Topper's Answers (Issued by CBSE)	C.B.S.E. 2019 Class–X	Mathematics

Time : 3 Hours **Max. Marks : 80**

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of **30** questions divided into four sections — A, B, C and D.*

(iii) *Section A contains **6** questions of **1** mark each. Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **8** questions of **4** marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark each, **two** questions of **2** marks each, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternatives in all such questions.*

(v) *Use of calculators is **not** permitted.*

Delhi Set **Code No. 30/1**

SECTION-A

Question numbers 1 to 6 carry 1 mark each.

1. If $\tan\alpha = \frac{5}{12}$, find the value of $\sec\alpha$.

Sol. $\tan\alpha = \frac{5}{12}$.

Using identity; $\sec^2\alpha - \tan^2\alpha = 1$

$\sec^2\alpha = 1 + \tan^2\alpha$

$\Rightarrow \sec^2\alpha = 1 + \left(\frac{5}{12}\right)^2$

$= 1 + \frac{25}{144}$

$= \frac{144+25}{144}$

$\Rightarrow \sec^2\alpha = \frac{169}{144} \Rightarrow \sec\alpha = \sqrt{\frac{13^2}{12^2}}$

$\sec\alpha = \frac{13}{12}$

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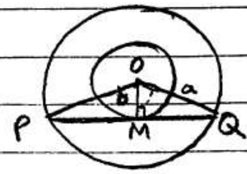
2. Two concentric circles of radii a and b $(a > b)$ are given. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. given, 2 concentric circles



$OP = OQ = a$

$OM = b$

To find - PQ

$PM = \sqrt{PO^2 - OM^2} \Rightarrow PM = \sqrt{a^2 - b^2}$

$PQ = 2PM \Rightarrow PQ = 2\sqrt{a^2-b^2}$ units

3. Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$.

Sol. 3. $(x+5)^2 = 2(5x-3)$

$\Rightarrow x^2 + 25 + \cancel{10x} = \cancel{10x} - 6$

$\Rightarrow x^2 + 31 = 0.$

$a = 1,\ b = 0,\ c = 31$

Discriminant $= b^2 - 4ac$

$= 0^2 - 4 \times 1 \times 31$

$= 0 - \cancel{36}\,124$

$= \cancel{-73}\ \cancel{-76}.\ \underline{-124}$

4. Express 429 as a product of its prime factors.

Sol. 4. 429 can be expressed as -

$429 = 3 \times 11 \times 13.$

$$\begin{array}{r|l} 3 & 429 \\ 11 & 143 \\ 13 & 13 \\ & 1 \end{array}$$

5. Find the sum of the first 10 multiples of 6.

Sol. 5. First 10 multiples of 6 form AP $\rightarrow$ $\underset{a}{6}, 12, 18 \ldots\ldots \underset{l}{60}.$

Sum of 1^{st} 10 multiples $= \frac{n}{2}[a + l]$

$= \frac{\cancel{10}^{5}}{\cancel{2}}[6 + 60]$

$= 330$

6. The distance between point A (5, – 3) and B (13, m) in 10 units. Calculate the value of m.

Sol. 6. given, $A = 5, -3$

$B = 13, m$

$AB = 10$ units

Using distance formula;

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$$\sqrt{(13-5)^2+(m+3)^2} = 10$$

On squaring;

$$8^2+(m+3)^2 = 100$$

$$\Rightarrow (m+3)^2 = 100-64$$

$$\Rightarrow \sqrt{(m+3)^2} = \sqrt{36}$$

$$\Rightarrow (m+3) = \pm 6$$

Considering only positive value;

$$m = 6-3$$

$$\Rightarrow \boxed{m = 3}$$

SECTION-B

Question numbers 7 to 12 carry 2 marks each.

7. A die is thrown once. Find the probability of getting (i) a composite number, (ii) a prime number.

Sol. 7. Event : Dice is thrown.

Outcomes : 1, 2, 3, 4, 5, 6 (6 outcomes)

Favourable events :-

i) Composite number : 4, 6

Probability of getting a composite no. = $\frac{\text{no. of favourable outcomes}}{\text{Total outcomes}}$

$= \frac{2}{6} = \frac{1}{3}$

ii) prime no. : 2, 3, 5.

Probability = $\frac{\text{no. of outcomes favourable to the event}}{\text{Total possible outcomes}}$

or $\left(\frac{\text{no. of prime nos.}}{\text{Total outcomes}}\right)$

$= \frac{3}{6} = \frac{1}{2}$

8. Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7 ?

Sol. 8. Event : cards ~~no~~ numbered from 7-40 are chosen

Total possible outcomes : (7, 8, 9 - - 40) = 34 cards.

