CHAPTER-1
RATIONAL AND IRRATIONAL NUMBERS

TOPIC-1
Properties of Rational and Irrational Numbers

Revision Notes

- **Rational Number :** A number that can be expressed in the form $\frac{p}{q}$, where p and q both are integers and $q \neq 0$, is called a rational number.
- The word 'rational' comes from the word 'ratio'. Thus, every rational number can be written as the ratio of two integers.
- In other words, $\frac{p}{q}$ is a rational number. If

  (1) $q \neq 0$

  (2) p and q have no common factor other than 1(one) *i.e.* p and q are co-primes.

  It is called in the lowest terms or simplest form or irreducible form.

  (3) q is usually positive, whereas p may be positive, negative or zero.
- In general, the set of rational number is denoted by the letter Q.

  $$\therefore \quad Q = \left\{\frac{p}{q} : p, q \in z \text{ and } q \neq 0\right\}$$
- Every integer (Positive, negative or zero) and every decimal number is a rational number.
- Corresponding to every rational number $\frac{p}{q}$, its negative rational number is $\frac{-p}{q}$.
- **Properties of rational numbers**

  (1) It a, b are any two rational numbers, then $a + b$ is also a rational numbers.

  (2) If a, b are any two rational numbers, then $a - b$ and $b - a$ is also a rational number.

  (3) If a, b are any two rational numbers, then $a \times b$ is also a rational number.

  (4) If a, b are any two rational numbers and $b \neq 0$, then $\frac{a}{b}$ is also a rational number.

  (5) The collection of rational number is ordered *i.e.* If a, b are any two rational numbers, then either $a < b$ or $a > b$ or $a = b$.

  (6) Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, if and only if : $a \times d = b \times c$

  Also, $$\frac{a}{b} > \frac{c}{d} \Rightarrow a \times d > b \times c$$

  and $$\frac{a}{b} < \frac{c}{d} \Rightarrow a \times d < b \times c$$

  (7) If a, b are any two different rational numbers, then $\frac{a+b}{2}$ is also a rational number and it lies between a and b.

  i.e. if $$a > b \Rightarrow a > \frac{a+b}{2} > b$$

  and if $$a < b \Rightarrow a < \frac{a+b}{2} < b$$

- **Irrational Number :** A number that cannot be expressed in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$, p and q have no common factors (except), is called an irrational number.
- The square roots, cube roots, etc, of natural numbers are irrational numbers, if their exact values cannot be obtained.

  $\sqrt{m}$ or $\sqrt[3]{m}$ is irrational, if exactly square root or cube root of m does not exist.
- A non-terminating and non-recurring decimal is an irrational number.

  Ex - (i) 0·26561987 ..... (ii) 238·56575859 .....
- The number π is also an irrational number.
- **Properties of irrational numbers**

  (1) For any two positive rational number x and y if $\sqrt{x}$ and $\sqrt{y}$ are irrationals then :

  $$\sqrt{x} > \sqrt{y} \Rightarrow x > y$$

  and

  $$\sqrt{x} < \sqrt{y} \Rightarrow x < y$$

  (2) $a + b\sqrt{x} = c + d\sqrt{x} \Rightarrow a = c$ and $b = d$

  (3) The negative of an irrational number is always irrational.

  (4) If x is rational number and y is irrational number, then $x + y$, $x - y$ and $y - x$ are irrational numbers.

  (5) If x is a non-zero rational number and y is an irrational number, then xy, $\frac{x}{y}$ and $\frac{y}{x}$ are irrational numbers.

  (6) The sum of two irrational numbers may or may not be irrational.

  Ex. - (i) $(6 + 2\sqrt{5}) + (9 - 2\sqrt{5}) = 15$, which is not an irrational number.

  (ii) $(2\sqrt{3} + 5) + (7\sqrt{2} - 5) = 2\sqrt{3} + 7\sqrt{2}$, which is an irrational number.

  (7) The difference of two irrational number may or may not be irrational.

  Ex- (i) $(9 - \sqrt{5}) - (3 - \sqrt{5}) = 6$, which is not an irrational number.

  (ii) $(8 + \sqrt{5}) - (5 + \sqrt{2}) = 3 + \sqrt{5} - \sqrt{2}$, which is an irrational number.

  (8) The perfect of two irrational numbers, may or may not be irrational.

  Ex- (i) $(2 + \sqrt{7}) \times (2 - \sqrt{7}) = 4 - 7 = -3$, which is not an irrational number.

  (ii) $(2 + \sqrt{7}) \times (3 - \sqrt{6}) = 6 - 2\sqrt{6} + 3\sqrt{7} - \sqrt{42}$, which is an irrational number.

  (9) The quotient of two irrational number may or may not be irrational.

  Ex- (i) $4\sqrt{75} \div 7\sqrt{3} = \frac{4\sqrt{25}}{7} = \frac{4}{7} \times 5 = \frac{20}{7}$,

  which is not an irrational number.

  (ii) $4\sqrt{25} \div 2\sqrt{5} = \frac{4\sqrt{25}}{2\sqrt{5}} = 2\sqrt{5}$, which is an irrational number.
- **Real Number :** The collection of all rational numbers together with all irrational numbers forms the collection of real numbers. This collection is denoted by R.

  i.e. $R = Q \cup \overline{Q}$

  where Q is the set of rational number and $\overline{Q}$ is the set of irrational numbers.
- Rational number (Q) is the set of all terminating or recurring decimals :
- Irrational number $(\overline{Q})$ is the set of all non-terminating and non-recurring decimals.

Real Numbers (R)

↓ ↓

Rational Numbers (Q) Irrational Numbers

$-3, 0, 5, \frac{-7}{2}, \frac{3}{5}$ ..... etc

$\sqrt{2}, \sqrt{3}, -\sqrt{6}$, ... etc.

↓

Integers (I or Z)
.... – 3, – 2, – 1, 0, 1, 2, 3, ...

↓

Non-integer Rational Numbers
$\frac{5}{14}, \frac{7}{2}, \frac{-3}{5}$, ... etc

↓ ↓ ↓

Negative Integers
....., – 3, – 2, – 1

→

Whole Numbers (W)
0, 1, 2, 3, ...

↓ ↓

Zero/Null integer
0

Natural Number (N)
1, 2, 3, 4, ...

- **Properties of real numbers :**

(1) If a, b are any two real numbers, then $a + b$ is also a real number.

(2) If a, b are any two real numbers, then $a - b$ is also a real number.

(3) If a, b are any two real numbers, then $a \times b$ is also a real number.

(4) If a, b are any two real numbers and $b \neq 0$, then $\frac{a}{b}$ is also a real number.

(5) The set of real numbers is ordered *i.e.* if a, b are any two real numbers, then either $a > b$ or $a < b$ or $a = b$. This is called trichotomy law.

(6) If a, b are any two real number, then $\frac{a+b}{2}$ is a real number and it lies between a and b.

i.e. if $a > b \Rightarrow a > \frac{a+b}{2} > b$

and if $a < b \Rightarrow a < \frac{a+b}{2} < b$

- Every real number (rational or irrational) can be represented by a unique point on the number line. Conversely, every point on the number line represents a unique real number.
- When numerator of a rational number divide by its denominator and remainder becomes zero after some steps, then such decimal expansion, are called terminating decimal.

Ex- $\frac{1}{2} = 0.5, \frac{7}{8} = 0.875, \frac{9}{80} = 0.1125$

If the denominator of a rational number can be expressed as 2^m or 5^n or $2^m \times 5^n$, where m and n both are whole numbers, then the rational number is a terminating decimal.

Ex- $\frac{9}{80}$

Since, $80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5^1$

i.e. 80 can be expressed as $2^m \times 5^n$

$\therefore$ Rational number $\frac{9}{80}$ is a terminating decimal.

- When numerator of a rational number divide by its denominator and remainder never becomes zero, then such decimal expansions are called non-terminating decimal.

Ex- (i) $\frac{10}{3} = 3.333$ ..... (ii) $\sqrt{2} = 1.41421356237300$ .....

(i) When remainder never becomes zero and they repeat after a certain stage which force the decimal expansion to go forever, then such decimal expansion are called non-terminating recurring (repeating) decimal.

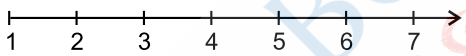
Ex- (i) $\frac{1}{7}$ = 0.142857142857 ..... (ii) $\frac{67}{13}$ = 5.153846153846 .....

(ii) When remainder never becomes zero and they do not repeat after a certain stage and force the decimal expansion to go forever, then such decimal expansion are called non-terminating non-recurring decimal.

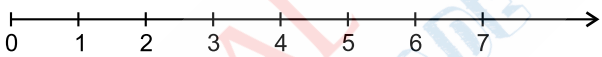
Ex- (i) π = 3·14159265358979323846 ..... (ii) 0·1010010001100 .....

- All integers (Positive, zero or negative) are terminating decimals.
- The decimal expansion of a rational number is either terminating or non-terminating recurring (repeating). Conversely, a number whose decimal expansion is terminating or non-terminating recurring is rational.
- The decimal expansion of a rational number $\frac{p}{q}$, where p and q are integers, $q > 0$, p and q have no common factors except 1 is :
  (i) terminating if prime factors of q are 2 or 5 or both.
  (ii) non-terminating recurring if q has a prime factor other then 2 or 5.
- The decimal expansion of a irrational number is non-terminating non-recurring. Conversely, a number whose decimal expansion is non-terminating non-recurring is irrational.
- All decimal numbers (terminating, recurring or non-terminating and non-recurring) are real numbers.
- **Representation of rational numbers on the number line :** We know that the rational numbers include natural numbers, whole numbers, integers and fractional numbers. All these numbers can be represented on the number line as shown below :

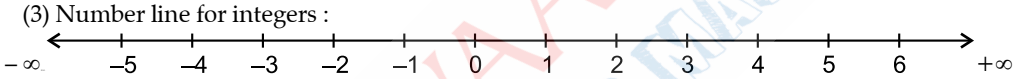
(1) Number line for natural numbers :



(2) Number line for whole numbers :

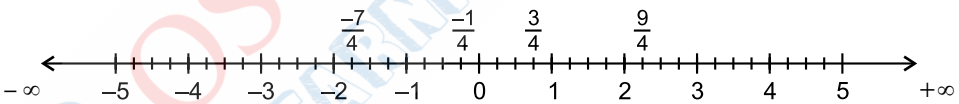


(3) Number line for integers :

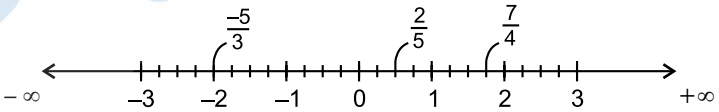


(4) Number line for fractions :

(i) For fractions $\frac{3}{4}, \frac{9}{4}, \frac{-1}{4}$ and $\frac{-7}{4}$

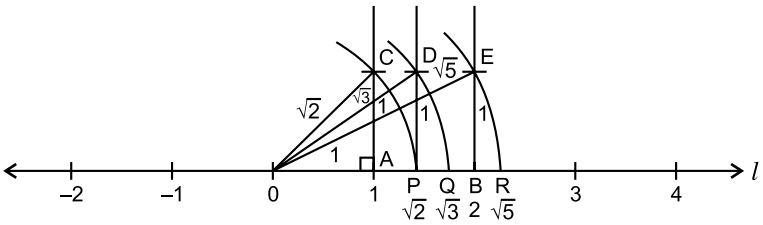


(ii) For fractions $\frac{2}{5}, \frac{7}{4}$ and $\frac{-5}{3}$



- **Representation of irrational number on the number line :** To represent the irrational number $\sqrt{2}$ on the number line l, construct a right angled triangle OAC, right angled at A, such that $OA = AC = 1$, then by pythagoras theorem,

$$OC^2 = OA^2 + AC^2$$
$$OC^2 = (1)^2 + (1)^2 = 2$$
$$OC = \sqrt{2}$$



Now mark a point, say P, on l on the right of O such that $OP = OC = \sqrt{2}$, then the point P represents the irrational number $\sqrt{2}$.

To represent the irrational number $\sqrt{3}$ on the line l, construct a right angled triangle OPD, right angled at P such that $DP = 1$, then by pythagoras theorem,

$$OD^2 = OP^2 + DP^2$$
$$OD^2 = (\sqrt{2})^2 + (1)^2$$
$$OD^2 = 2 + 1$$
$$OD^2 = 3$$
$$OD = \sqrt{3}$$

Now mark a point, say Q, on l to the right of O such that $OQ = OD = \sqrt{3}$, then the point Q represents the irrational number $\sqrt{3}$.

To represent the irrational number $\sqrt{5}$ on the line l, construct a right angled triangle OBE, right angled at B such that $OB = 2$ and $BE = 1$. Then by pythagoras theorem

$$OE^2 = OB^2 + BE^2$$
$$OE^2 = (2)^2 + (1)^2$$
$$OE^2 = 4 + 1$$
$$OE^2 = 5$$
$$OE = \sqrt{5}$$

Now mark a point, say R, on l to the right of O such that $OR = OE = \sqrt{5}$, then the point R represents the irrational number $\sqrt{5}$.

Thus, geometrical constructions can be devised to identify the points on the number line l which correspond to the irrational numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc. and so on.

➢ **Proofs of irrationality of $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$.**

(1) To prove $\sqrt{2}$ is irrational :

Let $\sqrt{2}$ be a rational number, then $\sqrt{2} = \frac{a}{b}$, where a, b are integers, $b \neq 0$ and a and b have no common factors (except 1)

$$2 = \frac{a^2}{b^2} \quad \text{(Squaring both the sides)}$$
$$a^2 = 2b^2$$

As a^2 is divisible by 2 and so a is also divisible by 2 ...(i)

Let
$$a = 2c$$
$$a^2 = 4c^2 \quad \text{(Squaring both sides)}$$
$$2b^2 = 4c^2 \quad (\because a^2 = 2b^2)$$
$$b^2 = 2c^2$$

As b^2 is divisible by 2 and so b is also divisible by 2 ...(ii)

From (i) and (ii), we get a and b both are divisible by 2

i.e. a and b have 2 as their common factor.

This contradicts our assumption that a and b have no common factors (except 1)

Hence, $\sqrt{2} = \frac{a}{b}$ is not a rational number. So, we conclude that $\sqrt{2}$ is an irrational number.

(2) To prove $\sqrt{3}$ is irrational :

Let $\sqrt{3}$ be a rational number, then $\sqrt{3} = \frac{a}{b}$, where a and b are integers, $b \neq 0$ and a and b have no common factors (excepts 1)

$$3 = \frac{a^2}{b^2}$$ (Squaring both sides)

$$a^2 = 3b^2$$

As a^2 is divisible by 3 and so a is also divisible by 3 ...(i)

Let $a = 3c$

$a^2 = 9c^2$ (Squaring both sides)

$3b^2 = 9c^2$ ($\because a^2 = 3b^2$)

As b^2 is divisible by 3 and so b is also divisible by 3 ...(ii)

From (i) and (ii), we get a and b both are divisible by 3.

i.e. a and b have 3 as their common factor.

This contradicts our assumption that a and b have no common factors (except 1)

Hence, $\sqrt{3} = \frac{a}{b}$ is not a rational number. So, we conclude that $\sqrt{3}$ is an irrational number.

(3) To prove $\sqrt{5}$ is irrational :

Let $\sqrt{5}$ be a rational number, then $\sqrt{5} = \frac{a}{b}$, where a and b are integers, $b \neq 0$ and a and b have no common factors (except 1)

$$5 = \frac{a^2}{b^2}$$ (Squaring both sides)

$$a^2 = 5b^2$$

As a^2 is divisible by 5 and so a is also divisible by 5. ...(i)

Let $a = 5c$

$a^2 = 25c^2$ (Squaring both sides)

$5b^2 = 25c^2$ ($\because a^2 = 5b^2$)

$b^2 = 5c^2$

As b^2 is divisible by 5 and so b is also divisible by 5. ...(ii)

From (i) and (ii), we get a and b both are divisible by 5.

i.e. a and b have 5 as their common factor.

