UNIT – I: NUMBER SYSTEMS

CHAPTER-1

REAL NUMBERS

Topic-1 Rational Numbers

Revision Notes

- **Rational Number :** A number 'r' is called a rational number, if it can be written in the form p/q, where p and q are integers and $q \neq 0$, denoted by 'Q'.

  e.g., $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, -\frac{2}{3}$ etc. are all rational numbers.

  Symbolically, $Q = \left\{\frac{p}{q}, q \neq 0 \text{ and } p, q \in I\right\}$

- **Decimal Expansion of Real Numbers :** The decimal expansion of real number is used to represent a number on the number line.

  If the decimal expansion of a real number is either terminating or non-terminating (recurring), then the real number is called a rational number.

- **Cases in Rational Number:**

  Case 1 : When Remainder becomes Zero : Every rational number $\frac{p}{q}, (q \neq 0)$ can be expressed as a decimal. On dividing p by q, when the remainder becomes zero, then the decimal is called a terminating decimal.

  e.g. **(i)** $\frac{1}{2} = 0.5$

  On dividing 1 by 2, we get value 0.5 *i.e.*, remainder equal to zero, so 0.5 is a terminating decimal.

  (ii) $\frac{52}{100} = 0.52$

  On dividing 52 by 100, we get value 0.52 *i.e.*, remainder equal to zero, so 0.52 is a terminating decimal.

- **Terminating decimal :** If a rational number ($\neq$ integer) can be expressed in the form $\frac{P}{2^n \times 5^m}$ $P \in Z, (n, m) \in W$, the rational number will be a terminating decimal.

  For Example : (i) $\frac{5}{8} = \frac{5}{2^3 \times 5^0}, \frac{5}{8}$ is terminating decimal.

  (ii) $\frac{9}{1280} = \frac{9}{2^8 \times 5^1}$, so $\frac{9}{1280}$ is terminating decimal.

  (iii) $\frac{4}{45} = \frac{4}{3^2 \times 5^1}$. Since it is not the form $\frac{P}{2^n \times 5^m}$

  so $\frac{4}{45}$ is non-terminating recurring decimal. Not terminating decimal.

- **Non-terminating decimal :** If in a rational number $\frac{p}{q}$, prime factors of q are other than 2 and 5, then the rational number is Non-terminating decimal.

For Example : (i) $\frac{4}{45}=\frac{4}{3^2\times2^0\times5^1}, \frac{4}{45}$ is non terminating decimal.

(ii) $\frac{5}{21}=\frac{5}{3^1\times7^1}, \frac{5}{21}$ is non terminating decimal.

$\frac{5}{21}=.\overline{238095}, \frac{4}{45}=.0\overline{8}$

Case 2 : When remainder never becomes Zero — A rational number expressed in the form of $\frac{p}{q}$ or division of p by q, when remainder never becomes zero and set of digits repeat periodically then the decimal is called non-terminating recurring or repeating decimal. It is denoted by the bar over it.

e.g. **(i)** $\frac{1}{3}$ = 0.333.... = $0.\overline{3}$

On dividing 1 by 3, we get 3 again and again in the decimal part of the quotient *i.e.*, remainder never becomes zero, so $0.\overline{3}$ is a non-terminating repeating decimal.

(ii) $\frac{3}{11}$ = 0.272727.... = $0.\overline{27}$

On dividing 3 by 11, we get 27 again and again in the decimal part of the quotient *i.e.*, remainder never becomes zero. So, $0.\overline{27}$ is a non-terminating repeating decimal.

- Every integer is a rational number.
- There are infinitely many rational numbers between any two given rational numbers.
- If x and y are any two rational numbers, then :
  (i) $x + y$ is a rational number
  (ii) $x - y$ is a rational number
  (iii) $x \times y$ is a rational number
  (iv) $x \div y$ is a rational number, ($y \neq 0$).

Example 1

Express $0.\overline{5}$ in the form of $\frac{p}{q}$.

Solution:

Step I : Assume the given decimal expansion as x and count the number of digits which are repeated.

Let $x = 0.\overline{5}$

or $x = 0.555......$ ...(i)

Thus 1 digit is repeated that is, 5.

Step II : Multiply both sides by 10 (because one digit is repeating)

On multiplying eqn. (i) by 10, we get

$10x = 5.555.....$ ...(ii)

Step III : Solving eqn. (i) and (ii) we get the value of x.

Subtracting eqn. (i) from (ii), we get

$10x - x = 5.555....... - 0.555......$

or $9x = 5$

or $x = \frac{5}{9}$

Hence $0.\overline{5} = \frac{5}{9}$

Mnemonics

1. We use R symbol. Think R as in 'Real'. The quick way to remember real numbers is that they're the numbers that truly exist and can be represented on number line.

N : Naturally (Natural numbers)
I : Involve (Integers)
R : Relation (Rational Numbers)
I : Insanity (Irrational Numbers)
Rational Numbers
These numbers can be expressed as a ratio of two integers hence the name rational numbers.
Denoted by symbol 'Q' - as in quotient so ratio means on dividing we get quotient.

Topic-2 Irrational Numbers

Revision Notes

- **Irrational Number :** If a number cannot be written in the form of p/q, where $q \neq 0$ and $p, q \in$ I, then it is called an irrational number.

  e.g., $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{2}+\sqrt{5}, \sqrt{3}-\sqrt{7}, \pi$ etc. are all irrational numbers.
- The decimal expansion of an irrational number is non-terminating and non-recurring.
- The addition, subtraction, multiplication and division of rational and irrational number is an irrational number. *i.e.,* If x and y are two real numbers where x is rational and y is an irrational, then

  (i) $x + y$ is an irrational number.

  (ii) $x - y$ is an irrational number.

  (iii) $x \times y$ is an irrational number.

  (iv) $x \div y$ is an irrational number.

Example 2

Locate $\sqrt{17}$ on the number line.

Solution:

Step I : Write the given number (without root) as the sum of the squares of two natural numbers.

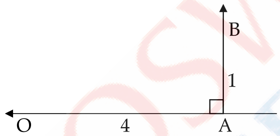
Here $17 = 16 + 1 = 4^2 + 1^2$

Step II : Draw these two natural numbers on the number lines in which one is perpendicular to other

Draw OA = 4 unit

and AB = 1 unit

Such that AB $\perp$ OA

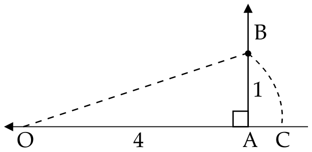


StepIII : By using Pythagoras theorem, find OB

$$OB = \sqrt{OA^2 + AB^2}$$
$$= \sqrt{4^2 + 1^2} = \sqrt{16+1}$$
$$= \sqrt{17}$$

Step IV : Having O as centre and radius OB, draw an arc, which cuts the number line at C. OC corresponds to $\sqrt{17}$.

Hence OC, represents $\sqrt{17}$.



Topic-3 n^{th} Root of a Real Number

Revision Notes

- **Definition:** In $a^n = b$, a and b are real numbers and n is a positive integer,.

  (i) a is an n^{th} root of b.

  (ii) It can also be written as $\sqrt[n]{b} = a$.

  (iii) It is also known as radical.

  Example : (i) 3 is fourth root of 81 *i.e.,* $3^4 = 81$ or $3 = \sqrt[4]{81}$

  (ii) 2 is sixth root of 64 *i.e.,* $2^6 = 64$ or $2 = \sqrt[6]{64}$
- **Square root:** The "2^{nd}" root is the square root.
- **Cube root:** The "3^{rd}" root is the cube root.
- $\sqrt{a} \times \sqrt{a} = a$: Square root is used two times in a multiplication to get the original value.

- $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$: Cube root is used three times in a multiplication to get the original value.
- $\underbrace{\sqrt[n]{a} \times \sqrt[n]{a} \times ....\sqrt[n]{a}}_{n \text{ terms}} = a$: The n^{th} root is used n times in a multiplication to get the original value.
- **Identities used for radicals :** Identities for two positive real numbers r and s :

  (i) $\sqrt{rs} = \sqrt{r} \cdot \sqrt{s}$ **(ii)** $\sqrt{\frac{r}{s}} = \frac{\sqrt{r}}{\sqrt{s}}$

  (iii) $(\sqrt{r} + \sqrt{s})(\sqrt{r} - \sqrt{s}) = r - s$ **(iv)** $(r + \sqrt{s})(r - \sqrt{s}) = r^2 - s$

  (v) $(\sqrt{r} - \sqrt{s})^2 = r - 2\sqrt{r}\sqrt{s} + s$
- **Laws of radicals :** Laws for two positive real numbers a and b :

  (i) $\sqrt[n]{a^n} = a$ **(ii)** $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}}$

  (iii) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, ($a, b > 0$ be real number) **(iv)** $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

  (v) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$ **(vi)** $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$

  (vii) $\sqrt[p]{(a^n)^m} = \sqrt[p]{a^{nm}}$ **(viii)** $a^{-m} = \frac{1}{a^m}$

  Examples :

  (i) $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$

  (ii) $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = (\sqrt{2})^2 - (\sqrt{3})^2 = 2 - 3 = -1$

  (iii) $(2 + \sqrt{2})(2 - \sqrt{2}) = 2^2 - (\sqrt{2})^2 = 4 - 2 = 2$

  (iv) $(\sqrt{5} + \sqrt{7})^2 = (\sqrt{5})^2 + (\sqrt{7})^2 + 2\sqrt{5}\sqrt{7} = 5 + 7 + 2\sqrt{35} = 12 + 2\sqrt{35}$

  (v) $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

Example 3

Simplify : $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

Solution:

Step I : Write the exponents in the form of powers

i.e., $(81)^{\frac{1}{4}} - 8.(216)^{\frac{1}{3}} + 15.(32)^{\frac{1}{5}} + (225)^{\frac{1}{2}}$

Step II: Factorize the radical

Here $(3^4)^{\frac{1}{4}} - 8.(6^3)^{\frac{1}{3}} + 15(2^5)^{\frac{1}{5}} + (15^2)^{\frac{1}{2}}$

Step III : Multiplying the powers

i.e. $(3)^{4 \times \frac{1}{4}} - 8.(6)^{3 \times \frac{1}{3}} + 15.(2)^{5 \times \frac{1}{5}} + (15)^{2 \times \frac{1}{2}}$

$= 3 - 8\,(6) + 15\,(2) + 15$

Step IV : Solving the expression

$3 - 48 + 30 + 15 = 0$

Topic-4 Laws of Exponents with Integral Powers

Revision Notes

- Let $a > 0$ be a real number and 'r' and 's' be rational numbers, then

  (i) $a^r.a^s = a^{r+s}$ **(ii)** $(a^r)^s = a^{rs}$

  (iii) $\frac{a^r}{a^s} = a^{r-s}, r > s$ **(iv)** $a^r\,b^r = (ab)^r$

(v) $a^{-r} = \frac{1}{a^r}$

(vi) $a^{\frac{r}{s}} = (a^r)^{1/s} = (a^{1/s})^r$

(vii) $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

(viii) $\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r$

(ix) $a^0 = 1$

Examples :

(i) $(3)^4 \times (3)^3 = 3^{4+3} = 3^7$

(ii) $\frac{(4)^7}{(4)^2} = (4)^{7-2} = 4^5$

(iii) $(3)^2 \times (4)^2 = (12)^2$

(iv) $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2$

(v) $\left(\frac{1}{3}\right)^{-7} = 3^7$

(vi) $(9)^{-2} = \frac{1}{9^2}$

Example 4

Find the value of $\frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}}$

Solution:

Step I : Taking common factor from numerator and denominator as possible we can.

i.e., $\frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}} = \frac{3^{38}(3^2+3^1+1)}{3^{39}(3^2+3^1-1)}$

Step II : Shifting the common factor which in denominator and solving the expression which are in bracket.

i.e., $\frac{3^{38-39}(9+3+1)}{(9+3-1)}$

Step III : Solving the expression

i.e., $= \frac{3^{-1}\times 13}{11}$

$= \frac{13}{3\times 11} = \frac{13}{33}$

Topic-5 Rationalization of Real Numbers

Revision Notes

- **Rationalization:** If a given number is transformed into an equivalent form, such that the denominator is a rational number then the process is known as rationalization.
- **Rationalizing the denominator :** If the denominator of a fraction contains a term with root (a number under a radical sign), the process of converting it to an equivalent expression with rational denominator is called as rationalizing the denominator.
- To rationalize the denominator of $\frac{1}{\sqrt{r}+s}$, we multiply this by $\frac{\sqrt{r}-s}{\sqrt{r}-s}$, where r and s are integers.

e.g.; **(i)**

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}$$

$$= \frac{5-\sqrt{3}}{(5)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

Here, to rationalize the denominator of $\frac{1}{5+\sqrt{3}}$, we should multiply & divide it by $\left(5-\sqrt{3}\right)$.

➢ List of Rationalization Factors.

