OSWAAL BOOKS®

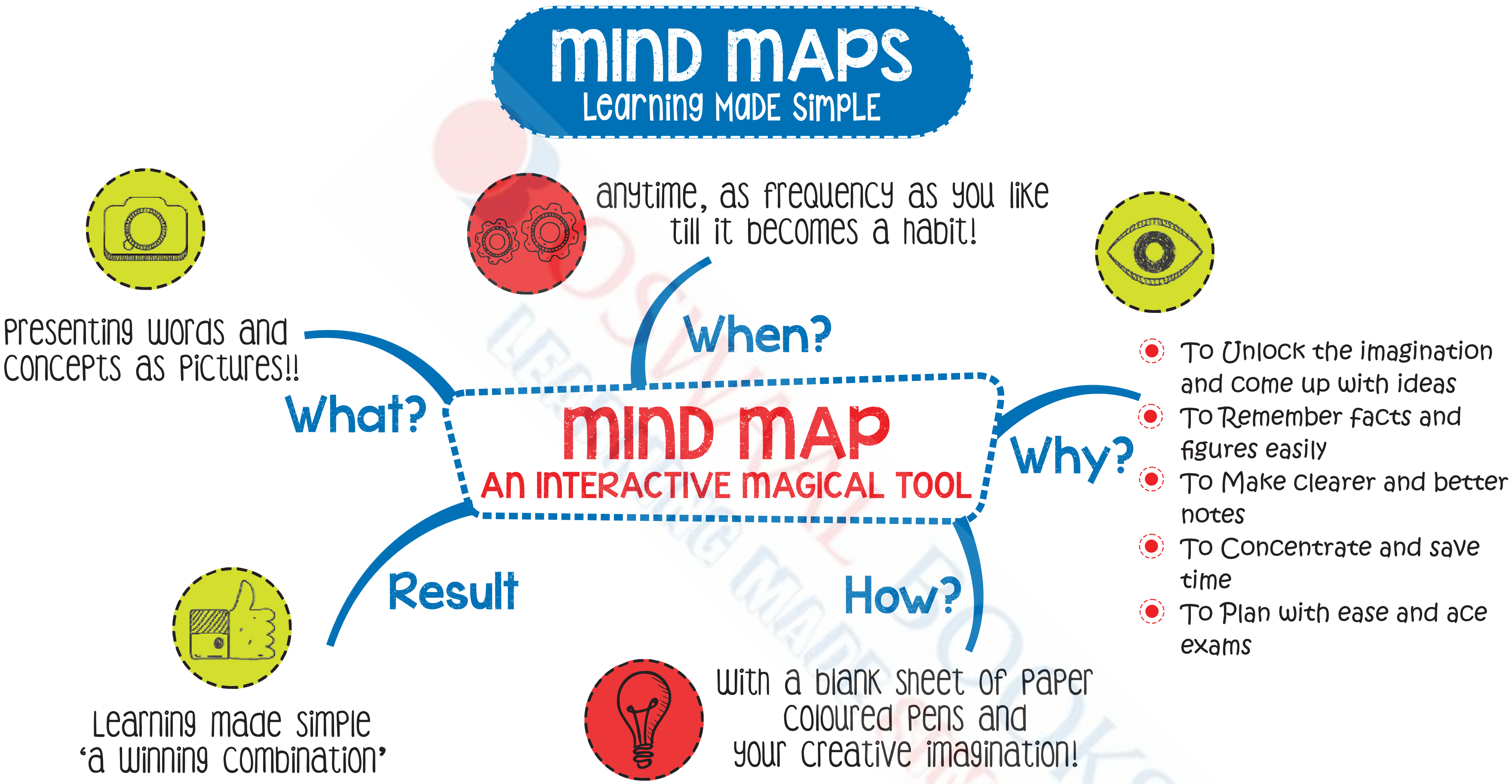
LEARNING MADE SIMPLE

CBSE

Mind Maps

CLASS 12

APPLIED MATHEMATICS



What are Associations?

It's a technique connecting the core concept at the Centre to related concepts or ideas. Associations spreading out straight from the core concept are the First Level of Association. Then we have a Second Level of Association emitting from the first level and the chronology continues. The thickest line is the First Level of Association and the lines keep getting thinner as we move to the subsequent levels of association. This is exactly how the brain functions, therefore these Mind Maps. Associations are one powerful memory aid connecting seemingly unrelated concepts, hence strengthening memory.