This contradicts our assumption that a and b have no common factors (except 1)

Hence, $\sqrt{5} = \frac{a}{b}$ is not a rational number. So we conclude that $\sqrt{5}$ is an irrational number.

TOPIC-2
Surds or Radicals

Revision Notes

- **Surds (Radicals) :** It a is any positive rational number and n (> 1) is a positive integer and if a cannot be written as the nth power of any rational number, then $\sqrt[n]{a}$ {or $(a)^{1/n}$} is called a Surd of order n.
- In other words, $\sqrt[n]{a}$ is a Surd (of orders) if

  (i) a is a positive rational number,

  (ii) n (> 1) is a positive integer, and

  (iii) $\sqrt[n]{a}$ is not a rational number.

Ex : $\sqrt{2}, \sqrt{3}$, ... are surds of order 2

$\sqrt[3]{2}, \sqrt[3]{5}$, ... are surds of order 3 and so on

- All surds are irrational numbers.
- Every surds is an irrational numbers, but every irrational numbers is not a surd.
  Ex- π is an irrational number but not a surd.
- **Some laws of Surds :** If a, b are positive rational numbers and n (> 1) is a positive integer, then

  (i) $(\sqrt[n]{a})^n = a$ (ii) $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$ (iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

- **Some facts about Surds of order 2**

  (1) If a, b are positive rational numbers, then

  (i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

  (iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

  (iv) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

  (v) $(\sqrt{a}+\sqrt{b})^2 = a+2\sqrt{ab}+b$

  (vi) $(\sqrt{a}-\sqrt{b})^2 = a-2\sqrt{ab}+b$

  (2) If a, b, c are d are positive real numbers, then $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

  (3) If a, b, c are d are rational numbers, $\sqrt{p}$ is an irrational numbers and $a+b\sqrt{p} = c+d\sqrt{p}$, then $a = c$ and $b = d$.

  (4) If $\sqrt{a}$ and $\sqrt{b}$ are, positive irrational numbers, then

  (i) $\sqrt{a}<\sqrt{b}$ if $a < b$

  (ii) $\sqrt{a}>\sqrt{b}$ if $a > b$

  (5) If a is a positive rational number and $\sqrt{b}$ is an irrational number, then

  (i) $a<\sqrt{b}$ if $a^2 < b$

  (ii) $a>\sqrt{b}$ if $a^2 > b$

- The process of multiplying a surd by another surd to get a rational number is called rationalization. Each surd is called rationalizing factor of the other surd.

  Ex - Since $2\sqrt{3}\times 7\sqrt{3} = 14 \times 3 = 42$, which is a rational number, therefore $2\sqrt{3}$ and $7\sqrt{3}$ are rationalizing factors of each other.

Know the terms

- **Pure Surd :** A surd is a pure surd, if it does not contain any other rational coefficient except unity.

  Ex - $\sqrt{3}, \sqrt{2}$, etc.

- **Mixed Surd :** A surd is a mixed surd, if it has some rational coefficient other than unity.

  Ex - $2\sqrt{3}, 4\sqrt{7}$, etc.

- **Similar Surd :** Similar surds are surds of same order and same radiants.

  Ex - $\sqrt{5}, 7\sqrt{5}, 10\sqrt{5}$, etc.

CHAPTER-2
COMPOUND INTEREST

TOPIC-1
Compound interest (Repeated Simple Interest Formula)

Revision Notes

- To calculate compound interest by using simple interest method, generally use calculate the simple interest for first period and add them to the original amount and again calculate the simple interest on the accumulated amount for the next period and so on upto the amount for the last period.
- Interest is the additional money besides the original money paid by the borrower for using lender's money.
- The money borrowed (or the money lent) is called principal.
- The sum of the principal and the interest is called amount.

OR

- The total money, paid by the borrower to the lender at the end of the specified period is called the amount.

  Thus, Amount = Principal + Interest

  i.e., $A = P + I$
- The interest paid on ₹ 100 for a specified period is called the 'Rate'.
- Time is the period for which the money is borrowed.
- **Simple Interest :** Simple interest is the interest calculated on the original money (principal) at the given rate of interest for any given period.

  $\therefore$ $$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

  i.e., $$\text{S.I.} = \frac{P \times R \times T}{100}$$
- In the simple interest; (i) the principal remains constant for the whole loan period.
  (ii) The interest of every year is same on the same sum and at the same rate of interest per annum.
- Compound interest is the interest calculated on the initial principal, which also includes all of the accumulated interest of previous period of a deposit or loan.
- The difference between the final amount and the original principal is called compound interest.

  $\therefore$ Compound interest = Final Amount – Original Principal

  i.e., $\text{C. I.} = A - P$
- In the compound interest, the principal goes on changing every year (or any other fixed period) *i.e.*, If the interest is compounded annually, the principal changes after every year and if the interest is compounded half-yearly, the principal changes after every six month (or any other fixed period).
- The period (time), after which the principal changes, is called the conversion period.
- When the interest is compounded annually, then there is one conversion period in a year. If the interest is (compounded semi-annually, then there are two) conversion period in a year and so on.
- Simple interest (S.I.) and compound interest (C.I.) are equal of first year (first conversion Period) on the same sum and at the same rate.
- C.I. of 2nd year (second conversion period) is always more than the C.I. or S.I. of first year (first conversion period)
- C.I. of 3rd year (third conversion period) is more than the C.I. or S.I. of second year (second conversion period) and so on.
- In every conversion period, C.I. increases but the S.I. remains same.
- The difference between the compound interest for any two consecutive conversion periods (year or half-year) is the interest of one period on the C.I. of the preceding conversion period.

  For example : If ₹ 1000 and ₹ 1200 are the C.I. for any two consecutive years or half-years, then their difference ₹ 1200 – ₹ 1000 = ₹ 200 is the interest of one year or half-year on ₹ 1000.
- Rate of interest (When C.I. of any two consecutive conversion periods are given) :

$$\text{Rate of interest} = \frac{\text{Difference in the interest of two consecutive conversion periods}}{\text{C.I. of Preceding Period} \times \text{Time}} \times 100\%$$

- For any two consecutive conversion period if the C.I. of a particular period is ₹ x, then the C.I. for the next period, on the same sum and at the same rate = ₹ x + interest on ₹ x for one period.

Know the terms

- **Constant** = A value that does not change.
- **Consecutive Periods** = Periods of time or events happen one after the other without interruption.
- **Preceding** = Existing or happening before someone or something.

TOPIC-2
Compound Interest (Using Formula)

Revision Notes

- When the interest is compounded yearly, the formula for finding the amount is :

$$\boxed{A = P\left(1+\frac{r}{100}\right)^n \text{ or } P\left(\frac{100+r}{100}\right)^n}$$

where, A = Final amount, P = Principal, r = rate of interest compounded yearly, and n = number of years.

- $$\text{C.I.} = A - P$$
$$= P\left(1+\frac{r}{100}\right)^n - P$$
$$\boxed{\text{C.I.} = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]}$$

- When the rates of interest for the successive fixed periods are different, then :

$$\boxed{A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right) \text{ ..... and so on}}$$

where, r_1, r_2, r_3 ..... and so on are the rate percent of interests for successive fixed periods.

- When the interest is compound half-yearly, then :

$$\boxed{A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}}$$

i.e., the rate percent is divided by 2 and the number of years is multiplied by 2.

- When the time is not an exact number of years and the interest is compounded yearly, then

$$\boxed{A = P\left(1+\frac{r}{100}\right)^{n_1}\left(1+\frac{r}{2\times 100}\right)^{n_2\times 2}}$$

where, n_1 is the whole number of years and n_2 is the fraction part of year.

For example : Find the amount when ₹ 5000 is invested for $3\frac{1}{2}$ years at 15% interest compounded yearly.

Then, $$\text{Amount in } 3\frac{1}{2} \text{ years} = P\left(1+\frac{r}{100}\right)^{n_1}\left(1+\frac{r}{2\times 100}\right)^{n_2\times 2}$$
$$= 5000\left(1+\frac{15}{100}\right)^3\left(1+\frac{15}{2\times 100}\right)^{\frac{1}{2}\times 2}$$

Note : Here, 3 is the whole number and $\frac{1}{2}$ is the fraction part of years.

- **Life span** = The number of years a machine can be effectively used is called its 'life span', after which it is sold as waste or scrap.

CHAPTER-3
EXPANSIONS

Revision Notes

- **Identity :** An equation, which is true for all values of its variables, is called an identity.

Special Products

(1) $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$

(3) $(a + b)(a - b) = a^2 - b^2$

(4) $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

(5) $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

(6) $\left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = a^2 - \frac{1}{a^2}$

(7) (i) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) $(x + a)(x - b) = x^2 + (a - b)x - ab$

(iii) $(x - a)(x + b) = x^2 - (a - b)x - ab$

(iv) $(x - a)(x - b) = x^2 - (a + b)x + ab$

(8) (i) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(ii) $(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$

(iii) $(a - b + c)^2 = a^2 + b^2 + c^2 + 2(-ab - bc + ca)$

$= a^2 + b^2 + c^2 + 2(ca - ab - bc)$

(iv) $(a - b - c)^2 = a^2 + b^2 + c^2 + 2(-ab + bc - ca)$

$= a^2 + b^2 + c^2 + 2(bc - ab - ca)$

(9) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$= a^3 + b^3 + 3a^2b + 3ab^2$

(10) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$= a^3 - b^3 - 3a^2b + 3ab^2$

(11) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(12) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(13) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(14) $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc.$

CHAPTER-4
FACTORISATION

Revision Notes

- When an algebraic expression can be written as the product of two or more algebraic expressions, then each of these expression is called a factor of the given expression.
- The process of writing an expression in the form of terms or brackets multiplied together, is called factorization.
- Each term and each bracket is called a factor of the expression.
- Factorization is the reverse of Multiplication.
- Different methods of factorisation

(i) Factorization by taking out common factors.

(ii) Factorization by grouping of terms.

(iii) Factorization by using identities

* $a^2 - b^2 = (a + b)(a - b)$
* $a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
* $a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
* $a^2 + b^2 + c^2 + 2(ab + bc + ca) = (a + b + c)^2$

(iv) Factorization of trinomials

* $x^2 + (a + b)x + ab = (x + a)(x + b)$

Factorization of trinomials by splitting the middle terms : In factorization of $ax^2 \pm bx \pm c$, we split the coefficient of middle term b into two parts such that the sum or difference of two parts is equal to b and the product of the two parts is equal to the product of a and c.

So, $x^2 - 5x + 6$

Here, $a = 1, b = -5, c = 6$

$$ac = 1 \times 6 = 6$$

Again,

$$ac = (-3) \times (-2) = +6$$

$$b = -5 = (-3) + (-2)$$

$$\therefore \quad x^2 - 5x + 6 = x^2 - (3 + 2)x + 6$$

$$= x^2 - 3x - 2x + 6$$

$$= x(x - 3) - 2(x - 3)$$

$$= (x - 3)(x - 2)$$

(v) Factorization by using the identities :

* $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
* $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
* $a^3 + b^3 + 3ab(a + b) = (a + b)^3$
* $a^3 - b^3 - 3ab(a - b) = (a - b)^3$
* $a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

CHAPTER-5

SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES

TOPIC-1

Solving Simultaneous equations by different methods

Revision Notes

- An equation of the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are non-zero, is called a general linear equation in the two variable x and y.
- $x = \alpha$ and $y = \beta$ is a solution of the linear equation $ax + by + c = 0$ if and only if $a\alpha + b\beta + c = 0$, where α, β are real numbers.
- Every linear equation in two variables has an unlimited number of solution.
  For example : $x = 0, y = 7, x = 2, y = 3$, and $x = 7, y = -7$ etc are all solutions of the equation $2x + y - 7 = 0$.
- A set of equations in two or more variables for which there are values that can satisfy all the equations simultaneously. Such equations are called simultaneous linear equation.
  For example : $2x + y = 0$ and $3x - 7y = 9$
- The various methods of solving a pair of linear equations in two variables are :

  (i) Substitution method : In this method, values of one variable (x or y) can be transformed in terms of another variable (y or x, and this variable is substituted in another equation and simplify it to get the value of one variable further put the value of this variable in any one of the given equation and get the value of another variable.

  (ii) Elimination method : In this method any are of the variable in eliminated by making the coefficients of that variable some in both the equations. After eliminating the variable, equation is one variable simplify it and get the value. Further put the value of the variable in any are of the given equation and get the value of another variable.

  (iii) Cross-multiplication method : Suppose system of linear equations is :

  $$a_1x + b_1y + c_1 = 0$$

  and

  $$a_2x + b_2y + c_2 = 0$$

  By using cross multiplication method this can be determined by

  $$\frac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \frac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$$

  $$\Rightarrow \quad \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Know the terms

- **Consistent :** If simultaneous linear equations has one and only one solution which satisfy the given equations, then the system of linear equations is said to be consistent and independent.
- **In consistent :** If the values of x and y obtained from two equations do not satisfy the third, equation, then the three equations cannot hold simultaneously and we conclude that the three equations are inconsistent.

TOPIC-2
Word Problems

Revision Notes

- Problems Stated in words are called word problem.
- Solving word problems involves two steps. First translating the words of the problem into algebraic equations. Second, solving the resulting equations by any one of the method.

CHAPTER-6
INDICES (Exponents)

Revision Notes

- If a is any real number and n is a natural number, then $a^n = a \times a \times a \times$ ..... n times
  where 'a' is called the base, n is called the exponent or power or index and a^n is the exponential form.
- a^n is read as 'a power n' or 'a raised to the power n'.
- The plural form of index is indices.
- Laws of exponents for real numbers :

(1) $a^m \times a^n = a^{m+n}$

(2) $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$

(3) $(a^m)^n = a^{mn}$

(4) $(a \times b)^m = a^m \times b^m$

(5) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$

(6) $a^0 = 1, a \neq 0$

(7) $a^{-m} = \dfrac{1}{a^m}, a \neq 0$

or $a^m = \dfrac{1}{a^{-m}}, a \neq 0$

(8) $\sqrt[m]{a} = a^{1/m}, a \neq 0$

(9) $a^{m/n} = a^{m \times 1/n} = \sqrt[n]{a^m}$, where $a \neq 0$

(10) $\left(\dfrac{a}{b}\right)^n = \left(\dfrac{b}{a}\right)^{-n}, n \in Z$

Know the formulas

(1) If $x^n = y^n, n \neq 0$

$\Rightarrow x = y, x > 0, y > 0$

(2) If $x^n = x^m$

$\Rightarrow n = m$ and $x \neq 1$

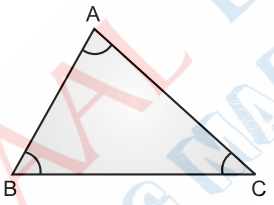
(3) $(-a)^m = a^m$, if m is an even number.

$(-a)^m = -a^m$ if m is an odd number.

CHAPTER-7
TRIANGLES

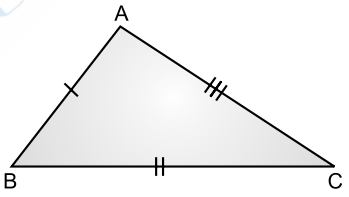
Revision Notes

- A plane closed figure formed by three intersecting line is called a triangle.
- A triangle has
  - **(i)** Three sides as AB, BC and CA
  - **(ii)** Three vertices as A, B and C.
  - **(iii)** Three angles as $\angle A$, $\angle B$ and $\angle C$.



- Three sides and three angles are called its six elements
- **Types of triangles on the basis of sides**
  - **(i) Scalene triangle :** If all the sides of a triangle are unequal, it is called a scalene triangle.

    In the figure, $AB \neq BC \neq CA$

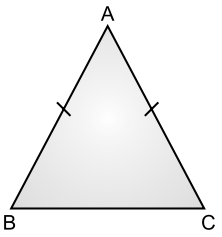


Scalene triangle

  - **(ii) Isosceles triangle :** It any two sides of a triangle are equal, it is called an isosceles triangle.