Term	Rationalizing Factor
$\frac{1}{\sqrt{r}}$	$\sqrt{r}$
$\frac{1}{\sqrt{r}-s}$	$\sqrt{r}+s$
$\frac{1}{\sqrt{r}+s}$	$\sqrt{r}-s$
$\frac{1}{\sqrt{r}-\sqrt{s}}$	$\sqrt{r}+\sqrt{s}$
$\frac{1}{\sqrt{r}+\sqrt{s}}$	$\sqrt{r}-\sqrt{s}$

Example 5

Rationalize the denominator of $\frac{7}{\sqrt{5}-\sqrt{2}}$

Solution:

To rationalize the denominator, we will multiply the numerator and denominator by its conjugate to remove the radical sign from the denominator.

Step I : Assume the given fraction as x and write the denominator.

Let $x = \frac{7}{\sqrt{5}-\sqrt{2}}$ and denominator $= \sqrt{5}-\sqrt{2}$

Step II : Find the conjugate of denominator.

Here, the conjugate of denominator $(\sqrt{5}-\sqrt{2})$ is $(\sqrt{5}+\sqrt{2})$

Step III : Multiply the numerator and denominator of x by the conjugate of denominator and rationalize it.

$$x = \frac{7}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$

$$= \frac{7(\sqrt{5}+\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{7(\sqrt{5}+\sqrt{2})}{5-2}$$

$$= \frac{7}{3}(\sqrt{5}+\sqrt{2})$$

1. If $x+\sqrt{y} = (2-\sqrt{3})^2$, then find the values of x and y.
2. Represent $\sqrt{10}$ on the number line.

UNIT – II: ALGEBRA

CHAPTER-2

POLYNOMIALS

Topic-1 Polynomials

Revision Notes

➢ **Polynomial :** The algebraic expression in which the variables involved have only non-negative integral exponent is called 'Polynomial'.

A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0.$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and $a_n \neq 0$. Here, $a_0, a_1, a_2, \ldots + a_n$ are respectively the co-efficients of $x^0, x^1, x^2, \ldots, x^n$ and n is called the **degree of the polynomial.**

This form of polynomial is known as the "Standard form of Polynomial".

e.g., **: (i)** $2x^3 - 4x^2 + 5x - 7$ is a polynomial in one variable (x).

(ii) $3y^3 - 12y^2 + 7y - 9$ is a polynomial in one variable (y).

➢ **Constant Polynomial :** A polynomial of degree zero is called a constant polynomial.

e.g.,: $4, -\frac{7}{5}, \frac{3}{4}$ are constant polynomials.

➢ **Zero Polynomial :** Zero is also known as zero polynomial and the degree of the zero polynomial is not defined.

➢ **Degree of a Polynomial :** Highest power of variable in a polynomial is called the 'degree of a polynomial'.

- In polynomial of one variable, the highest power of the variable is called the degree of the polynomial.

e.g., **: (i)** $4x^7 - 3x^3 + 2x^2 + 3x - 6$ is a polynomial in x of degree 7.

(ii) $\sqrt{3}x^2 + 3\sqrt{3}x + 3$ is a polynomial in x of degree 2.

(iii) $\frac{3}{4}x^4 + \frac{2}{5}x^2 + 7x - 3$ is a polynomial in x of degree 4.

- In a polynomial of more than one variable, the sum of the powers of variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

e.g.,: **(i)** $7x^3 - 4x^2y^2 + 3x^2y - 3y + 9$ is a polynomial in x and y of degree 4.

(ii) $\sqrt{3}x^5 - 4y^3 + 7x^3y + 2x - 3$ is a polynomial in x and y of degree 5.

➢ **Types of Polynomials :**

- **On the basis of terms**

Term : In a polynomial $x^2 + 3x + 4$, the expressions x^2, $3x$ and 4 are called terms.

e.g.,: **(i)** Polynomial $x^2 + 3x + 7$ has three terms.

(ii) Polynomial $x^2 - 4$ has two terms.

(I) Monomial : A polynomial of one non-zero term, is called a monomial.

e.g.,: $2x, -4x^2, 7x^3, 10x$ are monomials.

(II) Binomial : A polynomial of two non-zero terms, is called a binomial.

e.g.,: $(4x^2 + 8), (7y^2 - 3y), (3x - 6), (10x^2 - 4)$ are binomials.

(III) Trinomial : A polynomial of three non-zero terms, is called a trinomial.

e.g.,: $(x^2 + 2x + 4), (4x^2 + \frac{7}{5}x + 14), (3x^2 + 3\sqrt{3}x + \sqrt{3})$ are trinomials.

- **On the basis of degree**

(I) Linear Polynomial : A polynomial of degree 1 is called a linear polynomial. It is expressed in the form of $ax + b$, where a and b are real constants and $a \neq 0$.

e.g.,: **(i)** $3x + 6$ is a linear polynomial in x.

(ii) $\sqrt{3}x - 3$ is a linear polynomial in x.

(iii) $\frac{7}{5}y - 10$ is a linear polynomial in y.

(II) Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial. It is expressed in the form of $ax^2 + bx + c$, where a, b and c are real constants and $a \neq 0$.

e.g.,: **(i)** $3x^2 + 4x + 1$ is a quadratic polynomial in x.

(ii) $\sqrt{3}x^2 + 3x + 3\sqrt{3}$ is a quadratic polynomial in x.

(iii) $\frac{7}{5}y^2 + 3y - 1$ is a quadratic polynomial in y.

(III) Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial. It is expressed in the form of $ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants and $a \neq 0$.

e.g.,: **(i)** $2x^3 - 4x^2 + 8x - 3$ is a cubic polynomial in x.

(ii) $4y^3 + 3y^2 + 7y - 14$ is a cubic polynomial in y.

➢ **Zeroes of a Polynomial :**

Zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

(i) '0' may be a zero of a polynomial.

(ii) Every linear polynomial in one variable has a unique zero of a polynomial.

(iii) A non-zero constant polynomial has no zero of a polynomial.

(iv) Every real number is a zero of the zero polynomial.

(v) Maximum number of zeroes of a polynomial is equal to its degree.

Example 1 : Find whether –1 and 1 are the zeroes of the polynomial $x - 1$.

Solution : Let $p(x) = x - 1$

Then, $p(1) = 1 - 1 = 0$, $p(-1) = -1 - 1 = -2$

Therefore, 1 is a zero of the polynomial, but –1 is not.

Example 2 : Find the zero of the polynomial $p(x) = 3x + 1$.

Solution : Finding the zero of $p(x)$, is the same as solving the equation

$$p(x) = 0$$

Now, $3x + 1 = 0, \Rightarrow x = \frac{-1}{3}$. So, $\frac{-1}{3}$ is the zero of the polynomial $3x + 1$.

Example 1

Verify whether 3 and 0 are zeroes of polynomial $x^2 - 3x$.

Solution:

Step I : Write the given polynomial equal to $p(x)$

$$p(x) = x^2 - 3x \qquad \text{...(i)}$$

Step II : Putting 3 in place of x and solve it to find the value of $p(3)$. If the value of $p(3)$ is zero, then 3 will be zero of the given polynomial.

Putting $x = 3$ in (i), we get

$$\begin{aligned} p(3) &= (3)^2 - 3(3) \\ &= 9 - 9 \\ &= 0 \end{aligned}$$

Step III : Similarly, putting 0 in place of x and solving it.

Putting $x = 0$ in (i), we get

$$\begin{aligned} p(0) &= 0^2 - 3(0) \\ &= 0 \end{aligned}$$

So, 3 and 0 are zeroes of the given polynomial $p(x)$.

Mnemonics

1. Memorising by 'SOAP' Mnemonic

Same opposite Always Positive

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

same

opposite

Always Positive

2. We can check the long division using mnemonic

Dirty Monkeys Smell Bad

(i) Divide the leading term of the dividend by the leading term of the divisor. Write this quotient directly above the term you just divided into.

(ii) **Multiply:** Multiply the quotient from step by the entire divisor and write it under the dividend so the like terms are lined up.

(iii) **Subtract:** Change the sign of the subtrahend and subtract.

(iv) Bring down the next term and Repeat steps 1 to 4.

Topic-2 Remainder Theorem

Revision Notes

➤ We know the property of division which follows in the basic division *i.e,*

Dividend = Divisor × Quotient + Remainder

This same follows the division of polynomial.

According to Remainder Theorem, if $p(x)$ and $g(x)$ are two polynomials in which the degree of $p(x) \geq$ degree of $g(x)$ and $g(x) \neq 0$ are given, then we can get polynomials $q(x)$ and $r(x)$, so that :

$$p(x) = g(x) \times q(x) + r(x)$$

where, $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

It says that $p(x)$ divided by $g(x)$, gives $q(x)$ as quotient and $r(x)$ as remainder.

Example 1. Divide $p(x)$ by $g(x)$,

where, $p(x) = 3x^2 + x - 1$

and $g(x) = x + 1$

Solution :

$$\begin{array}{r|l}
x+1 & 3x^2 + x - 1 \;(\,3x - 2 \\
 & \underline{\overset{(-)}{3x^2} + \overset{(-)}{3x}} \\
 & -2x - 1 \\
 & \underline{\overset{(+)}{-2x} \overset{(+)}{- 2}} \\
 & 1
\end{array}$$

$\therefore \quad 3x^2 + x - 1 = (x + 1)(3x - 2) + 1$

i.e., Dividend = **(Divisor × Quotient) + Remainder**

Here, Dividend = $3x^2 + x - 1$, Divisor = $x + 1$, Quotient = $3x - 2$, Remainder = 1.

Example 2. Divide $p(x) = x^3 + 1$ by $x + 1$.

Solution :

$$\begin{array}{r|l}
x+1 & x^3 + 1 \;(\,x^2 - x + 1 \\
 & \underline{\overset{(-)}{x^3} + \overset{(-)}{x^2}} \\
 & -x^2 + 1 \\
 & \underline{\overset{(+)}{-x^2} \overset{(+)}{- x}} \\
 & x + 1 \\
 & \underline{\overset{(-)}{x} + \overset{(-)}{1}} \\
 & 0
\end{array}$$

$\therefore \; x^3 + 1 = (x+1)(x^2 - x + 1)$

Here, Dividend = $x^3 + 1$, Divisor = $x + 1$, Quotient = $x^2 - x + 1$, Remainder = 0.

Example 2

Using remainder theorem, find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

Solution:

Step-I : Consider the given polynomials as $p(x)$ and $g(x)$.

Given polynomial is $p(x)$

$= x^4 + x^3 - 2x^2 + x + 1$ ...(i)

and $g(x) = x - 1$

Step-II : Find the zero of $g(x)$ by which we have to divide the polynomial $p(x)$.

Putting $g(x) = 0$, to get an equation

i.e., $g(x) = 0$

or, $x = 1$

So, $x = 1$ is zero of $g(x)$.

Step-III : Put the value of zero of $g(x)$ in the polynomial $p(x)$, which have to be divided by $g(x)$.

On putting $x = 1$ in eqn. (i), we get

$$\begin{aligned} p(1) &= (1)^4 + (1)^3 - 2(1)^2 + 1 + 1 \\ &= 1 + 1 - 2 + 1 + 1 \\ &= 2 \end{aligned}$$

Here, the value of $p(1)$ is 2, which is the required remainder obtained by dividing $x^4 + x^3 - 2x^2 + x + 1$ by $(x - 1)$.

Topic-3 Factor Theorem

Revision Notes

- If $p(x)$ is a polynomial of degree $x \geq 1$ and a is any real number, then
  - **(i)** Graph of linear equation is a straight line, while graph of quadratic equation is a parabola.
  - **(ii)** Degree of polynomial $\geq$ Number of zeroes of polynomial.
  - **(iii)** If remainder $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
  - **(iv)** $(x + a)$ is a factor of polynomial $p(x)$, if $p(-a) = 0$.
  - **(v)** $(x - a)$ is a factor of polynomial $p(x)$, if $p(a) = 0$.
  - **(vi)** $(x - a)(x - b)$ is a factor of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$.
  - **(vii)** $(ax + b)$ is a factor of polynomial $p(x)$, if $p(-b/a) = 0$.
  - **(viii)** $(ax - b)$ is a factor of polynomial $p(x)$, if $p(b/a) = 0$.
- **Factorization of a Polynomial :**

• By Splitting the Middle Term :

Let a quadratic polynomial be $x^2 + lx + m$, where l and m are constants.

Factorize the polynomial by splitting the middle term lx as $ax + bx$, so that $ab = m$. Then,

$$\begin{aligned} x^2 + lx + m &= x^2 + ax + bx + ab \\ &= x(x + a) + b(x + a). \\ &= (x + a)(x + b). \end{aligned}$$

• By using Factor Theorem :

Consider a quadratic polynomial $ax^2 + bx + c$, where a, b and c are constants. It has two factors $(x - \alpha)$ and $(x - \beta)$.