Numbers and Quantification

Arithmetic function

An arithmetic function is a function whose domain is positive integer. *i.e.*, $f : c \to Z^+$

Congruence modulo

Let $a, b \in Z$, $m \in Z^+$
Then,
a is congruent to b modulo m written as '$a \cong b$ (mod)m' iff
1. a mod m = b mod m
2. $m \mid (a-b)$ *i.e.*, $(a-b)$ mod $m = 0$

Properties of congruence modulo

- If $a \cong b$ (mod m), then $b \cong a$ (mod m)
- If $a \cong b$ (mod m), then $(a \pm c) \cong (b \pm c)$ (mod m)
- If $a \cong b$ (mod m), then $ac \cong bc$ (mod m)
- If $a \cong b$ (mod m) and $c \cong d$ (mod m), then $(a \pm c) \cong (b \pm d)$ (mod m)
- If $a \cong b$ (mod m) and $c \cong d$ (mod m), then $ac \cong bd$ (mod m)
- If x_1 is a solution of $ax \cong b$ (mod m), then any other integer $x_2 \cong x_1$ is also a solution.

Types of arithmetic functions

Divisor function

$\tau(n)$ or $d(n)$ = number of divisors of $n = \sum_{d/n} 1$

e.g., $\tau(4)$ = number of divisors of 4
Divisors of 4 = 1, 2, 4
So, $\tau(4) = 3$

Sigma function

$\sigma(n)$ = sum of number of divisors of $n = \sum_{d/n} d$

e.g., $\sigma(4)$ = sum of divisors of 4 = 1+2+4=7

Sigma k function

$\sigma_k(n)$ = Sum of k^th powers of divisors of $n = \sum_{d/n} d^k$

e.g., $\sigma_2(4)$ = Sum of divisors of 2^nd powers of 4.
$= 1^2 + 2^2 + 4^2$
$= 1 + 4 + 16 = 21$

Omega function

$\omega(n)$ = Number of prime divisors of $n = \sum_{p/n} 1$ where is prime

e.g., $\omega(4)$ = Number of prime divisor of 4 = 1, as 1 is the only prime divisors of 4.

Mobius function

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } p^2/n, \text{ for some prime } p \\ (-1)^k & \text{if } n = p_1, p_2, \ldots\ldots p_k \text{ where } p_1, p_2 \ldots\ldots\ldots p_k \text{ are distinct prime} \end{cases}$$

when $n = 1$, $\mu(1) = 1$
when $n = 2$, $m(2) = (-1)^1 = -1$
when $n = 4$, $m(4) = 2^2 / 2^2 = 0$

Euler's totient f (n) function

If n is a positive integer, then f function counts the positive integers less than or equal to n that are relatively prime to n.
$\phi(n)$ = number of integers $\leq n$ coprime to n.
e.g., f (10) = 4
(as 1, 3, 7, 9 are coprime to 10)
f (7) = 6 (as 1, 2, 3, 4, 5, 6 are coprime to 7)

Trace the Mind Map
- First Level
- Second Level
- Third Level

Numerical Applications

Alligation or mixture

If ingredients are mixed in a ratio, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{C.P. of the dearer- Mean price}}{\text{Mean price - C.P. of the cheaper}}$$

We represent it as writer:
C.P. of unit quantity of cheaper C.P. of unit quantity of dearer

(c) (d)
Mean Price
(m)
(d-m) (m-c)

Races and games

- A race is a contest which can measure the speed in driving running, sailing, rowing, jumping etc. This is calculated over a centre distance.
- In every race end point is known as winning point
- In a dead heat race, every participant reach at the winning point
- The person that comes first wins the race and called winner.
- A game of leo in a race is the game where participants agree that whosoever scores the 100 point first wins the race.

Pipes and gistern

- If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y>x$), then on opening both pipes, the net part filled in 1 hour $= \left(\frac{1}{x}-\frac{1}{y}\right)$
- If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x>y$), then on opening both pipes, the net part emptied in 1 hour $= \left(\frac{1}{y}-\frac{1}{x}\right)$

Boat and streams

Boat (u km/h) →, Stream (v km/h) → } Down stream ($u+v$)km/h

Boat (u km/h) →, Stream (v km/h) ← } Up stream ($u-v$)km/h

If the speads of downstream and upstream are a km/h and b km/h, respectively, then speed in still water $= \frac{1}{2}(a+b)$km/h

Rate of stream $= \frac{1}{2}(a-b)$km/h

Partnership

There are basically 2 types of partners in a business. One in the active partner who participates actively in the business and the other the **sleeping partners** who invests his capital and stays away from the day to day running of the business.

Amount and Time period same : When people invest the same amount of money for the same period then the profit is shared in 1 : 1 : 1 ratio (*i.e.*) equally among the partners.