    In the figure, $AB = AC \neq BC$

    and $\angle B = \angle C$

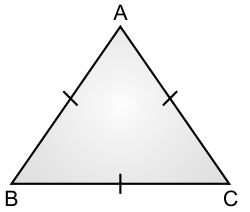


Isosceles triangle

(iii) Equilateral triangle : If all the three sides of a triangle are equal, it is called an equilateral triangle.

In the figure, $AB = BC = CA$

$\angle A = \angle B = \angle C = 60°$

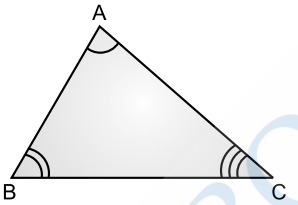


Equilateral triangle

➢ **Types of triangles on the basis of angles**

(i) Acute angled triangle : A triangle in which all the three angles are acute (less than 90°), is called an acute angle triangle.

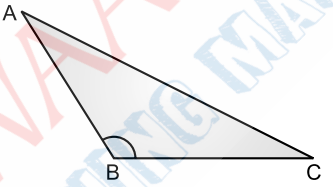
In the figure, $\angle A < \angle 90°$, $\angle B < \angle 90°$, $\angle C < \angle 90°$,



Acute angled triangle

(ii) Obtuse angled triangle : A triangle whose one angle is obtuse (greater than 90° but less than 180°), is called an obtuse angled triangle.

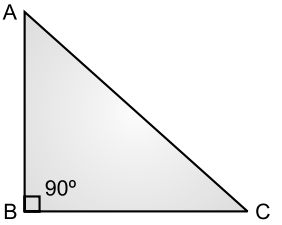
In the figure, $90° < \angle B < \angle 180°$



Obtuse-angled triangle

(iii) Right-angled triangle : A triangle whose one angle is a right angle (equal to 90°), is called a right angled triangle.

In the figure, $\angle B = 90°$

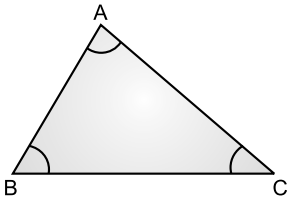


Right-angled triangle

➢ **Angle sum property of a triangle**

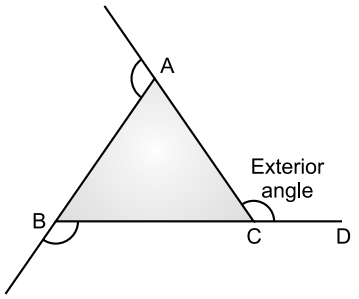
The sum of the interior angles of a triangle is equal to 180°.

In the given triangle ABC, $\angle A + \angle B + \angle C = 180°$



➢ **Exterior angle Property of a triangle**

It one side of a triangle is produced, the exterior angle so formed is equal to the sum of two opposite interior angles.



In the given triangle ABC,

(i) Exterior angle of $A = \angle B + \angle C$

(ii) Exterior angle of $B = \angle A + \angle C$

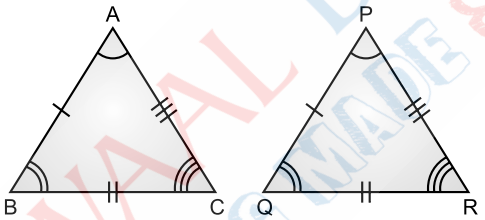
(iii) Exterior angle of $C = \angle A + \angle B$

- **Congruent Triangle**

Two triangles are called congruent of and only if they have exactly the same shape and the same size. *i.e.*, all the angles and all the sides of one triangle are equal to the corresponding angles and the corresponding sides of the other triangle each to each.

In the given triangle ABC and POR :

(i) $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$



(ii) $AB = PQ$, $BC = QR$ and $AC = PR$

$\therefore$ ΔABC is congruent to ΔPQR.

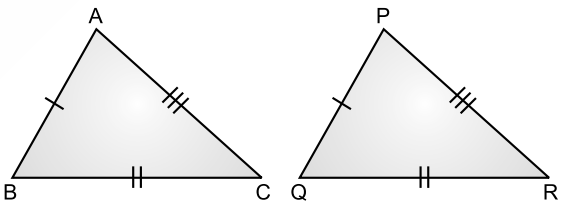
- The symbol of two congruent triangle is $\cong$ and read as is "congruent to".

i.e., $\Delta ABC \cong \Delta PQR$

- The abbreviation *CPCT* is used for corresponding parts of congruent triangles.
- **Criteria for congruence of triangles**

(i) SSS (Side-Side-Side) Congruence rule

If the three sides of one triangle are equal to the three sides of other triangle, then two triangles are congruent



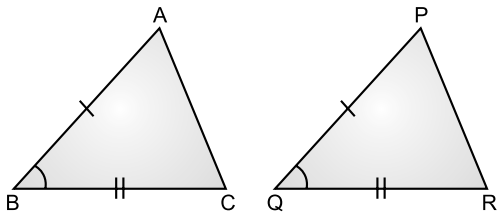
In ΔABC and ΔPQR

$AB = PQ$, $BC = QR$ and $CA = RP$

$\therefore$ $\Delta ABC \cong \Delta PQR$

(ii) SAS (Side-Angle-Side) Congruence rule

If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

$AB = PQ, BC = QR$

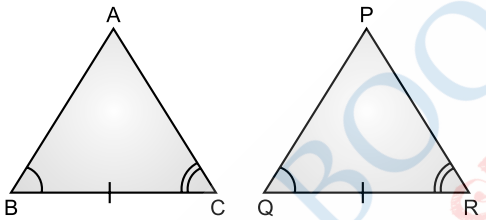
and $\angle B = \angle Q$

$\therefore$ $\Delta ABC \cong \Delta PQR$

The equality of the 'included angle' is essential

(iii) ASA (Angle-side-Angle) Congruence rule

If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

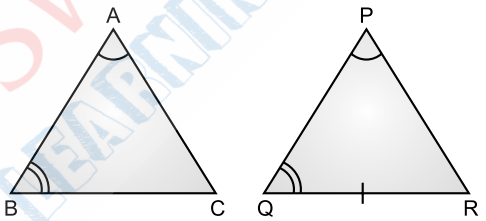
$\angle B = \angle Q, \angle C = \angle R$

and $BC = QR$

$\therefore$ $\Delta ABC \cong \Delta PQR$

(iv) AAS (Angle-Angle-Side) Congruence rule

If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

$\angle A = \angle P, \angle B = \angle Q$

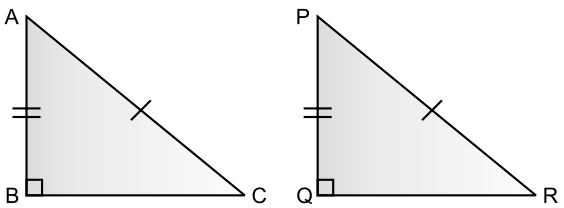
and $BC = OR$

$\therefore$ $\Delta ABC \cong \Delta PQR$

The equality of 'corresponding sides' is essential.

(v) RHS (Right angle-Hypotenuse-Side) Congruence rule

If the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then two right triangles are congruent.

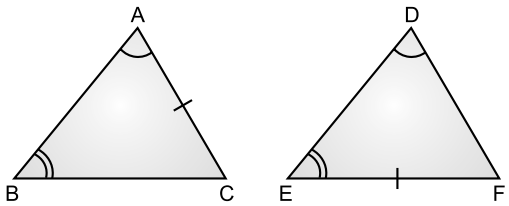


In ΔABC and ΔPQR

$AB = PQ, AC = PR$

and $\angle B = \angle Q$

$\therefore$ $\Delta ABC \cong \Delta PQR$



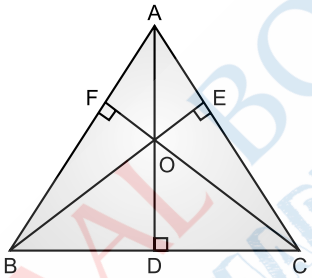
The the given figure, the equal sides are AC and EF, and these two sides (*i.e.*, AC and EF) are not opposite to equal angles.

$\therefore$ ΔABC and ΔDEF are not congruent by AAS.

Know the Terms

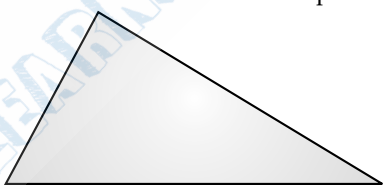
- **Altitude :** The length of perpendicular from a vertex of a triangle to the opposite side is called an altitude of the triangle.

  In the given figure, AD, BE and CF are the three altitude to the corresponding sides BC, CA and AB respectively.



- **Orthocentre :** All the three altitudes are always intersect each other at one point only. This point of intersection is called the orthocentre of the triangle.
- **Median :** The straight line joining the mid-point of side to the opposite vertex at a triangle is called a median of the triangle.

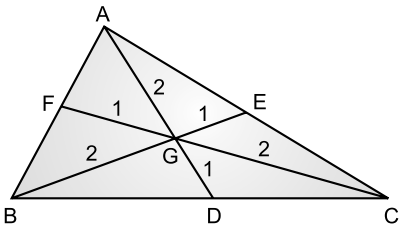
  In the given figure, AD, BE and CF are three Medians to the corresponding sides BC, CA and AB respectively.



- **Centroid :** All the three medians are always intersect each other at one point only. This point of intersection is called the centroid of the triangle.

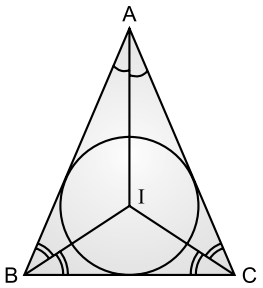
  The centroid of a triangle divides each median in the ratio 2 : 1.

  i.e., in the given figure, $AG : GD = 2 : 1 = BG : GE = CG : GF$



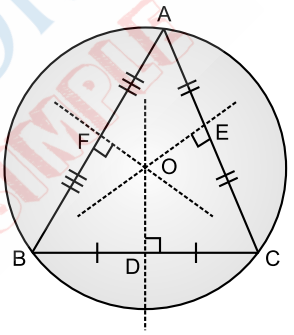
- **Bisector :** Line bisecting an interior angle of a triangle is called the bisector of the angle of the triangle.

  In the figure, AI, BI and CI are three internal bisector of $\angle A$, $\angle B$ and $\angle C$ respectively.



- **Incentre :** All the three internal bisectors of the angles of a triangle are always intersect each other at one point only. This point of intersection is called the incentre of the triangle.
- **Incircle :** Incentre is the centre of the circle which touches all the sides of ΔABC and this circle is called incircle of ΔABC.
- **Right bisector :** Line bisecting a side of a triangle and perpendicular to it is called the right bisector of the side of the triangle.

  In the given figure, OD, OE and OF are the right bisectors of sides BC, CA and AB respectively.



- **Circumcentre :** All the three right bisectors of the sides of a triangle are always intersect each other at one point only. This point of intersection is called the circumcentre of the circle.
- **Circumcircle :** Circumcircle is the centre of a circle which passes through the vertices of ΔABC and this circle is called the circumcircle of ΔABC.

Know the Facts

- If one side of a triangle is produced, the exterior angle so formed is greater than each at the interior opposite angles.
- A triangle cannot have more than one right angle.
- A triangle cannot have more than one right angle.
- A triangle cannot have more than one obtuse angle.
- In a right angled triangle, the sum of the other two angles (acute angles) is 90°.
- In every triangle, at least two angles are acute.
- If two angles of a triangle are equal to two angles of any other triangle, each to each, ten the third angles of both the triangles are also equal.
- Congruent triangle are 'equal in all respect' *i.e.*, they are the exact duplicate of each other.
- If two triangles are congruent, then any one can be superposed on the other to cover it exactly.
- In congruent triangles, the sides and the angles which coincide by superposition are called corresponding sides and corresponding angles.
- The corresponding sides lie opposite to the equal angles and the corresponding angle lie opposite to the equal sides.
- The order of letters in the name of congruent triangles displays the corresponding relationship between the two triangles.
- The other of letters in the name of congruent triangles displays the corresponding relationship between the two triangles.
  When $\Delta ABC \cong \Delta PQR$, it means that $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $AB = PQ$, $BC = QR$, $CA = RP$,
  Writing any other correspondence *i.e.*, $\Delta ABC \cong \Delta PRQ$, $\Delta ABC \cong \Delta RPQ$ etc. will be incorrect.

●●

CHAPTER-8
MID-POINT THEOREM

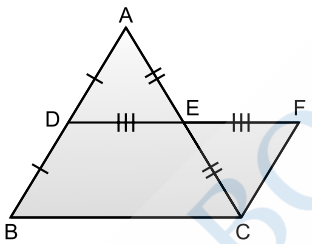
Revision Notes

- **Mid-Point Theorem :** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is equal to half of it.

  Given : D and F are the mid-point of side AB and AC respectively of ΔABC

  To Prove : $DE \parallel BC$ and $DE = \frac{1}{2}BC$

  Construction : Produce DE upto F such that $DE = EF$ and join CF.



  Proof : In ΔADE and ΔCFE

$AE = CE$ (E is the Mid-Point of AC)

$DE = FE$ (by construction)

$\angle AED = \angle CEF$ (vertically opposite angle)

$\Delta ADE \cong \Delta CFE$ (by SAS criteria)

$\therefore$ $AD = CF$ (by CPCT) ...(i)

But $BD = AD$ (D is the Mid-Point of AB) ...(ii)

from eqn. (i) and (ii)

$BD = CF$ ...(iii)

and $\angle EAD = \angle ECF$ (by CPCT)

But these are alternate interior angles as BA and CF are two lines and AC is a transversal line

$\therefore$ $BA \parallel CF$ or $BD \parallel CF$ ...(iv)

From eqn. (iii) and (iv)

$BCFD$ is a parallelogram

$\therefore$ $DF = BC$ (opposite side of a parallelogram)

$DE + EF = BC$

$DE + DE = BC$ [DE = EF, by construction]

$2DE = BC$

$DE = \frac{1}{2}BC$

and $DF \parallel BC$ ($DCFD$ is a parallelogram)

or $DE \parallel BC$

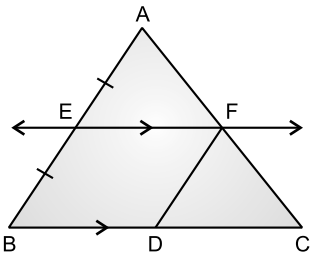
$\therefore$ $DE = \frac{1}{2}BC$ and $DE \parallel BC$ **Hence Proved**

- **Converse of mid-point theorem :** The line draws through the mid-point of one side of a triangle parallel to another side bisects the third side.

  Given : A triangle ABC, E is mid-point of AB. Line l drawn through E and Parallel to BC meeting AC at F.

  To Prove : $AF = FC$

  Construction : Take a point D on BC such that $EF = BD$. Join DF



Proof : $\because$ $BD \parallel EF$ (given)

and $BD \parallel EF$ (by construction)

$\therefore$ $BDFE$ is a parallelogram

$\therefore$ $BE = DF$ (opposite side of parallelogram)

But $BE = AE$ (E is the mid-point of AB)

from (i) and (ii)

$AE = DF$ (iii)

$\angle EBD = \angle FDC$ ...(iv)

(corresponding angle, $BE \parallel DF$ and BC is a transversal)

$\angle EBD = \angle AEF$ ...(v)

(corresponding angle, $BD \parallel EF$ and BA is a transversal)

from (iv) and (v)

$\angle AEF = \angle FDC$ ...(vi)

$\angle AFE = \angle FCD$ ...(vii)

(corresponding angle, $BC \parallel EF$ and AC is a transversal)

In ΔAEF and ΔFDC

$AE = FD$ [from (iii)]

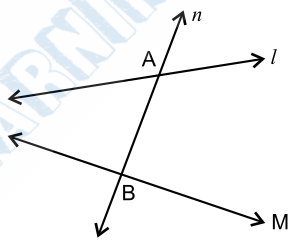
$\angle AEF = \angle FDC$ [from (vi)]

$\angle AFE = \angle FCD$ [from (vii)]

$\therefore$ $\Delta AEF \cong \Delta FDC$ (by AAS criteria)

$\therefore$ $AF = FC$ (by CPCT) **Hence Proved**

- If a line n intersects two lines l and m at points A and B respectively, then the line segment AB is called the intercept made on line n by the lines l and m.