$\therefore$ $\quad ax^2 + bx + c = a(x - \alpha)(x - \beta)$

or, $\quad ax^2 + bx + c = ax^2 - a(\alpha + \beta)x + a\alpha\beta$

On equating the coefficient of x and constant term, we get $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

On simplifying, we get the value of α and β.

Example 1. Factorize $6x^2 + 17x + 5$ by splitting the middle term, and by using the Factor Theorem.

Solution :

(i) By Splitting the Middle Term :

If we find the two numbers a and b, such that $a + b = 17$ and $ab = 6 \times 5 = 30$, then we can get the factors. Factors of 30 are 1 and 30, 2 and 15, 3 and 10, 5 and 6, of these pairs, 2 and 15 will give us $a + b = 17$.

So,
$$\begin{aligned} 6x^2 + 17x + 5 &= 6x^2 + (2 + 15)x + 5 \\ &= 6x^2 + 2x + 15x + 5 \\ &= 2x(3x + 1) + 5(3x + 1) \\ &= (2x + 5)(3x + 1) \end{aligned}$$

(ii) **By Factor Theorem :**

$$6x^2 + 17x + 5 = 6\left(x^2 + \frac{17}{6}x + \frac{5}{6}\right) = 6p(x).$$

If a and b are the zeroes of $p(x)$, then, new line $6x^2 + 17x + 5 = 6(x - a)(x - b)$, and $ab = \frac{5}{6}$.

Then, see some possibilities for a and b.

They could be $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm 1$.

Now, $p\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{17}{6}\left(\frac{1}{2}\right) + \frac{5}{6} \neq 0,$

But, $p\left(\frac{-1}{3}\right) = 0$. So, $\left(x + \frac{1}{3}\right)$ is a factor of $p(x)$.

Similarly, we will get $\left(x + \frac{5}{2}\right)$ as a factor of $p(x)$.

$$\therefore \quad 6x^2 + 17x + 5 = 6\left(x+\frac{1}{3}\right)\left(x+\frac{5}{2}\right)$$

$$= 6\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right) = (3x + 1)(2x + 5).$$

In this example, the use of the splitting method appears more efficient.

Example 3

Factorize the cubic polynomial $x^3 + 6x^2 + 11x + 6$.

Solution:

Step I : Consider the given cubic polynomial as $p(x)$ and find the constant term

$$p(x) = x^3 + 6x^2 + 11x + 6$$

Here,constant term = 6

Step II : Find all the factors of constant term of $p(x)$.

All possible factor of 6 are $\pm 1, \pm 2, \pm 3$ and ± 6.

Step III : Check at which factor, $p(x)$ is zero by trial method and get one factor of $p(x)$.

At $\quad x = -1$

$$\begin{aligned} p(-1) &= (-1)^3 + 6(-1)^2 + 11(-1) + 6 \\ &= -1 + 6 - 11 + 6 \\ &= -12 + 12 = 0 \end{aligned}$$

So, $(x + 1)$ is a factor of $p(x)$.

Step IV : Now, write $p(x)$ as the product of this factor and a quadratic polynomial.

On dividing $p(x)$ by $(x + 1)$, we get quotient $x^2 + 5x + 6$

So, $\quad p(x) = (x + 1)(x^2 + 5x + 6)$

Step V : Now, use splitting method or factor theorem to find the factor of $p(x)$.

Now, by splitting the middle term, we get

$$\begin{aligned} p(x) &= (x + 1)[x^2 + 3x + 2x + 6] \\ &= (x + 1)[x(x + 3) + 2(x + 3)] \\ &= (x + 1)(x + 2)(x + 3) \end{aligned}$$

Hence, the factors of given polynomial are $(x + 1)$, $(x + 2)$ and $(x + 3)$.

Topic-4 Algebraic Identities

Revision Notes

- **Algebraic Identities :**
  An algebraic identity is an algebraic equation that is true for all values of the variables occurring in it.
- **Some useful algebraic identities:**

(i) $(x + y)^2 = x^2 + 2xy + y^2$
(ii) $(x - y)^2 = x^2 - 2xy + y^2$
(iii) $x^2 - y^2 = (x + y)(x - y)$
(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
(v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
(vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
(vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
(viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
(ix) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
(x) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Example 1. Factorize the following :

(i) $x^2 + 4xy + 4y^2$ **(ii)** $x^3 - 8$
(iii) $4x^2 - 12xy + 9y^2$ **(iv)** $x^3 + 8y^3 + 6x^2y + 12xy^2$

Solution :

(i) $x^2 + 4xy + 4y^2 = (x)^2 + 2 \times (x) \times (2y) + (2y)^2 = (x + 2y)^2 = (x + 2y)(x + 2y)$

(ii) $x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 2^2) = (x - 2)(x^2 + 2x + 4)$

(iii) $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2 = (2x - 3y)^2$
$= (2x - 3y)(2x - 3y)$

(iv) $x^3 + 8y^3 + 6x^2y + 12xy^2$
$= (x)^3 + (2y)^3 + 3(x)(2y)[x + 2y]$
$= (x + 2y)^3 = (x + 2y)(x + 2y)(x + 2y)$

Example 4

Evaluate $(102)^3$ by using suitable identities.

Solution:

Step I : Express the given number without power as the sum or difference of two numbers

Given number without power is 102. Since, it is greater than 100, so it can be written as 100 + 2

$\therefore \quad (102)^3 = (100 + 2)^3$

Step II : Compare the expression with $(x + y)^3$.

On comparing $(100 + 2)^3$ with $(x + y)^3$, we get

$x = 100$ and $y = 2$

Step III : Use the identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ to expand it.

By using the identity,

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$, we get

$(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

Step IV : Simplify the above expression.

$(102)^3 = 1000000 + 8 + 60000 + 1200$

$= 1000000 + 8 + 61200$

$= 1061208$

CHAPTER-3

LINEAR EQUATIONS IN TWO VARIABLES

Topic-1 Introduction of Linear Equation

Revision Notes

- Linear equations in one variable. $x + 1 = 0$, $x + \sqrt{2} = 0$ etc. are we know that such equations have unique solution and solution of these type of equations can be represented on number line.
- Linear equation in two variables :

  An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are both non zero, is called a linear equation in two variables.

  e.g. $x + y = 16$, $p + 4q = 7$, $3 = \sqrt{7}x - y$ and $21 + m = 3$

  All are linear equations in two variables.
- **Solution of linear equation in two variables :**

  Any pair of values of x and y which satisfies the equation $ax + by + c = 0$, is called its solution. This solution can be written as an ordered pair (x, y), first writing value of x and then value of y.
- Linear equation in two variables has infinitely many solutions.

  For finding the solution of linear equation in two variables (*i.e.,* $ax + by + c = 0$), we use following steps :

  Step 1: Write the given equation in two variables, if not present.

  Step 2: Put an arbitrary value (for convenience put $x = 0$ or $y = 0$) of x (or y) in the given equation and then it reduces into linear equation of one variable, which gives a unique solution. Thus, we get one pair of solution of given equation.

  Step 3: Repeat step 2 for another arbitrary value of x (or y) and get another pair of solution of given equation.

Example 1

Find four different solutions of the equation $2x + y = 7$.

Solution:

Step I : Write the given linear equation.

Given, linear equation in two variables is

$$2x + y = 7 \quad \text{...(i)}$$

Step II : Put an arbitrary value of x (or y) in the given equation and find corresponding value of y (or x).

or, On putting $x = 0$ in *eq.* (i), we get

$$2(0) + y = 7$$

or, $y = 7$

So, (0, 7) is a solution of the given equation

or, On putting $y = 0$ in eq. (i), we get

$$2x + 0 = 7$$

or, $x = \frac{7}{2}$

So, $(\frac{7}{2}, 0)$ is also a solution of the given equation.

Step III : Repeat step 2 for other solutions.

or, On putting $x = 1$ in eq. (i), we get

$$2(1) + y = 7$$

or, $y = 5$

So, (1, 5) is also a solution of the given equation.

or, On putting $y = 1$ in eq. (i), we get

$$2x + 1 = 7$$

or, $2x = 6$

or, $x = 3$

So, (3, 1) is also a solution of the given equation.

Step IV : Write all the solutions.

(0, 7), $(\frac{7}{2}, 0)$, (1, 5) and (3, 1) are four solutions of the given equation.

Mnemonics

1. Solving equations

"Don't Call Me After Midnight"

(i) **D**istribute (multiply term outside parentheses by what's inside)

$$x + 4(2 + 3x) = 21$$

(ii) Combine like terms

$$x + 8 + 12x = 21$$

(iii) Move variable

$$x + 12x$$

(iv) Add or subtract

$$21 - 8$$

(v) Multiply or divide

$$13x = 13$$

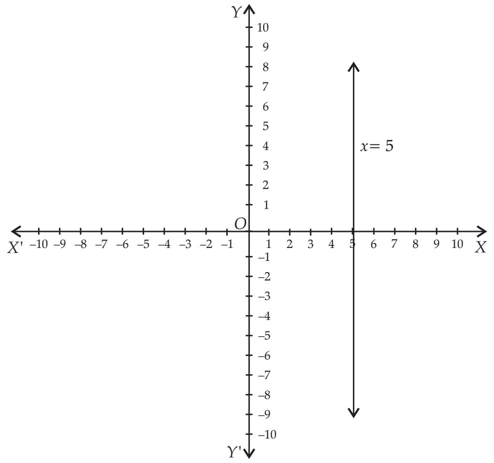
$$x = \frac{13}{13} = 1$$

Topic-2 Plotting the Equations

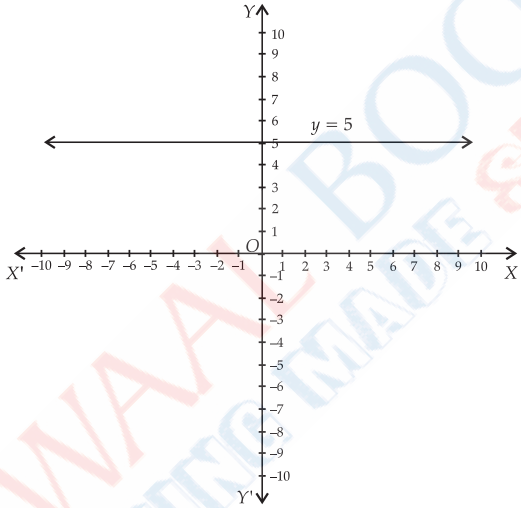
Revision Notes

- An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are both non-zero, is called a linear equation in two variables.
- A linear equation in two variables is represented graphically by a straight line, the points of which make up the collection of solutions of equation. This is called the graph of the linear equation.
- A linear equation in two variables has infinitely many solutions.
- $x = 0$ is the equation of the Y-axis and $y = 0$ is the equation of the X-axis.
- The graph of $x = k$ is a straight line parallel to the y-axis.

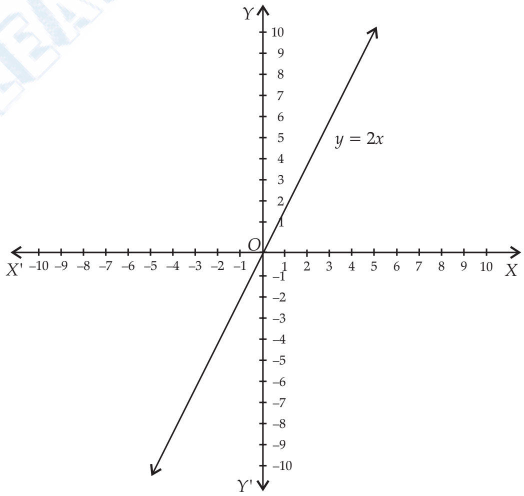
  (As shown in figure) Here, $k = 5$



- The graph of $y = k$ is a straight line parallel to the X-axis. (As shown in figure)
  Here, $k = 5$



- An equation of the type $y = mx$ represents a line passing through the origin, where m is a real number and is called slope of the line.
  (As shown in figure) Here, $m = 2$



- Every point on the line satisfies the equation of the line and every solution of the equation is a point on the line.
  Method of plotting the graph of linear equation in two variables :
  Let the linear equation in two variables be $ax + by + c = 0$, where $a \neq 0$ and $b \neq 0$. Then, to draw its graph, we use the following steps :

Step 1 : Write the given linear equation and express y in terms of x.

i.e., $by = -(ax + c)$

or, $y = \frac{-(ax+c)}{b}$ ...(i)

Step 2 : Put the different arbitrary values of x in *eq.* (i) and find the corresponding values of y.

Step 3 : Form a table as following, by writing the values of y below the corresponding values of x.

x	x_1	x_2	x_3	x_4
y	y_1	y_2	y_3	y_4

Step 4 : Draw the co-ordinate axes on graph paper and take a suitable scale to plot points (x_1, y_1), (x_2, y_2), (x_3, y_3) ... on graph paper.

Step 5 : Join the points (x_1, y_1), (x_2, y_2), (x_3, y_3) ... by a straight line and produce it on both sides.

Hence, the line so obtained is the required graph of the given linear equation.