Amount same and Time Period Different : When the money invested is the same but the time period for which it is invested is different, then the profit is shared in the ratio of the time period for which the amount is invested.

Time Period same and Amount Different : When the time period of the amounts invested is the same but the amounts are different then the profit is shared in the ratio of the amounts invested respectively.

Inequalities

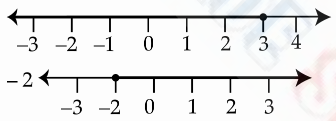
Definition

- An inequality is a relation that holds between two values, when they are different eg.: $x<5$, here, there is a relation between x & 5.
- Two real numbers or algebraic expressions related by symbol '<', '>', '≤' or '≥' form inequality.

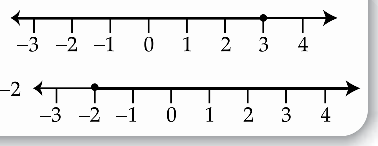
e.g.: $y \le 4$, $2x+3y \ge 5$, $3>x$.

Representation

To represt $x<a$ or $x>-a$ on a number line, put a circle on the number 'a' and dark line to the left (or right) of the number 'a'.
e.g.:



To represent $x \le a$ (or $x \ge a$) on a number line, put a dark circle on the number 'a' and dark the line to the left (or right) of the number 'a'.
e.g.:



Types of Inequalities

Numerical Inequalities : Relation between numbers
e.g.: $3<5, 7>4$
Literal or Variable Ineqalities : Relation between two variables or variable and numbers.
e.g.:$y<8$
Double Inequalities: Relationship from two sides
e.g.:$2<x<5$
Strict Inequalities: An inequality that uses the symbol <or> e.g.: $x<5$, $3<5$
Slack Inequality: An inequality that uses the symbols ≤ or ≥. e.g.: $y \le 5$
Linear Inequalities in One Variable: An inequality which involves a linear function in one variable
e.g.: $x<5$
Linear Inequalities in two variables: An inequality which involves a linear function in two variables,
e.g.: $3x+2y<5$
Quadratic Inequalities: An inequality which involves a quadratic function, eg.: $x^2+2x \le 5$.

Rules of inqualities

- Equal numbers may be added to (or subtracted from) both the sides of an inequality without affecting the sign of inequality:
  e.g.: $x<7$ is same as $x+2 < 7+2$
- Both sides of an inequality can be multiplied (or divided) by the same positive number without affecting the sign of inequality.
  e.g.: $x+y<7$ is same as $(x+y) \times 3 < 7 \times 3$
- When both sides are multiplied or divided by a – ve number, then sign of inequality is reversed.
  e.g.: $x+y<7$ is same as
  $(x+7) \times (-3) > 7 \times (-3)$

Trace the Mind Map
First Level · Second Level · Third Level

Matrices

What is Matrix ?

A matrix of order $m \times n$ is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A = [a_{ij}]_{m\times n}$ is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ a_{m1} & a_{m2} & \dots a_{mn} \end{bmatrix}_{m\times n}$$

Types of Matrices

Row matrix

It is of the form $A = [a_{ij}]_{1\times n}$

e.g. $[2\ 5\ -4]_{1\times 3}$, $[\sqrt{2}\ \ 4]_{1\times 2}$

Column matrix

It is of the form $A = [a_{ij}]_{m\times 1}$

e.g. $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}_{3\times 1}$, $\begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2\times 1}$

Square matrix

Here $m = n$ (no. of rows = no. of columns)

$A = [a_{ij}]_{n\times n}$

e.g. $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & -4 \\ 0 & -1 & -2 \end{bmatrix}_{3\times 3}$

Diagonal matrix

All non-diagonal entries are zero *i.e.*

$a_{ij} = 0 \ \forall\ i \neq j$

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$

Scalar matrix

$a_{ij} = 0,\ i \neq j$ and $a_{ij} = k$ (scalar), $i = j$

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Identity matrix

$a_{ij} = 0$ *when* $i \neq j$ and $a_{ij} = 1$ when $i = j$

e.g. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

Zero matrix

All entries are zero

e.g. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

Equality of two matrix

$A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are said to be equal, if $a_{ij} = b_{ij} \ \forall\ i, j$

e.g. $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 4/2 & 2-1 \\ \sqrt{9} & 0 \end{bmatrix}$

but $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

Transpose of matrix

A new matrix whose rows are columns of the original, it is denoted by A^T or A'

e.g. $A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$,

then $A^T = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 5 \\ 5 & 4 & 7 \end{bmatrix}$

