differentiable at sharp points and at point of discontinuity

$\Rightarrow f(x)$ is not differentiable at $x=2, 3, 4, 5$

$\therefore$ The number of points where $f(x)$ is not differentiable $= 4$

$\Rightarrow m=4$

Now, $$I=\int_{-2}^{2} f(x)\,dx$$

$$\Rightarrow I=\int_{-2}^{-1} f(x)\,dx+\int_{-1}^{2} f(x)\,dx$$

$$\Rightarrow I=\int_{-2}^{-1} x^3-3x)\,dx+\int_{-1}^{2} 2\,dx$$

$$\Rightarrow I=\left[\frac{x^4}{4}-\frac{3x^2}{2}\right]_{-2}^{-1}+[2x]_{-1}^{2}$$

$$\Rightarrow I=\left(\frac{1}{4}-\frac{3}{2}-\frac{16}{4}+6\right)+(4-(-2))$$

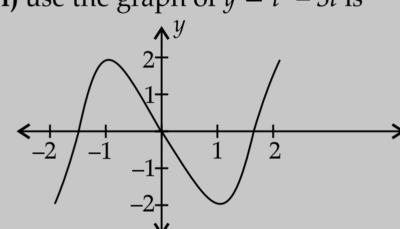
$$\Rightarrow I=\left(\frac{1}{4}-\frac{3}{2}+2\right)+6$$

$$\Rightarrow I=\frac{1-6+32}{4}$$

$$\Rightarrow I=\frac{27}{4}$$

$\therefore$ The ordered pair $(m, I)=(4, \frac{27}{4})$

Hint: (i) use the graph of $y=t^3-3t$ is



(ii) A function is not differentiable at sharp points and at the point of discontinuity.

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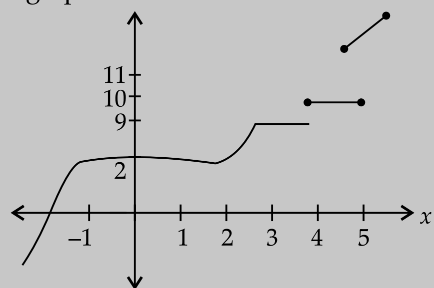
Shortcut: $f(x) = \begin{cases} x^3 - 3x & x \le -1 \\ 2 & -1 < x \le 2 \\ x^2 + 2x - 6 & 2 < x \le 3 \\ 9 & 3 < x \le 4 \\ 10 & 4 < x < 5 \\ 11 & x = 5 \\ 2x + 1 & x > 5 \end{cases}$

$\Rightarrow f(x)$ is not differentiable at $x = 2, 3, 4, 5$ from the graph



$$\Rightarrow \quad m = 4$$

$$I = \int_{-2}^{2} f(x)\,dx$$

$$\Rightarrow \quad I = \int_{-2}^{-1} x^3 - 3x)\,dx + \int_{-1}^{2} 2\,dx$$

$$\Rightarrow \quad I = \left[\frac{x^4}{4} - \frac{3x^2}{2}\right]_{-2}^{-1} + [2x]_{-1}^{2}$$

$$\Rightarrow \quad I = \frac{27}{4}$$

$$\therefore \quad (m, I) \equiv \left(4, \frac{27}{4}\right)$$

65. Option (A) is correct.

Explanation: Given: $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$

The projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$

As we know, the projection of $\vec{x}$ on $\vec{y}$ is given by $\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}$

$$\Rightarrow \quad \frac{\vec{a}\cdot\vec{b}}{|\vec{c}|} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(\alpha\hat{i} + 3\hat{j} - \hat{k})(\hat{i} + 2\hat{j} - 2\hat{k})}{|\hat{i} + 2\hat{j} - 2\hat{k}|} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{\alpha + 6 + 2}{\sqrt{1 + 4 + 4}} = \frac{10}{3}$$

$$\Rightarrow \quad \alpha + 8 = 10$$

$$\Rightarrow \quad \alpha = 2$$

Also, given that $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$

$$\Rightarrow (3\hat{i} - \beta\hat{j} + 4\hat{k}) \times (\hat{i} + 2\hat{j} - 2\hat{k}) = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$\Rightarrow \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$\Rightarrow (2\beta - 8)\ \hat{i} - \hat{j}\,(-6 - 4) + \hat{k}\,(6 + \beta)\,\hat{k}$$

$$= -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$$\Rightarrow \quad 2\beta - 8 = -6 \text{ or } 6 + \beta = 7$$

$$\Rightarrow \quad \beta = 1$$

$$\therefore \quad \alpha + \beta = 2 + 1 = 3$$

Hint: (i) The projection of $\vec{x}$ on $\vec{y}$ is $\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}$

(ii) If $\vec{x} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{y} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

then $\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

Shortcut: The projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$

$$\Rightarrow \quad \frac{\vec{a}\cdot\vec{b}}{|\vec{c}|} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{\alpha + 6 + 2}{\sqrt{1^2 + (2)^2 + (-2)^2}} = \frac{10}{3}$$

$$\alpha = 2$$

and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = -6\hat{i} + 10\hat{j} + 7\hat{k}$

$$\Rightarrow \quad 2\beta - 8 = -6,\ 6 + \beta = 7$$

$$\Rightarrow \quad \beta = 1$$

$$\Rightarrow \quad \alpha + \beta = 3$$

66. Option (C) is correct.

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Explanation: Given curves are $y^2 = 8x$ and

$y = \sqrt{2}x$

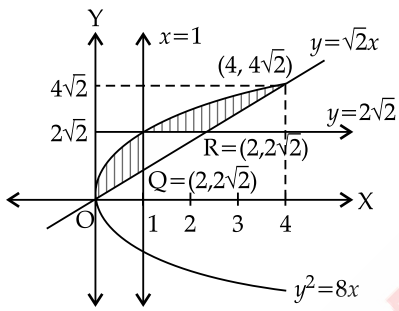
Let us find the intersection point of both the curves:

$$\begin{aligned}
(\sqrt{2}x)^2 &= 8x \\
\Rightarrow \quad 2x^2 &= 8x \\
\Rightarrow \quad 2x^2 - 4x &= 0 \\
\Rightarrow \quad x(x-4) &= 0 \\
\Rightarrow \quad x &= 0, 4 \\
\Rightarrow \quad y &= 0,\ 4\sqrt{2}
\end{aligned}$$

So intersection points are (0, 0) and $(4,\ 4\sqrt{2})$

Let PQR be the triangle formed by $y = \sqrt{2}x$, $x = 1$ and $y = 2\sqrt{2}$

$\Rightarrow P = (1,\ 2\sqrt{2}),\ Q = (1,\ \sqrt{2}),\ R = (2,\ 2\sqrt{2})$



$$\text{Area of } OPSRQ = \int_0^4 (\sqrt{8x} - \sqrt{2}x)\,dx$$

$$= \left[\sqrt{8}\left(\frac{2}{3}x^{3/2}\right) - \frac{\sqrt{2}}{2}x^2\right]_0^4$$

$$= \frac{2\sqrt{8}}{3}(4)^{3/2} - \frac{1}{\sqrt{2}}(4)^2$$

$$= \frac{4\sqrt{2}}{3}(8) - \frac{16}{\sqrt{2}}$$

$$\Rightarrow \quad \text{Area of } OPSRQ = \left(\frac{32\sqrt{2}}{3} - 8\sqrt{2}\right) \text{sq. units}$$

$$\text{Now,} \quad \text{area of } \Delta PQR = \frac{1}{2} \times (PQ) \times (PR)$$

$$= \frac{1}{2} \times (2\sqrt{2} - \sqrt{2}) \times 1$$

$$= \frac{1}{2} \times \sqrt{2}$$

$$\Rightarrow \quad \text{Area of } \Delta PQR = \frac{\sqrt{2}}{2} \text{ sq. units}$$