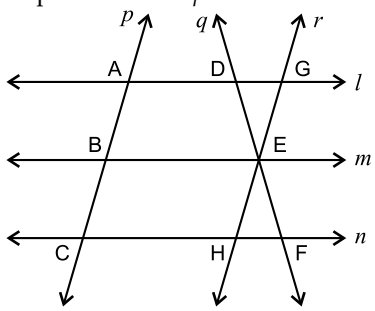


Equal Intercept Theorem : If a transversal makes equal intercepts on three (or more) parallel lines, then any other line cutting them also makes equal intercepts.

Given : Three parallel lines l, m and n. A transversal p cutting them at points A, B and C respectively such that $AB = BC$. Any other line q cuts them at points D, E and F respectively.

To prove : $DE = EF$

Construction : Through E, draw a line r parallel to line p to meet line l at G and line n at H.



Proof : In quadrilateral $ABEG$

$AG \parallel BE$ (given)

and $AB \parallel GE$ (by construction)

$\therefore$ Quadrilateral $ABEG$ is a parallelogram

$\therefore$ $AB = GE$ ...(i)

Similarly, in quadrilateral $BCHE$

$BE \parallel CH$ (given)

$BC \parallel EH$ (by construction)

$\therefore$ Quadrilateral $BCHE$ is a parallelogram

$\therefore$ $BC = EH$ ...(ii)

But $AB = BC$ (given) ...(iii)

$\therefore$ from (i), (ii) and (iii)

$GE = EG$ ...(iv)

In ΔDEG and ΔFEH

$\angle DEG = \angle FEH$ (vertically opposite angle)

$GE = EH$ [from (iv)]

$\angle DGE = \angle FHE$ (alternate interior angle, $l \parallel n$ and r is a transversal line)

$\therefore$ $\Delta DEG \cong \Delta FEH$ (by ASA criteria)

$\therefore$ $DE = EF$ (by CPCT) **Hence Proved**

CHAPTER-9
PYTHAGORAS THEOREM

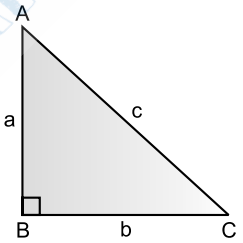
Revision Notes

- **Pythagoras Theorem :** In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of two sides.

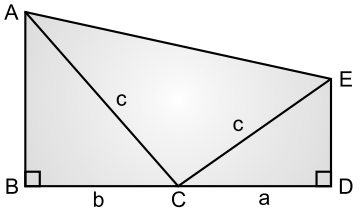
  Given : A triangle ABC in which $\angle ABC = 90°$. and $AB = a$, $BC = b$, $AC = c$

  To Prove : $AC^2 = AB^2 + BC^2$

  or $c^2 = a^2 + b^2$



Construction : Produce the side BC to a point D such that $CD = AB = a$. At D, draw $DE \perp AD$ and cut off $DE = BC = b$. Join AE.



Proof : In ΔABC and ΔCDE

$AB = CD$ (by construction)

$BC = DE$ (by construction)

$$\angle ABC = \angle CDE = 90°$$

$\therefore$ $\Delta ABC \cong \Delta CDE$ (by SAS criteria)

$\therefore$ $AC = CE = c$ (by CPCT)

and $\angle ACB = \angle DCE$ (by CPCT) ...(i)

In ΔABC $\angle ABC + \angle ACB + \angle BAC = 180°$ (angle sum property)

$$90° + \angle ACB + \angle BAC = 180°$$

$$\angle ACB + \angle BAC = 90°$$

$$\angle ACB + \angle DCE = 90°$$ ...(ii) [from (i)]

Now, $\angle ACB + \angle ACE + \angle DCE = 180°$ [BCD is a straight line]

$$\angle ACB + \angle DCE + \angle ACE = 180°$$

$$90° + \angle ACE = 180°$$ [from (ii)]

$$\angle ACE = 90°$$

i.e., ACE is a right angle triangle at C.

Now, $\angle ABC + \angle CDE = 90° + 90° = 180°$

$\therefore$ $AB \parallel DE$ ($\because$ sum of co-interior angle is 180°)

$\therefore$ $ABDE$ is a trapezium

$$\text{Area of trapezium} = \text{area of } \Delta ABC + \text{area of } \Delta CDE + \text{area of } \Delta ACE$$

$$= \frac{1}{2} \times BC \times AB + \frac{1}{2} \times CD \times ED + \frac{1}{2} \times CE \times AC$$

(by construction area of right triangle $= \frac{1}{2} \times$ base $\times$ height)

$$= \frac{1}{2} b \times a + \frac{1}{2} \times a \times b + \frac{1}{2} \times c \times c$$

$$= \frac{1}{2} ab + \frac{1}{2} ab + \frac{1}{2} c^2$$

$$= ab + \frac{c^2}{2}$$ ...(iii)

Now, $\text{Area of trapezium} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{height}$

$$= \frac{1}{2} \times (AB + DE) \times BD$$

$$= \frac{1}{2} (a + b) \times (BC + CD)$$

$$= \frac{1}{2} (a + b)(a + b)$$ ...(iv)

from (iii) and (iv)

$$\frac{1}{2} .(a + b)(a + b) = ab + \frac{c^2}{2}$$

$$(a + b)^2 = 2ab + c^2$$

$$a^2 + b^2 + 2ab = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

i.e., $AB^2 + BC^2 = AC^2$ **Hence Proved**

- **Converse of Pythagoras Theorem :** If in any triangle, the square of the largest side of the triangle is equal to the sum of the squares on remaining two sides, then the triangle is a right-angled triangle and the angle opposite to the largest side is a right angle.

Know the Terms

- **Pythagorean triplets :** Consider three positive numbers a, b and c with c as the largest number of a, b and c. If $a^2 + b^2 = c^2$, a, b and c are called pythagorean triplets.
  For example : 5, 12 and 13 are Pythagorean triplets as $5^2 + 12^2 = 13^2$

Know the Facts

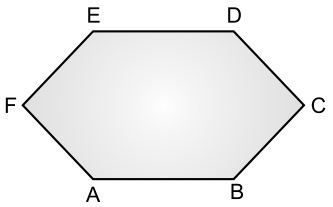
(1) In a right-angled triangle, the hypotenuse is the largest side.
(2) If AB is the largest side of a triangle ABC, then :
(i) $AB^2 = AC^2 + BC^2$
- ABC is a right-angled triangle with AB as hypotenuse and $\angle ACB = 90°$
  (ii) $AB^2 > AC^2 + BC^2$
- ABC is an obtuse-angled triangle with angle ACB greater than 90°.
  (iii) $AB^2 < AC^2 + BC^2$
- ABC is an acute-angled triangle.

CHAPTER-10
RECTILINEAR FIGURES
[Quadrilaterals : Parallelogram, Rectangle, Rhombus, Square and Trapezium]

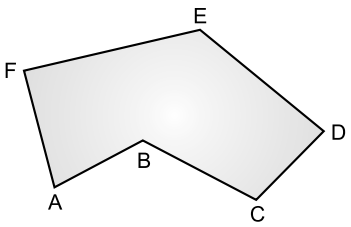
TOPIC-1
Polygons

Revision Notes

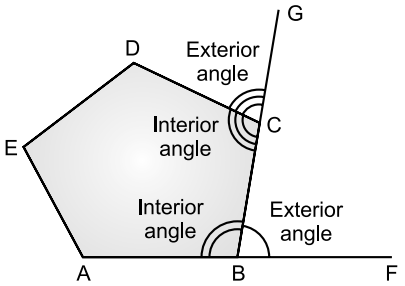
- Rectilinear means along a straight line or in a straight line or forming a straight line.
- A plane figure whose boundaries and line segments is called a rectilinear figure.
- A polygon is a simple closed rectilinear figure *i.e.*, a polygon is a simple closed plane figure, bounded by at least three segments.
- The line segments are called the sides.
- The points of intersection of consecutive sides are called its vertices.
- An angle formed by two consecutive sides of a polygon is called an interior angle or simple angle.
- Line segments joining any two non-consecutive. Vertices of a polygon is called its diagonals.
- **Convex Polygon :** It all the interior angles of a polygon are less then 180°, then the polygon is called a convex polygon.



- **Concave Polygon :** If one or more interior angles of a polygon is greater then 180°, *i.e.*, reflex, then the polygon is called concave polygon.



- If the sides of a polygon are produced, then angles it makes with the next sides is called the exterior angles. In figure, $\angle CBF$ or $\angle DCG$ etc are exterior angle.



- The corresponding to each interior angle, there is an exterior angle. Also, as an exterior angle and its adjacent interior angle make a line. So, we have :

$$\text{an exterior angle} + \text{adjacent interior angle} = 180°$$

- Regular Polygon : A polygon is called regular polygon. If all its sides have equal length and all its angles have equal size.

  In a regular polygon, all its exterior angle are also equal in size.
- Sum of interior angles of an (n) sided polygon (whether it is regular or not) = $(2n - 4) \times$ right angles
- Sum of exterior angles of an (n) sided polygon (whether it is regular or not) = 360°
- Each interior angle of a regular polygon

$$= \frac{(2n-4)\times \text{ right angles}}{4}$$

$$= \frac{(2n-4)\times 90°}{n}$$

- Each exterior angles of a regular polygon

$$= \frac{4\times \text{right angles}}{n}$$

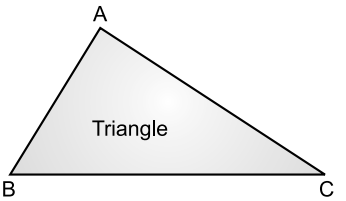
$$= \frac{360°}{n}$$

- At each vertex of every polygon, Exterior angle + Interior angle = 180°
- If each exterior angle of a regular polygon is $x°$, the number of sides in it = $\frac{360°}{x}$
- Greater the number of sides in a regular polygon greater is the value of its each interior angle and smaller is the value of its each exterior angle.

Know the Terms

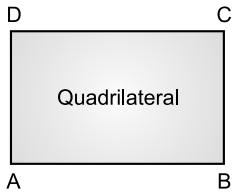
Triangles

- A simple closed plane figure, bounded by three line segments is called a triangle.



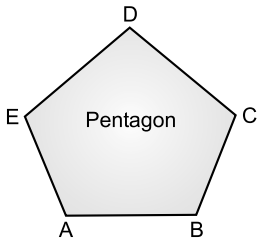
Quadrilateral

- A simple closed plane figure, bounded by four line segments is called a quadrilateral.



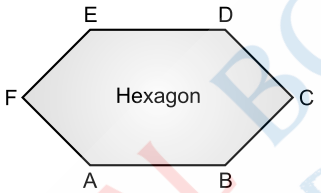
Pentagon

- A simple closed plane figure, bounded by five line segments is called a pentagon.



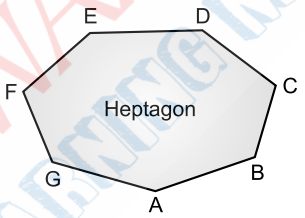
Hexagon

- A simple closed plane figure, bounded by six line segments is called a hexagon.



Heptagon

- A simple closed plane figure, bounded by seven line segments is called a heptagon.



TOPIC-2
Quadrilaterals and its properties

Revision Notes

- A simple closed plane figure bounded by four line segments is called a quadrilateral.

  It has :

  Four sides— AB, BC, CD and DA

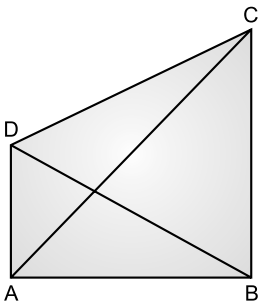
  Four (interior) angles — $\angle A$, $\angle D$, $\angle C$ and $\angle D$.

  Four vertices — A, B, C and D

  Two diagonals— AC and BD

  Pairs of adjacent sides—(AB, BC), (BC, CD), (CD, DA) and (DA, AB)

  Pairs of adjacent angles—($\angle A$, $\angle B$), ($\angle B$, $\angle C$) ($\angle C$, $\angle D$), and ($\angle D$, $\angle A$)

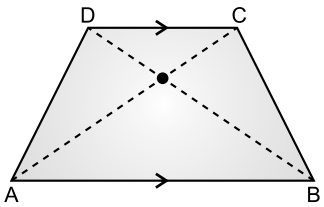


- Sum of (interior) angles of a quadrilateral is 360°.

  i.e. $\angle A + \angle B + \angle C + \angle D = 360°$

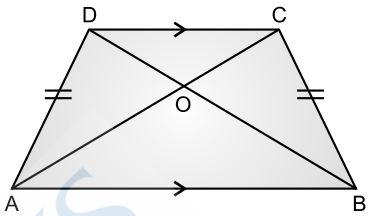
➢ **Types of quadrilaterals**

(i) Trapezium : A quadrilateral in which one pair of opposite sides if parallel but the other pair of opposite sides is non-parallel, is called a trapezium.



In figure, $AB \parallel DC$ where as BC and AD are non-parallel sides.

In trapezium, diagonals are not equal and diagonals are not bisect each other. If non-parallel sides AD and BC of the trapezium $ABCD$ are equal, then it is called an isosceles trapezium.



In this case :

(i) $\angle C = \angle D$ and $\angle A = \angle B$

(ii) diagonal, $AC = BD$

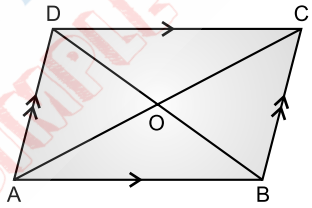
(iii) If diagonal intersect at point O, then $OA = OB$ and $OC = OD$.

(iv) $\angle A + \angle D = 180°$ or $\angle A + \angle C = 180°$
and $\angle B + \angle C = 180°$ or $\angle B + \angle D = 180°$

(v) diagonal are not bisect each other.

2. Parallelogram : A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram. It is usually written as '||gm'.

In this case :



(i) opposite side are parallel *i.e.* $AB \parallel DC$ and $BC \parallel AD$.

(ii) opposite sides are equal *i.e.* $AB = DC$ and $BC = AD$.

(iii) opposite angles are equal *i.e.* $\angle A = \angle C$ and $\angle B = \angle D$.

(iv) consecutive angles are supplementary.

i.e. $\angle A + \angle B = 180°, \quad \angle B + \angle C = 180°$
$\angle C + \angle D = 180°, \quad \angle A + \angle D = 180°$

(v) diagonals bisect each other.

i.e. $OA = OC = \frac{1}{2} AC$ and $OB = OD = \frac{1}{2} BD$

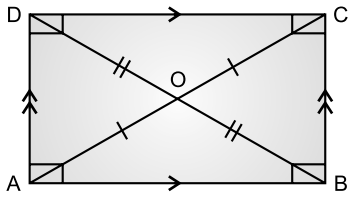
(vi) each diagonal divides the parallelogram into two congruent triangles.

i.e. $\Delta ABC \cong \Delta CDA$ and $\Delta ABD \cong \Delta CDB$

(vii) diagonals divide the parallelogram into four triangles of equal area.

i.e. $\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA = \frac{1}{4}$ (Parallelogram $ABCD$)

3. Rectangle : If one of the angles of a parallelogram is a right angle, then it is called a rectangle.



In this case :

(i) each angle is a right angle *i.e.* $\angle A = \angle B = \angle C = \angle D = 90°$

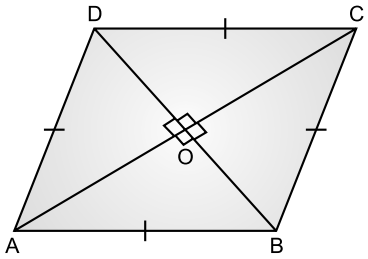
(ii) diagonal are equal *i.e.* $AC = BD$

(iii) diagonal bisect each other.

i.e. $OA = OC$ and $OB = OD$.