Symmetric and Skew-Symmetric Matrix

Symmetric

If $A^T = A$, *e.g.* $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A$

Skew-Symmetric

If $A^T = -A$, $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$,

$A^T = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = A$

Operation on Matrices

Addition

Let $A = [a_{ij}]_{m\times n}$ –A and $B = [b_{ij}]_{m\times n}$, than $A+B = [a_{ij} + b_{ij}]$

e.g. $A = \begin{bmatrix} 1 & 0 \\ 2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}$

$A+B = \begin{bmatrix} 2 & -2 \\ 2 & 10 \end{bmatrix}$

Multiplication

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$, then $C = AB = [C_{ik}]_{m\times p}$

when $C_{ik} = \sum_{f=1}^{n} a_{ij} \cdot b_{jk}$

e.g. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

then, $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

Multiplication of a Matrix by a Scalar

If $A = [a_{ij}]_{m\times n}$, then $kA = [ka_{ij}]_{m\times n}$

e.g. $A = \begin{bmatrix} 2 & -4 \\ 5 & 6 \end{bmatrix}$, $3A = \begin{bmatrix} 6 & -12 \\ 15 & 18 \end{bmatrix}$

Existence of non - zero matrices whose product is the zero matrix

Product of two matrices is zero matrix, as it is not necessary that one of the matrices is a zero matrix

e.g. $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$, $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Trace the Mind Map

- First Level
- Second Level
- Third Level

Determinants

Expansion of a Determinant

of Order 2

$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}.a_{22} - a_{21}.a_{12}$

of Order 3

$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}.a_{33} - a_{32}.a_{23}) - a_{12}(a_{21}.a_{33} - a_{31}.a_{23}) + a_{13}(a_{21}.a_{32} - a_{31}.a_{22})$

Minors and Cofactors

Minors

If $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then minor of $a_{11} = M_{11}$

$= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

Cofactors

If the minor M_{ij} is multiplied by $(-1)^{i+j}$, then we obtain the factor of a_{ij}. It is denoted by A_{ij} or C_{ij}, i.e., $A_{ij} = (-1)^{i+j}$ M_{ij}, cofactor of $a_{11} = (-1)^{1+1} M_{11}$ or $A_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

Properties of Determinants

- If each element of a row or column is zero, then determinant becomes zero. *e.g.* $A = \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{bmatrix} \therefore |A| = 0$
- The value of a determinant remains unchanged if its row and columns are interchanged. *e.g.* $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$
- If any two adjacent rows or columns are interchanged then the determinant retains its absolute value, but its sign is changed. *e.g.* $\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \Delta_1 = \begin{bmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $\Rightarrow \Delta_1 = -1\Delta$
- If each element of a row or a column of a determinant is multiplied by a constant k, then the value of new determinant are k times the value of the original determinant. *e.g.* $\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \Delta_1 = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = k\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \Rightarrow \Delta_1 = k\Delta$
- If each element of a row (or column of determinant is the sum of the two more terms, then the determinants can be expressed as the sum of the two or more determinants. *e.g.* $\begin{bmatrix} a_1+a_2 & b & c \\ p_1+p_2 & q & r \\ u_1+u_2 & v & w \end{bmatrix} = \begin{bmatrix} a_1 & b & c \\ p_1 & q & r \\ u_1 & v & w \end{bmatrix} + \begin{bmatrix} a_2 & b & c \\ p_2 & q & r \\ u_2 & v & w \end{bmatrix}$

Leontief Model

$(I - B)x = D$
x = variable vector
D = demand vector
I – B = Leontief Matrix
I – B is non singular
$x = (I - B)^{-1} D$

Adjoint of a Matrix

Statement

Let $A = [a_{ij}]_{n \times n}$ be a square matrix of order n and C_{ij} (or A_{ij}) be the cofactor of a_{ij} in the determinant $|A|$. Then, the adjoint of A is defined as the transpose of the cofactor matrix and it is denoted by adj (A).

Example

If $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then adj $A = \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}$,

where A_{ij} 's are coafactors of A.

Singular and Non-Singular Matrix

Singular: If A is a square matrix, and $|A| = 0$, then A is known as singular matrix.

Non-Singular: If A is a square matrix and $|A| \neq 0$, then A is known as non - singular matrix.

Properties of Adjoint of a Matrix

If A is a square matrix of order $n \times n$, then
(i) $A(\text{adj } A) = (\text{adj } A)A = |A|I_n$
(ii) adj $(A^T) = (\text{adj } A)^T$
(iii) $|\text{adj } A| = |A|^{n-1}$, provided $|A| \neq 0$

Inverse of Square Matrix

Statement

Let A be a non-singular (*i.e.* $|A| \neq 0$) square matrix. Then, a square matrix B, such that $AB = BA = I$ is called of inverse of A and it is denoted by A^{-1}

i.e. $A^{-1} = \frac{1}{|A|} \text{adj}(A)$.