Note : It is advisable to choose integral values of x for step 2 in such a way that the corresponding values of y are also integers.

Example 2

Draw the graph of the equation $2x + y = 3$.

Solution:

Step I : Write given linear equation and express in terms of x.

Given linear equation is $2x + y = 3$

or, $y = 3 - 2x$ ...(i)

Step II : Put different arbitrary values of x and find the corresponding values of y.

when $x = 0$, then $y = 3 - 2(0) = 3$

when $x = 1$, then $y = 3 - 2(1) = 1$

when $x = -1$, then $y = 3 - 2(-1) = 5$

Step III : Form the table.

From the above step, we get the following table :

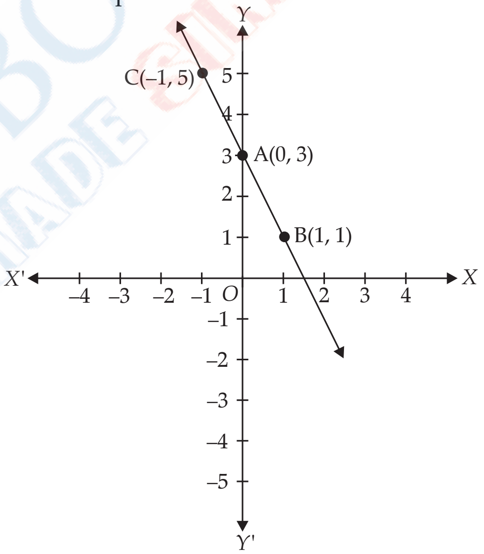
x	0	1	–1
y	3	1	5

Step IV : Draw the co-ordinate axes and plot the points on graph paper.

Draw the co-ordinate axes XOX′ and YOY′ and plot the points A(0, 3), B(1, 1) and C(–1, 5) by taking suitable scale.

Step V : Join the points by a straight line.

On joining the points A, B and C, we get a straight line BC and produce it on both sides.



Hence, line BC represents the required graph of given linear equation.

UNIT – III: CO-ORDINATE GEOMETRY

CHAPTER-4

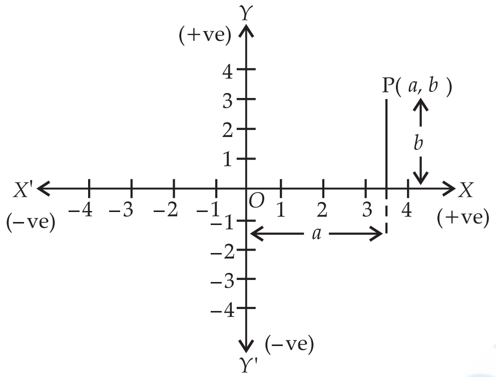
CO-ORDINATE GEOMETRY

Topic-1 Cartesian System

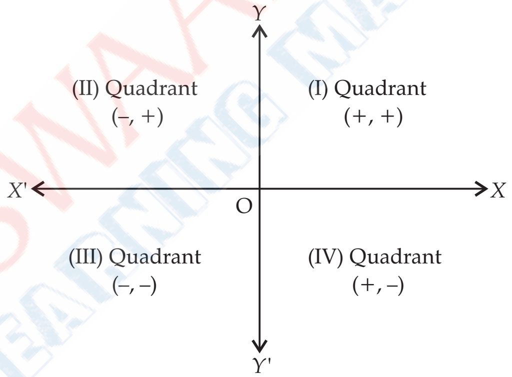
Revision Notes

➢ **Cartesian System :** The system by which we can describe the position of a point in a plane is called Cartesian System.

- In Cartesian System, two mutually perpendicular lines (one horizontal and other vertical) are required to locate the position of a point or an object.
- The plane is called the Cartesian or co-ordinate plane and the lines are called the co-ordinate axes.
- The horizontal line XOX' is called the X–axis and vertical line YOY' is called Y–axis.



- The point where XOX' and YOY' intersect is called the origin, and is denoted by O.
- Location of a point P in cartesian system, written in the form of ordered pair say $P(a, b)$ in above figure.
- a is the length of perpendicular of $P(a, b)$ from Y-axis and is called abscissa of P.
- b is the length of perpendicular of $P(a, b)$ from X-axis and is called ordinate of P.
- Positive numbers lie on the directions OX and OY, are called the positive directions of the X-axis and the Y-axis, respectively. Similarly, OX' and OY' are called the negative directions of the X-axis and the Y-axis respectively.
- The co-ordinate axes divide the plane into four parts called quadrants (one-fourth part) numbered I, II, III and IV anti-clockwise from OX.



- **How to write the co-ordinates of a point :**

- x – co-ordinate (or abscissa) = perpendicular distance of a point from Y-axis.
- y – co-ordinate (or ordinate) = perpendicular distance of a point from X-axis.
- If abscissa of a point is x and ordinate is y, then the co-ordinates of the point are (x, y).
- The abscissa of every point on Y-axis is zero.
- The ordinate of every point on X-axis is zero.
- Co-ordinate of a point on X-axis are of the form $(x, 0)$.
- Co-ordinate of a point on Y-axis are of the form of $(0, y)$.
- X-axis and Y-axis intersect at origin, represented by O and its co-ordinates are (0, 0).

Example. On which axes do the given points lie.

A (0, 2), B(– 3, 0), C(0, – 3), D(0, 4), E(6, 0), F(3, 0).

Solution : On X-axis : B(–3, 0), E(6, 0), F(3, 0)

On Y-axis : A(0, 2), C(0, – 3), D(0, 4)

Example 1

Write the quadrant in which each of the following points lie :

(i) (– 2, – 3), **(ii)** (3, – 4), **(iii)** (– 1, 2)

Sol. If both (abscissa and ordinate) co-ordinates are positive, then point lie in I quadrant. If abscissa is negative and ordinate is positive, then point lies in II quadrant. If both co-ordinates are negative then point lies in III quadrant. If abscissa is positive and ordinate is negative, then point lies in IV quadrant.

Hence (i) the point (– 2, – 3) lies in III quadrant because its both co-ordinates are negative.

(ii)The point (3, – 4) lies in IV quadrant because its x-co-ordinate is positive and y-co-ordinate is negative.

(iii) The point (– 1, 2) lies in II quadrant because its x-co-ordinate is negative and y-co-ordinate is positive.

Topic-2 Co-ordinate Plane

Revision Notes

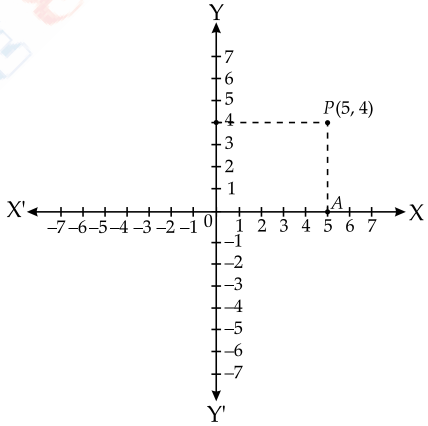
➢ **Plotting a Point in the plane if its Co-ordinates are given:** Let us suppose the co-ordinates of a point be (5, 4). To plot this point in the co-ordinate plane, following steps are followed :

Step 1 : Draw the co-ordinate axes and choose units such that one centimetre represents one unit on both the axes.

Step 2 : Starting from the origin O, count the 5 units on the positive X-axis and mark the corresponding point as A.

Step 3 : Starting from the point A, count the 4 units on the positive Y-axis and mark the corresponding point as P.

Step 4 : Point P is the position of the point (5, 4), as distance of point P from Y-axis is 5 units and distance from X-axis is 4 units.



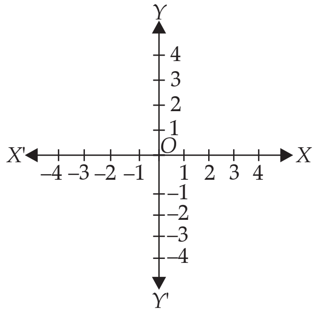
Example 2

Plot the points (3, 4) and (– 3, 4) on a graph paper.

Solution:

Step I : Draw the co-ordinate axes and write unit as both the axes at the same distance as 1 unit on all four directions of both axes.

Here XOX' = X axis and YOY' = Y-axis.



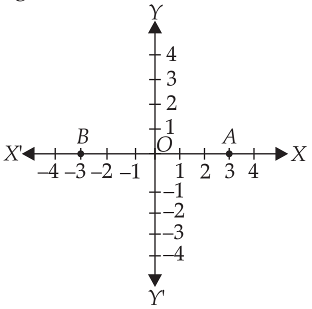
Step II : Find the direction and distance of given points corresponding to axes.

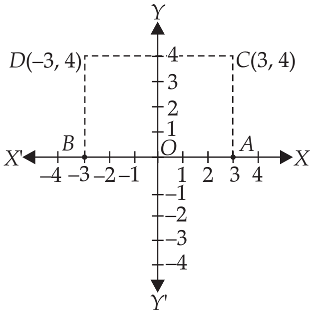
Step IV : Show the y-co-ordinates of both point Start from the point A count 4 units in the positive direction of y-axis and mark it C. Similarity from point B count 4 units in the positive direction of y-axis and mark it D.

Hence C is the point (3, 4) and D is the point (– 3, 4) on the graph paper.

Here the co-ordinates of the point (3, 4) shows that the distance of this point from Y-axis is 3 units in positive direction and from X-axis is 4 units in positive direction. Similarity, co-ordinates of the point (– 3, 4) show that distance of this point from y-axis (–3) units in negative direction and from x-axis is 4 units in positive direction

Step III : Show the x-co-ordinates of both points.
Count 3 units from origin on the positive x-axis and Mark the point as A. Also count 3 units from origin on the negative x-axis and mark it B.





UNIT – IV: GEOMETRY

CHAPTER-5

INTRODUCTION TO EUCLID'S GEOMETRY

Topic-1 Euclid's Geometry

Revision Notes

- **Axiom :** Axioms are the assumptions which are obvious universal truths. They are not proved.
- **Euclid's Axioms :**

- Things which are equal to the same thing are equal to one another.

  e.g., If $\overrightarrow{AB} = \overrightarrow{PQ}$ and $\overrightarrow{PQ} = \overrightarrow{XY}$, then $\overrightarrow{AB} = \overrightarrow{XY}$.

- If equals are added to equals, the wholes are equal.

  e.g., If $m\angle 1 = m\angle 2$, then

  $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

- If equals are subtracted from equals, the remainders are equal.

  e.g., If $m\angle 1 = m\angle 2$, then

  $m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3$.

- Things which coincide with one another are equal to one another.

  e.g., If $\overrightarrow{AB}$ coincides with $\overrightarrow{XY}$, such that A falls on X and B falls on Y, then $\overrightarrow{AB} = \overrightarrow{XY}$

- The whole is greater than the part.

  e.g., If $m\angle 1 = m\angle 2 + m\angle 3$, then $m\angle 1 > m\angle 2$ & $m\angle 1 > m\angle 3$.

- Things which are double of the same thing are equal to one another.

e.g., If $a = 2c$ and $b = 2c$, then $a = b$.

- Things which are halves of the same thing are equal to one another.

  e.g., If $a = \frac{c}{2}$ and $b = \frac{c}{2}$, then $a = b$

Mnemonics

Postulates are true assumptions specific to Geometry (PG-Post Graduate) Axioms are true assumption, not specifically linked to geometry (Requires no proof) (AP–Andhra Pradesh).

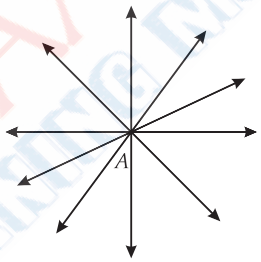
Topic-2 Euclid's Postulates

Revision Notes

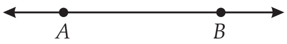
- **Postulates :** The basic facts which are taken for granted, without proof and which are specific to geometry are called **postulates**.
- **Plane :** A plane is a surface such that the line obtained by joining any two points in it will be entirely in the plane.
- **Incidence Axioms on lines :**

  (i) A line contains infinitely many points.

  (ii) Through a given point A, (infinite) lines can be drawn.



  (iii) One and only one line can be drawn to pass through two given points A and B.



- **Collinear points :** Three or more points are said to be **collinear**, if there is a straight line which passes through all of them.

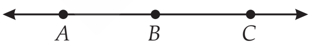


Figure I

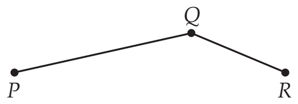
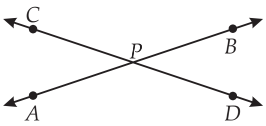


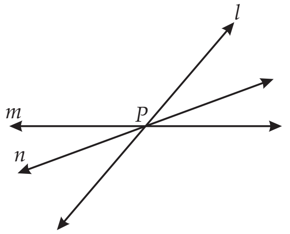
Figure II

In figure I; A, B, C are collinear points, while in figure II; P, Q, R are non-collinear points.