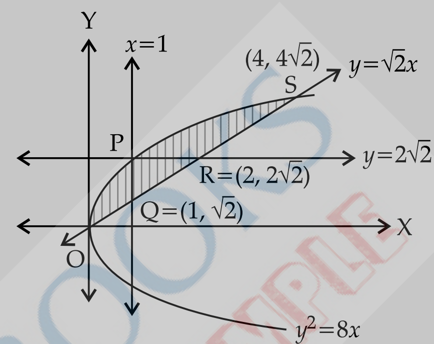
So, Required area = area of shaded region
= Area of $OPSRQ$ – Area of ΔPQR

$$= \frac{32\sqrt{2}}{3} - 8\sqrt{2} - \frac{\sqrt{2}}{2}$$

$$= \frac{64\sqrt{2} - 48\sqrt{2} - 3\sqrt{2}}{6}$$

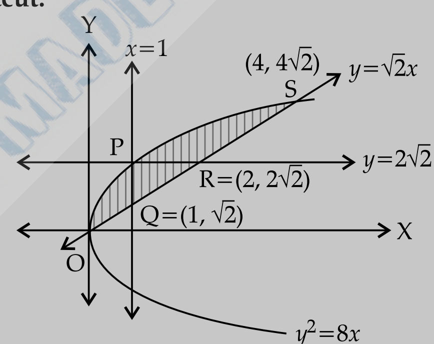
$$= \frac{13\sqrt{2}}{6} \text{ sq. units}$$

Hint:



Required area = Area of $OPSRQ$ – Area of ΔPQR

Shortcut:



Required area = Area of $OPSRQ$ – Area of ΔPQR

$$= \int_0^4 (\sqrt{8x} - \sqrt{2}x) - \frac{1}{2} \times (2\sqrt{2} - \sqrt{2}) \times 1$$

$$= \frac{32\sqrt{2}}{3} - 8\sqrt{2} - \frac{\sqrt{2}}{2}$$

$$= \frac{13\sqrt{2}}{6} \text{ sq. units}$$

67. Option (B) is correct.

Explanation: Given: A system of linear equation has infinitely many solution.

$$\begin{aligned}
2x + y - z &= 7 \\
x - 3y + 2z &= 1 \\
x + 4y + \delta z &= k
\end{aligned}$$

As we know, if a system of linear equations

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$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_3z = d_2$$
$$a_3x + b_2y + c_3z = d_3$$

has infinitely many solutions then, $D = D_1 = D_2 = D_3 = 0$

where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$,

$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

So, $D = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$

$\Rightarrow 2(-3\delta - 8) - 1(\delta - 2) - 1(4 + 3) = 0$

$\Rightarrow -6\delta - 16 - \delta + 2 - 7 = 0$

$\Rightarrow \quad 7\delta = -21$

$\Rightarrow \quad \delta = -3$

Also, $D_3 = \begin{vmatrix} 2 & 1 & 7 \\ 1 & -3 & 1 \\ 1 & 4 & k \end{vmatrix} = 0$

$\Rightarrow 2(-3k - 4) - 1(k - 1) + 7(4 + 3) = 0$

$\Rightarrow -6k - 8 - k + 1 + 49 = 0$

$\Rightarrow \quad 7k = 42$

$\Rightarrow \quad k = 6$

$\Rightarrow \quad \delta + k = -3 + 6 = 3$

Hint: If a system of linear equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_3z = d_2$$
$$a_3x + b_2y + c_3z = d_3$$

has infinite solutions, the $D = D_1 = D_2 = D_3 = 0$ where

$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$,

$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Shortcut: If a system of linear equations has infinite solutions, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$

$\Rightarrow \quad \delta = -3$

and $\begin{vmatrix} 2 & 1 & 7 \\ 1 & -3 & 1 \\ 1 & 4 & k \end{vmatrix} = 0$

$\Rightarrow \quad k = 6$

$\therefore \quad \delta + k = 3$

68. Option (A) is correct.

Explanation: Given: α and β are roots of $x^2 + (2i - 1) = 0$

$\Rightarrow \quad x^2 + (2i - 1) = 0$

$\Rightarrow \quad x^2 = 1 - 2i$

$\Rightarrow \quad \alpha^2 = 1 - 2i$ and $\beta^2 = 1 - 2i$

$\Rightarrow \quad \alpha^2 = \beta^2$

$\Rightarrow \quad (\alpha^2)^4 = (\beta^2)^4$

$\Rightarrow \quad \alpha^8 = \beta^8$

$\therefore \quad \alpha^8 + \beta^8 = 2\alpha^8$

$\therefore \quad \alpha^8 + \beta^8 = 2(\alpha^2)^4$

$\Rightarrow \quad \alpha^8 + \beta^8 = 2(1 - 2i)^4$

$\Rightarrow \quad |\alpha^8 + \beta^8| = 2\,|1 - 2i|^4$

$\Rightarrow \quad |\alpha^8 + \beta^8| = 2\left(\sqrt{(1)^2 + (-2)^2}\right)^4$

$\Rightarrow \quad |\alpha^8 + \beta^8| = 2\left(\sqrt{5}\right)^4$

$\Rightarrow \quad |\alpha^8 + \beta^8| = 2(25) = 50$

Hint: (1) The modulus of a complex number $a + bi = \sqrt{a^2 + b^2}$

69. Option (C) is correct.

Explanation:

p	q	$p \vee q$	$(p \vee q) \Rightarrow q$	$p \wedge q$	$(p \wedge q) \wedge (p \vee q) \Rightarrow q)$	$(p \wedge q) \vee ((p \vee q) \Rightarrow q)$	$(p \wedge q) \Rightarrow ((p \vee q) \Rightarrow q)$	$(p \wedge q) \Leftrightarrow ((p \vee q) \Rightarrow q)$
T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T
F	T	T	T	F	T	T	T	F
F	F	F	T	F	F	T	T	F

Clearly, $(p \wedge q) \Rightarrow ((p \vee q) \Rightarrow q)$ is a tautology.

Hint: Tautology is a statement which is always true.

Shortcut: $(p \vee q) \Rightarrow q$

$\equiv \sim (p \vee q) \vee q$

$\equiv (\sim p \wedge \sim q) \vee q$

{By De Morgan's law}

$\equiv (\sim p \vee q) \wedge (\sim q \vee q)$

$\equiv (\sim p \vee q) \wedge \text{T}$

$\equiv (\sim p \vee q)$

And $(p \wedge q) \Rightarrow ((p \vee q) \Rightarrow q)$

$\equiv (p \wedge q) \Rightarrow (\sim p \vee q)$

$\equiv \sim (p \wedge q) \vee (\sim p \vee q)$

$\equiv \text{T}$

So option '*C*' is correct.

70. Option (A) is correct.

Explanation: Given: $a_{ij} = 2^{j-i}$

$$\Rightarrow \quad A = \begin{bmatrix} 2^{1-1} & 2^{2-1} & 2^{3-1} \\ 2^{1-2} & 2^{2-2} & 2^{3-2} \\ 2^{1-3} & 2^{2-3} & 2^{3-3} \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+1+1 & 2+2+2 & 4+4+4 \\ 1/2+1/2+1/2 & 1+1+1 & 2+2+2 \\ 1/4+1/4+1/4 & 1/2+1/2+1/2 & 1+1+1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 3 & 6 & 12 \\ 3/2 & 3 & 6 \\ 3/4 & 3/2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = 3A$$

Also, $A^3 = A^2.A = 3A.A = 3A^2 = 3(3A) = 3^2A$

Similarly $A^4 = A^3.A = 3^2A.A = 3^2(A^2) = 3^2(3A) = 3^3A$

$\Rightarrow A^2 + A^3 + A^4 + \ldots\ldots + A^9 + A^{10} = 3A + 3^2A + 3^3A + \ldots\ldots + 3^8A + 3^9A$

$= 3A\,(1 + 3 + 3^2 + \ldots\ldots + 3^7 + 3^8)$

$$= 3A\left(\frac{3^{9-1}}{3-1}\right)$$

$$= 3A\left(\frac{3^{9-1}}{2}\right)$$

$$= \left(\frac{3^{10}-3}{2}\right)A$$

Hint: (i) Simplify using multiplication of matrices.