Properties

Let A and B be square matrices of same order, then
(i) A square matrix is invertible if and only if it is non-singular
(ii) $(A^{-1})^{-1} = A$
(iii) $(A')^{-1} = (A^{-1})'$
(iv) $(AB)^{-1} = B^{-1}A^{-1}$
(v) $|A^{-1}| = |A|^{-1}$
(vi) $AA^{-1} = A^{-1}A = I$
(vii) $(kA)^{-1} = \frac{1}{k}A^{-1}$, if $k \neq 0$

Cramer's Rule

The solution of the system $a_1x + b_1y + c_1z + d$, $a_2x + b_2y + c_2z + d_2$ and $a_3x + b_3y + c_3z + d_3$ *is* given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$ and $z = \frac{D_z}{D}$, where

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Provided that $D \neq 0$

Determinant of Square Matrix

A unique number (real or complex) can be associated, to every square matrix $A=[a_{ij}]$ of order m, is called determinant of matrix A. It is denoted by $|A|$ or det (A)

Trace the Mind Map: First Level, Second Level, Third Level

Differentiation and its Applications

Higher Order Derivatives

Because the derivative of a function $y = f(x)$ is itself a function $y' = f'(x)$, we can take the derivative of $f'(x)$, which is generally referred to as the *second derivative of f(x)* and written $f''(x)$ or $f^2(x)$. This differentiation process can be continued to find the third, fourth, and successive derivatives of $f(x)$, which are called higher order derivatives of $f(x)$.

e. g. : Find the first, second and third derivatives of $f(x) = 5x^4 - 3x^3 + 7x^2 - 9x + 2$.

Solution:

$$f'(x) = 20x^3 - 9x^2 + 14x - 9$$
$$f''(x) = f^{(2)}(x) = 60x^2 - 18x + 14$$
$$f'''(x) = f^{(3)}(x) = 120x - 18$$

Increasing and decreasing functions

A function f is said to be **(i)** increasing on (a,b) if $x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \leq f(x_2) \forall x_1, x_2 \in (a, b)$,
and **(ii)** decreasing on (a, b) if $x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \geq f'(x_2)$ $\forall x_1, x_2 \in (a, b)$

If $f'(x) \geq 0 \forall x \in (a, b)$ then f is increasing in (a,b) and if $f'(x) \leq 0 \forall x \in (a,b)$, then f is decreasing in (a,b)

eg: Let $f(x) = x^3 - 3x^2 + 4x, x \in R$, then $f'(x) = 3x^2 - 6x + 4 = 3(x-1)^2 + 1 > 0 \forall x \in R$. So, the function f is strictly increasing on R.

Maxima and Minima

A point C in the domain of 'f' at which either $f'(C)=0$ or is not differentiable is called a critical point of f.

First derivative test

Let f be continuous at a critical point C in open interval. Then **(i)** if $f'(x) > 0$ at every point left of C and $f'(x) < 0$ at every point right of C, then 'C' is a point of local maxima.

(ii) If $f'(x) < 0$ at every point left of C and $f'(x) > 0$ at every point right of C, then 'C' is a point of local minima.

(iii) If $f'(x)$ does not change sign as 'x' increases through C, then 'C' is called the point of inflection.

Second derivative test

Let f be a function defined on given interval, f is twice differentiable at C. Then

(i) $x=C$ is a point of local maxima. If $f'(C)=0$ and $f''(C) < 0$, $f(C)$ is local maxima. of f.

(ii) $x=C$ is a point of local minima if $f'(C)=0$ and $f''(C)>0$. $f(C)$ is local minima of f.

(iii) The test fails if $f'(C)=0$ and $f''(C)=0$

Marginal Cost and Marginal Revenue

Marginal refers to an instantaneous rate of change, i.e., a derivative.

If x is the number of units of a product produced in some time interval, then

If Total Cost = C(x), then Marginal Cost = C′(x)

Similarly, marginal revenue is the rate of change of total revenue w.r.t. total number of unit sold.