~~For~~ Favourable event : cards multiple of 7.

Favourable outcomes : (7, 14, 21, 28, 35) 5 cards

Probability of selecting a cards multiple of 7 = $\frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$

$= \frac{5}{34}$

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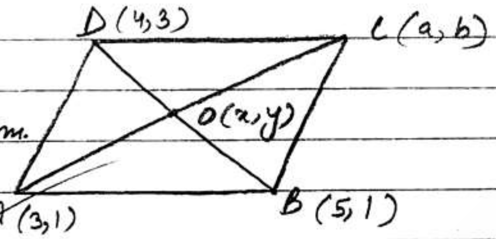
9. In parallelogram ABCD, A(3, 1), B(5, 1) C(a, b) and D(4, 3) are the vertices. Find vertex C(a, b).

Sol. 9. Points A, B, C, D are vertices of a parallelogram.



We know that diagonals of a parallelogram bisect each other.

$\therefore$ O is the midpoint of both AC and BD.

Using section formula for mid-point.

on BD,

$$x = \frac{4+5}{2}, \quad y = \frac{3+1}{2}$$

$$\Rightarrow x = \frac{9}{2}, \quad y = 2.$$

on AC

$$x = \frac{3+a}{2}, \quad y = \frac{b+1}{2}$$

$$\Rightarrow \frac{9}{\cancel{2}} = \frac{3+a}{\cancel{2}}, \quad \cancel{y}\ 2 = \frac{b+1}{2} \Rightarrow \boxed{a=6}, \boxed{b=3}$$

$$\Rightarrow a = 6, \quad b = 3.$$

10. Solve below simultaneous equations for x and y,. $3x - 5y = 4$ and $9x - 2y = 7$.

Sol. 10. Given –

$$3x - 5y = 4 \quad \text{—— (1)}$$

$$9x - 2y = 7 \quad \text{—— (2)}$$

To find - 'x' ~~ang~~ and 'y'

Multiplying (1) × 3, ~~and~~ (2) × 1 and ~~adding~~ subtracting (2) from (1); we get :

$$(3x-5y) \times 3 - (9x-2y) \times 1 = 4 \times 3 - 7 \times 1$$

$$\Rightarrow \cancel{9x} - 15y - \cancel{9x} + 2y = 12 - 7$$

$$\Rightarrow -13y = 5 \quad \Rightarrow \boxed{y = -\frac{5}{13}}$$

Then, Putting $y = -\frac{5}{13}$ in (1);

$$3x = 4 + 5y$$

$$\Rightarrow 3x = 4 + 5 \times -\frac{5}{13}$$

$\Rightarrow 3x = \frac{52-25}{13} \Rightarrow x = \frac{\cancel{27}^{9}}{13 \times \cancel{3}}$

$\Rightarrow \boxed{x = \frac{9}{13}}$

11. If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of n.

Sol. 11. Using Euclid's Division Lemma (states that $a = bq + r$, $0 \le r < b$) we can find the HCF of 65 and 117.

$117 = 65 \times 1 + 52$
$65 = 52 \times 1 + 13$
$52 = 13 \times 4 + 0$

$\therefore$ HCF of $65, 117 = 13$

But,
$65n - 117 = 13$
$\Rightarrow 65n = 13 + 117$
$\Rightarrow n = \frac{\cancel{130}^{10}}{\cancel{65}_{5}}$
$\Rightarrow \boxed{n = 2}$

12. For what value of k, the given quadratic equation $kx^2 - 6x - 1 = 0$ has no real roots ?

Sol. 12) given, quadratic equation $\Rightarrow kx^2 - 6x - 1 = 0$.
where $a = k$, $b = -6$, $c = -1$.

For no real roots (you imaginary roots), discriminant must be less than 0.

That is, $D < 0$
$\Rightarrow b^2 - 4ac < 0$
$\Rightarrow (-6)^2 - 4 \times k \times -1 < 0$
$\Rightarrow 36 + 4k < 0$
$\Rightarrow 4k < -36 \Rightarrow \boxed{k < -9}$

$\therefore$ k should be less than -9. ($k = -10, -11 \ldots$)

SECTION-C

Question numbers 13 to 22 carry 3 marks each.