(ii) Sum of *G.P.* with a as first term, r as common ratio and n as number of terms is given by $\dfrac{a\,(r^n-1)}{r-1}$.

$$\textbf{Shortcut: } A = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 1/2 & 1 & 2 \\ 1/4 & 1/2 & 1 \end{bmatrix}$$

$\Rightarrow \quad A^2 = 3A$

$A^3 = A^2A = 3^2A$

$\therefore A^2 + A^3 + \ldots\ldots + A^{10} = 3A + 3^2A + \ldots\ldots + 3^9A$

$$= A\left\{\frac{3\,(3^9-1)}{3-1}\right\}$$

$$= \left(\frac{3^{10}-3}{2}\right)A$$

71. Option (D) is correct.

Explanation: Set $A = A_1 \cup A_2 \cup A_3 \ldots \cup A_x$, where $A_i \cap A_j = \phi;\ i \neq j,\ 1 \leq i, j \leq k$

And relation $R = \{(x, y) : y \in A_i : iff\ x \in A_i;\ 1 \leq i \leq k\}$

(1) Symmetric: If $(x, y) \in R$, then $(y, x) \in R$

$\therefore$ Given relation is symmetric.

(2) Reflexive: $\because (a, a) \in R$ for all $a \in A_i$

$\therefore$ Given relation is reflexive

(3) Transitive: If $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow y \in A;\ iff\ x \in A_i$ and $z \in A_i\ iff\ y \in A_i$

$\Rightarrow z \in A;\ iff\ x \Rightarrow A_i$

$\Rightarrow (x, z) \in R$

$\therefore$ Given relation is transitive.

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Since, given relation is symmetric, reflexive and transitive

$\therefore$ It is an equivalence relation.

> **Hint: (i)** Recall the definition of symmetric, reflexive and transitive relation.
>
> **(ii)** A relation is said to be equivalence relation if it is symmetric, reflexive and transitive.

> **Shortcut:** Let $A = =\{1, 2, 3, 4\}$
>
> $\Rightarrow R = \{(1\,1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1)$ $(3, 2), (3, 3)\}$
>
> $\because$ R is reflexive transitive and symmetric.
>
> $\therefore$ It is an equivalence relation.

72. Option (B) is correct.

Explanation:

Given, $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1 \ \forall \ n \geq 0$

$\Rightarrow \quad a_2 = 2a_0 - a_0 + 1 = 1$

and $\quad a_3 = 2a_2 - a_1 + 1 = 3$

and $\quad a_4 = 2a_3 - a_2 + 1 = 6$

And $\quad a_5 = 2a_4 - a_3 + 1 = 10$

$$\therefore \quad a_n = \frac{n(n-1)}{2}$$

$$\text{Let} \quad p = \sum_{n=2}^{\infty} \frac{a_n}{7^n}$$

$$\Rightarrow \quad p = \sum_{n=2}^{\infty} \frac{n(n-1)}{2 \cdot 7^n}$$

$$\Rightarrow \quad p = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \ldots\ldots \quad \ldots\text{(i)}$$

$$\Rightarrow \quad \frac{p}{7} = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \frac{10}{7^6} + \ldots\ldots \quad \ldots\text{(ii)}$$

Equation (i) – equation (ii), we get

$$\frac{6p}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \ldots\ldots \quad \ldots\text{(iii)}$$

$$\Rightarrow \quad \frac{6p}{7^2} = \frac{1}{7^3} + \frac{2}{7^4} + \frac{3}{7^5} + \frac{4}{7^6} + \ldots\ldots \quad \ldots\text{(iv)}$$

Equation (iii) – equation (iv), we get

$$\frac{6p}{7} \cdot \left(1 - \frac{1}{7}\right) = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \frac{1}{7^5} + \ldots\ldots$$

$$\Rightarrow \quad \frac{6p}{7}\left(\frac{6}{7}\right) = \frac{\frac{1}{7^2}}{1 - \frac{1}{7}}$$

$$\Rightarrow \quad \left(\frac{6}{7}\right)^2 p = \frac{1}{(7)(6)}$$

$$\Rightarrow \quad p = \frac{7}{216}$$

> **Hint:**
>
> **(i)** Use $an = \dfrac{n(n-1)}{2}$
>
> **(ii)** Simplify given sequence and try to convert it into the form of infinite G.P. and solve further.
>
> **(iii)** Sum of infinite G.P. with first term a and common ratio r ($r < 1$) is given by $\dfrac{a}{1-r}$.

73. Option (C) is correct.

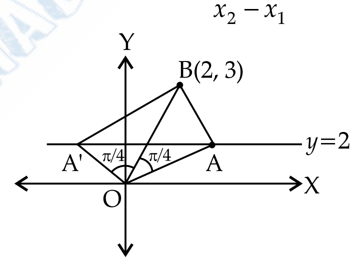
Explanation: Given: A and A' lies on $y = 2$

Let coordinates of $A \equiv (x_1, 2)$ and $A' \equiv (x_2, 2)$

Let slope of OA be m_1 and OB be m_2

As we know slope of line passing through (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$$\Rightarrow \quad m_1 = \frac{2}{x_1} \text{ and } m_2 = \frac{3}{2}$$

Also, we know that if angle between two lines having slope m_1 and m_2 is θ, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \left|\frac{\frac{2}{x_1} - \frac{3}{2}}{1 + \left(\frac{2}{x_1}\right)\left(\frac{3}{2}\right)}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{\frac{2}{x_1} - \frac{3}{2}}{1 + \frac{3}{x_1}}\right|$$

$$\Rightarrow \quad 1 = \left|\frac{(4 - 3x_1)}{2(x_1 + 3)}\right|$$

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$$\Rightarrow \quad \pm 1 = \frac{4-3x_1}{2(x_1+3)}$$

$$\Rightarrow \quad 4-3x_1 = \pm(2x_1+6)$$

$$\Rightarrow \quad 4-3x_1 = 2x_1+6$$

and $\quad 4-3x = -2x_1 - 6$

$$\Rightarrow \quad 5x_1 = -2 \text{ and } x_1 = 10$$

$$\Rightarrow \quad x_1 = \frac{-2}{5} \text{ and } x_1 = 10$$

$\because$ x is positive for A and negative for A'

$$\therefore \quad x_1 = 10 \text{ and } x_2 = \frac{-2}{5}$$

$$\Rightarrow \quad A \equiv (10, 2) \;\&\; A' \equiv \left(\frac{-2}{5}, 2\right)$$

$\therefore$ by distance formula,

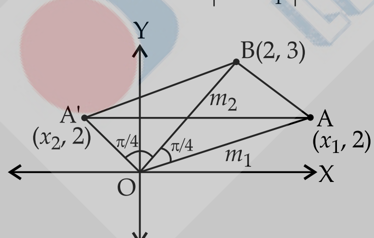
$$AA' = \sqrt{\left(10+\frac{2}{5}\right)^2 + (2-2)^2}$$

$$\Rightarrow \quad AA' = \frac{52}{5} \text{ units}$$

Hints: (1) The angle between two lines having slope m_1 and m_2 is θ and $\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

Shortcut: $m_1 = \dfrac{2}{x_1}$ and $m_2 = \dfrac{3}{2}$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \left|\frac{\frac{2}{x_1}-\frac{3}{2}}{1+\frac{3}{x_1}}\right|$$



$$\Rightarrow \quad \pm 1 = \frac{4-3x_1}{2(x_1+3)}$$

$$\Rightarrow \quad x_1 = \frac{-2}{5} \text{ and } x_1 = 10$$

$$\therefore \quad A \equiv (10, 2) \text{ and } A' = \left(\frac{-2}{5}, 2\right)$$

$$\Rightarrow \quad AA' = \sqrt{\left(10+\frac{2}{5}\right)^2 + (2-2)^2}$$

$$AA' = \frac{52}{5} \text{ units}$$

74. Option (B) is correct.

Explanation: Given: A wire of length 22 m

Let length of the side of triangle be x and the length of the side of square be y and p be the length of wire formed into triangle

$$\therefore \quad p = 3x$$

and $\quad 22-p = 4y$

$$\Rightarrow \quad x = \frac{p}{3} \text{ and } y = \frac{1}{4}(22-p)$$

Now, area of triangle $= \dfrac{\sqrt{3}}{4}x^2 = \dfrac{\sqrt{3}}{4}\left(\dfrac{p}{3}\right)^2$

and $\quad$ area of square $= y^2 = \left[\dfrac{1}{4}(22-p)\right]^2$

$$\therefore \quad \text{Total area} = \frac{\sqrt{3}}{4}\frac{p^2}{9} + \frac{1}{16}(22-p)^2$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{36}p^2 + \frac{1}{16}(22^2 + p^2 - 44p)$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{36}p^2 + \frac{p^2}{16} - \frac{22}{8}p + \frac{22^2}{16}$$