If Total Revenue = R(x), where x is the number of unit sold then Marginal Revenue = R′(x).

e.g. : If $C(x) = 9.5x + 4500$ $R(x) = 36x - .03x^2$

- Find the marginal cost function.
  - $C'(x) = 9.5$
- Find the marginal revenue function.
  - $R'(x) = 36 - .06x$
- Find the marginal profit function.
  - $P'(x) = R'(x) - C'(x)$
    $= 36 - .06x - 9.5$
    $= 26.5 - .06x$

Trace the Mind Map

- First Level
- Second Level
- Third Level

Integrals and It's Applications-1

Integration by Parts

For functions $f(x)$ and $g(x)$, we have

$\int[f(x).\ g(x)]\ dx = f(x).\int g(x).\ dx - \int\{f'(x).\int g(x).\ dx\}.\ dx$

Here, we can choose first function according to its position in ILATE, where

- I = Inverse trigonometric function
- L = Logarithmic function
- A = Algebraic function
- T = Trigonometric function
- E = Exponential function

Integration by Substitution

$\int f(x).\ dx = \int f\{g(t)\}.\ g'(t)$, if we substitute $x=g(t)$, such that $dx = g'(t)\ dt$.

(i) $\int \{f(x)\}^n . f'(x).\ dx = \frac{\{f(x)\}^{n+1}}{n+1} + C,\ n \neq -1$

(ii) $\int \frac{f'(x)}{f(x)} .\ dx = \log|f(x)| + C,\ f(x) \neq 0$

Some Examples

Integration by parts

Evaluate $\int e^{-2x} \sin x\ dx$

Method:-

(i) Use integration by part method i.e. consider $\sin x$ as 1st function and e^{-2x} as 2nd function constant

(ii) Use the formula: $\int \underset{\text{I}}{u}.\underset{\text{II}}{v}\ dx = u\int v.dx - \int\left[\frac{d}{dx}(u).\int v\,dx\right]dx$

(iii) Again further use the integration by parts and simplify it

Integration by substitution

Evalute $\int \frac{1}{x+\sqrt{x}}\ dx$

Method:- To solve this problem, we use the concept of method of substitution. Here first of all put $x = t^2$, then apply the formula i.e.

$\int \frac{1}{x} dx = \log|x| + C$

Integration as process of differentiation

$\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x).dx = F(x)+C$

Basic Formulae of Integrals

1. $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
2. $\int e^x\ dx = e^x + C$
3. $\int e^{ax}\ dx = \frac{e^{ax}}{a} + C$
4. $\int a^x\ dx = \frac{a^x}{\log a} + C$
5. $\int \frac{1}{x} dx = \log|x| + C$
6. $\int \sin x\, dx = -\cos x + C$
7. $\int \cos x\, dx = \sin x + C$
8. $\int \tan x\, dx = -\log|\cos x| + C = \log|\sec x| + C$
9. $\int \cot x\, dx = \log|\sin x| + C = -\log|\text{cosec}\, x| + C$
10. $\int \sec x\, dx = \log|\sec x + \tan x| + C = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C$
11. $\int \text{cosec}\, x\, dx = \log|\text{cosec}\, x - \cot x| + C = \log\left|\tan\frac{x}{2}\right| + C$
12. $\int \sec x \tan x\ dx = \sec x + C$
13. $\int \text{cosec}\, x \cot x\ dx = -\text{cosec}\, x + C$
14. $\int \sec^2 x\, dx = \tan x + C$
15. $\int \text{cosec}^2 x\, dx = -\cot x + C$
16. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
17. $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
18. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$
19. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$
20. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
21. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
22. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$
23. $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$
24. $\int (ax+b)^n\, dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C,\ n \neq -1$

Trace the Mind Map
- First Level
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- Third Level

Integrals and It's Applications-2

Evaluation of Definite Integrals by Substitution

Step-I: Consider a definite integral: $\int_a^b f.[g(x)].g'(x)\,dx$ Substitute $g(x) = t$ $\Rightarrow g'(x)\,dx = dt$

Step-II: Find the limits of integration in new system of variable i.e. the lower limit $g(a)$ and the upper limit $g(b)$ and the given integral becomes $\int_{g(a)}^{g(b)} f(t).dt$.

Step-III: Evaluate the integral, so obtained by usual method

Example:

$\int_0^1 x e^{x^2}\,dx$

let $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

Substituting, when $x = 0, t = 0$

$x = 1, t = 1$

$I = \int_0^1 e^t \frac{dt}{2} = \frac{1}{2}\int_0^1 e^t dt$

$= \frac{e^t}{2}\Big|_0^1 = \frac{1}{2}\left[e^1 - e^0\right] = \frac{1}{2}[e-1] = \frac{e}{2} - \frac{1}{2}$