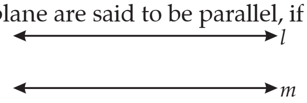
- **Intersecting lines :** Two lines which cut at one point are said to be **intersecting lines**. The point P common to two given line segments AB and CD is called their **point of intersection.**



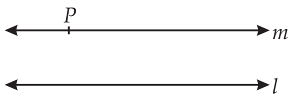
- **Concurrent lines :** Three or more lines intersecting at a same point are said to be concurrent.



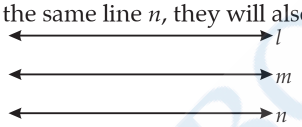
- **Parallel lines :** Two lines l and m in a plane are said to be parallel, if they have no point in common and we write $l \,||\, m$.



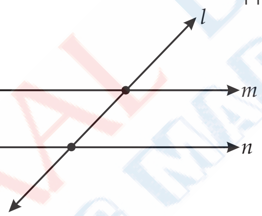
  The distance between two parallel lines always remains the same.
- Two distinct lines cannot have more than one point in common.
- **Parallel Line Axiom :** If l is a line and P is a point not on the line l, there is one and only one line (say m) which passes through P and parallel to l.



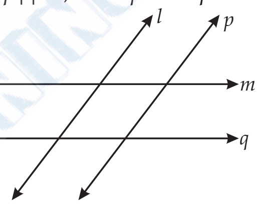
- If two lines l and m are both parallel to the same line n, they will also be parallel to each other.



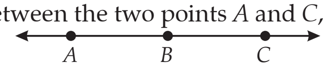
- If l, m, n are lines in the same plane such that l intersects m and $n \,||\, m$, then l also intersects n.



- If l and m are intersecting lines, $l \,||\, p$ and $q \,||\, m$, then p and q also intersect.



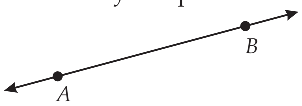
- If line segments AB, AC, AD and AE are parallel to a line l, then points A, B, C, D and E will be collinear.
- **Betweenness :** Point B is said to lie between the two points A and C, if :



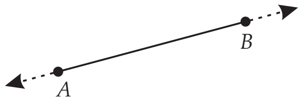
  (i) Points A, B and C are collinear, and
  (ii) $AB + BC = AC$.
- **Mid-point of a Line-segment :** For a given line segment AB, a point M is said to be the mid-point of AB, if :

  (i) M is an interior point of AB, and
  (ii) $AM = MB$.
- **Euclid's Five Postulates :**

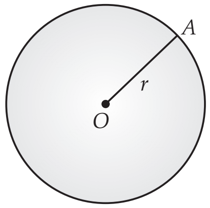
  Postulate 1 : A straight line can be drawn from any one point to another point.



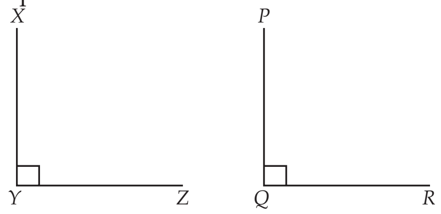
  Postulate 2 : A terminated line can be produced indefinitely *i.e.*, 'A line segment can be extended on either side to form a line'.



Postulate 3 : A circle can be drawn with any centre and any radius.



Postulate 4 : All right angles are equal to one another.



If $\angle XYZ = 90°$ and $\angle PQR = 90°$, then $\angle XYZ = \angle PQR$. [congruent angles]

Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less then two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

- Two equivalent versions of the Euclid's 5$^{\text{th}}$ postulates :
  - **(i)** For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.
  - **(ii)** Two distinct intersecting lines cannot be parallel to the same line.
- Non-Euclidean geometries :

  All the attempts to prove the Euclid's fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.
- **Theorems :** Theorems are statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.

CHAPTER-6

LINES AND ANGLES

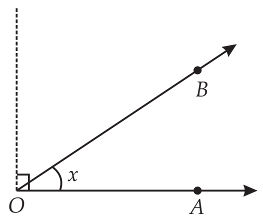
Topic-1 Types of Angles

Revision Notes

- **Line :** Line is a collection of points which has only length neither breadth nor thickness.
- **Line Segment :** A line with two end points.
- **Ray :** A part of line with one end point.
- **Angle :** An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms and the end point is called the vertex.
- **Types of Angles :**

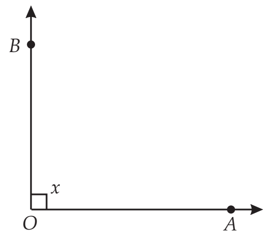
  (i) Acute Angle : $0° < x < 90°$

  An angle whose measure is more than 0° but less than 90° is called an Acute angle.



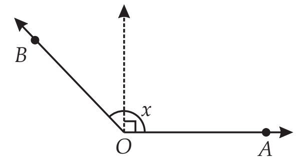
(ii) Right Angle : $x = 90°$

An angle whose measure is 90°, is called a right angle.



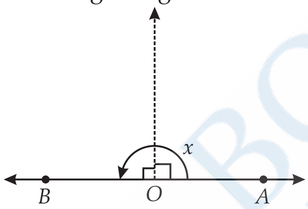
(iii) Obtuse Angle : $90° < x < 180°$

An angle whose measure is more than 90° but less than 180° is called an obtuse angle.



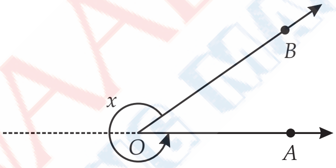
(iv) Straight Angle : $x = 180°$

An angle whose measure is 180° is called a straight angle.



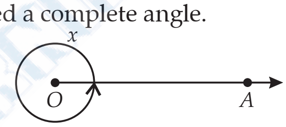
(v) Reflex Angle : $180° < x < 360°$

An angle whose measure is more than 180° but less than 360° is called a reflex angle.

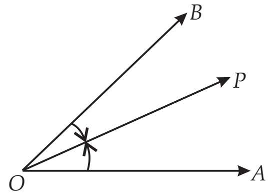


(vi) Complete Angle : $x = 360°$

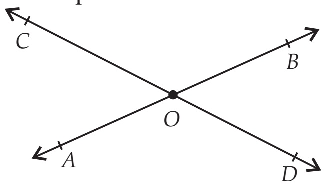
An angle whose measure is 360° is called a complete angle.



- **Complementary Angles :** Two angles whose sum is 90° are called complementary angles.
  e.g. **:** Complement of 30° angle is 60° angle.
- **Supplementary Angles :** Two angles whose sum is 180° are called supplementary angles.
  e.g. **:** Supplement of 70° angle is 110° angle.
- **Adjacent Angles :** Two angles are called adjacent angles, if :
  (i) they have the same vertex,
  (ii) they have a common arm, and
  (iii) uncommon arms on opposite side of the common arm.
  In the figure, $\angle AOP$ and $\angle BOP$ are adjacent angles.



- **Vertically Opposite Angles :** When two straight lines intersect each other four angles are formed. The pair of angles which lie on the opposite sides of the point of intersection are called vertically opposite angles.



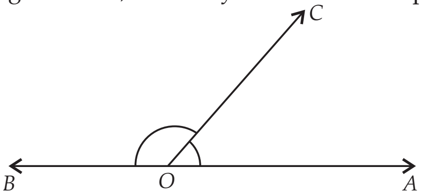
In figure, $\angle AOC$ and $\angle BOD$ are vertically opposite angles and $\angle AOD$ and $\angle BOC$ are also vertically opposite angles.

Vertically opposite angles are always equal.

➢ **Linear Pair of Angles :** Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.

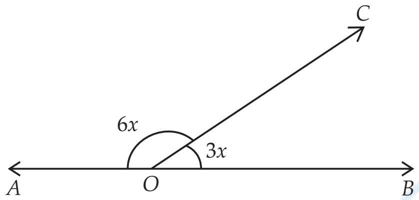
OR

When the sum of two adjacent angles is 180°, then they are called linear pair of angles.



In figure, $\angle AOC$ and $\angle BOC$ form a linear pair of angles.

Examples: Find *x* in the figure given below



Solution :

$$6x + 3x = 180° \quad \text{[Linear pair]}$$
$$9x = 180°$$
$$x = \frac{180°}{9} = 20°$$

Mnemonics

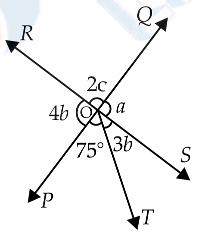
1. To draw an angle with the help of protractor we use the mnemonic 'LIRO'—left inner right outer to draw angle on left side of a line we use inner scale of protractor to draw from right side we use outer scale.
2. Easy way to learn supplementary and complementary angles.

Supplementary = 180° (It makes 8)

Complementary = 90° (It makes 9)

Example 1

In the given figure, two straight lines *PQ* and *RS* intersect each other at *O*. If $\angle POT = 75°$, find the values of *a, b, c*.



Solution:

Step-I : Identify the straight line and use the suitable property to find the value of *b*.

Here *ROS* is a straight line. So by property that sum of all angles on a straight line is 180°, we get

$\therefore \quad \angle ROP + \angle POT + \angle TOS = 180°$

$4b + 75° + 3b = 180°$

or, $\quad 7b + 75° = 180°$

or, $\quad 7b = 180° - 75° = 105°$

or, $\quad b = \frac{105°}{7} = 15°$

and $\quad a + 2c = 180° \qquad$ ...(i)

Step-II : Use the property which gives relation for *a* and find the value of *a*.

Since, vertically opposite angles are equal

$\therefore \quad a = 4b$

or, $\quad a = 4 \times 15°$

or, $\quad a = 60° \qquad$ ...(ii)

Step-III : Solve eqn. (i) and (ii) to find the value of *c*.

From eqn. (i) we get

$a + 2c = 180°$

or, $\quad 60° + 2c = 180°$

or, $\quad 2c = 180° - 60° = 120°$

or, $\quad c = \frac{120°}{2}$

$= 60°$

Hence, $\quad a = 60°, b = 15°, c = 60°$

Topic-2 Transversal Line

Revision Notes

- **Intersecting Lines :** Two lines are said to be intersecting when the perpendicular distance between the two lines is not same everywhere. They meet at one point.
- **Non-Intersecting lines :** Two lines are said to be non-intersecting lines when the perpendicular distance between them is same every where. They do not meet. If these lines are in the same plane these are known as **parallel lines.**
- **Transversal Line :** A straight line which intersects two or more given lines at distinct points is called a transversal of the given lines.

  In figure, straight lines l and m are intersected by transversal r. Following angles are formed :

  Exterior Angles : $\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$.

  Interior Angles : $\angle 2$, $\angle 3$, $\angle 5$ and $\angle 8$.

  Corresponding Angles : Two angles on the same side of a transversal are known as corresponding angles if both lie either above the lines or below the lines.

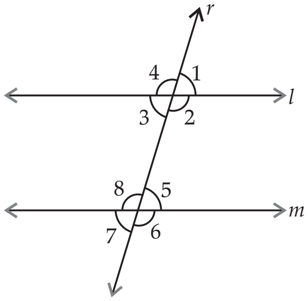
  In figure, $\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$, $\angle 4$ & $\angle 8$ are the pairs of corresponding angles.

  Alternate Interior Angles : $\angle 2$ & $\angle 8$, $\angle 3$ & $\angle 5$ are the pairs of alternate interior angles.

  Alternate Exterior Angles : $\angle 1$ & $\angle 7$, $\angle 4$ & $\angle 6$ are the pairs of alternate exterior angles.

  Consecutive Interior Angles : The pair of two interior angles on the same side of the transversal are called the pairs of consecutive interior angles.

  In figure, $\angle 2$ & $\angle 5$, $\angle 3$ & $\angle 8$ are the pairs of consecutive interior angles.



  If a transversal intersects two parallel lines, then :

  (i) each pair of corresponding angles is equal.

  (ii) each pair of alternate interior angles is equal.

  (iii) each pair of interior angles on the same side of the transversal is supplementary.

  If a transversal intersects two lines such that, either :

  (i) any one pair of corresponding angles is equal, or

  (ii) any one pair of alternate interior angles is equal, or

  (iii) any one pair of co-interior angles is supplementary, then the lines are parallel.

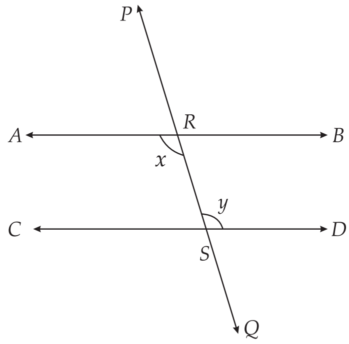
THEOREM

- **Theorem 1 :**

  Statement : If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

  Given : $AB||CD$ and PQ is the transversal, x and y are alternate interior angles.