4. Rhombus : It all four sides of a parallelogram are equal, then it is called a rhombus.

In this case :



(i) all four sides are equal *i.e.* $AB = BC = CD = DA$

(ii) diagonal bisect each other. *i.e.* $OA = OC$ and $OB = OD$.

(iii) diagonal intersect each other at right angle *i.e.*

$$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$$

(iv) each diagonal bisects angles at the vertices *i.e.* diagonal AC bisects angle A and C and diagonal BD bisects angle B and D.

i.e. $\angle BAC = \angle DAC$, $\angle ABD = \angle CBD$, $\angle BCA = \angle DCA$ and $\angle CDB = \angle ADB$

5. Square : If two adjacent sides of a rectangle are equal, then it is called a square. 'or' If one angle of a rhombus is a right angle, then it is called a square.

In this case :

(i) all four sides are equal *i.e.* $AB = BC = CD = DA$

(ii) each angle is right angle *i.e.* $\angle A = \angle B = \angle C = \angle D = 90°$

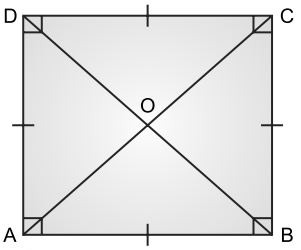
(iii) diagonal are equal. *i.e.* $AC = BD$

(iv) diagonals bisect each other at right angle *i.e.* $OA = OC$ and $OB = OD$

and $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

(v) each diagonal bisects angles at the vertices. *i.e.*

$\angle BAC = \angle DAC = \angle ABD = \angle CBD = \angle BCA = \angle DCA = \angle CDB = \angle ADB = 45°$



6. **Kite :** A quadrilateral in which two pairs at adjacent sides are equal is called a kite (or diamond).

In this case :

(i) Two pairs of adjacent sides are equal. *i.e.* $AB = AD$ and $BC = DC$

(ii) angle between unequal sides are equal.

i.e. $\angle ADC = \angle ABC$

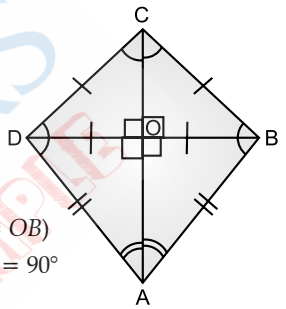
(iii) one diagonal bisects a pair of opposite angle which are unequal.

i.e. $\angle DCA = \angle BCA$

and $\angle DAC = \angle BAC$ $(\because OD = OB)$

(iv) diagonals are perpendicular to each other *i.e.* $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

(v) $OD = OB$



- **Some other properties of quadrilateral**

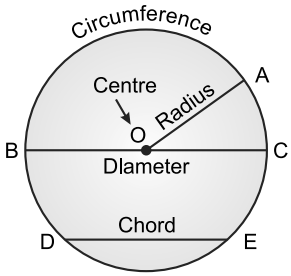
(i) If one pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.
(ii) If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
(iii) If the opposite sides of a quadrilateral are equal, the quadrilateral is parallelogram.
(iv) If the opposite angles of a quadrilateral are equal, the quadrilateral is parallelogram.
(v) If the diagonals of a parallelogram are equal, the parallelogram is a rectangle.
(vi) If the diagonals of a parallelogram are equal and intersect each other at right angle, the parallelogram is a square.
(vii) If the diagonals of a rectangle intersect each other at right angles, the rectangle is a square.
(viii) A diagonals of a square makes an angle of 45° with the sides of the square.
(ix) A diagonals of a rhombus bisect the angles at the vertices.
(x) The diagonals of a rhombus intersect each other at right angles.
(xi) If the diagonals of a parallelogram intersect each other at right angles, the parallelogram is a rhombus.
(xii) If the diagonals of a rhombus are equal, the rhombus is a square.

CHAPTER-11
CIRCLES

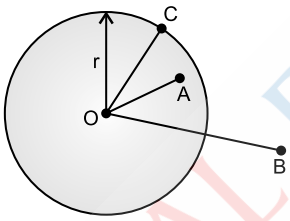
TOPIC-1
Circle and its related terms with chord properties

Revision Notes

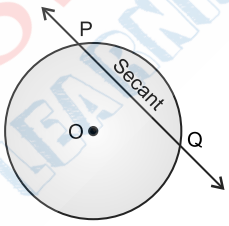
- **Circle :** The collection of all those points in a plane, each of which is at a constant distance from a fixed point in that plane is called a circle.
- **Centre :** The fixed point 'O' is called the 'centre' of the circle.



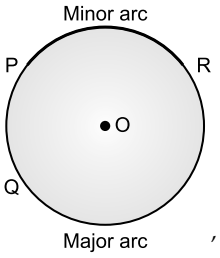
- **Radius :** The fixed distance between the centre and a point on the circle is called the 'radius' of the circle.
- **Circumference :** The perimeter (or length) of the circle is called the circumference of the circle.
- **Chord :** A line segment joining any two points of a circle is called a chord of the circle.
- **Diameter :** A chord which passes through the centre of the circle is called the diameter of the circle.
  In other words, The longest chord of the circle is called the diameter of the circle.
- Length of a diameter = 2 × radius.
- **Interior point :** A point is called an interior point, if its distance from the centre of the circle is less then the radius of the circle, *i.e.* or $OA < r$.



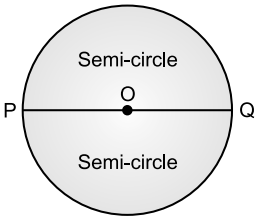
- **Exterior point :** A point is called an exterior point, if its distance from the centre of the circle is greater than the radius of the circle, *i.e.* or $OB > r$.
- **Point of the circle :** A point is called on the circumference of the circle, it its distance from the centre of the circle is equal to the radius of the circle. *i.e.* $OC = r$.
- **Secant of a circle :** A line which meets a circle in two points is called a secant of the circle. In figure, PQ is a secant of the circle with centre O.



- **Arc of a circle :** A part of the circumference of a circle is called its arc. The arc of a circle is denoted by the symbol '$\frown$'

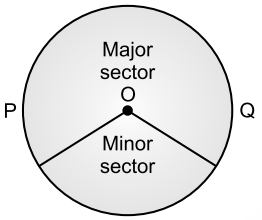


- **Minor arc :** An arc which is less than one-half of the whole arc of the circle (arc $\widehat{PSQ}$) is called a minor arc of the circle.
- **Major arc :** An arc which is greater than one-half of the whole arc (arc $\widehat{PQR}$) is called a major arc of the circle.
- **Semi circle :** When both the arcs are equal, then each arc is called a semi-circle.

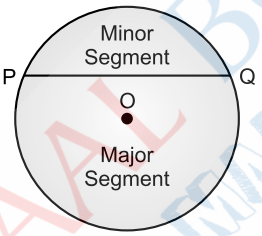


The diameter of the circle is divide into two equal part of the circle. Each part is called a semi-circle.

- **Sector of a circle :** The region bounded by an arc and two radii, joining the centre to the end points of the arc, is called a sector.

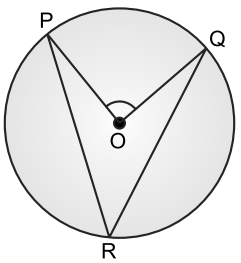


- **Minor sector :** The part containing the minor arc is called minor sector.
- **Major sector :** The part containing the major arc is called major sector.
- **Segment of a circle :** The part of the circle, bounded by an arc and a chord, is called a segment.

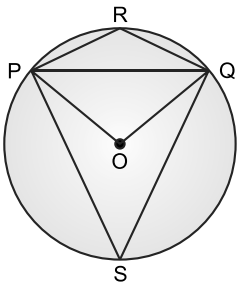


- **Minor segment :** The part containing the minor arc is called a minor segment.
- **Major segment :** The part containing the major arc is called a major segment.
- **Angle subtended by an arc :** The angle subtended by the two bounding radii of an arc at the centre of the circle is called the angle subtended by the arc.

  In figure, $\angle POQ$ is the angle subtended by the major arc PQ and reflex $\angle POQ$ is the angle subtended by the major arc PQ of a circle with centre O. $\angle PRQ$ is called the angle subtended by the minor arc PQ at the point R.

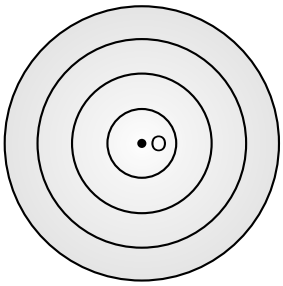


- **Angle subtended by a chord :** Let PQ be the chord of a circle with centre O. Then, $\angle POQ$ is the angle subtended by the chord PQ at the centre O.

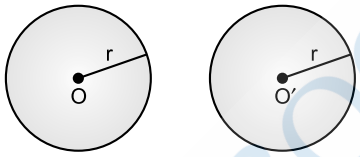


$\angle PRQ$ and $\angle PSQ$ are the angles subtended by the chord PQ at the point R and S respectively on the minor arc and major arc.

- **Concentric circles :** Two or more circles are called concentric circles if and only if they have same centre but different radii.

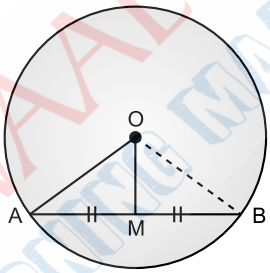


- **Equal (or congruent) circle :** Two or more circle are called equal (or congruent) circle if and only if they have same radius.



- **Chord properties :**

1. A straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is at right angles on the chord.

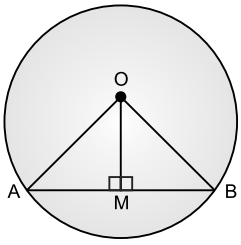


Given : A circle with centre O and OM bisects the chord AB.
To prove : $OM \perp AB$
Construction : Join OA and OB
Proof : In ΔAMO and ΔBMO

	$AM = BM$	(OM bisects AB)
	$OM = OM$	(Common)
	$OA = OB$	(radii of same circle)
$\therefore$	$\Delta AMO \cong \Delta BMO$	(by SSS criteria)
$\therefore$	$\angle AMO = \angle BMO$	(by CPCT) ...(i)
But	$\angle AMO + \angle BMO = 180°$	($\because$ AMB is a straight line)
	$\angle AMO + \angle AMO = 180°$	[from (i)]
	$2\angle AMO = 180°$	
	$\angle AMO = 90° = \angle BMO$	

Hence, $OM \perp AB$.

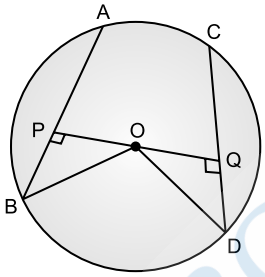
2. The perpendicular to a chord, from the centre of the circle, bisects the chord.



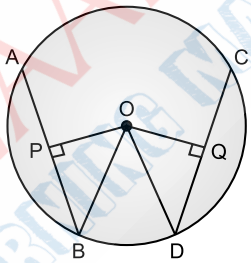
Given : A circle with centre O and OM perpendicular to the chord AB.
To prove : $AM = BM$
Construction : Join OA and OB.
Proof : In ΔAMO and ΔBMO

$$\angle AMO = \angle BMO = 90^\circ \quad (\because OM \perp AB)$$
$$OA = OB \quad \text{(radii of same circle)}$$
$$OM = OM \quad \text{(common)}$$
$$\therefore \quad \Delta AMO \cong \Delta BMO \quad \text{(by RHS Criteria)}$$
$$\therefore \quad AM = BM \quad \text{(by CPCT)}$$

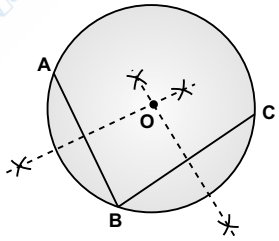
3. Equal chords of a circle are equidistant from the centre. Let AB and CD are two equal chords. OP and OQ are perpendicular distance from the centre of the circle to the chord AB and CD respectively.
then $OP = OQ$



4. Chords of a circle, equidistant from the centre of the circle are equal. Let AB and CD are two chords of the circle. OP and OQ are the perpendicular distance from the centre of the circle to the chord AB and CD respectively
Such that $OP = OQ$
then, $AB = CD$

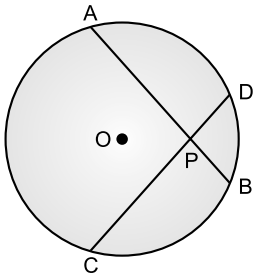


5. There is one and only one circle, which passes through three given points not in a straight line.



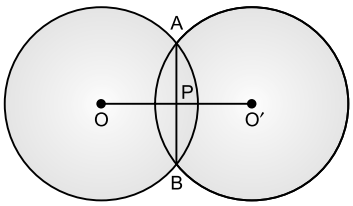
- The perpendicular bisector of every chord of a circle always passes through its centre.
- The perpendicular bisectors of two (non-parallel) chords of a circle intersect at the centre of the circle.
- If the two equal chords AB and CD meet at point P, inside the circle, then also we have :

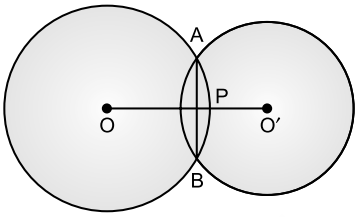
$$PA = PC \text{ and } PB = PD$$



In other ways, if two equal chords of a circle intersect each other at a point, inside or outside the circle, the corresponding parts of the chords are equal.

- Whenever two circles, equal or unequal, intersect each other, the line joining their centres bisect the common chord.



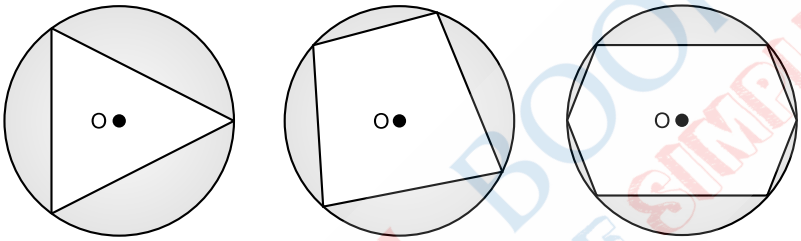


OO' is perpendicular bisector of AB

i.e. $PA = PB$ and $\angle OPA = \angle OPB = 90°$

Know the Terms

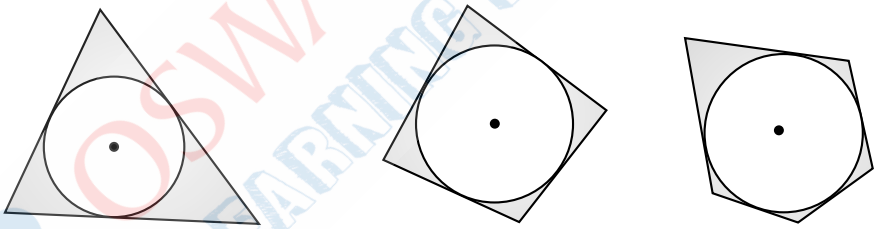
- **Circumscribed circle :** A circle that passes through all the vertices of a polygon is called a circumscribed circle.



The centre of circumscribed circle is called circumcentre.

The polygon whose all the vertices are on the circumference of the circle is called inscribed polygon.

- **Inscribed circle :**



A circle that touches all the sides of a polygon is called an inscribed circle of the polygon.

The centre of inscribed circle is called incentre.