13. If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$, $A > B$, then find the values of A and B.

Sol. 13. Given, $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A+B) = 1$.
$\Rightarrow \tan(A+B) = \tan 45°$
$\Rightarrow A+B = 45°$ —— (1)

Now taking,

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^\circ$

$\Rightarrow A-B = 30^\circ$ —— ②

Adding ① and ②;

$A+B+A-B = 45^\circ + 30^\circ$

$\Rightarrow 2A = 75^\circ \Rightarrow A = \frac{75^\circ}{2} \Rightarrow A = 37.5^\circ$

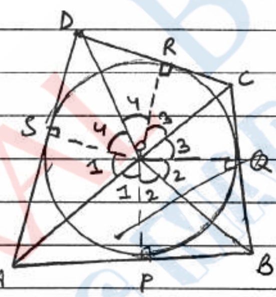
$B = 45^\circ - A \Rightarrow B = 45^\circ - 37.5^\circ \Rightarrow B = 7.5^\circ$

$\therefore \boxed{A = 37.5^\circ,\ B = 7.5^\circ}$

14. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol.

14. To prove: opposite sides of a quadrilateral circumscribing a circle subtend equal angles at the centre.



Construction: Constructed a quadrilateral ABCD, circumscribing a circle (centre O). Circle touches AB, BC, CD, DA at P, Q, R, S respectively.

To prove: $\angle AOB + \angle COD = 180^\circ$

Or $\angle AOD + \angle BOC = 180^\circ$

We know, that tangents from same exterior point subtend equal angle at the centre of circle with radius.

$\therefore \angle AOP = \angle AOS = \angle 1$ (say)

Similarly, $\angle BOP = \angle BOQ = \angle 2$

$\angle COQ = \angle COR = \angle 3$

$\angle DOR = \angle DOS = \angle 4$.

$\therefore \angle AOP + \angle BOP + \angle BOQ + \angle COQ + \angle COR + \angle DOR + \angle DOS + \angle AOS = 360^\circ$ [Complete angle around a point]

$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 3 + 2\angle 4 = 360^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^\circ$.

$\Rightarrow \angle AOB + \angle COD = 180^\circ$

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Or $(\angle 1+\angle 4)+(\angle 2+\angle 3)=180°$

$\Rightarrow \angle AOD+\angle BOC=180°$

Hence, proved!

15. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	30 – 36	36 – 42
Number of students :	10	11	7	4	4	3	1

Sol. 15. Calculating mean using stepdeviation method.

No. of days (Class interval)	No. of students (f_i)	x_i $\left(\frac{\text{Upper+lower limit}}{2}\right)$	$u_i=\frac{x_i-A}{h}$	$f_i \times u_i$
0 - 6	10	3	$\frac{3-21}{6}=-3$	$10\times-3=-30$
6 - 12	11	9	$\frac{9-21}{6}=-2$	$11\times-2=-22$
12 - 18	7	15	$\frac{15-21}{6}=-1$	$7\times-1=-7$
18 - 24	4	A 21	$\frac{21-21}{6}=0$	$4\times0=0$
24 - 30	4	27	$\frac{27-21}{6}=1$	$4\times1=4$
30 - 36	3	33	$\frac{33-21}{6}=2$	$3\times2=6$
36 - 42	1	39	$\frac{39-21}{6}=3$	$1\times3=3$
Total :	$\Sigma f_i=40$			$\Sigma f_iu_i=-46$

Class size $(h)=6-0=\underline{\underline{6}}$

Assumed mean $(A)=21$

$$\text{Mean}=A+\frac{\Sigma f_iu_i}{\Sigma f_i}\times h$$

$$\Rightarrow \text{Mean}=21+\left(\frac{-46}{40}\right)\times 6$$

$$=21-\frac{23}{\cancel{20}\,10}\times\cancel{6}\,3$$

$$=21-6.9$$

$$=14.1 \text{ days}$$

Mean of no. of days student remains absent = 14.1 days

16. A wiper blade has length 21 cm, sweeps 120°. Calculate the area swept by two blades.

Sol. 16. Given, each wiper blade has length $(r)=21$ cm.

and sweeps angle $=120°$

Area swept by one blade $=\frac{\theta}{360}\times\pi r^2$ unit square

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$= \frac{\cancel{120}^{\circ}}{\cancel{360}_{3}} \times \frac{22}{\cancel{7}} \times \cancel{21} \times 21\ cm^2.$

$= \underline{462 cm^2}.$

Blades donot overlap.