For A to be minimum, $\dfrac{dA}{dp} = 0$

$$\Rightarrow \quad \frac{dA}{dp} = 2\left(\frac{\sqrt{3}}{36}p^2\right) + 2\left(\frac{p}{16}\right) - \frac{22}{8} = 0$$

$$\Rightarrow \quad \frac{dA}{dp} = \frac{\sqrt{3}}{18}p + \frac{p}{8} - \frac{22}{8} = 0$$

$$\Rightarrow \quad p\left(\frac{\sqrt{3}}{18} + \frac{1}{8}\right) = \frac{22}{8}$$

$$\Rightarrow \quad p = \frac{22/8}{\frac{4\sqrt{3}+9}{72}}$$

$$\Rightarrow \quad p = \frac{22}{8} \times \frac{72}{4\sqrt{3}+9}$$

$$\Rightarrow \quad x = \frac{p}{3} = \frac{22 \times 9}{(4\sqrt{3}+9)\,3}$$

$$\Rightarrow \quad x = \frac{66}{4\sqrt{3}+9}$$

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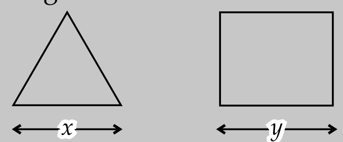
Hint: (i) The area of equilateral triangle with side a is $\dfrac{\sqrt{3}}{4}a^2$

(ii) Find total area in terms of length of wire of triangle and find critical points.

Shortcut: Let p be the length of wire formed into triangle.



$$\Rightarrow \quad x = \frac{p}{3} \text{ and } y = \frac{(22-p)}{4}$$

$$\therefore \quad \text{Total area} = \frac{\sqrt{3}}{4}\left(\frac{p^2}{9}\right) + \frac{(22-p)^2}{16}$$

$$\Rightarrow \quad A = \left(\frac{\sqrt{3}}{18} + \frac{1}{16}\right)p^2 + \frac{22^2}{16} - \frac{22}{8}p$$

$$\Rightarrow \quad \frac{dA}{dp} = \left(\frac{\sqrt{3}}{18} + \frac{1}{8}\right)p - \frac{22}{8}$$

For minimum A, $\dfrac{dA}{dp} = 0$

$$\Rightarrow \quad p\left(\frac{\sqrt{3}}{18} + \frac{1}{8}\right) = \frac{22}{8}$$

$$\Rightarrow \quad p = \frac{22 \times 9}{(4\sqrt{3}+9)}$$

$$\Rightarrow \quad x = \frac{66}{(4\sqrt{3}+9)}$$

75. Option (D) is correct.

Explanation: Let $f(x) = \cos^{-1}\left(\dfrac{2\sin^{-1}\left(\dfrac{1}{4x^2-1}\right)}{\pi}\right)$

As we know the domain of $\cos^{-1} y$ is $[-1, 1]$

$$\Rightarrow \quad -1 \le \left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right) \le 1$$

$$\Rightarrow \quad -\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$$

$$\Rightarrow \quad -1 \le \frac{1}{4x^2-1} \le 1$$

$$\text{So,} \quad \frac{1}{4x^2-1} \ge -1 \text{ and } \frac{1}{4x^2-1} \le 1$$

$$\Rightarrow \quad \frac{1}{4x^2-1} + 1 \ge 0 \text{ and } \frac{1}{4x^2-1} - 1 \le 0$$

$$\Rightarrow \quad \frac{4x^2}{4x^2-1} \ge 0 \text{ and } \frac{2-4x^2}{4x^2-1} \le 0$$

$$\Rightarrow \quad \frac{x^2}{(2x-1)(2x+1)} \ge 0$$

$$\text{and} \quad \frac{(1-\sqrt{2}x)(1+\sqrt{2}x)}{(2x-1)(2x+1)} \le 0$$

$$\Rightarrow \quad x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

$$\text{and } x \in \left(-\infty, \frac{1}{\sqrt{2}}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$$

$$\Rightarrow \quad x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

Hint: (i) Domain of $\cos^{-1} y$ is $[-1, 1]$

(ii) Recall wavy curve method for solving rational inequalities.

Shortcut:

$$\text{Let} \quad f(x) = \cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$$

For domain of $f(x)$,

$$\Rightarrow \quad -1 \le \left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right) \le 1$$

$$\Rightarrow \quad -1 \le \frac{1}{4x^2-1} \le 1$$

$$\Rightarrow \quad x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

76. Option (D) is correct.

Explanation: We have to find the constant term in expansion of $\left(3x^3 - 2x^2 + \dfrac{5}{x^5}\right)^{10}$

$$= \left(\frac{3x^8 - 2x^7 + 5}{x^5}\right)^{10}$$

$$= x^{-50}(3x^8 - 2x^7 + 5)^{10}$$

For constant term in the expansion of $x^{-50}(3x^8 - 2x^7 + 5)^{10}$ we will find the coefficient of x^{50} in $(3x^8 - 2x^7 + 5)^{10}$

As we know, the coefficient of x^r in the expansion of $(a + b + c)^n$ is given by $\dfrac{n!}{r_1!\,r_2!\,r_3!}\,(a)^{r_1}\,(b)^{r_2}\,(c)^{r_3}$

where $r^1 + r^2 + r^3 = n$

So here coefficient of x^{50} in the expansion of $(3x^8 - 2x^7 + 5)^{10}$

$$= \frac{10!}{r_1!\,r_2!\,r_3!}\left(3x^8\right)^{r_1}\left(-2x^7\right)^{r_2}(5)^{r_3}$$

$$= \frac{10!}{r_1!\,r_2!\,r_3!}(3)^{r_1}(-2)^{r_2}(5)^{r_3}(x)^{8r_1+7r_2}$$

Here $r_1 + r_2 + r_3 = 10$ and $8r_1 + 7r_2 = 50$

Let $r_1 = 1 \Rightarrow r_2 = 6$ and $r_3 = 3$

$\therefore$ Coefficient is $\dfrac{10!}{1!\;6!\;3!}(3)^1(-2)^6(5)^3$

$$= \frac{10\times9\times8\times7}{3\times2}\times3\times(2)^6\times(5)^3$$

$$= 2 \times (5)^4 \times (2)^2 \times 7 \times (2)^6 \times (3)^2$$

$$= (2^9)\,(3)^2\,(5)^4\,(7)$$

Now, $(2^K)\,l = (2^9)\,(3)^2\,(5)^4\,(7)$

$\Rightarrow K = 9$

Hint: $\left(3x^3 - 2x^2 + \dfrac{5}{x^5}\right)^{10} = x^{-50}(3x^8 - 2x^7 + 5)^{10}$.

So find the coefficient of x^{50} in $(3x^8 - 2x^7 + 5)^{10}$

Shortcut: The general term of $\left(3x^3 - 2x^2 + \dfrac{5}{x^5}\right)^{10}$

is $T_{r+1} = \dfrac{10!}{r_1!\,r_2!\,r_3!}(3)^{r_1}\,(-2)^{r_2}\,(5)^{r_3}\,(x)^{3x_1+2r_2-5r_3}$

Where $r_1 + r_2 + r_3 = 10$

and $3r_1 + 2r_2 - 5r_3 = 0$ (for constant term)

Solving above two equations, we get,

$r_1 = 1, r_2 = 6, r_3 = 3$

$\therefore$ Constant term $= \dfrac{10!}{6!\;3!}(3)^1\,(-2)^6(5)^3$

$= 2^9\,(3)^2\,(5)^4\,(7)$

$\therefore \quad k = 9.$

77. Option (D) is correct.

Explanation: Let $I = \displaystyle\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right)dx$

Now,

$$\cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) = \cos\left(\pi x - \pi\left[\frac{x}{2}\right]\right)$$

$$\cos\left(\pi x - \pi\left[\frac{x}{2}\right]\right) = \begin{cases}\cos(\pi x - 0) & 0 < x < 2\\ \cos(\pi x - \pi) & 2 \le x < 4\\ \cos(\pi x - 2\pi) & 4 \le x < 5\end{cases}$$

$$\therefore I = \int_0^2 \cos(\pi x)\,dx + \int_2^4 \cos(\pi x - \pi)\,dx + \int_4^5 \cos(\pi x - 2\pi)\,dx$$

$$\Rightarrow I = \left[\frac{\sin(\pi x)}{\pi}\right]_0^2 + \left[\frac{\sin(\pi x - \pi)}{\pi}\right]_2^4 + \left[\frac{\sin(\pi x - 2\pi)}{\pi}\right]_4^5$$

$$\Rightarrow I = \frac{1}{\pi}\,(\sin 2\pi - \sin 0 + \sin(4\pi - \pi)) - \sin(2\pi - \pi) + \sin(5\pi - 2\pi) - \sin(4\pi - 2\pi)$$

$$\Rightarrow I = \frac{1}{\pi}\,(0)$$

$\Rightarrow I = 0$

Hint: Break the integral into 3 parts, $x \in [0, 2)$ and $x \in [2, 4)$ and $x \in [4, 5)$ and solve it.