Steps for Calculating Definite Integral

Step-I: There is no need to keep integration constant C

$\int_a^b f(x).dx = [F(x)+C]_a^b$

$= [F(b)+C] - [F(a)+C]$

$= F(b) - F(a)$

Step-II: Evaluate $\int_a^b f(x).dx = [F(x)]_a^b$

$= F(b) - F(a)$

Step-III: Calculate $F(b) - F(a)$

Properties of Definite Integral

(i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $a < c < b$

(iv) $\int_a^a f(x)\,dx = 0$

(v) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(vi) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dt$

(vii) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(viii) $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

or $\int_a^b f(x)\,dx = \begin{cases} 0, & \text{if } f(a+x) = -f(b-x) \\ 2\int_a^{\frac{a+b}{2}} f(x)\,dx, & \text{if } f(a+x) = f(b-x) \end{cases}$

(ix) $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even i.e. } f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is odd i.e. } f(-x) = -f(x) \end{cases}$

Trace the Mind Map: First Level, Second Level, Third Level

Integrals and It's Applications-3

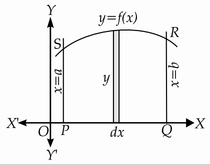
Area under simple curves

The area of the region bounded by the curve $y = f(x)$, x- axis and the lines $x=a$ and $x=b\,(b>a)$ is given by

$A = \int_a^b y\,dx$ or $\int_a^b f(x)dx$

eg : the area bounded by $y=x^2$, x- axis in I quadrant and the lines $x=2$ and $x=3$ is -

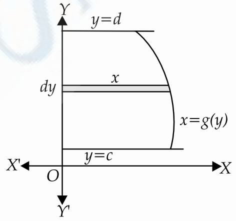
$$A = \int_2^3 y\,dx = \int_2^3 x^2\,dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{1}{3}(27-8) = \frac{19}{3} \text{ Sq.units.}$$



The area of the region bounded by the curve $x = f(y)$, $y-axis$ and the lines $y=c$ and $y=d\,(d>c)$ is given by $A = \int_c^d x\,dy$ or $\int_c^d f(x)dy$

eg : the area bounded by $x = y^3$, $y-axis$ in the I quadrant and the lines $y=1$ and $y=2$ is

$$\int_a^b f(x)dx = \int_a^b f^3\,dy = \left[\frac{1}{4}y^4\right]_1^2 = \frac{1}{4}(2^4-1^4) = \frac{15}{4} \text{ Sq. units}$$



Consumer's Surplus

The consumers surplus is the difference between what the consumers would be willing to pay for a commodity and what they actually pay for them.

The consumers' surplus is given by

$$CS = \int_0^{Q_e} D(x)\,dx - Q_e P_e$$

where D is the demand function, Q_e is the equilibrium quantity and P_e is equilibrium price.

Example

Example : Suppose that the demand function for producing a can of tennis balls is $P(x)=20-0.05x$ and that the current price level is P_e = Rs.8. Find the consumers surplus.

Sol.: First, we need to find the value of Q_e that corresponds to P_e =Rs.8. Setting P = 8 and solving for x gives

$8 = 20 - 0.05x \Rightarrow 0.05x = 12 \Rightarrow x = 240$ or $Q_e = 240$

Using the integral 'formula' for consumer surplus, we find that

$$CS = \int_0^{240} (20-0.05x)\,dx - 240.8$$

$$= 20x\Big|_0^{240} - 0.025x^2\Big|_0^{240} - 1920 = \text{Rs.}1,440.$$

Producer's Surplus

Producer's surplus is the difference between what producers are willing and able to supply a good for and the price they actually receive.

The producers surplus is given by

$$PS = Q_e P_e - \int_0^{Q_e} S(x)\,dx$$

Where S is the demand function, Q_e is the equilibrium quantity and P_e is equilibrium price.

Trace the Mind Map
- First Level
- Second Level
- Third Level

Differential Equations and Modeling

Ordinary Differential Equations

An equation involving derivatives of the dependent variable with respect to independent variable (variables) and constant e.g.: $xy\frac{d^2y}{dx^2}+x\frac{dy}{dx}+k=0$. If there is only one independent variable, then we call it as an ordinary differential equation. e.g.: $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2=0$

Order of Differential Equations

It is the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

e.g.: the order of $\frac{dy}{dx}=e^x$ is one and order of $\frac{d^2y}{d^2x}+x=0$ is two.

Degree of Differential Equations

The highest exponent of the highest order derivative.

e.g.: The degree of $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=0$ is three.

Order and degree (if defined) of a differential equation are always positive integers.