$\therefore$ Area swept by 2 blades $= 462 \times 2\ cm^2 = \underline{924 cm^2}$

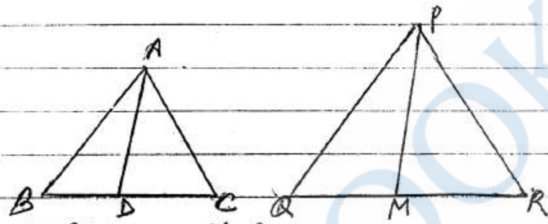
17. In similar triangle, ΔABC and ΔPQR, AD and PM are the medians respectively Prove that $\frac{AD}{PM} = \frac{AB}{PQ}$

Sol. 17. Given,

$\Delta ABC \sim \Delta PQR.$

~~AB~~ AD and PM

are medians of ΔABC and ΔPQR respectively



Since, $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}.$ —①

D is the midpoint of BC (AD is median)

M is the midpoint of QR (PM is median)

$\therefore BC = 2BD$
$QR = 2QM$ } ②

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ [from ①]

$\Rightarrow \frac{AB}{PQ} = \frac{\cancel{2}BD}{\cancel{2}QM}$ [from ②]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \Rightarrow$ $\therefore \Delta ABD \sim \Delta PQM$] ⓐ
That is, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Similarly, $\frac{BC}{QR} = \frac{AC}{PR}$ [from ①]

$\Rightarrow \frac{\cancel{2}BD}{\cancel{2}QM} = \frac{AC}{PR}$

$\Rightarrow \frac{AC}{PR} = \frac{BD}{QM}.$

$\therefore \Delta ACD \sim \Delta PRM$ — ⓑ That is, $\frac{AC}{PR} = \frac{AD}{PM} = \frac{CD}{RM}$

From both ⓐ and ⓑ, we get that

$\frac{AB}{PQ} = \frac{AD}{PM}$ Hence proved!

18. Verify $g(x) = x^3 - 3x + 1$ in a factor of $P(x) = x^5 - 4x^3 + x^2 + 3x + 1$ or not.

Sol. 18. Given: $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$.

$g(x) = x^3 - 3x + 1$

To check: if $g(x)$ is a factor of $p(x)$ or not.

Method: simply divide.

$\Rightarrow$

$$x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1 } (x^2 - 1$$

$$x^5 - 3x^3 + x^2$$

$$- \quad + \quad -$$

$$-x^3 + 3x + 1$$

$$-x^3 + 3x - 1$$

$$+ \quad - \quad +$$

$$2$$

We get remainder $= 2$

Therefore, $p(x)$ is not completely divisible by $g(x)$

$g(x)$ is not a factor of $p(x)$

19. Prove that $\sqrt{3}$ is an irrational number.

Sol. 19. Let us assume, if possible, that $\sqrt{3}$ is rational

Then, $\sqrt{3}$ can be expressed as $\frac{p}{q}$ where $(q \neq 0)$ and

p, q are coprimes [HCF$(p, q) = 1$]

$\therefore \sqrt{3} = \frac{p}{q}$ [$p, q \in Z$; HCF$(p, q) = 1$]

On squaring both sides,

$$3 = \frac{p^2}{q^2}$$

$$\Rightarrow p^2 = 3q^2. \text{ ——— } (1)$$

3 divides p^2

$\therefore$ 3 divides p.

Then, p can be written as;

$p = 3a$ for some integer 'a'.

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On squaring,

$p^2 = 9a^2$

Put $p = 3q$ from ①

$\Rightarrow \cancel{9}q^2 = \cancel{9}^{3}a^2$

$\Rightarrow q^2 = 3a^2.$

3 divides q^2

$\therefore$ 3 divides q.

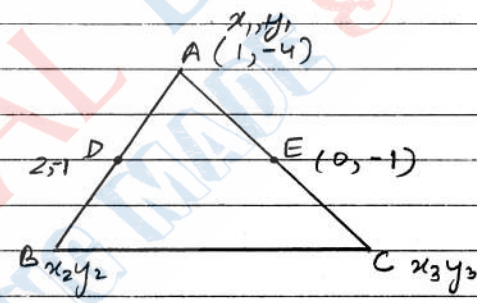
$\therefore$ 3 divides both p and q, 3 is a common factor of p and q.
But, p and q are co-primes.

Therefore, our assumption is wrong

$\therefore \sqrt{3}$ is irrational.

20. In ΔABC, A is (1, – 4). E(0, – 1) and D (2, – 1) are the midpoints of AB and AC. Calculate the area of ΔABC.

Sol. 20. Given: Triangle ABC with
$A(x_1, y_1) = A(1, -4)$



D and E are midpoints of AB and AC.