Shortcut: Let $I = \displaystyle\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right)dx$

$\Rightarrow I =$

$$\int_0^2 \cos(\pi x)\,dx + \int_2^4 \cos(\pi x - \pi)\,dx + \int_4^5 \cos(\pi x - 2\pi)\,dx$$

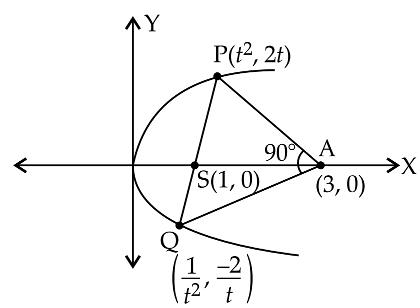
$\Rightarrow I = 0$

78. Option (B) is correct.

Explanation: Given: Focal chord of $y^2 = 4x$ is PQ

and it subtends an angle of $\dfrac{\pi}{2}$ at point (3, 0)

Let parametric coordinates of point P be $(t^2, 2t)$ then parametric coordinates of point Q be $\left(\dfrac{1}{t^2}, \dfrac{-2}{t}\right)$



$\because \qquad AP \perp AQ$

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$\therefore \quad (m_{AP})(m_{AQ}) = -1$

$\Rightarrow \left(\frac{2t}{t^2-3}\right)\left(\frac{\frac{-2}{t}}{\frac{1}{t^2}-3}\right) = -1$

$\Rightarrow \frac{-4t^2}{(t^2-3)(1-3t^2)} = -1$

$\Rightarrow 4t^2 = -3t^4 + 10t^2 - 3$

$\Rightarrow 3t^4 - 6t^2 + 3 = 0$

$\Rightarrow (t^2-1)^2 = 0$

$\Rightarrow t = 1$

$\therefore$ Coordinates of point P and Q are (1, 2) and (1, –2) respectively.

Since, line segment PQ is also a focal chord of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a^2 > b^2$.

$\therefore$ P and Q must be end points of latus rectum

$\Rightarrow \frac{2b^2}{a} = 4$ and $ae = 1$

As we know, $b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = a^2 - a^2e^2$

$\Rightarrow b^2 = a^2 - 1$

$\Rightarrow \frac{a^2-1}{a} = 2$

$\Rightarrow a^2 - 2a - 1 = 0$

$\Rightarrow a = 1 + \sqrt{2}$

$\Rightarrow e = \frac{1}{a} = \frac{1}{1+\sqrt{2}}$

$\Rightarrow e^2 = \frac{1}{3+2\sqrt{2}}$

$\Rightarrow \frac{1}{e^2} = 3+2\sqrt{2}$

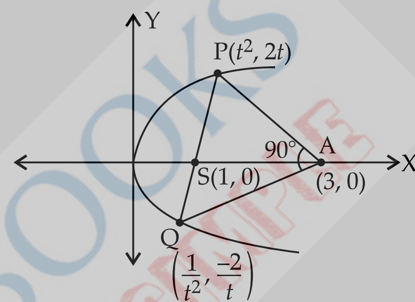
Hint:

(i) Coordinates of end point of any focal chord of parabola $y^2 = 4ax$ are $P(at^2, 2at)$ and $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

(ii) P and Q must be end points of latus rectum of ellipse.

(iii) The end points of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a^2 > b^2$ are $\left(ae, \frac{b^2}{a}\right)$ and $\left(ae, \frac{-b^2}{a}\right)$

Shortcut:



$\therefore AP \perp AQ$

$\therefore (m_{AP})(m_{AQ}) = -1$

$\Rightarrow \left(\frac{2t}{t^2-3}\right)\left(\frac{\frac{-2}{t}}{\frac{1}{t^2}-3}\right) = -1$

$\Rightarrow (t^2-1)^2 = 0$

$\Rightarrow t = 1$

So, coordinates of point P and Q are (1, 2) and (1, –2) respectively

$\therefore$ P and Q must be end point of latus rectum.

$\Rightarrow \frac{2b^2}{a} = 4$ and ae = 1

$\Rightarrow \frac{b^2}{a} = 2$ and $a^2e^2 = a^2 - b^2 = 1$

$\Rightarrow a = 1 + \sqrt{2}$

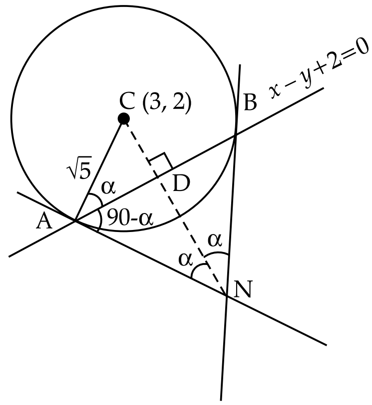
$\Rightarrow \frac{1}{e^2} = 3+2\sqrt{2}$

79. Option (C) is correct.

Explanation: Circles $C_1 : x^2 + y^2 = 2$ and $C_2 : (x-3)^2 + (y-2)^2 = 5$

Now, equation of tangent to the circle C_1 and M (–1, 1) is given by $x(-1) + y(1) = 2$

$$\Rightarrow \qquad x - y + 2 = 0$$



Let $\qquad \angle ANB = 2\alpha$

$\therefore \qquad \angle CAD = \alpha$

Now, $\qquad CD = \left|\dfrac{3-2+2}{\sqrt{1^2+1^2}}\right| = \dfrac{3}{\sqrt{2}}$

Apply Pythagoras' theorem in ΔACD, we get

$$(CD)^2 + (AD)^2 = (AC)^2$$

$$\Rightarrow \qquad AD = \sqrt{5-\frac{9}{2}}$$

$$\Rightarrow \qquad AD = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \qquad \tan\alpha = \frac{\text{CD}}{\text{AD}} = 3$$

$$\Rightarrow \qquad \sin\alpha = \frac{\text{CD}}{\text{AC}} = \frac{3}{\sqrt{10}}$$

Now, in ΔADN,

$$\sin\alpha = \frac{\text{AD}}{\text{AN}} = \frac{3}{\sqrt{10}}$$

$$\Rightarrow \qquad AN = \frac{\sqrt{10}}{3}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{5}}{3}$$

Now, area of $\Delta ANB = \dfrac{1}{2}(AN)^2 \sin 2\alpha$

$$= \frac{1}{2}\left(\frac{5}{9}\right)(2\sin\alpha\cos\alpha)$$

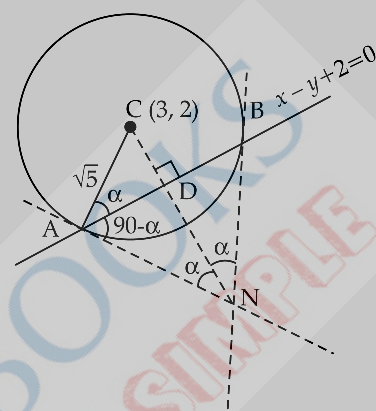
$$= \frac{5}{9}\times\frac{3}{\sqrt{10}}\times\frac{1}{\sqrt{10}}$$

$$= \frac{1}{6} \text{ square unit}$$

Hint: (i) Equation of the tangent to the circle $x^2 + y^2 = r^2$ at point (x_1, y_1) is given by $xx_1 + yy_1 = r^2$

(ii) Draw the diagram as per given question and solve further using the concert of circle.