  To Prove :



  Proof :

  In the given figure $\angle PRB = y$ ...(i) (Corresponding Angles)

  $\angle PRB = x$ ...(ii) (Vertically Opposite Angles)

  From equation (i) and (ii), we get

  $x = y$, **Hence Proved.**

- **Theorem 2 :**

  Statement : If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.
- **Theorem 3 :**

  Statement : If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.
- **Theorem 4 :**

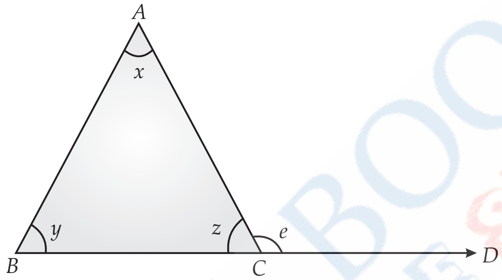
  Statement : If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.
- **Theorem 5 :**

  Statement : Lines which are parallel to the same line are parallel to each other.
- **Theorem 6 :**

  Statement : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

  Given : A Triangle ABC with interior angles x, y and z, and exterior angle 'e'.

  To Prove :



Proof : In the figure above :

$x + y + z = 180°$ ...(i) (Angle Sum Property)

$e + z = 180°$ ...(ii) (Linear Pair)

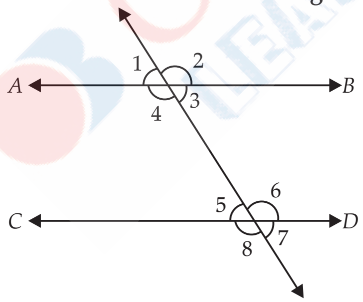
Comparing equations (i) and (ii),

$x + y + z = e + z$

therefore, $x + y = e$, **Hence Proved.**

Example 2

In the given figure AB | | CD and ∠1 and ∠2 are in the ratio 4 : 5. Determine all the angles from 1 to 8.



Solution:

Step I : Write the given ratio of angles in terms of variable (say x)

Given ratio = 4 : 5

Let $\angle 1 = 4x$ and $\angle 2 = 5x$

Step II : Use the suitable property to find the value of x.

We know that, sum of angles on a straight line is 180°.

$\therefore \quad \angle 1 + \angle 2 = 180°$ [AB is a straight line]

$\Rightarrow \quad 4x + 5x = 180°$

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = \dfrac{180°}{9} = 20°$

Step III : Put the value of x and get the measure of ∠1 and ∠2.

We have,

$\angle 1 = 4x = 4 \times 20° = 80°$

and $\angle 2 = 5x = 5 \times 20° = 100°$

Step IV : Find ∠3 and ∠4 by using suitable property.

We know that vertically opposite angles are equal.

$\therefore \quad \angle 3 = \angle 1 = 80°$

and $\angle 4 = \angle 2 = 100°$

Step V : Find ∠5 and ∠6 by using suitable property.

We know that, alternate interior angles are equal.

$\therefore \quad \angle 5 = \angle 3 = 80°$

and $\angle 6 = \angle 4 = 100°$

Step VI : Find ∠7 and ∠8 using suitable property.

We know that, vertically opposite angles are equal.

$\therefore \quad \angle 7 = \angle 5 = 80°$

and $\angle 8 = \angle 6 = 100°$

Hence, $\angle 1 = \angle 3 = \angle 5 = \angle 7 = 80°$

$\angle 2 = \angle 4 = \angle 6 = \angle 8 = 100°$

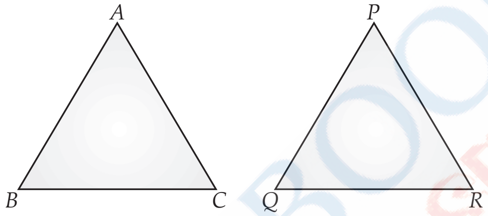
CHAPTER-7

TRIANGLES

Topic-1 Criteria for Congruence of Triangles

Revision Notes

- **Congruence of Triangle** : The geometrical figures of same shape and size are congruent to each other *i.e.*, two triangles ΔABC and ΔPQR are congruent if and only if their corresponding sides and the corresponding angles are equal.



If two triangles ΔABC and ΔPQR are congruent under the correspondence $A \longrightarrow P$, $B \longrightarrow Q$ and $C \longrightarrow R$, then symbolically it is expressed as

$$\Delta ABC \cong \Delta PQR$$

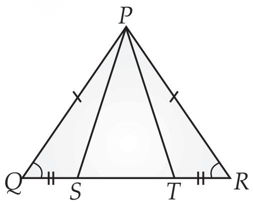
- ***SAS* Congruence Rule :** Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.
- ***ASA* Congruence Rule :** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.
- ***AAS* Congruence Rule :** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.
- ***SSS* Congruence Rule :** If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- ***RHS* Congruence Rule :** If in two right triangles, the hypotenuse and one side of a triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Example 1

Q. In an isosceles ΔPQR with $PQ = PR$, S and T are points on QR such that $QT = RS$ show that $PS = PT$.

Solution:

Step I : Read the question carefully and write the given conditions.



ΔPQR is an isosceles triangle in which

$PQ = PR$ ...(i)

S and T are points on QR such that

QT = RS ...(ii)

Step II : Apply the theorems related to given conditions to find other information.

Since $PQ = PR$

So $\angle R = \angle Q$ ...(iii)

[Property of isosceles triangle]

From eqn. (ii) we have $QT = RS$

$\Rightarrow$ $QT - ST = RS - ST$

[Subtracting ST from both sides]

$\Rightarrow$ $QS = RT$ ...(iv)

Step III : Apply the suitable congruence rule in two triangles.

In ΔPQS and ΔPRT, $PQ = PR$ [From eqn. (i)]

$\angle Q = \angle R$ [From eqn. (iii)]

and $QS = RT$ [From eqn. (iv)]

$\therefore$ $\Delta PQS \cong \angle PRT$

(By SAS congruence rule)

Step IV : Apply the property CPCT *i.e.*, corresponding part of congruent triangles, to get the required result.

As $\Delta PQS \cong \Delta PRT$

Then $PS = PT$ (CPCT)

Mnemonics

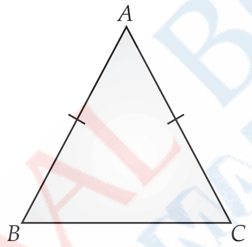
SSS has filled the form with SAS and AAS.
Concept: Congruency of a triangle
Interpretation

Side Side Side	SSS (3 sides are equal)
Side Angle Side	SAS (2 sides and included angles)
Angle Angle Side	AAS (2 angles and corresponding side)

Topic-2 Properties of Triangles

Revision Notes

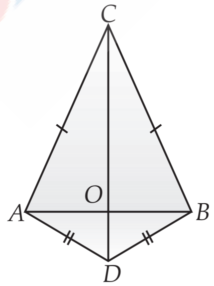
- A triangle is isosceles if its any two sides are equal. Here, we will discuss some properties related to isosceles triangle.
  - **(i)** Angles opposite to equal sides of a triangle are equal.
    In figure, $\angle B = \angle C$



  - **(ii)** The sides opposite to equal angles of a triangle are equal.
    In figure, $AB = AC$
- In an isosceles triangle, bisector of the vertical angle of a triangle bisect the base.
- The medians of an equilateral triangle are equal in length.
- A point equidistant from two intersecting lines lies on the bisector of the angles formed by the two lines.

Example 2

AB is a line segment. C and D are points on opposite side of AB such that each of them is equidistant from the points A and B. Show that line CD is the perpendicular bisector of AB.



Solution:

Step I : Read the question carefully and write all given conditions.

AB is a line segment. C and D are points on opposite sides of AB such that

$CA = CB$ ...(i)

and $DA = DB$ ...(ii)

CD intersects AB at point O.

Step II : Find what is given to show.

CD is the perpendicular bisector of AB.

Step III : Show that ΔCAD and ΔCBD are congruent, further use CPCT to find relation between angles.

In ΔCAD and ΔCBD

$CA = CB$ [from eqn. (i)]

$AD = BD$ [from eqn. (ii)]

and $CD = CD$ [Common side]

$\therefore$ $\Delta CAD \equiv \Delta CBD$

[By SSS Congruence rule]

or, $\angle ACD = \angle BCD$ [By CPCT]

or $\angle ACO = \angle BCO$ ...(iii)

Step IV : Show that ΔCAO and ΔCBO are congruent and further use CPCT to find relation between angles and sides.

In ΔCAO and ΔCBO

$CA = CB$ [from eqn. (i)]

$\angle ACO = \angle BCO$ [from eqn. (iii)]

and $CO = CO$ [Common side]

$\therefore \quad \Delta CAO \equiv \Delta CBO$ [By SAS Congruence rule]

or, $AO = BO$ ...(iv) [By CPCT]

and $\angle AOC = \angle BOC$ ...(v) [By CPCT]

Step V : Since AB is a line segment, so we use the property of linear pair and find the measure of $\angle AOC$ or $\angle BOC$.

AB is a line segment. So,

$\angle AOC + \angle BOC = 180°$

or, $\angle AOC + \angle AOC = 180°$ [from eqn. (v)]

or, $2\angle AOC = 180°$

or, $\angle AOC = \frac{180°}{2}$

or, $\angle AOC = 90°$

$AO = BO$ [From eqn. (iv)]

$\angle BOC = \angle AOC = 90°$ each

Hence, CD is the perpendicular bisector of AB.

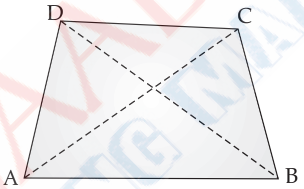
CHAPTER-8

QUADRILATERALS

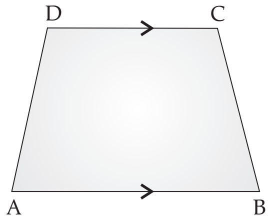
Topic-1 Types of Quadrilaterals

Revision Notes

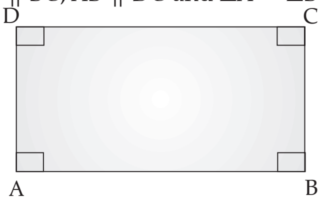
- A quadrilateral is a closed figure obtained by joining four points (with no three points collinear) in an order.



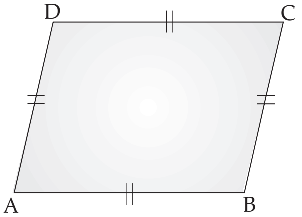
- A diagonal is a line segment obtained on joining the opposite vertices of a quadrilateral.
- Two sides of a quadrilateral having no common end point are called its opposite sides.
- Two angles of a quadrilateral having common arm are called its adjacent angles.
- Two angles of a quadrilateral not having a common arm are called its opposite angles.
- A trapezium is a quadrilateral in which one pair of opposite sides is parallel. In fig. below, $ABCD$ is a trapezium with sides $AB \parallel DC$ and non-parallel sides AD and BC.



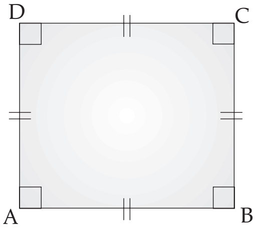
- If the non-parallel sides of a trapezium are equal, then it is known as isosceles trapezium.
- A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.
- A rectangle is a parallelogram in which opposite sides are equal and each angle is 90°. In fig. below, $ABCD$ is a rectangle with $AD = BC$, $AB = DC$, $AD \parallel BC$, $AB \parallel DC$ and $\angle A = \angle B = \angle C = \angle D = 90°$.



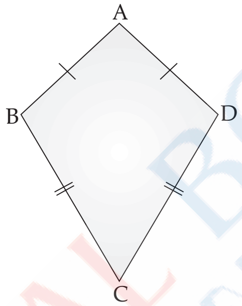
- A rhombus is a parallelogram all the sides of whose are equal. In fig. below, $ABCD$ is a rhombus with $AB = BC = CD = DA$.



➢ A square is a parallelogram in which all sides are equal and each angle is 90°. In fig. below, $ABCD$ is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.

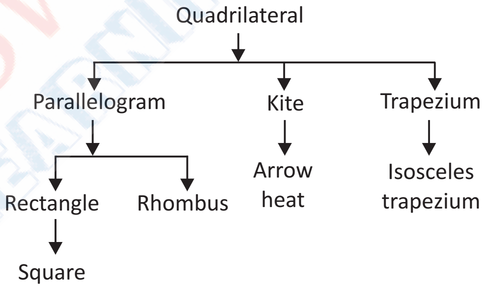


➢ A kite is a quadrilateral in which two pairs of adjacent sides are equal. In fig below, $ABCD$ is a kite with $AB = AD$ and $BC = CD$.



Key Facts for Quadrilateral

➢ 1. 'Quad' means four so quadrilateral is a 4 sided figure.
2. How special parallelograms are related



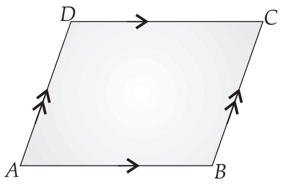
3. The sum of interior angles of a quadrilateral is 360°.
4. The diagonals of a parallelogram bisect each other.
5. The diagonals of a rhombus bisect each other at 90°
6. The diagonals of a rectangle are equal
7. The diagonals of a square are equal and bisect each other at 90°.