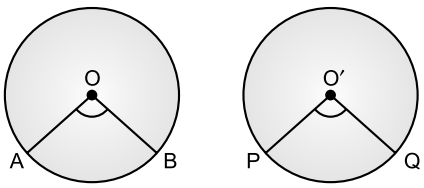
The polygon whose all the sides touches by the circumference of the circle is called circumscribed polygon.

TOPIC-2
Properties of arc and chord

Revision Notes

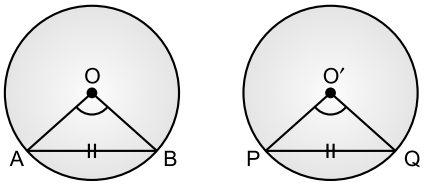
- **Axiom and equal arcs**

  In equal circles (or in the same circle), if two arcs subtend equal angles at the centres (or centre) then they are equal.



Conversely, In equal circles (or in the same circle). If two arcs are equal they subtend equal angles at the centre (or centre)

- In equal circles (or in the same circle), equal chords cut off equal arcs.



Given : AB and PQ are chords of two equal circles with centre O and O' respectively, and $AB = PQ$

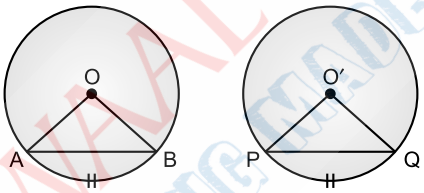
To prove : Arc AB = Arc PQ

Construction : Join OA, OB, $O'P$ and $O'Q$

Proof : In ΔOAB and $O'PQ$

	$OA = O'P$	(radii of equal circles)
	$OB = O'Q$	(radii of equal circles)
	$AB = PQ$	(given)
$\therefore$	$\Delta OAB \cong \Delta O'PQ$	(by SSS criteria)
$\therefore$	$\angle AOB = \angle PO'Q$	(by CPCT)
$\therefore$	Arc AB = Arc PQ	(axioms of equal arcs)

- In equal circles (or in the same circles), if two arcs are equal then their chords are equal.



Given : Arc AB and arc PQ of two equal circles with centres O and O' respectively, and arc AB = arc PQ

To prove : $AB = PQ$

Construction : Join OA, OB, $O'P$ and $O'Q$

Proof :

	Arc AB = Arc PQ	(given)
$\therefore$	$\angle AOB = \angle PO'Q$	(Axioms of equal arcs) ...(i)

In ΔAOB and $\Delta PO'Q$

	$OA = O'P$	(radii of equal circles)
	$OB = O'Q$	
	$\Delta AOB = \Delta PO'Q$	(by SAS criteria)
$\therefore$	$AB = PQ$	(by CPCT)

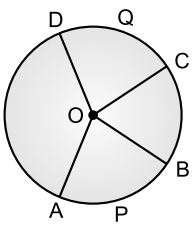
- Equal chords of the same circle (or of equal circles) subtend equal angels at the centre (or centres) of the circle (or circles).
- In equal circles (or on the same circle), equal angles at the centres (or centre) make equal chords.
- In a circle with centre O, for arc APB and arc CQD, if

  (i) arc APB **= arc** $CQD \Rightarrow \angle AOB = \angle COD$

  (ii) arc APB **= 2 × arc** $CQD \Rightarrow \angle AOB = 2 \times \angle COD$

  (iii) arc APB **: arc** CQD **= 5 : 7**

  $\Rightarrow \angle AOB : \angle COD = 5 : 7$.



CHAPTER-12
MENSURATION (Plane Figure)

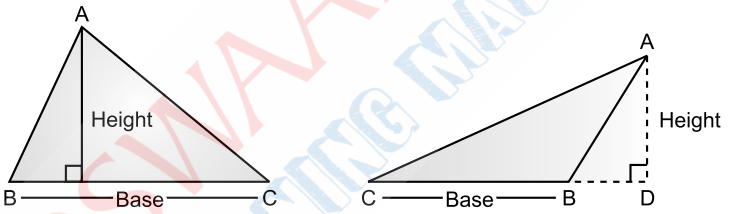
TOPIC-1
Area and Perimeter of Triangles

Revision Notes

- **Perimeter :** The Perimeter of a closed plane figure is the length of its boundary *i.e.*, the sum of lengths of its sides.
- The unit of measurement of perimeter is the unit of length *i.e.*, cm. m, etc.
- **Area :** The area of a closed plane figure is the measurement of the region (surface) enclosed by the boundary (sides).
- The unit of area is square unit. *i.e.*, square centimetre (cm^2), square metre (m^2), square kilometre (km^2) etc.
- A triangle is a closed figure bounded by three line segments.
- **Area of triangle :**

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{corresponding height (altitude)}$$

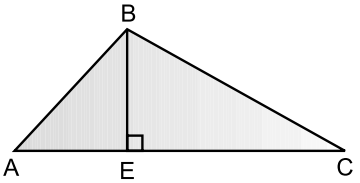
$$= \frac{1}{2} \times BC \times AD$$



- Any side of the triangle can be taken as its base, then the length of perpendicular (altitude) from the vertex opposite to this side is called its corresponding height. Which may be inside the triangle or outside the triangle.
- In this triangle, AC is taken as base and BE is the corresponding height (altitude)

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{corresponding height}$$

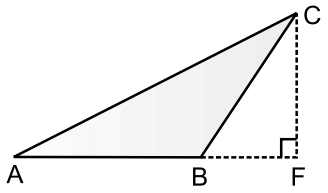
$$= \frac{1}{2} \times AC \times BE$$



- In this triangle, *AB* is taken as base and *CF* is the corresponding height (altitude),

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{corresponding height}$$

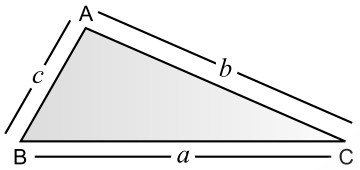
$$= \frac{1}{2} \times AB \times CF$$



- It a, b and c are three sides of a triangle ABC, then

$$\text{Perimeter of triangle} = \text{Sum of all sides}$$

$$\boxed{2S = a + b + c}$$



- **Heron's Formula**

It a, b and c are three sides of a triangle,

Then its $\quad$ Perimeter $(2s) = a + b + c$

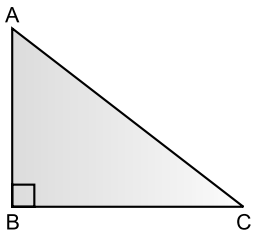
$$\text{Semi-Perimeter } (s) = \frac{a+b+c}{2}$$

$\therefore$ $\quad$ Area of triangle by Heron's $= \sqrt{s(s-a)(s-b)(s-c)}$

- Some special types of triangles

(i) Right-angled triangle :

If ABC is a triangle in which angle B is right angle.



Then,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times BC \times AB$$

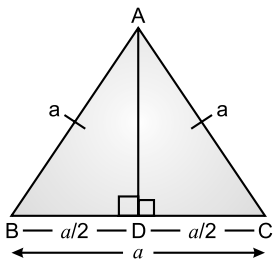
$$= \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times (\text{Product of sides containing right angle})$$

(ii) Equilateral triangle

Let ABC be an equilateral triangle with side a and AD be the perpendicular from A to BC.

We known that, In equilateral triangle, Perpendicular bisect the base.



$\therefore$ $$BD = CD = \frac{a}{2}$$

In right ΔADB,

$$AD^2 = AB^2 - BD^2$$

(by pythagoras theorem)

$$AD^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$AD^2 = \frac{3a^2}{4}$$

$$AD = \frac{\sqrt{3}a}{2}$$

$\therefore$ $$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}$$

$$= \frac{\sqrt{3}a^2}{4}$$

Perimeter of equilateral of triangle = Sum of all sides

$= a + a + a = 3a$

(iii) Isosceles triangle :

Let ABC be an isosceles triangle with $AB = AC = a$ and $BC = b$ and AD be the perpendicular from A to BC. We know that, In isosceles triangle, perpendicular from the common vertex of equal sides to the base, bisects the base.

$\therefore$ $$BD = CD = \frac{b}{2}$$

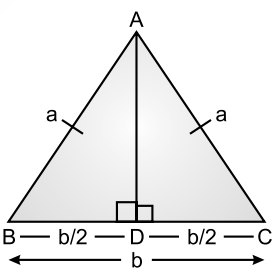
In right ΔADB,

$$AD^2 = AB^2 - BD^2 \quad \text{(by pythagoras theorem)}$$

$$= a^2 - \left(\frac{b}{2}\right)^2$$

$$= a^2 - \frac{b^2}{4}$$

$$AD^2 = \frac{4a^2 - b^2}{4}$$



$\therefore$ $$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times b \times \frac{\sqrt{4a^2 - b^2}}{2}$$

$$= \frac{b}{4} \times \sqrt{4a^2 - b^2}$$

Perimeter of isosceles triangle = sum of all sides

$= a + a + b$

$= 2a + b$

TOPIC-2
Area and Circumference of a Circle

Revision Notes

- The collection of all those points in a plane, each of which is at a constant distance from a fixed point in that plane is called a circle.
- The length of the boundary of circle is called the circumference of circle.
- The ratio of circumference of any circle to its diameter is always constant, and this constant ratio is represented by greek letter π(pie)

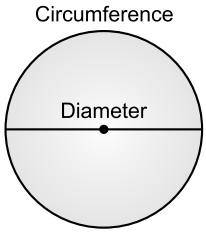
$\therefore \quad \dfrac{\text{Circumference}}{\text{diameter}} = \pi$

$\text{Circumference} = \pi \times \text{diameter}$

$C = \pi \times d \quad (d = \text{diameter} = 2 \times \text{radius})$

$C = \pi \times 2r$

$C = 2\pi r$



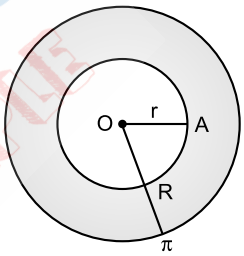
The value of $\pi = \dfrac{22}{7} = 3.14$ (approximately)

- Area of circle $= \pi r^2$, where r is the radius of the circle.

Know the formulas

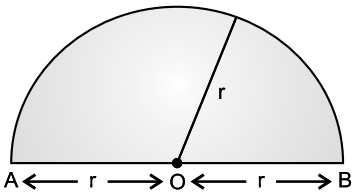
- Let $OA = r$ and $OB = R$ are the radii of smaller and bigger concentric circles. Then

$$\text{Area of ring (shaded region)} = \text{Area of bigger circle} - \text{Area of smaller circle}$$
$$= \pi R^2 - \pi r^2$$
$$= \pi(R^2 - r^2)$$
$$\text{Circumference of smaller circle} = 2\pi r$$
$$\text{Circumference of bigger circle} = 2\pi R$$



- If r is the radius of a circle then,

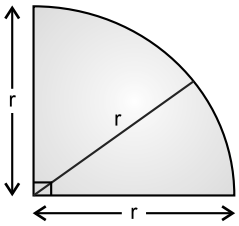
$$\text{Area of semicircle} = \frac{1}{2} \times \text{Area of ccircle}$$
$$= \frac{1}{2} \times \pi r^2$$
$$= \frac{\pi r^2}{2}$$



$$\text{Perimeter of semi circle} = \frac{1}{2} \times 2\pi r + 2r$$
$$= \pi r + 2r$$
$$= (\pi + 2)r$$

- If r is the radius of a circle then

$$\text{Area of the quadrant} = \frac{1}{4} \times \text{Area of circle}$$
$$= \frac{1}{4} \times \pi r^2$$
$$= \frac{\pi r^2}{4}$$

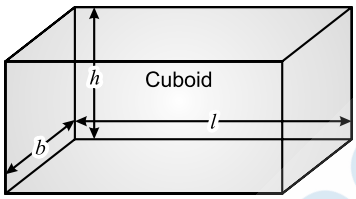


$$\text{Perimeter of the quadrant} = \frac{1}{4} \times 2\pi r + r + r$$
$$= \frac{\pi r}{2} + 2r$$
$$= \left(\frac{\pi}{2} + 2\right)r$$

TOPIC-3
Surface area and volume of 3D-Solids

Revision Notes

- **Solid :** A figure which occupies a portion of space enclosed by plane or curved surfaces is called a solid.
- **Volume :** The measurement of the space enclosed by a solid is called its volume. It is measured in cubic units.
- **Surface area of a solid :** The sum of the areas of the plane or curved surfaces (faces) of a solid is called its total surface area. It is measured in square units.
- **Cuboid :** A rectangular solid which has six faces, each at which is a rectangle, is called a cuboid.



The length, breadth and height of a cuboid are usually denoted by l, b and h respectively.

- A cuboid has 12 edges, 8 vertices and 6 surfaces (faces).
- Surface Area of a cuboid : The sum of areas of all the six faces of a cuboid is called its (total) surface Area.

$$\text{Surface area of cuboid} = 2 \times l \times b + 2 \times b \times h + 2 \times h \times l$$
$$= 2(lb \times bh + hl) \text{ square units}$$

- $$\text{Lateral surface area of a cuboid} = \text{The sum of area of the four walls of a cuboid}$$
$$= 2 \times l \times h + 2 \times b \times h$$
$$= 2(l + b)\, h \text{ sq. units}$$

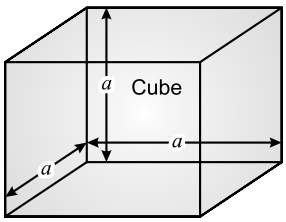
- The space enclosed by a cuboid is called its volume.

$$\text{Volume of a cuboid} = l \times b \times h \text{ cubic units}$$

- The line joining opposite corners of a cuboid is called its diagonal. A cuboid has four diagonals

$$\text{Length of a diagonal} = \sqrt{l^2 + b^2 + h^2} \text{ units.}$$

- It is the length of the longest rod that can be placed in the cuboid.
- **Cube :** A rectangular solid, in which each face is a square, is called a cube.
  Here, length = breadth = height = edge = a (say)
- Surface area of a cube = $6a^2$ sq. units
- Lateral surface of cube = $4a^2$ sq. units
- Volume of cube = a^3 cubic units



- Length of a diagonal = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$ units
- When a body has uniform cross-section, its :

$$\text{Volume} = \text{Area of cross-section} \times \text{length}$$
$$\text{Surface area (excluding cross-section)} = \text{perimeter of cross section} \times \text{length}$$

- If water is flowing through a pipe or a canal, etc, of uniform cross section.

$$\text{the volume of water that flows in unit time} = \text{Area of cross section} \times \text{speed of flow of water}$$

Know the formulas

- The volume of material in a hollow body = Its external volume – Its internal volume
- If the external dimensions (length, breadth and height) of a box are l, b and h and if each side is of thickness x, then the internal dimensions of the

  (i) Closed box are l-2x, b-2x and h-2x

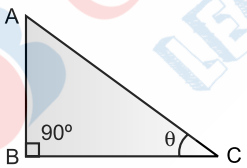
  (ii) open box are l-2x, b-2x and h-x
- Speed = $\frac{\text{Distance}}{\text{time}}$
- x km/h = $x \times \frac{5}{18}$ m/s
- x m/sec = $x \times \frac{18}{5}$ km/h

CHAPTER-13
TRIGONOMETRY

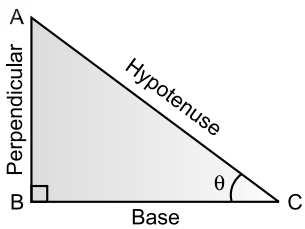
TOPIC-1
Trigonometric Ratios

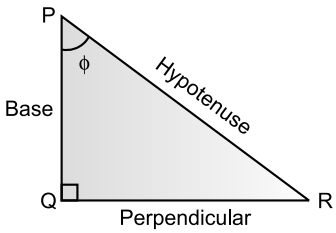
Revision Notes

- The wood 'Trigonometry' is derived from Greek words 'tri' (means three), 'gon' (means sides) and 'metron' (means measure). In fact, trigonometry is the study of relationship between the sides and the angles of a triangle.
- The ratio between the lengths of a pair of two sides of a right-angled triangle is called a trigonometry ratio.
- **Right-angled triangle :** A triangle whose one angle is exactly a right angle (90°) is called a right-angled triangle.