Shortcut:



Let $\qquad \angle ANB = 2\alpha$

$\therefore \qquad \angle CAD = \alpha$

Now, $\qquad CD = \left|\dfrac{3-2+2}{\sqrt{1^2+1^2}}\right| = \dfrac{3}{\sqrt{2}}$

And $\qquad AD = \sqrt{(\text{AC})^2-(\text{AD})^2}$

$$= \frac{1}{\sqrt{2}}$$

$$\Rightarrow \qquad \sin\alpha = \frac{\text{CD}}{\text{AC}} = \frac{3}{\sqrt{10}}$$

Now, in ΔADN, $\sin\alpha = \dfrac{\text{AD}}{\text{AN}}$

$$\Rightarrow \qquad AN = \frac{\sqrt{5}}{3}$$

Now, area of $\Delta ANB = \dfrac{1}{2}(AN)^2\sin 2\alpha$

$$= \frac{1}{2}\left(\frac{5}{9}\right)\left(2.\frac{3}{\sqrt{10}}.\frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{6} \text{ square unit}$$

80. Option (B) is correct.

Explanation: The observations are x_1, x_2, x_3, x_4, x_5 mean of $x_1, x_2, x_3, x_4, x_5 = \frac{24}{5}$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 24 \quad \text{...(i)}$$

and mean of $x_1, x_2, x_3, x_4 = \frac{7}{2}$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14 \quad \text{...(2)}$$

From eq. (1) and (2)

$$x_5 = 24 - 14 = 10$$

variance of $x_1, x_2, x_3, x_4, x_5 = \frac{194}{25}$

$$\Rightarrow \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} = \frac{194}{25} + \left(\frac{24}{5}\right)^2$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = \frac{194}{25} + \frac{576}{5} = 154$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 154 - (10)^2$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$$

variance of $x_1, x_2, x_3, x_4 = a$

$$\Rightarrow \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \frac{49}{4} = a$$

$$\Rightarrow \frac{54}{4} - \frac{49}{4} = a$$

$$\Rightarrow a = \frac{5}{4}$$

$$\Rightarrow 4a + x_5 = 5 + 10 = 15$$

Hint : Mean $= \frac{\Sigma x}{n}$

Variance $= \frac{\Sigma x^2}{n} - \frac{\Sigma x}{n}$

Shortcut : $\frac{\Sigma x_1}{5} = \frac{24}{5}$

$$\Rightarrow \Sigma x_1 = 24$$

$$\sigma_1^2 = \frac{\Sigma x_1^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$$\Rightarrow \Sigma x_1^2 = 154$$

and $x_1 + x_2 + x_3 + x_4 = 14$

$$\Rightarrow x_5 = 10$$

and $\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \frac{49}{4} = a$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 4a + 49$$

$$\Rightarrow x_5^2 = 154 - 4a - 49$$

$$\Rightarrow 100 = 154 - 4a - 49$$

$$\Rightarrow 4a = 5$$

$$\Rightarrow 4a + x_5 = 15$$

Section B

81. Correct answer is [26].

Explanation: Given: $S = \{Z \in C : |z - 2| \leq 1, z(1 + i) + \bar{z}(1 - i) \leq 2\}$

Let $z = x + iy$

Now, $|z - 2| \leq 1$

$$\Rightarrow |(x - 2) + iy| \leq 1$$

$$\Rightarrow (x - 2)^2 + y^2 \leq 1$$

$\Rightarrow$ It represents the region inside circle whose centre is (2, 0) and radius is 1.

Now, $z(1 + i) + \bar{z}(1 - i) \leq 2$

$$\Rightarrow (x + iy)(1 + i) + (x - iy)(1 - i) \leq 2$$

$$\Rightarrow x - y - 1 \leq 0$$

$\Rightarrow$ It represents the all points which lies on and above the line $x - y - 1 = 0$

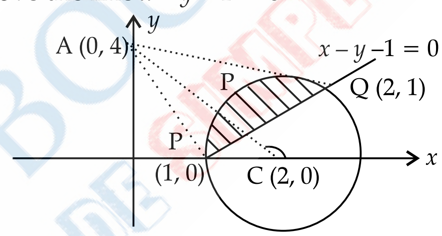


Fig.

Now, $|z - 4i|$ represents distance of a point A (0, 4) from z.

Now, $AP = \sqrt{17}$ and $AQ = \sqrt{13}$

$\therefore |z - 4i|_{max} = AP$ and $|z - 4i|_{min} = AD$

Let coordinates of point D be $(\cos\theta + 2, \sin\theta)$

Now, $(m)_{AC} = \tan\theta = -2$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{5}} \text{ and } \sin\theta = \frac{2}{\sqrt{5}}$$

$\therefore$ coordinates of point D is $\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

So, $z_1 = 2 - \frac{1}{\sqrt{5}} + i\frac{2}{\sqrt{5}}$ and $z_2 = 1$

Now, $$|z_1| = \sqrt{\left(2 - \frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2}$$

$$\Rightarrow |z_1| = \sqrt{4 + \frac{1}{5} - \frac{4}{\sqrt{5}} + \frac{4}{5}}$$

$$\Rightarrow |z_1| = \sqrt{\frac{5\sqrt{5} - 4}{5}}$$

$$\Rightarrow |z_1|^2 = \frac{5\sqrt{5} - 4}{\sqrt{5}}$$

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Now, $5\left(|z_1|^2 + |z_2|^2\right) = 5\left(\dfrac{5\sqrt{5}-4}{\sqrt{5}}+1\right)$

$= 30 - 4\sqrt{5}$

$\Rightarrow \alpha + \beta\sqrt{5} = 30 - 4\sqrt{5}$

$\Rightarrow \quad \alpha = 30, \beta = -4$

$\therefore \quad \alpha + \beta = 26$

Hint :

(i) $|z-2| \le 1$, represents the region inside the circle whose centre is (2, 0) and radius is 1.

(ii) $z(1+i) + \bar{z}(1-i) \le 2$, represents all the points which lies on and above the line $x - y - 1 = 0$

(iii) Find the required region using above points and solve further.

82. Option (B) is correct.

Explanation: Given: $\dfrac{dy}{dx} + \dfrac{\sqrt{2}}{2\cos^4 x - \cos 2x}$

$y = x\, e^{[\tan^{-1}(\sqrt{5}\cot 2x)]}$; $x \in \left(0, \dfrac{\pi}{2}\right)$

It is linear differential equation.

Comparing above differential equation with

$\dfrac{dy}{dx} + py = Q,$

we get, $\quad p = \dfrac{\sqrt{2}}{2\cos^4 x - \cos 2x}$

and $\quad Q = xe^{\tan^{-1}(\sqrt{5}\cot 2x)}$

Now, $\quad I.F. = e$

So, $\quad \displaystyle\int p.dx = \int \frac{\sqrt{2}}{2\cos^4 x - \cos 2x}\,dx$

$\Rightarrow \displaystyle\int p.dx = \int \frac{\sqrt{2}}{\frac{1}{2}(2\cos^4 x)^2 - \cos 2x}\,dx$

$\Rightarrow \displaystyle\int p.dx = \int \frac{\sqrt{2}}{\frac{1}{2}(1+\cos 2x)^2 - \cos 2x}\,dx$

$\Rightarrow \displaystyle\int p.dx = \int \frac{2\sqrt{2}}{1+\cos^2 2x}\,dx$

$\Rightarrow \displaystyle\int p.dx = \int \frac{2\sqrt{2}\sec^2 2x}{2+\tan^2 2x}\,dx$

Let $t = \tan 2x \Rightarrow dt = 2\sec^2 2x\, dx$

$\Rightarrow \displaystyle\int p.dx = \sqrt{2}\int \frac{dt}{(\sqrt{2}) + t^2}$

$\Rightarrow \displaystyle\int p.dx = \sqrt{2}, \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$

$\Rightarrow \displaystyle\int p.dx = \tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)$

$\therefore IF = e$

So, solution of the given differential equation is given by $y\,(IF) = \int Q.(I.F.).dx$

$$\Rightarrow y\, e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)} = \int x\, e^{\tan-1(\sqrt{2}\cot 2x)}.\, e^{\tan-1\left(\frac{\tan 2x}{\sqrt{2}}\right)}\, dx \quad ...(i)$$

Now, $\tan^{-1}\left(\sqrt{2}\cot 2x\right) + \tan^{-1}\left(\dfrac{\tan 2x}{\sqrt{2}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2}\cot 2x + \tan 2x/\sqrt{2}}{1-1}\right) = \dfrac{\pi}{2}$