Let coordinates of B be (x_2, y_2) and that of C be (x_3, y_3).
Using section formula for mid-point;

$$\frac{1+x_2}{2} = 2 \quad , \quad \frac{-4+y_2}{2} = -1$$

$$\Rightarrow x_2 = 3 \quad , \quad y_2 = 2$$

$$\underline{(x_2, y_2) = (3, 2)}$$

Similarly, $\dfrac{1+x_3}{2} = 0$, $\dfrac{-4+y_3}{2} = -1$

$$\Rightarrow x_3 = -1 \quad , \quad y_3 = 2.$$

$\underline{\cancel{(x_3, y_3) = (-1, 2)}}$ $\quad x_3, y_3 = -1, 2$

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Area of triangle $= \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$ unit square

$= \frac{1}{2}|1(2-2) + 3(2+4) + (-1)(-4-2)|$ unit square

$= \frac{1}{2}|1\times0 + 3\times6 + (-1\times-6)|$ sq. units

$= \frac{1}{2}|18+6|$ sq. units $= 12$ sq. units

$\therefore$ Area of Δ $= 12$ sq. units

21. Two numbers are in the ratio of 5 : 6. If 7 is subtracted from each there ratio becomes 4 : 5. Find the numbers.

Sol. 21. Let the required two numbers be $5x$ and $6x$.

given, if 7 is subtracted from both nos, ratio becomes 4 : 5

New nos. $= (5x-7)$ and $(6x-7)$.

According to the question;

$$\frac{5x-7}{6x-7} = \frac{4}{5} \quad \text{——} (1)$$

Solving (1);

$5(5x-7) = 4(6x-7)$

$\Rightarrow 25x - 35 = 24x - 28$

$\Rightarrow x = 35 - 28$

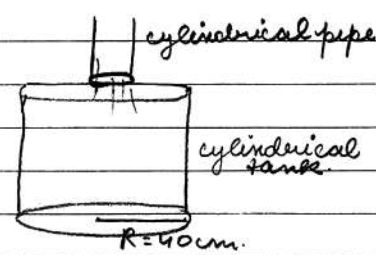
$\Rightarrow x = 7$

$\therefore$ The required nos. are : $5x = 35$

$6x = 42$

22. A Cylindrical task of radius 40 cm in filled upto height 3.15 *m* by an other cylindrical pipe with the rate of 2.52 km/h in $\frac{1}{2}$ hour. Calculate the diameter of cylindrical pipe ?

Sol. 22. Let the radius of cylindrical pipe be 'r' metres



given –

Radius (R) of cylindrical tank $= 40$ cm $= \frac{2}{5}$ m

Height of tank filled $= 3.15$ m

Time taken $= \frac{1}{2}$ h $= 30$ minutes $= 30\times60$ s.

Rate of flow of water in pipe $= 2.52$ km/h $= \frac{\cancel{252}^{\cancel{126}\,14}}{\cancel{100}_{20}} \times \frac{\cancel{5}}{\cancel{18}_{9}}$ m/s.

$= 0.7$ m/s.

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~~Volume of~~

To find - internal diameter of pipe $(2r)$.

Solution:

Volume of water passed through pipe in $\frac{1}{2}$ hours $= \pi r^2 \times h$ unit cube

$= \pi r^2 \times$ rate of flow $\times$ time

$= \pi r^2 \times 0.7 \times 30 \times 60 \; m^3$.

Volume of water in tank in $\frac{1}{2}$ hours $= \pi R^2 \times h$

$= \pi \left(\frac{2}{5}\right)^2 \times 3.15 \; m^3$

But, volume of water passed through pipe = Volume of water collected in tank

$\therefore \cancel{\pi} r^2 \times \frac{7}{1\cancel{0}} \times 3\cancel{0} \times 60 = \cancel{\pi} \left(\frac{2}{5}\right)^2 \times \frac{315}{100}$

$\Rightarrow r^2 = \frac{4}{5 \times 5} \times \frac{315}{5100} \times \frac{1}{7} \times \frac{1}{3} \times \frac{1}{60}$

$\Rightarrow r^2 = \frac{1}{2500} \Rightarrow r = \sqrt{\frac{1}{50^2}} \Rightarrow r = \pm \frac{1}{50}$

Radius is always positive, so $r = -\frac{1}{50}$ can be ignored.

$\Rightarrow r = \frac{1}{50} m$.

$\Rightarrow r = \frac{1}{50} \times 100 \, cm \Rightarrow r = 2 \, cm$

Internal diameter of pipe $= d = 2r = 4 \, cm$
or $0.04 \, m$.

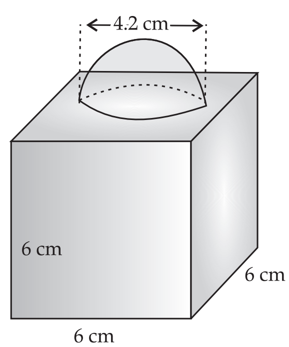
SECTION-D

Question numbers 23 to 30 carry 4 marks each.

23. In Figure, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4·2 cm. Find

(a) the total surface area of the block.