Topic-2 Properties of a Parallelogram

Revision Notes

➢ Opposite sides of a parallelogram are parallel.

- Opposite sides of a parallelogram are equal.
- Opposite angles of a parallelogram are equal.
- Consecutive angles (conjoined angles) of a parallelogram are supplementary.
- A diagonal of a parallelogram divides it into two congruent triangles.
- Diagonals of a parallelogram bisect each other.
- If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.
- If in a quadrilateral each pair of opposite angles is equal, then it is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
- If one pair of opposite side is equal and parallel then it is a parallelogram.
- A quadrilateral is a parallelogram, if a pair of opposite sides is equal and parallel. In fig. below, $ABCD$ is a parallelogram in which $AB \parallel DC$ and $AD \parallel BC$.



- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.
- Every rectangle is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are :
  - All the interior angles of a rectangle are right angles.
  - The diagonals of a rectangle are equal.
- Every rhombus is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :
  - All the sides of a rhombus are equal.
  - Diagonals of a rhombus bisect at right angles.
- Every square is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a square are :
  - All sides are equal.
  - All angles are equal to 90°.
  - Diagonals are equal.
  - Diagonals bisect each other at right angle.
  - Diagonals bisect the angles of vertex.

Topic-3 Mid-Point Theorem

Revision Notes

- **Mid-Point Theorem :** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
- **Converse of mid-point theorem :** The line drawn through the mid-point of one side of a triangle parallel to the another side, bisects the third side.
- If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

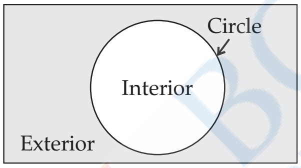
CHAPTER-9

CIRCLES

Topic-1 Basic Properties of Circles

Revision Notes

- A circle is a collection (set) of all those points in a plane, each one of which is at a constant distance from a fixed point in the plane.
- The fixed point is called the centre and the constant distance is called the radius of the circle.
- All the points lying inside a circle are called its interior points and all those points which lie outside the circle are called its exterior points.
- The collection (set) of all interior points of a circle is called the interior of the circle while the collection of all exterior points of a circle is called the exterior of the circle.



- In a circle, equal chords subtend equal angles at the centre.
- The chords corresponding to congruent arcs are equal.
- If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
- If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
- One and only one circle can be drawn through three non-collinear points.
- An infinite number of circles can be drawn through a given point P.
- An infinite number of circles can be drawn through the two given points.
- Perpendicular bisectors of two chords of a circle intersect each other at the centre of the circle.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

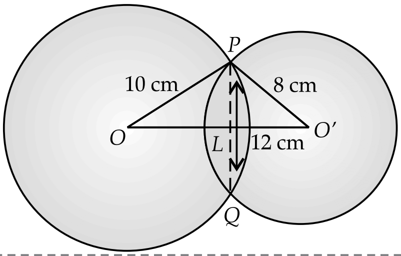
Example 1

Q. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find distance between their centres.

Solution:

Step I : Draw figure as per given information.

Let O and O' be the centres of the circles of radii 10 cm and 8 cm, respectively. Let PQ be their common chord.



Step II :Write the given information.

Given, OP = 10 cm, O'P = 8 cm and PQ = 12 cm.

Find OO'.

Step III : For finding OO', first find PL.

$$\text{PL} = \frac{1}{2}\text{PQ} = 6 \text{ cm}$$

($\because$ Perpendicular from the centre of a circle to a chord bisects the chord)

Step IV : Apply Pythagoras theorem in ΔOLP.

In right angle ΔOLP, we have

$$OP^2 = OL^2 + PL^2$$

or, $$OL = \sqrt{OP^2 - PL^2}$$

$$= \sqrt{(10)^2 - (6)^2}$$

$$= \sqrt{64}$$

$$= 8 \text{ cm}$$

Step V : Apply Pythagoras theorem in $\Delta O'LP$.

In right angle $\Delta O'LP$, we have

$$O'P^2 = PL^2 + O'L^2$$

$$O'L = \sqrt{(OP)^2 - (PL)^2}$$

$$= \sqrt{8^2 - 6^2}$$

$$= \sqrt{64 - 36}$$

$$= \sqrt{28}$$

$$= 5.29 \text{ cm.}$$

$\therefore$ $$OO' = OL + O'L$$

$$= 8 + 5.29$$

$$= 13.29 \text{ cm}$$

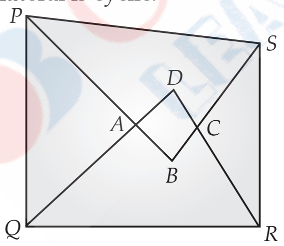
Topic-2 Cyclic Quadrilaterals

Revision Notes

- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, *i.e.*, lie on the same circle.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- **Concentric Circles :** Circles with a common centre are called concentric circles.
- The degree measure of a semi-circle is 180°.
- The degree measure of a circle is 360°.
- The degree measure of a major arc is (360°– θ), where θ is the degree measure of the corresponding minor arc.
- Area of a circle = πr^2 sq. units

Example 2

Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.



Solution:

Step I : Draw a figure according to given information.

Let PQRS be a quadrilateral in which the angle bisectors PB, QD, RD and SB of internal angles P, Q, R and S, respectively form a quadrilateral ABCD

Step II : Write the proving statement.

ABCD is a cyclic quadrilateral

i.e., $\angle A + \angle C = 180°$

or, $\angle B + \angle D = \mathbf{180°}$

Step III : For proving the result, first find the angles $\angle BAD$ and $\angle BCD$.

Since, $\angle BAD = \angle \mathbf{PAQ}$

(Vertically opposite angles)

Also, $\angle \mathbf{PAQ} = 180° - (\angle APQ + \angle AQP)$

($\because$ In ΔPAQ, using angle sum property

i.e., $\angle PAQ + \angle APQ + \angle AQP = 180°$)

$$= 180° - \frac{1}{2}(\angle APQ + \angle AQP)$$

$$= 180° - \frac{1}{2}(\angle P + \angle Q) \quad \text{...(i)}$$

[Q PB and QD are bisectors of $\angle P$ and $\angle Q$, respectively]

Similarly,

$$\angle BCD = \angle RCS$$

$$= 180° - (\angle CRS + \angle RSC)$$

$$= 180° - \frac{1}{2}(\angle R + \angle \mathbf{S}) \quad \textbf{...(ii)}$$

Step IV : Adding the results obtained in step III and further use the property of a quadrilateral, which prove the required results.

On adding eqs (i) and (ii), we get

$$\angle BAD + \angle BCD = 180° - \frac{1}{2}(\angle P + \angle Q) + 180° - \frac{1}{2}(\angle R + \angle S)$$

$$= 360° - \frac{1}{2}(\angle P + \angle Q + \angle R + \angle S)$$

$$= 360° - \frac{1}{2} \times 360°$$

($\because$ sum of angles of a quadrilateral is 360°)

$$= \mathbf{360° - 180°}$$

$$= \mathbf{180°}$$

ABCD is a cyclic quadrilateral because sum of a pair of opposite angles of quadrilateral ABCD is 180°. **Hence Proved**

UNIT – V: MENSURATION

CHAPTER-10

AREAS

Topic-1 Area of Triangle

Revision Notes

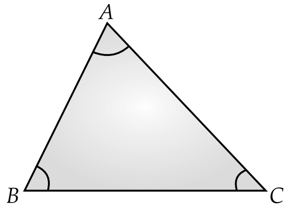
- **Parts of a Triangle**

  In ΔABC, there are :

  (i) three vertices, namely A, B and C.

  (ii) three angles, namely $\angle A$, $\angle B$ and $\angle C$.

  (iii) three sides, namely AB, BC and CA.



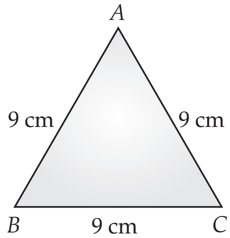
- Area $= \frac{1}{2} \times$ base $\times$ corresponding height.

- For an equilateral triangle of side 'a'.

  (i) Area $= \frac{\sqrt{3}}{4}a^2$

  (ii) Perimeter $= 3a$

  (iii) Altitude $= \frac{\sqrt{3}}{2}a$



Example: Find the area of an equilateral triangle with side 9 cm.

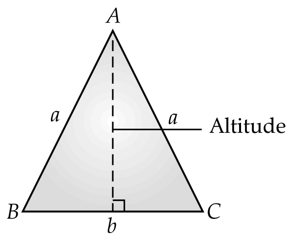
Solution: Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times a^2 = \frac{\sqrt{3}}{4} \times (9)^2$

$$= \frac{81\sqrt{3}}{4} \text{ cm}^2$$

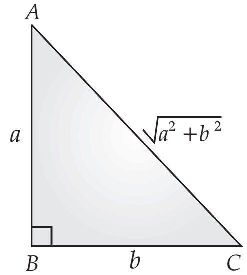
- For an isosceles triangle with length of two equal sides as 'a' and base 'b'.

  (i) Area $= \frac{b}{4}\left|\sqrt{4a^2 - b^2}\right|$

  (ii) Perimeter $= 2a + b$



(iii) Altitude $= \frac{1}{2}\left|\sqrt{4a^2 - b^2}\right|$



➢ For right angled triangle, with 'a' and 'b' are the sides that includes the right angle.

(i) Area $= \frac{1}{2} \times a \times b$

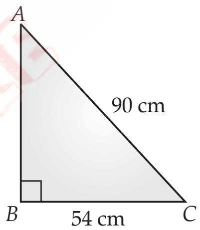
(ii) Perimeter $= (a + b + \sqrt{a^2 + b^2})$

(iii) Altitude $= a$

Example: The longest side of a right triangle is 90 cm and one of the remaining two sides is 54 cm. Find its area.

Solution: By Pythagoras Theorem

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{90^2 - 54^2}$$
$$= \sqrt{8100 - 2916}$$
$$= \sqrt{5184} = 72 \text{ cm}$$
$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 54 \times 72$$
$$= 1944 \text{ cm}^2$$

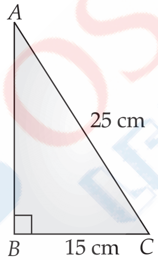


Example 1

The base of a right triangle is 15 cm and its hypotenuse is 25 cm, then calculate its area.

Solution:

Step I : We find the height (perpendicular) of the right angled triangle by using pythagoras theorem.



$$AB^2 = AC^2 - BC^2$$
$$= (25)^2 - (15)^2$$
$$= 625 - 225 = 400$$
$$AB = 20 \text{ cm}$$

Step II : Now we find area of triangle by using

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\left[A = \frac{1}{2} \times b \times h\right]$$

$$= \frac{1}{2} \times 15 \times 20$$
$$= 150 \text{ cm}^2$$

Topic-2 Heron's Formula

Revision Notes

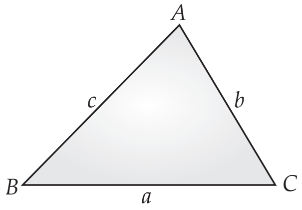
➢ **Heron's Formula**

Consider a triangle with sides a, b and c

Let $AB = c$, $BC = a$ and $CA = b$

So, Its perimeter $= a + b + c$

Semi-perimeter, $s = \frac{a+b+c}{2}$



$$\text{Area of triangle} = \left|\sqrt{s(s-a)(s-b)(s-c)}\right|$$

This formula is known as 'Heron's formula'.

This formula is applicable to all type of triangles whether it is a right triangle or an isosceles or an equilateral triangle.

Example: Find the area of a triangle when its two sides are 24 cm and 10 cm and the perimeter of the triangle is 60 cm.

Solution: Let, third side = x cm

$\therefore$ $24 + 10 + x = 60$

or, $x = 60 - 34$

$= 26$ cm

$$s = \frac{60}{2} = 30 \text{ cm} \qquad \left[s = \frac{a+b+c}{2}\right]$$

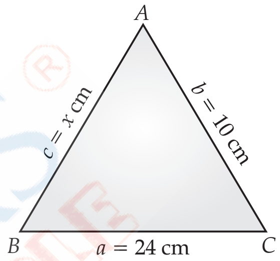
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{30(30-24)(30-10)(30-26)}$$

$$= \sqrt{30\times 6\times 20\times 4} = \sqrt{10\times 3\times 3\times 2\times 10\times 2\times 2\times 2}$$

$$= 10\times 3\times 4$$

$$= 120 \text{ cm}^2$$



Example 2

The side of a triangle are 12 cm, 16 cm, and 20 cm. Find its area.