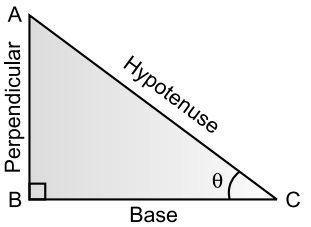


- **Base :** A side adjacent to given acute angle of a right angled triangle is called a base. Here side BC or side PQ is a base.





- **Perpendicular :** A side opposite to given acute angle of a right-angled triangle is called a perpendicular or altitude. Here side AB or side QR is a perpendicular or altitude.
- **Hypotenuse :** A side opposite to a right angle is called a hypotenuse. In other words,' Longest side of a right angled triangle is called a hypotenuse. Here side AC or side PR is a hypotenuse.
- In a right-angled triangle ABC, for acute angle Q, the six trigonometrical ratio can be defined as :



(i) Since (sin) is defined as the ratio between the lengths of perpendicular (altitude/height) and hypotenuse.

$$\therefore \quad \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

(ii) Cosine (cos) is defined as the ratio between the lengths of base and hypotenuse.

$$\therefore \quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

(iii) tangent (tan) is defined as the ratio between the lengths of perpendicular and base.

$$\therefore \quad \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC}$$

(iv) Cotangent (cot) is defined as the ratio between the lengths of base and perpendicular.

$$\therefore \quad \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB}$$

(v) Secant (sec) is defined as the ratio between the lengths of hypotenuse and base.

$$\therefore \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BC}$$

(vi) cosecant (cosec) is defined as the ratio between the lengths of hypotenuse and perpendicular.

$$\therefore \quad \text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{AB}$$

- Each trigonometrical ratio is a real number and has no unit.
- **Reciprocal relations**

(i) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$ and $\text{cosec}\,\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

So, $\sin\theta$ and $\text{cosec}\,\theta$ are reciprocal of each other

$\therefore \sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ and $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

and $\sin\theta \times \text{cosec}\,\theta = 1$

(ii) Similarly, $\cos\theta = \dfrac{1}{\sec\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cos\theta \times \sec\theta = 1$

(iii) $\tan\theta = \dfrac{1}{\cot\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$ and $\tan\theta \times \cot\theta = 1$

- **Quotient-rections**

(i) Since,

$$\frac{\sin\theta}{\cos\theta} = \frac{\dfrac{\text{Perpendicular}}{\text{Hypotenuse}}}{\dfrac{\text{Base}}{\text{Hypotenuse}}} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$\therefore$ $$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{...(i)}$$

(ii) $$\frac{\cos\theta}{\sin\theta} = \frac{\frac{\text{Base}}{\text{Hypotenuse}}}{\frac{\text{Perpendicular}}{\text{Hypotenuse}}}$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

$\therefore$ $$\cot\theta = \frac{\cos\theta}{\sin\theta} \quad \text{...(ii)}$$

Know the formulas

➢ In triangle ABC, angle B is right angle.

Now, $\angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\angle A + 90° + \angle C = 180°$

$\angle A + \angle C = 90°$

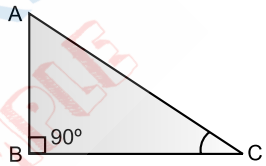


Fig.

i.e., $\angle A < 90°$ and $\angle C < 90°$

Therefore, $\angle A$ and $\angle C$ are acute angle.

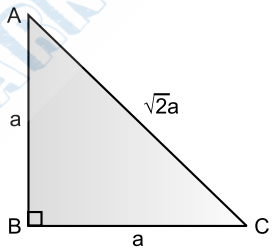
TOPIC-2
Trigonometric Ratios of Standard Angles

Revision Notes

➢ **Trigonometrical Ratio of 45°**

Let ABC be an isosceles triangle right-angled at B with side $AB = AC = a$ (say) then, $\angle C = \angle A = 45°$

(Angle opposite to equal side)



In right triangle ABC.

$$AC^2 = AB^2 + BC^2 \quad \text{(by pythagoras theorem)}$$
$$AC^2 = a^2 + a^2 = 2a^2$$
$$AC = \sqrt{2}a$$

$\therefore$ $$\sin 45° = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{AB}{BC} = \frac{a}{a} = 1$$

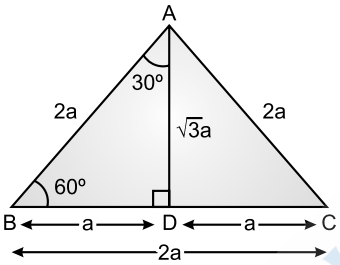
$$\cot 45° = \frac{BC}{AB} = \frac{a}{a} = 1$$

$$\sec 45° = \frac{AC}{BC} = \frac{\sqrt{2}a}{a} = \sqrt{2}$$

$$\text{cosec } 45° = \frac{AC}{AB} = \frac{\sqrt{2}a}{a} = \sqrt{2}$$

➢ **Trigonometrical Ratios of 30° and 60°**

Let ABC de an equilateral triangle with each side $= 2a$ (say), and let $AD \perp BC$.



In equilateral triangle, perpendicular bisects the base

$\therefore$ $BD = CD = a$

Since each angle of an equilateral triangle is 60°.

$\therefore$ $\angle ABD = 60°$

$\therefore$ In right ΔADB, $\angle BAD = 30°$ (by angle sum property of triangle)

In right ΔADB,

$$AD^2 = AB^2 - BD^2 \quad \text{(by pythagoras theorem)}$$
$$= (2a)^2 - (a)^2$$
$$AD^2 = 3a^2$$
$$AD = \sqrt{3}a$$

$\therefore$

$$\sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$\cot 30° = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

$$\sec 30° = \frac{AB}{AD} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$$

$$\text{cosec } 30° = \frac{AB}{BD} = \frac{2a}{a} = 2$$

Now,

$$\sin 60° = \frac{AB}{BD} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60° = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

$$\cot 60° = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$\sec 60° = \frac{AB}{BD} = \frac{2a}{a} = 2$$

$$\text{cosec } 60° = \frac{AB}{AD} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$$

➢ **Trigonometrical-Ratios of 0° and 90°**

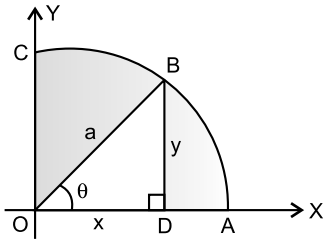
Consider the arc ABC of a circle with centre O and radius $= a$

Let $B(x, y)$ be any point on this arc.

Draw BD perpendicular to OX, then

$OD = x$, $BD = y =$ and $OB = a$.

Let $\angle BOD = \theta$. then



$$\sin\theta = \frac{y}{a}, \qquad \cos\theta = \frac{x}{a}, \qquad \tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}, \qquad \sec\theta = \frac{a}{x}, \qquad \text{cosec}\,\theta = \frac{a}{y}$$

(i) When $\theta = 0$, B coincides with A so that $x = a$ and $y = 0$

$\therefore$ $\sin 0° = \frac{0}{a} = 0$ $\qquad$ $\cot 0° = \frac{a}{0}$, which is not define

$\cos 0° = \frac{a}{a} = 1$ $\qquad$ $\sec 0° = \frac{a}{a} = 1$

$\tan = \frac{0}{a} = 0$ $\qquad$ $\text{cosec}\,0° = \frac{a}{0}$, which is not define

(ii) When $\theta = 90°$, B coincides which C so that $x = 0$, and $y = a$

$\therefore$ $\sin 90° = \frac{a}{a} = 1$ $\qquad$ $\cot 90° = \frac{0}{a} = 0$

$\cos 90° = \frac{0}{a} = 0$ $\qquad$ $\sec 90° = \frac{a}{0}$, which is not define

$\tan 90° = \frac{a}{0}$, which is not define $\qquad$ $\text{cosec}\,90° = \frac{a}{a} = 1$

➢ Trigonometric table for standard angles of 0°, 30°, 45°, 60° and 90°

Angle θ → Ratio ↓	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

➢ We can be seen from the above table, that as the angle increases from 0° to 90°,

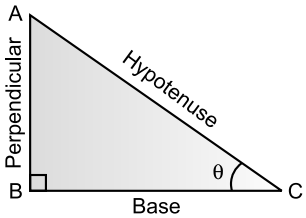
(i) Value of sin increases from 0 to 1

(ii) Value of cos decreases from 1 to 0

(iii) Value of tan increase from 0 to ∞ and so on.

➢ **Square relations :**

From the right-angled ΔABC, by pythagoras theorem, we get



$$AB^2 + BC^2 = AC^2 \qquad \text{...(i)}$$

(i) Dividing both sides of eqn. (i) by AC^2, we get

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

or $$\boxed{\sin^2\theta = 1 - \cos^2\theta}$$

or $$\boxed{\cos^2\theta = 1 - \sin^2\theta}$$

(ii) Dividing both sides of eqn. (i) by BC^2, we get

$$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

or $$\boxed{\tan^2\theta = \sec^2\theta - 1}$$

or $$\boxed{\sec^2\theta - \tan^2\theta = 1}$$

(iii) Dividing bot sides of eqn. (i) by AB^2, we get

$$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

$$\boxed{1 + \cot^2\theta = \text{cosec}^2\theta}$$

or $$\boxed{\cot^2\theta = \text{cosec}^2\theta - 1}$$

or $$\boxed{\text{cosec}^2\theta - \cot^2\theta = 1}$$

TOPIC-3
Solution of Right Triangles

Revision Notes

- To solve a right angled triangle means, to find the values of remaining angles and remaining sides, when :
  (i) One side and one acute angle are given.
  (ii) Two side of the triangle are given.
  In general, when one side and one acute angle of a right-angled triangle are given. We take

$$\frac{\text{unknow side}}{\text{known side}} = \text{corresponding trigonometrical Ratio of the given angle}$$

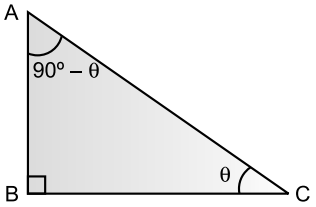
TOPIC-4
Complementary Angles

Revision Notes

- **Complementary angles :** Two angles are called complementary if the sum of their measure is 90°
  Ex. (i) 70° and 20° are complementary as 70° + 20° = 90°
  (ii) 40° and 50° are complementary as 40° + 50° = 90°
  (iii) $x°$ and $(90° - x°)$ are complementary as $x + 90° - x = 90°$
- Let ABC be a right angle triangle right angled at B. and $\angle C = \theta$, then

$$\angle A = 180° - \angle B - C$$

$$\angle A = 180° - 90° - \theta$$
$$\angle A = 90° - \theta$$



(1) $\sin(90° - \theta) = \dfrac{BC}{AC} = \cos\theta$

$\sin(90° - \theta) = \cos\theta$

(2) Similarly, $\cos(90° - \theta) = \sin\theta$

(3) $\tan(90° - \theta) = \cot\theta$

(4) $\cot(90° - \theta) = \tan\theta$

(5) $\sec(90° - \theta) = \text{cosec}\,\theta$

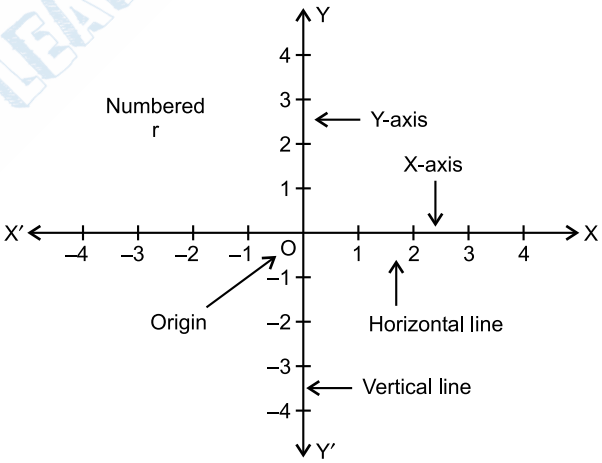
(6) $\text{cosec}(90° - \theta) = \sec\theta$

CHAPTER-14
CO-ORDINATE GEOMETRY

TOPIC-1
Representation of Co-cordinates in the Cartesian Plane

Revision Notes

- Co-ordinate geometry is the study of geometrically representing order pair of numbers.
- **Ordered Pair :** An ordered pair is a pair of objects taken in a specific order.
- An ordered pair is written as (a, b) where a and b are called component.
- The ordered pair (a, b) and (b, a) and different unless $a = b$.
- The two components of an ordered pair may be equal.
- Two ordered pairs (a, b) and (c, d) are called equal, written as $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.
- **Cartesian plane :** When two numbered lines perpendicular to each other (Usually horizontal and vertical) are placed together so that the two origins (the points corresponding to zero) coincide then the resulting configuration is called a cartesian place or co-ordinate plane.



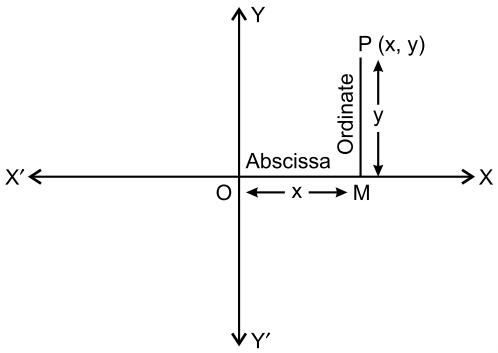
(i) The horizontal number line XOX' is called the X-axis.

(ii) The vertical number line YOY' is called the Y-axis.

(iii) XOX' and YOY' taken together are called coordinate axes.

(iv) The point of intersection where both lines are intersect to each other, is called the origin.

- **Co-ordinates of a point :** Let P be any point in the Co-ordinate plane. From P draw $PM \perp XOX$, then

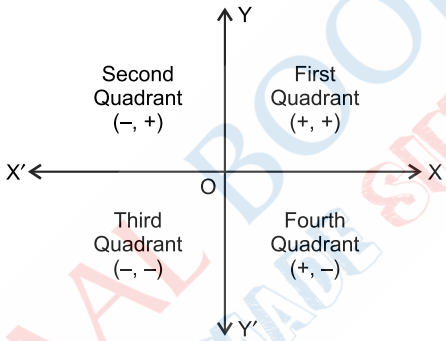


(i) OM is called x-coordinate or abscissa of P and is usually denoted by x.

(ii) PM is called y-coordinate or ordinate of P and usually denoted by y.

(iii) x and y taken together are called cartesian coordinates or simply coordinates of P and are denoted by (x, y)

➢ **Quadrants :** The co-ordinate axes divide a co-ordinate plane into four parts, which are known as quadrants.



(i) XOY is called the first quadrant. In this quadrant abscissa and ordinate are both positive.

(ii) YOX' is called the second quadrant. In this quadrant, abscissa is negative and ordinate is positive.

(iii) $X'OY'$ is called the third quadrant. In this quadrant, abscissa and ordinates are both negative.

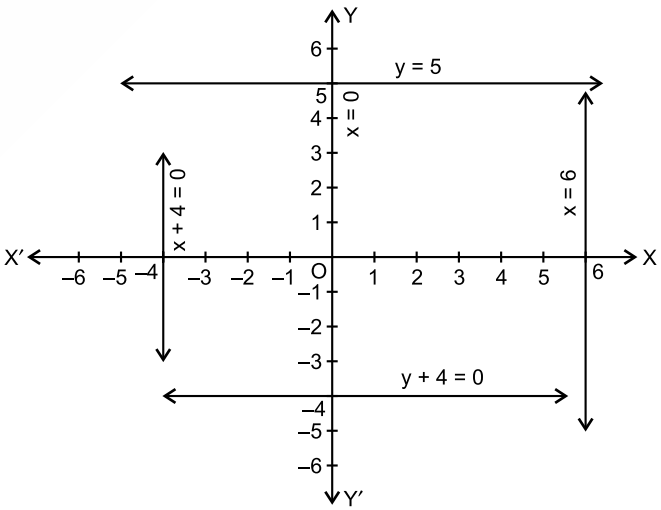
(iv) $Y'OX$ is called the fourth quadrant. In this quadrant, abscissa is positive and ordinate is negative.