(b) the volume of the block formed. (Take $\pi = \frac{22}{7}$)



OR

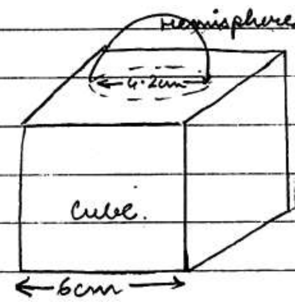
A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308·8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use π = 3·14)

Sol.

23. Radius of the hemisphere $(r) = \frac{d}{2}$

$= \frac{4.2 \text{ cm}}{2}$

$= 2.1 \text{ cm}$



Side of cube (a) = 6 cm.

a) Total surface area of block = Total surface area of cube + Curved surface area of hemisphere − area enclosed by base of hemisphere

$= 6a^2 + \frac{4\pi r^2}{2} - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6 \times 6^2 + \frac{22}{7} \times (2.1)^2 \text{ cm}^2$

$= 216 + \frac{22}{7} \times 2.1 \times 2.1 \text{ cm}^2$

$= [216 + 13.86] \text{ cm}^2$

$= 229.86 \text{ cm}^2$.

b) Volume of block formed = volume of cube + volume of hemisphere

$= a^3 + \frac{2}{3}\pi r^3$

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times 2.1^3 \text{ cm}^3$

$= 216 + \frac{2}{3} \times \frac{22}{7} \times \frac{21 \times 21 \times 21}{1000} \text{ cm}^3$

$= 216 + 19.404 \text{ cm}^3$

$= \underline{235.404 \text{ cm}^3}$.

24. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Sol. 24. To prove : Square of hypotenuse, in a right triangle, is equal to the sum of squares of other two sides. (Pythagoras theorem)

That is, $AC^2 = AB^2 + BC^2$.

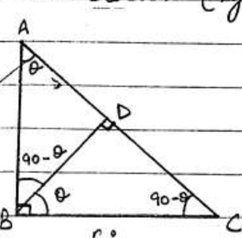


fig.

Construction : Construct $BD \perp AC$

Name $\angle BAC = \theta$.

Then, $\angle BCA = 90 - \theta$, $\angle ABD = 90 - \theta$, $\angle DBC = \theta$

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It is clear that,

$\Delta ABD \sim \Delta ACB \sim \Delta BCD.$

Using $\Delta ABD \sim \Delta ACB$, we get :

$\frac{AB}{AC} = \frac{AD}{AB} \Rightarrow AB^2 = AC \times AD$ —— ①

Similarly, using $\Delta BCD \sim \Delta ACB$, we get :-

$\frac{BC}{AC} = \frac{CD}{CB} \Rightarrow \frac{BC}{AC} = \frac{CD}{BC} \Rightarrow BC^2 = AC \times CD$ —— ②

Adding ① and ②, gives :

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC(AD + CD)$ [AD+CD = AC] Figure

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow \boxed{AB^2 + BC^2 = AC^2.}$

Hence, proved that in a right triangle, sum of square of ~~by~~ other 2 sides is equal to the square of hypotenuse

25. Change the following distribution to a 'more than type' distribution. Hence draw the 'more than type' ogive for this distribution.

Class interval :	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Frequency :	10	8	12	24	6	25	15

Sol. 25. More than series

More than or equal to 20 - 100

More than 30 - 90

More than 40 - 82

More than 50 - 70

More than 60 - 46

More than 70 - 40

More than 80 - 15

More than 90 - 0

Class Interval	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	10	8	12	24	6	25	15

$\Sigma f_i = 100$

$n = 100.$ $\frac{n}{2} = 50$

For more than ogive, we plot the point A (20, 100), B (30, 90), C (40, 82) D (50, 70), E (60, 46), F (70, 40), G (80, 15) and H (90, 0)