From equation (i),

$$y\, e^{\tan-1\left(\frac{\tan 2x}{\sqrt{2}}\right)} = \int e^{\pi/2} x\, dx$$

$$\Rightarrow \quad y\, e^{\tan-1\left(\frac{\tan 2x}{\sqrt{2}}\right)} = e^{\frac{\pi}{2}}.\frac{x^2}{2} + C \quad ...(ii)$$

$\because \quad y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^2}{32}$

$\therefore \quad \dfrac{\pi^2}{32} e^{\frac{\pi}{2}} = e^{\frac{\pi}{2}}.\dfrac{\pi^2}{32} + C$

$\Rightarrow \quad C = 0$

put $x = \dfrac{\pi}{3}$ in equation (ii), we get

$$y\left(\frac{\pi}{3}\right).e^{\tan^{-1}\left(-\sqrt{\frac{3}{2}}\right)} = e^{\frac{\pi}{2}}.\frac{\pi^2}{18}$$

$$\Rightarrow \frac{\pi^2}{18} e^{\tan^{-1}(\alpha)}.e^{\tan-1\left(-\sqrt{\frac{3}{2}}\right)} = e^{\frac{\pi}{2}}.\frac{\pi^2}{18}$$

$$\Rightarrow \quad e^{\tan^{-1}(-\alpha) + \tan^{-1}\left(-\sqrt{\frac{3}{2}}\right)} = e^{\frac{\pi}{2}}$$

$$\Rightarrow \quad \tan^{-1}(-\alpha) + \tan^{-1}\left(-\sqrt{\frac{3}{2}}\right) = \frac{\pi}{2}$$

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$$\Rightarrow \quad \alpha\sqrt{\frac{3}{2}} = 1$$

$$\Rightarrow \quad \alpha^2 = \frac{2}{3}$$

$$\Rightarrow \quad 3\alpha^2 = 2$$

Hint :

(i) Solution of linear differential equation $\frac{dy}{dx} + py = Q$, where p and Q are the function of x is given by y (I.F.) $= \int Q.\text{(I.F.)}\,dx$

where $I.F. = e^{\int p.dx}$

(ii) Use $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(iii) Use $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

83. Correct answer is [26].

Explanation: Given: Equation of plane is $-x + y + z = 1$

Points $p(1, 2, -1)$ and $Q(2, -1, 3)$ lie on same side of the plane.

Now, perpendicular distance of a point p from plane $-x + y + z - 1 = 0$ is

$$d_1 = \left|\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

And perpendicular distance of a point Q from plane $-x + y + z\,1 = 0$ is

$$d_2 = \left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

$$\because \quad d_1 = d_2$$

$\therefore$ $\overrightarrow{PQ}$ is parallel to given plane.

So, distance between P and Q = distance between their foot of perpendiculars

$$\Rightarrow \quad |\overrightarrow{PQ}| = \sqrt{(2-1)^2+(-1-2)^2+(3+1)^2}$$

$$= \sqrt{26}$$

$$\Rightarrow \quad d = \sqrt{26}$$

$$\Rightarrow \quad d^2 = 26$$

Hint :

(i) Perpendicular distance of a point P and Q from the plane is same.

(ii) Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Shortcut : Points P and Q lie on same side of plane. Perpendicular distance of a point P and Q from the plane $-x + y + z = 1$ is same. So, distance between the foot of perpendiculars $= PQ$

$$\Rightarrow \quad d = \sqrt{(2-1)^2+(-1-2)^2+(3+1)^2}$$

$$= \sqrt{26}$$

$$\Rightarrow \quad d^2 = 26$$

84. Correct answer is [32].

Explanation: Given: $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2 \theta + 5 = 0$; $\theta \in [-4\pi, 4\pi]$

$\Rightarrow 3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$\{\because 2\cos^2\theta = 1 + \cos 2\theta\}$

$$\Rightarrow \quad 3\cos^2 2\theta + \cos 2\theta = 0$$

$$\Rightarrow \quad \cos 2\theta(3\cos 2\theta + 1) = 0$$

$$\Rightarrow \quad \cos 2\theta = 0 \text{ or } \cos 2\theta = -\frac{1}{3}$$

Case-1 If $\cos 2\theta = 0$

$$\Rightarrow \quad 2\theta = 2n + 1)\frac{\pi}{2};\ n \in I$$

$$\Rightarrow \quad \theta = (2n+1)\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \pm\frac{\pi}{4}, \pm 3\frac{\pi}{4}, \pm 5\frac{\pi}{4}, \ldots\ldots\ldots\ldots \pm 15\frac{\pi}{4}$$

$\therefore$ For $\theta \in [-4\pi, 4\pi]$, 16 values of θ is possible for this case.

Case-2 If $\cos 2\theta = -\frac{1}{3}$

Let $\quad \cos\alpha = -\frac{1}{3}$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(-\frac{1}{3}\right);\ \alpha \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \quad 2\theta = 2n\pi \pm \alpha;\ \alpha \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \qquad \theta = n\pi \pm \frac{\alpha}{2}$$

$\therefore$ For $\theta \in [-4\pi, 4\pi]$, 16 values of θ is possible for this case.

So, number of elements in the set S is 32.

Hint :

(i) Simplify given trigonometric equation using $1 + \cos 2\theta = 2\cos^2\theta$ and solve further.

(ii) General solution of $\cos x = 0$ is $x = (2n + 1)\frac{\pi}{2};\ n \in I.$

(iii) General solution of $\cos x = \cos\alpha$ is $x = 2n\pi \pm \alpha;\ \alpha \in (0, \pi)$

Shortcut : $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$$\Rightarrow \quad 3\cos^2 2\theta + \cos 2\theta = 0$$

$$\Rightarrow \quad \cos 2\theta = 0 \text{ or } \cos 2\theta = -\frac{1}{3}$$

If $\qquad \cos 2\theta = 0$

$$\Rightarrow \quad \theta = (2n + 1)\frac{\pi}{4}, n \in \mathrm{I}$$

$\therefore$ For $\theta \in [-4\pi, 4\pi]$, 16 values of θ is possible.

If $\cos 2\theta = -\frac{1}{3}$

Similarly, 16 values of θ is possible for $\theta \in [-4\pi, 4\pi]$ for this case.

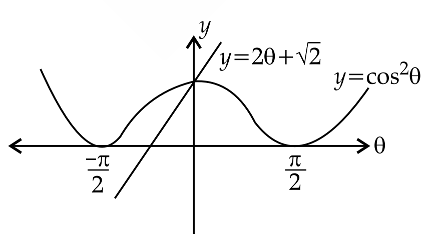
$\therefore$ Total solution $= 32$ for $\theta \in [-4\pi, 4\pi]$

85. Correct answer is [1].

Explanation: Given: $2\theta - \cos^2\theta\ \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

Lets draw the graph of $y = \cos^2\theta$ and $y = 2\theta + \sqrt{2}$



$\because$ Both graphs intersect at one point.

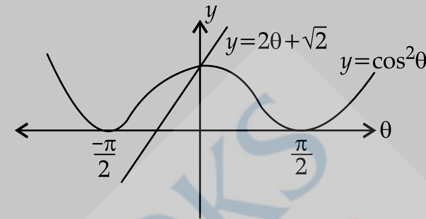
$\therefore$ Number of solution for given equation is 1.

Hint :

(i) Draw the graph of $y = \cos^2\theta$ and $y = 2\theta + \sqrt{2}$ and find intersection point of both graph.

Shortcut : $2\theta - \cos^2\theta + \sqrt{2} = 0$

$$\Rightarrow \quad \cos^2\theta = 2\theta + \sqrt{2}$$



Since, both graphs intersect at one point

So, Number of solution for given equation is 1.