Solution:

Step I : We find the semi-perimeter of a triangle by using Heron's formula

$$s = \frac{a+b+c}{2}$$

$$s = \frac{12+16+20}{2} = 24 \text{ cm}$$

Step II : Now we find the area of triangle by using Heron's formula

Area

$$= \left|\sqrt{s(s-a)(s-b)(s-c)}\right|$$

$$= \sqrt{24(24-12)(24-16)(24-20)}$$

$$= \sqrt{24\times 12\times 8\times 4}$$

$$= 96 \text{ cm}^2$$

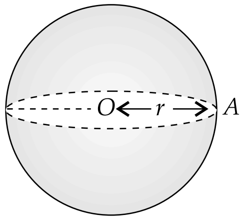
CHAPTER-11

SURFACE AREAS AND VOLUMES

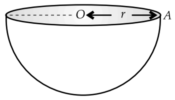
Topic-1 Surface Area and Volume of Sphere (Including Hemisphere)

Revision Notes

➢ **Sphere:** A sphere is a perfectly round geometrical object in three-dimensional space, such as the shape of a round ball.



- **Hemisphere:** A hemisphere is half of a sphere



- The total surface area of any object will be equal or greater than its lateral surface area.
- Volume is the capacity or the space occupied by a body.
- The unit of measurement of both volume and capacity is cubic unit such as cubic feet, cubic cm and cubic m, etc.
- When an object of certain volume is recast into a new shape, the volume of the new shape, formed will always be equal to the volume of the original object.
- The solids having the same curved surface do not necessarily occupy the same volume.
- When an object is dropped into a liquid, the volume of the displaced liquid is equal to the volume of the object that is dipped.

Important Formulae

- **Sphere :**

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

- **Hemisphere :**

$$\text{Curved surface area} = 2\pi r^2$$

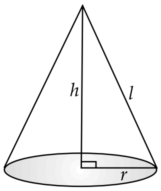
$$\text{Total surface Area} = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3}\pi r^3$$

Topic-2 Surface Area and Volume of Right Circular Cone

Revision Notes

- **Cone:** Cone is a pyramid with a circular base.



- **Right circular cone :**

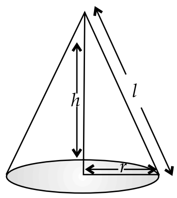
$$\text{Slant height } (l) = \sqrt{h^2 + r^2}$$

$$\text{Area of curved surface} = \pi rl = \pi r\sqrt{h^2 + r^2}$$

$$\text{Total surface area} = \text{Area of curved surface} + \text{Area of base}$$

$$= \pi rl + \pi r^2 = \pi r(l + r)$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$



Example 1

There are two cones. The ratio of their radii are 4 : 1 Also, the slant height of the second cone is twice that of the former. Find the relationship between their curved surface area.

Solution:

Step I : First consider the unknown variables.

Let r_1 and l_1 be the radius and slant height of first cone.

Let r_2 and l_2 be the radius and slant height of second cone.

Step II : Write the formula for curved surface area for both the cones.

Curved surface area of first cone (CSA_1) = $\pi r_1 l_1$ and curved surface area of second cone (CSA_2) = $\pi r_2 l_2$

Step III : Use the given condition and simplify it.

According to the question,

$$r_1 : r_2 = 4 : 1$$

or, $$\frac{r_1}{r_2} = \frac{4}{1} \text{ and}$$

$$l_2 = 2l_1 \text{ or, } \frac{l_1}{l_2} = \frac{1}{2}$$

$$\therefore \quad \frac{CSA_1}{CSA_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \left(\frac{r_1}{r_2}\right)\left(\frac{l_1}{l_2}\right)$$

$$= \left(\frac{4}{1}\right)\left(\frac{1}{2}\right)$$

$$= \frac{2}{1}$$

$\therefore$ $CSA_1 = 2CSA_2$ *i.e.*, curved surface area of first cone is twice of the second cone.

UNIT – VI: STATISTICS & PROBABILITY

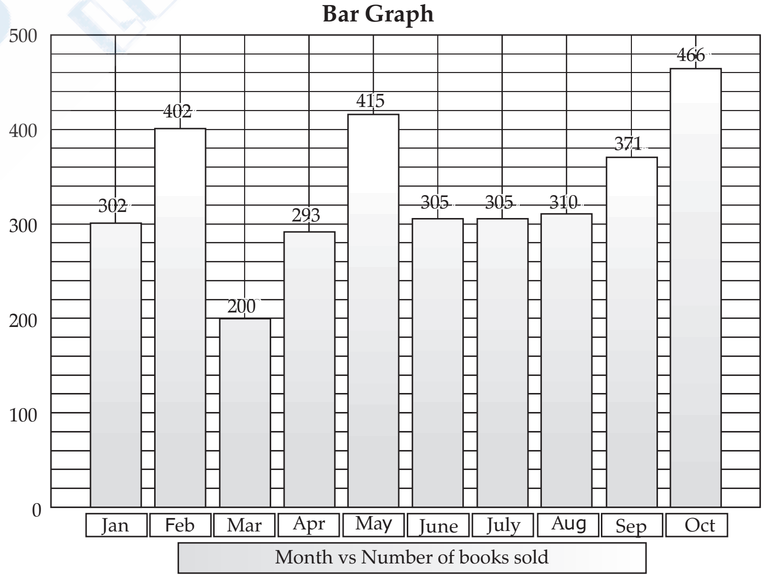
CHAPTER-12

STATISTICS

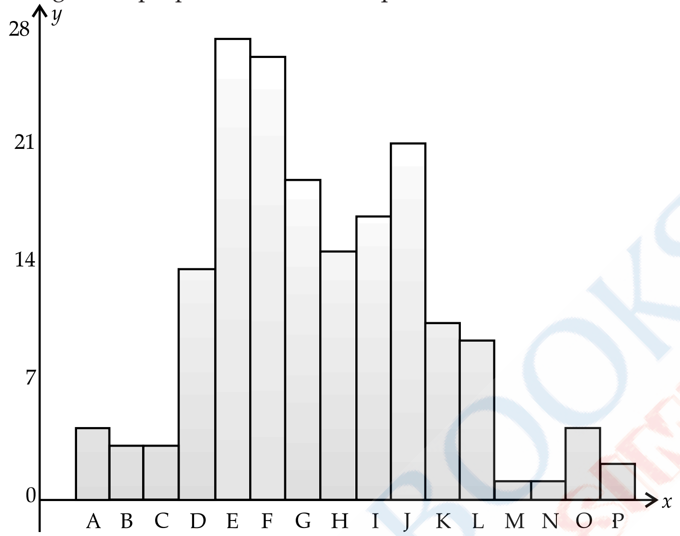
Revision Notes

- Graphical Representation of Data can be represented graphically in following ways :
  (a) Bar Graph **(b)** Histogram **(c)** Frequency polygon.
- **Bar Graph:** A bar graph is a pictorial representation of data in which rectangular bars of uniform width are drawn with equal spacing between them on one axis, usually the x-axis. The value of the variable is shown on the other axis that is the y-axis.
  Following Bar graph depicts number of books sold per month.

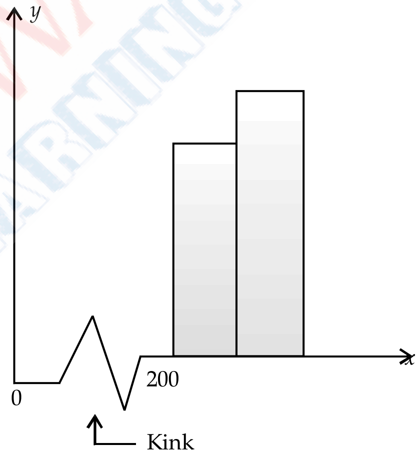
Bar Graph



- Bar charts are used for comparing two or more values.
- **Histogram:** A histogram is one of the most commonly used graphs. A histogram is a vertical bar-graph with no spacing between the bars. The histogram is constructed by the following steps :
  1. The values of the observations are taken on the x-axis with the class-limits clearly marked.
  2. The frequencies are taken along the y-axis.
  3. The base of the rectangle corresponding to a particular class is the line segment (on the x-axis) with the lower limit and the upper limit of the class as the end-points.
  4. The areas of the rectangles are proportional to the frequencies of the classes.



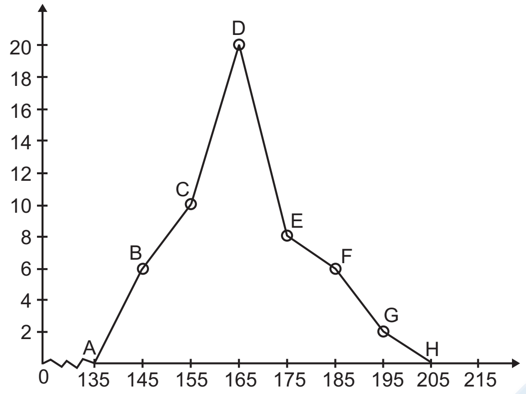
- A histogram is a set of adjacent rectangles whose areas are proportional to the frequencies of a given continuous frequency distribution. The height of rectangles corresponds to the numerical value of the data.
- The histogram is drawn only for exclusive/continuous frequency distributions.
- If classes are not of equal width, then the height of the rectangle is calculated by the ratio of the frequency of that class, to the width of that class.
- A histogram is different from a bar chart, as in the former case it is the area of the bar that denotes the value, not the height.
- When the scale on the x-axis starts at a higher value and not from the origin, a **kink** is indicated near the origin to signify that the graph is drawn to a scale beginning at a higher value and not at the origin.



- 'Kinks' are a tool used to express areas in a graph. In this case, the kink tells us that there is no observation which takes the value less than 200.
- **Frequency Polygon:** The frequency polygon of a frequency distribution is a line-graph drawn by plotting the class marks on the x-axis against the frequencies on the y-axis. In case of grouped data, where the classes are of equal width, the frequency polygon is obtained by joining the mid-points of the top edges of the rectangles in the histogram. Two extra lines are drawn by introducing two extra classes (or values).
- One class is introduced before the first class and the other is introduced after the last class. These classes have zero frequencies.
- Frequency polygons are used for understanding the shape of distributions.
- If both a histogram and a frequency polygon are to be drawn on the same graph, then first draw the histogram and then join the mid-points of the tops of the adjacent rectangles in the histogram with line-segments to get the frequency polygon.
- The cumulative frequency of a class-interval is the sum of frequencies of that class and the classes which preceed (come before) it.

➢ $$\text{Class size} = \frac{\text{Range}}{\text{Number of classes}}$$

➢ Class size = Upper limit – Lower Limit



Example 1

The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian Society is given below :

Sections of Society	Number of girls per thousand boys
Schedule Caste (SC)	940
Schedule Tribe (ST)	970
Non–SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph?

(iii) What step should be taken to improve the situation ?

Solution:

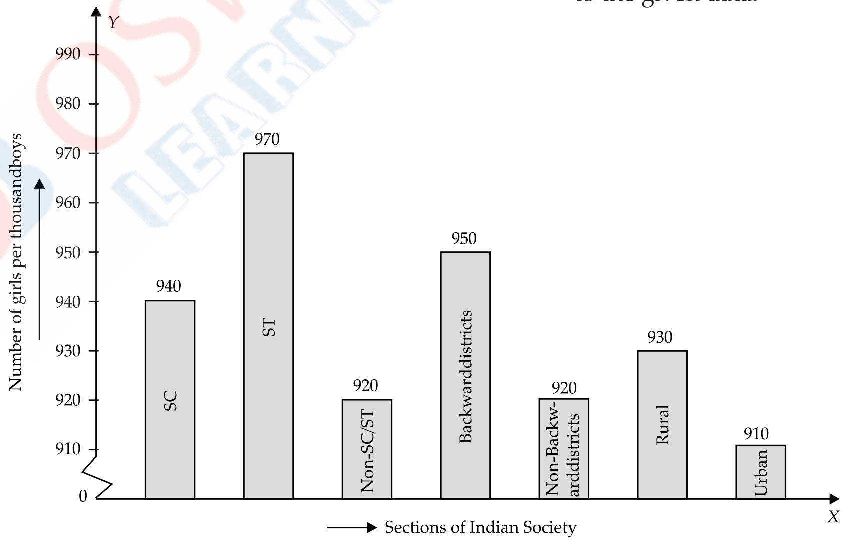
Step I : Choose the appropriate data for horizontal axis (*i.e.*, *x*-axis) and vertical axis (*i.e.*, *y*-axis).

Here, we represent the sections on horizontal axis choosing any scale, since width of bar is not important but for clarity, we take equal widths for all bars and maintain equal gap between then. Let one section be represented by one unit.

We represent the number of girls per thousand boys on vertical axis. Here, we can choose the scale as 1 unit = 10.

Step II : Draw the graph as per given information.

(i) How, the graph is as shown below according to the given data.



Step III : Draw the conclusion for part (ii).

(ii) From the graph, we observe that in Scheduled Tribe (ST), there is maximum number of girls per thousand boys among different sections of Indian Society, *i.e.*, 970 whereas there are minimum number of girls per thousand boys in urban area.

Step IV : Suggest one positive step to improve the situation.

(iii) Pre-natal sex determination should be strictly banned in urban areas.