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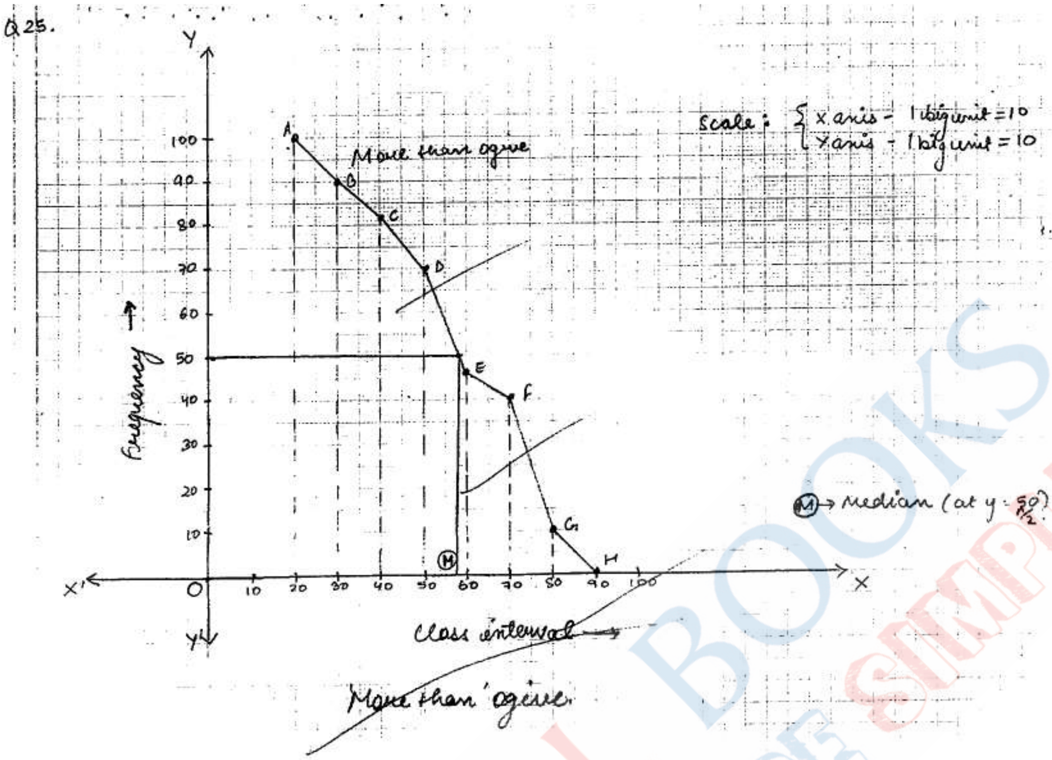
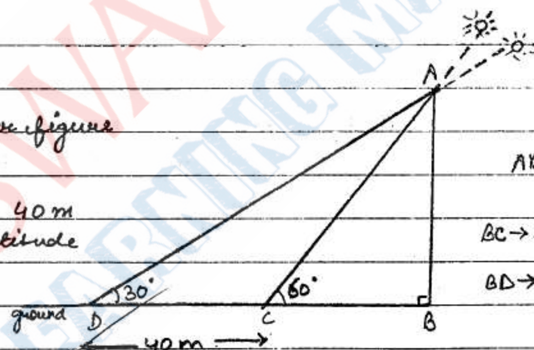
Q.25.



26. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Given $\sqrt{3}$ = 1·732)

Sol.

26. given – as per figure



AB → tower

BC → shadow of tower at 60°

BD → shadow of tower at 30°

Shadow of tower is 40 m longer at sun's altitude at 30°

$\therefore$ BD − BC = 40 m

$\Rightarrow$ CD = 40 m

In $\triangle$ ACB,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} \times BC = AB \Rightarrow BC = \frac{AB}{\sqrt{3}} \quad \text{—(1)}$$

In $\triangle$ ADB,

$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{40 + BC}$$

$$\Rightarrow 40 + BC = \sqrt{3} \times AB$$

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$\Rightarrow 40 + \frac{AB}{\sqrt{3}} = \sqrt{3} \times AB$ [Put $BC = \frac{AB}{\sqrt{3}}$ from ①].

$\Rightarrow AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB\left(\frac{3-1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB \times \frac{2}{\sqrt{3}} = 40 \Rightarrow AB = \frac{40 \times \sqrt{3}}{2}$ m

$\Rightarrow AB = \underline{20\sqrt{3}\text{ m}}$

given, use $\sqrt{3} = 1.732$ $\therefore AB = 20 \times 1.732$ m $= \underline{34.64\text{ m}}$

$\therefore$ Height of tower $= 34.64$ m.

27. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m+n)^{th}$ term of the A.P. is zero.

Sol. 27. Let the first term of given A.P. be 'a'

And, the common difference be 'd'.

and a_p dentotes p^{th} term.

given; $m(a_m) = n(a_n)$ $[m \neq n]$.