➢ **Dependent and Independent Variable :** If there is a formula between two variables, then the subject of the formula is called dependent variable and the other variable is called independent variable.

Ex. : In the given formula $y = 3x + 5$, y is dependent variable and x is independent variable.

By giving different values to the independent variable, we can find the corresponding values of the dependent variable.

➢ **Graph of $x = 0$, $y = 0$, $x = a$ and $y = b$**



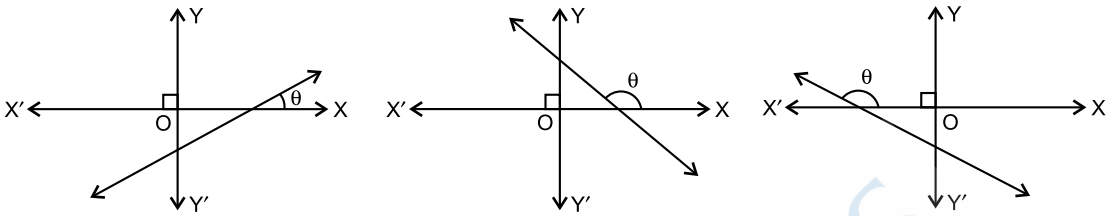
(i) $x = 0$ is the equation of the y-axis as the value of 'x' for every point (x, y) on the y-axis is '0'

(ii) $x = a$ is the equation of a line parallel to y-axis and at a distance of 'a' units from it.

(iii) $y = 0$, is the equation of the x-axis, as the value of 'y' for every point (x, y) on the x-axis is '0'.

(iv) $y = b$ is the equation at a line parallel to x-axis and at a distance of 'b' units from it.

- If the graph of an equation is a straight line, the equation is called a linear equation.
- **Inclination :** The angle which a straight line makes with the positive direction of x-axis (measured in the anti-clockwise direction) is called inclination of the line.



The inclination of a line is usually denoted by θ (theta).

- **Slope (gradient) :** It θ is the inclination of a line, the slope of the line is tan θ and is usually denoted by letter M.

$\therefore$ $$\text{slope} = M = \tan\theta$$

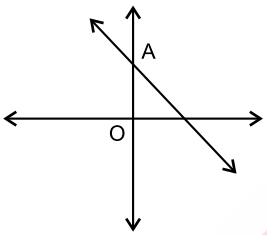
(i) For x-axis and every line parallel to x-axis, the inclination $\theta = 0°$

(ii) For y-axis and every line parallel to y-axis, the inclination $\theta = 90°$

$\therefore$ Slope $(M) = 90° =$ infinite (not defined)

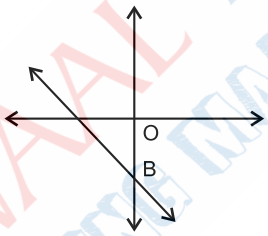
- ***Y*-Intercept :** If a straight line meets y-axis at a point,the distance of this point from the origin is called y-intercept and is usually denoted by C.

(i)



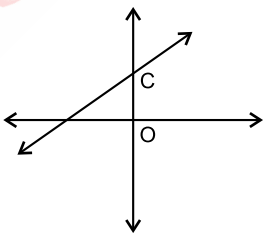
(Y-intercept $C = OA$)

(ii)



($C = OB$)

(iii)



($C = OC$)

(i) For x-axis, y-intercept $= 0$

(ii) For every line parallel to y-axis, y-intercept $= 0$

(iii) Y-intercept is positive, if measured above the origin *i.e.*, OA and OC from (i) and (iii)

(iv) Y-intercept is negative, if measured below the origin *i.e.*, OB from (ii).

- When a straight line makes an angle θ with the positive direction of x-axis and cut the intercept C on y-axis then the equation of the straight line is

$$y = mx + c, \text{ where } m = \tan\theta$$

- **Finding the slope and the *y*-intercept of a given line**

Steps :

(i) Let, $ax + by + c = 0$ is the general form of the equation of a line.

(ii) Make y, the subject of the equation. For this :

$$ax + by + c = 0$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

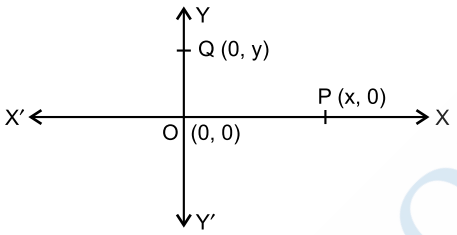
(iii) The coefficient of x is the slope and the constant term is the y-intercept of the given the.

$\therefore$ $$\text{Slope } (m) = -\frac{a}{b}$$

and $$y\text{-intercept } (c) = -\frac{c}{b}$$

Know the fact

- The word 'Ordered' implies that the ordered in which the two elements of the pair occur is meaningful.
- To know that the positive of point in a plane, we need two independent informations abscissa and ordinate of the point.
- The co-ordinates of a point indicate its position with reference to co-ordinate axes.
- In stating the co-ordinates of a point, the abscissa precedes the ordinate. The two are separated by a comma and are enclosed in the bracket ( ). Thus, a point P whose abscissa is 'x' and ordinate is 'y', is written as (x, y) or $P(x, y)$.
- The co-ordinates of the origin O are (0, 0).
- For any point on x-axis, its ordinate is always zero and so the co-ordinates of any point P on x-axis are $(x, 0)$



- For any point on y-axis, its abscissa is always zero and so the co-ordinates any point Q on y-axis are $(0, y)$.
- The x-co-ordinate (abscissa) of a point is positive if it is measured along OX and is negative if it is measured along OX'.
- The y-co-ordinate (ordinate) of a point is positive if it is measured along OY and is negative if it is measured along OY'.

TOPIC-2
Graphical Solution of Simultaneous Linear Equations

Revision Notes

- A equation of the form $ax + by + c = 0$ is called a linear equation in two variables with x and y as the variables and a, b, c as the constants.
- The graph of a linear equation in two variables is always a straight line.
- **Graph of linear equation in two variables :**
- Steps to draw the graph of a linear equation $ax + by + c = 0$
  - **(i)** Rewrite the given equation with y as the subject.
  - **(ii)** Select any three convenient values of x and find the corresponding values of y for each of the selected value of x.
  - **(iii)** Make table of values.
  - **(iv)** Draw the axes on the graph and choose suitable scale.
  - **(v)** Plot the points on the graph paper (co-ordinate plane)
  - **(vi)** Draw a straight line passing through the points plotted on the graph.
- **Graphical solution of a pair of linear equations :**
  In order to solve simultaneous linear equation graphically :
  - **(i)** Draw graph (straight line) for each of the given linear equation.
  - **(ii)** Find the co-ordinates of the point of intersection of the two lines drawn.
  - **(ii)** The co-ordinates of the point of intersection of the two lines will be the common solution of the given equations.
  - **(iv)** Write the values of x and y.

Know the facts

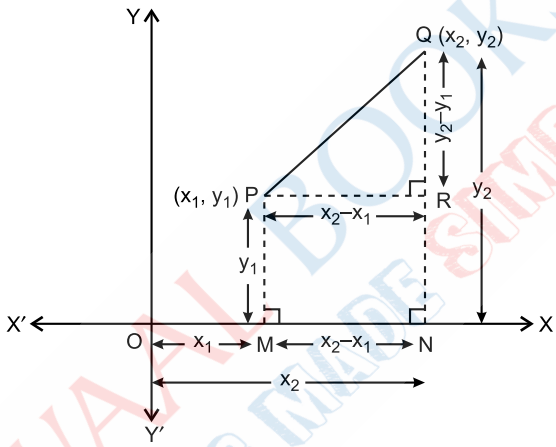
- If the two equations have a unique common solutions, then the equations are called consistent and independent. In this case, the lines have one and only one point in common.
- If the two equations have serval common solutions, then the equations are called consistent and dependent. In this case, the two lines will coincide.

- If the two equations have no common solution, then the equations are called inconsistent. In this case, the two lines will be parallel.

TOPIC-3
Distance Formula

Revision Notes

- **To find the distance between two given points :**
  Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points in the co-ordinate plane.
  Draw PM, QN perpendicular on x-axis and PR perpendicular on NQ
  From the figure,



$$PR = MN = ON - OM = x_2 - x_1$$

$$QR = QN - RN = QN - PM = y_2 - y_1$$

Now, in right triangle PRQ,

$$PQ^2 = PR^2 + QR^2 \qquad \text{(by Pythagoras Theorem)}$$

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore \qquad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We can also write, $$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad [\because (x - y)^2 = (y - x)^2]$$

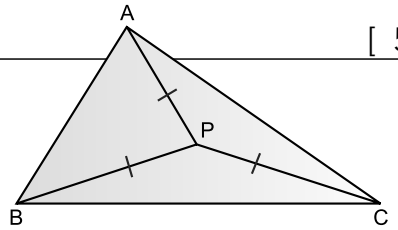
- The distance of the point (x, y) from the origin $(0, 0) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$
- To prove that a quadrilateral is a
  (i) Parallelogram, show that opposite sides are equal.
  (ii) rhombus, show that all sides are equal.
  (iii) rectangle, show that opposite sides are equal and diagonals are also equal.

  OR

  Show that opposite sides are equal, and one angle is 90°.
  (iv) Square, show that all sides are equal and diagonals are also equal.

  OR

  Show that all sides are equal and one angle is 90°.
- **Circumcentre of a triangle**



It is the point which is equidistance from the vertices of the triangle *i.e.*, if P is the circumcentre of the triangle ABC, then $PA = PB = PC$ = circumradius.

- If a circle is drawn with P as a centre and PA or PB or PC as radius, the circle will pass through all the three vertices of the triangle.

CHAPTER-15
STATISTICS AND GRAPH WORK

TOPIC-1
Data and its Representation

Revision Notes

- **Statistics :** Statistics is that branch of mathematics which deals with the collection, presentation, analysis and interpretation of data.
- **Data :** Facts and information collected with a definite purpose are called data.
- **Primary Data :** The information collected by the investigator himself or her self with a definite purpose in his or her mind is called primary data.
- **Secondary data :** The information gathered from a source which already had the information stored is called secondary data.
- **Raw data :** The numerical data recorded in its original form as it is collected by the investigator or received from some source is called raw data.
- **Arrayed data :** If the raw data is arranged in ascending or descending order, then it is called an arrayed data.
- **Variable :** A quantity which can very from one individual to another is called a variable. *e.g.*, height, weight, age etc.
  - **(i) Continuous variable :** A variable which can take any numerical value within a certain range is called a continuous variable. *e.g.*, height, age and weight of people are continuous variables.
  - **(ii) Discrete (Discontinuous variable) :** A variable which is incapable of taking all possible numerical values between two given values is called a discrete variable.
- **Range :** The difference between the maximum and minimum values of a variable is called its range.
- **Variate :** A particular value of a variable is called variate (observation).
- **Frequency :** The number of times a variable (observation) occurs in a given data is called frequency of that variate.
- **Frequency distribution :** A tabular arrangement of given numerical data showing the frequency of different variates is called frequency distribution and table itself is called frequency distribution table.
  There are two types of frequency distributions :
  - **(i) Inclusive distribution :** In an inclusive distribution, the upper limit of one class does no coincide with the lower limit of the next class. Let a mark 'x' belong to the class interval 1 — 10, then it mean that $1 \le x \le 10$.
  - **(ii) Exclusive distribution :** In an exclusive distribution, the upper limit of one class coincides with the lower limit of the next class. Let a mark 'x' belongs to the class interval 10 — 20, then means $10 \le x \le 20$.
- **Class Interval and class limit :** When the presentation of data are divided into groups, then these groups are called classes or class intervals. *e.g.*, 10 — 20, 20 — 30, 1 — 5, 6 — 10 etc are all class interval.
  Fro the class interval 10 — 20, 10 is the lower limit and 20 is the upper limit. Similarly for 1 — 5, 1 is the lower limit and 5 is the upper limit.
- **Converting discrete distribution to continuous distribution :** To convert discrete classes into continuous classes, we require some adjustment.
  $$\text{Adjustment factor} = \frac{\text{lower limit one class} - \text{upper limit of previous class}}{2}$$
  Then, subtract the adjustment factor from all the lower limits and add the adjustment factor all the upper limits.
- **True class limits :** In a continuous distribution, the class limits are called true or actual class limits,

In a discrete distribution, the class limits obtained after adjustment are the true or actual class limits. The actual class limits are also called class boundaries.

- **Stated class limits :** In discrete distribution , the original(given) class limits are called the stated class limits.
- **Class size :** The difference between the actual upper limit and the actual lower limit of a class is called its class size.
  e.g., class size of the class 10 — 20 = 20 — 10 = 10
- **Class Mark :** The mark of a class is the value midway between its actual lower limit and actual upper limit.

$$\text{Class mark} = \frac{\text{Actual lower limit} + \text{Actual upper limit}}{2}$$

e.g., Class mark of the class 10 — 20 $= \frac{10+20}{2} = 15$

- **Cumulative frequency and cumulative frequency table :** The sum of frequencies of all the previous classes and that particular class is called the cumulative frequency of the class.
  The table for cumulative frequencies is called the cumulative frequency table.
- The (numerical) data can be represented graphically.
- **Graphical representation of continuous frequency distribution.**

(i) Histogram : Histogram are used to represent continuous grouped data. Histogram consists of a set of adjacent rectangles. The height of rectangles correspond to the frequency of the class and the breadth of the rectangles corresponds to the class size. There is no gap between different rectangles.

(ii) Frequency Polygon : Continuous frequency distribution can also be represented by frequency polygons.
In a frequency polygon, we plot points on the graph paper representing frequencies of different classes against the corresponding mid-points of class intervals. On joining consecutive points by line segments, we get frequency curve and if the end points of frequency graph are joined to the mid points of immediate lower and upper class intervals outside the range with zero frequency, we get frequency polygon.

A frequency polygon can be draw : (i) Using histogram (ii) without using histogram.

TOPIC-2
Mean and Median

Revision Notes

- **Mean (or arithmetic mean or average) :** Mean of a number of observations is the sum of all values of all the observations divided by the total number of observations.

  The mean of n observations (variates) $x_1, x_2, x_3, \ldots\ldots x_n$ is given by the formula :

$$\text{Mean}(\bar{x}) = \frac{\text{Sum of observation}}{\text{Total number of observation}}$$

$$= \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_4}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i}{n}$$

Where, $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots\ldots + x_n$

- The greek letter Σ (read as sigma) represents the sum.
- If $\bar{x}$ is the mean of n number of observations x_1, x_2, them the sum of deviations of $\bar{x}$ from the observations $x_1, x_2, x_3, \ldots\ldots x_n$, is zero. *i.e* $\Sigma(x - \bar{x}) = 0$
- Let $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$.
  (1) If each observation under consideration is increased by quantity a, then their mean is also increased by the same quantity a. *i.e.,* new mean will be $\bar{x} + a$.

(2) If each observation under consideration is decreased by quantity a, then their mean is also decreased by the same quantity a. *i.e.*, new mean will be $\bar{x} - a$.

(3) If each observation under consideration is multiplied by quantity a, then mean is also multiplied by the same quantity a. *i.e.*, new mean will be $\bar{x} \times a$ or $a\bar{x}$.

(4) If each observation under consideration is divided by quantity a, then mean is also divided by the same quantity a. *i.e.*, new mean will be $\frac{\bar{x}}{a}$

Median : Median is the central value (or middle observation) of a statistical data if it is arranged in ascending or descending order.

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots\ldots x_n$. arranged in ascending or descending order, then

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term, if } n \text{ is odd}$$

and,

$$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ term}\right], \text{ if } n \text{ is even.}$$

●●