86. Correct answer is [29].

Explanation: Let $A = 50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$

Let $\qquad B = \tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$

Let $\qquad 2\theta = \tan^{-1}(2\sqrt{2});\ 2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\therefore \quad \tan 2\theta = 2\sqrt{2}$$

$$\Rightarrow \quad \frac{2\tan\theta}{1 - \tan^2\theta} = 2\sqrt{2} \qquad \text{...(i)}$$

$$\Rightarrow \quad 2\sqrt{2}\tan^2\theta + 2\tan\theta - 2\sqrt{2} = 0$$

$$\Rightarrow \quad 2\sqrt{2}\tan^2\theta + 4\tan\theta - 2\tan\theta - 2\sqrt{2} = 0$$

$$\Rightarrow \quad (\tan\theta + \sqrt{2})(2\sqrt{2}\tan\theta - 2) = 0$$

$$\Rightarrow \quad \tan\theta = -\sqrt{2} \text{ or } \frac{1}{\sqrt{2}}$$

$$\because \quad \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\therefore \quad \tan\theta = -\sqrt{2} \text{ is not possible}$$

So, $\quad \tan\theta = \frac{1}{\sqrt{2}}$

$$\therefore \quad B = \frac{1}{\sqrt{2}}$$

Now, $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \tan^{-1}(2)$

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Let $C = \tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$

$\Rightarrow C = \tan$
$$\left(\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}(2)\right)$$

$$\Rightarrow C = \tan\left(\tan^{-1}\left(\frac{1}{2}\right) + 2\left[\tan^{-1}\left(\frac{\frac{1}{2}+2}{1-1}\right)\right]\right)$$

$$\Rightarrow C = \tan\left(\tan^{-1}\left(\frac{1}{2}\right) + 2\left(\frac{\pi}{2}\right)\right)$$

$$\Rightarrow C = \tan\left(\pi + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$\Rightarrow C = \tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$\Rightarrow C = \frac{1}{2}$$

$$\therefore A = 50\left(\frac{1}{2}\right) + 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow A = 25 + 4 = 29$$

Hint :

(i) Convert $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ in terms of $\tan^{-1}$ and simplify further using $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(ii) Assume $2\theta = \tan^{-1}(2\sqrt{2})$ and solve further.

Shortcut : Let $A = 50$

$$\tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$$

Let $B = \tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$ and

$$\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$$

Let $2\theta = \tan^{-1}(2\sqrt{2}); 2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \tan 2\theta = 2\sqrt{2}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$

$\therefore B = \frac{1}{\sqrt{2}}$

Now, $C = \tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}(2)\right)$

$\Rightarrow C = \tan\left(\tan^{-1}\left(\frac{1}{2}\right) + \pi\right)$

$\Rightarrow C = \frac{1}{2}$

$\therefore A = 50\left(\frac{1}{2}\right) + 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow A = 29$

87. Correct answer is [3395].

Explanation:

$$f(x) = (c+1)x^2 + (1-c^2)x + 2k \quad \text{...(1)}$$

and $f(x+y) = f(x) + f(y) - xy \ \forall x, y \in \mathrm{R}$

$$\lim_{y\to 0}\frac{f(x+y)-f(x)}{y} = \lim_{y\to 0}\frac{f(y)-xy}{y}$$

$\Rightarrow f(x) = f(0) - x$

$$f(x) = -\frac{1}{2}x^2 + f(0).\,x + \lambda$$

but $f(0) = 0 \Rightarrow \lambda = 0$

$$f(x) = -\frac{1}{2}x^2 + (1-c).x \quad \text{...(2)}$$

as $f = 1 - c^2$

Comparing equation (1) and (2)

We obtain, $c = -\frac{3}{2}$

$$\therefore \quad f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$$

$$\left|2\Sigma_{x=1}^{20} f(x)\right| = \Sigma_{x=1}^{20} x^2 + \frac{5}{2}.\Sigma_{x=1}^{20} x$$

$$= 2870 + 525 = 3395$$

88. Correct answer is [88].

Explanation: Given: Equation of hyperbola is

$$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; a > 0, b > 0$$

And eccentricity of H is $e = \dfrac{\sqrt{11}}{2}$

And sum of length of transverse and conjugate axis is $2a + 2b = 4\left(2\sqrt{2} + \sqrt{14}\right)$

As we know, $e = \sqrt{1 + \dfrac{b^2}{a^2}}$

$$\Rightarrow \qquad e^2 = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow \qquad \frac{11}{4} = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow \qquad b^2 = \frac{7}{4}a^2$$

$$\Rightarrow \qquad b = \frac{\sqrt{7}}{2}a$$

$$\because \qquad 2a + 2b = 4\left(2\sqrt{2} + \sqrt{14}\right)$$

$$\Rightarrow \quad 2a + \sqrt{7}a = 4\left(2\sqrt{2} + \sqrt{14}\right)$$

$$\Rightarrow a\left(2 + \sqrt{7}\right) = 4\sqrt{2}\,(2 + \sqrt{7})$$

$$\Rightarrow \qquad a = 4\sqrt{2}$$

$$\Rightarrow \qquad b = 2\sqrt{14}$$

$$\therefore \qquad a^2 + b^2 = \left(4\sqrt{2}\right)^2 + \left(2\sqrt{14}\right)^2$$

$$= 32 + 56 = 88$$

89. Correct answer is [28].

Explanation: Given: $P_1 : \vec{r}\,.\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$

$\Rightarrow P_1 : 2x + y - 3z = 4$

Now, equation of plane passing through points $(2, -3, 2)$, $(2, -2, -3)$ and $(1, -4, 2)$ is given by

$$\begin{vmatrix} (x-2) & (y+3) & (z-2) \\ (2-2) & (-2+3) & (-3-2) \\ (1-2) & (-4+3) & (2-2) \end{vmatrix} = 0$$

$$\Rightarrow \qquad \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \ (x-2)(-5) - (y+3)(-5) + z - 2) = 0$

$\Rightarrow \qquad -5x + 5y + z + 23 = 0$

$\therefore \qquad P_2 : -5x + 5y + z + 23 = 0$

Let a, b, c be the direction ratios of the line of intersection of plane P_1 and P_2

$$\therefore \quad \frac{a}{1+15} = \frac{-b}{2-15} = \frac{c}{10+5} = \lambda$$

$\Rightarrow \qquad a = 16\lambda, b = 13\lambda, c = 15\lambda$

$\Rightarrow \qquad \alpha = 13, \beta = 15$

$\therefore \qquad \alpha + \beta = 28$

Hint :

(i) Equation of plane passing through (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(ii) Line of intersection of the planes is perpendicular to the both normal vector of planes.

Shortcut : Given : $P_1 : \vec{r}\,.\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$

$\Rightarrow \qquad P_1 : 2x + y - 3z = 4$

And $\quad P_2 : \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$

$\Rightarrow \qquad P_2 : -5x + 5y + z + 23 = 0$

Let a, b, c be direction ratios of the line of intersection of plane P_1 and P_2

$$\therefore \quad \frac{a}{1+15} = \frac{-b}{2-15} = \frac{c}{10+c} = \lambda$$

$\Rightarrow \qquad a = 16\lambda, b = 13\lambda, c = 15\lambda$

$\Rightarrow \qquad \alpha = 13, \beta = 15$ Þ $\alpha + \beta = 28$

90. Correct answer is [18915].

Explanation: $b_i \in \{1, 2, 3, \ldots\ldots\ldots, 100\}$

Let $\qquad P$ = set when b_1, b_2, b_3 are consecutive.

$$\therefore \qquad n(P) = \underbrace{97 + 97 + 97 + \ldots.\ 97}_{98 \text{ times}} = 97 \times 98$$

Let $\qquad \theta$ = set when b_2, b_3, b_4 are consecutive.

$$\therefore \qquad n(Q) = \underbrace{97 + 97 + \ \ldots.\ 97}_{98 \text{ times}} = 97 \times 98$$

Now, $\ P \cap Q$ = set when b_1, b_2, b_3, b_4 are consecutive.

So, $\ n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$= 97 \times 98 + 97 \times 98 - 97$

$= 97\,(98 + 98 - 1)$

$= 97\,(195)$

$= 18915$

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Hint :

(i) There are 98 sets of three consecutive integer and 97 sets of four consecutive integer.

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Shortcut : There are 98 sets of there consecutive integer and 97 sets of four consecutive integer.

Number of permutation of $b_1\ b_2\ b_3\ b_4$ = (Number of permutation when b_1, b_2, b_3 are consecutive) + (Number of permutations when $b_2\ b_3\ b_4$ are consecutive) – (Number of permutation when $b_1\ b_2\ b_3\ b_4$ are consecutive)

$$= 97 \times 98 + 97 \times 98 - 97$$
$$= 97\ (98 + 98 - 1)$$
$$= 18915$$

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