To show: $a_{(m+n)} = 0$

$m(a_m) = n(a_n)$

$\Rightarrow m[a + (m-1)d] = n[a + (n-1)d]$

$\Rightarrow am + md(m-1) = an + nd(n-1)$

$\Rightarrow am - an = nd(n-1) - md(m-1)$

$\Rightarrow a(m-n) = d[n(n-1) - m(m-1)]$

$\Rightarrow a(m-n) = d[n^2 - n - m^2 + m]$

$\Rightarrow a(m-n) = d[n^2 - m^2 + m - n]$

$\Rightarrow a(m-n) = d[(m+n)(n-m) + (m-n)]$

$\Rightarrow a(\cancel{m-n}) = d(\cancel{m-n})[-(m+n) + 1]$

$\Rightarrow a - d[-(m+n) + 1] = 0$

$\Rightarrow \boxed{a + (m+n-1)d = 0}$

$\Rightarrow \cancel{a+} \boxed{a_{m+n} = 0}$

$\therefore a_{m+n} = 0$

Hence, proved!

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28. A shopkeeper buy certain number of books in ₹ 80. If he buy 4 more books then new cost price of each book is reduced by ₹ 1. Find the number of books initially he buy.

Sol. 28. Let the no. of books bought by the shopkeeper be 'n'.

Total money spent = Rs 80

$\therefore$ Cost of each book = Rs $\frac{80}{n}$

Now, given : He buys 4 more books, no. of books bought = $n+4$ (for same amount)

New cost of each book = Rs $\frac{80}{n+4}$

given, new cost of each book is Rs. 1 less than earlier.

$\therefore \quad \frac{80}{n} - \frac{80}{n+4} = 1$

$\Rightarrow 80\left(\frac{1}{n} - \frac{1}{n+4}\right) = 1$

$\Rightarrow \frac{\cancel{n}+4-\cancel{n}}{n(n+4)} = \frac{1}{80} \Rightarrow 4 \times 80 = n(n+4)$

$\Rightarrow n^2 + 4n - 320 = 0$

Using quadratic formula; $\Rightarrow n = \frac{-4 \pm \sqrt{16 + 4 \times 320}}{2}$

~~$\Rightarrow n = \frac{-4 \pm \sqrt{676}}{2}$~~

~~$= \frac{-4 \pm 26}{2} \Rightarrow n = \frac{-30}{2}$ or $\frac{22}{2}$~~

$\Rightarrow n = \frac{-4 \pm \sqrt{1296}}{2}$

$\Rightarrow n = \frac{-4 \pm 36}{2}$

$\Rightarrow n = \frac{-40}{2}$ or $\frac{32}{2} \Rightarrow n = -20$ or 16

Since, no. of books is a whole no., it cannot be negative

$n = -20$ can be ignored.

$\therefore \boxed{n = 16}$

No. of books bought by the shopkeeper = 16.

29. Construct a pair of tangents to a circle of radius 4 cm from an external point at a distance 6 cm from the centre of the circle.

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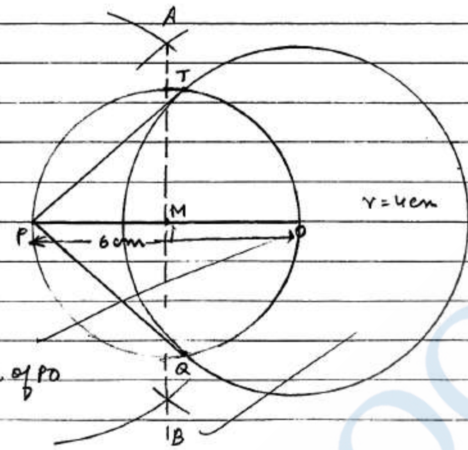
Sol. 29. To construct: a pair of tangents to a circle of radius = 4 cm, from a point at a distance 6 cm from centre.

Steps of construction:

1) Draw a circle of radius 4 cm with O as the centre.

2) Take a point P at PO = 6 cm.

3) Join PO. Construct a perpendicular bisector of PO at M (PM = MO, AB ⊥ PO)



4) With M as centre and PM (= MO) as radius, draw a circle touching the circle with centre O at T and Q

5) Join PT and PQ

∴ PT and PQ are required tangents.

30. Prove that :

$$\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}=2$$

Sol. 30. To prove: $\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}=2$

Taking from LHS,

$= \text{LHS}$

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}$

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}$ [Re-arranging]

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\left(\frac{1}{\sin^2\theta}\right)}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\left(\frac{1}{\cos^2\theta}\right)}$ $\left[\operatorname{cosec}\theta=\frac{1}{\sin\theta},\ \sec\theta=\frac{1}{\cos\theta}\right]$

$= \frac{1}{1+\sin^2\theta}+\frac{1\times\sin^2\theta}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1\times\cos^2\theta}{1+\cos^2\theta}$

$= \frac{1+\sin^2\theta}{1+\sin^2\theta}+\frac{1+\cos^2\theta}{1+\cos^2\theta}$

$= 1 + 1$

$= 2$

$= \text{RHS}$

LHS = RHS

Hence, proved!

❑❑❑

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