Solution of Differential Equations

- A function which satisfies the given differential equation is called its solution.
- The solution which contains as many arbitrary constants as the order of the differential equations is called a general solution.
- The solution free from arbitrary constants is called particular solution.

e.g. $y=e^x+1$ is a solution of $y''-y'=0$.
Since $y'=e^x$ and $y''=e^x \Rightarrow y''-y'=e^x-e^x=0$

Formations of Differential Equations

To form a Differential Equation from a given function, we differentiate the function successively as many times as the no. of arbitrary constants are given in the function.
e.g.: Let the function be $y = ax + b$, then we have to differentiate it two times, since there are 2 arbitrary constant a and b.
$\therefore y'=a \Rightarrow y''=0$. Thus, $y''=0$ is the required Differential Equation.

Linear Differential Equations

A Differential Equation of the form $\frac{dy}{dx}+Py=Q$, where P, Q are constants or functions or 'x' only is called a first order linear Differential of Equation.

Its solution is $ye^{\int P.dx}=\int Q.e^{\int P.dx}dx+c$

e.g. $\frac{dy}{dx}+3y=2x$ has solution

$ye^{\int 3.dx}=\int 2x.e^{\int 3.dx}dx+c \Rightarrow ye^{3x}=2\int xe^{3x}+c$

Growth and Decay Model

The differential equation for growth and decay is

$\frac{dx}{dt}=kx,\ x(t_0)=x_0$

If $k>0$, it models growth. If $k<0$, it models decay.

The above equation is solved using variable separation method as.

$\int\frac{dx}{x}=\int kdt \Rightarrow \log x = kt + \log c.$

$\Rightarrow \log\left(\frac{x}{c}\right)=kt \Rightarrow x = ce^{kt}$

Variable Separable Method

It is used to solve such an equation in which variables can be separated completely .

eg., $ydx = xdy$ can be solved as $\frac{dx}{x}=\frac{dy}{y}$

Integrating both sides, $\log x = \log y + \log c$

$\Rightarrow \frac{x}{y}=c \Rightarrow x = cy$ is the solution.

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Probability Distributions

Probability Distribution

The probability distribution of a random variable x is the system of numbers x: $x_1\, x_2 \,.......,\, x_n$

P (x): $p_1\, p_2 \,.......p_n$ where, $p_i > 0$,

$\sum_{i=1}^{n} p_i = 1,\ i = 1, 2.......n.$

Expectation

Let x be a random variable whose possible values $x_1 x_2 \,,.........,x_n$ occurs with probabilities $p_1\, p_2 \,........p_n$ resp. Then, mean of $x, \mu = \sum_{i=1}^{n} x_i p_i$, It is also called the expectation of x, denoted by E (x)

Variance

Let x be a random variable whose possible values $x_1 x_2 \,,.........,x_n$ occurs with probabilities $p(x_1), p(x_2),.......... p(x_n)$, respectively, Let, $\mu = E(x)$ be the mean of x. The variance of x, var (x) or

$\sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \ p(x_i)$ or $E[(x-\mu)^2]$

Binomial Theorem

$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$

where

n = the number of trials (or the number being sampled)

x = the number of successes desired

p = probability of getting a success in one trial

$q = 1 - p$ = the probability of getting a failure in one trial

Let

x = the number of successes = 15

n = the number of trials = 18

p = the probability of success = 0.85

$P(15) = {}^{18}C_{15}(0.85)^{15}(1-0.85)^{18-15}$

$P(15) = \left(\frac{18!}{15!\,(18-15)!}\right)(0.85)^{15}[(1-0.85)^3]$

$P(15) = (816)\,(0.87)\,(.0036)$

$P(15) = .239$

Poisson Distribution

$P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$

where

$x = 0, 1, 2, 3,....$

μ = mean number of occurrences in the interval

e = Euler's constant ≈ 2.71828

The mean number of power outages in the city or Brunswick is 4 per year. Find the probability that in a given year, there are exactly 3 outages,

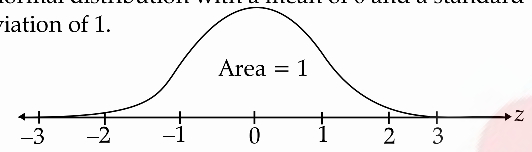
$\mu = 4, x = 3$

$P(3) = \frac{4^3(2.71828)^{-4}}{3!}$

≈ 0.195

Standard Normal Curve

- A normal distribution with a mean of 0 and a standard deviation of 1.



- Any x-value can be transformed into a z-score by using the formula

$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x-\mu}{\sigma}$

Normal Distribution

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

μ = mean of x

σ = Standard deviation of x

$\pi \approx 3.14159....$

$e \approx 2.71828....$

Trace the Mind Map

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Inferential Statistics

Sample

A **sample** is a smaller group of members of a population selected to represent the population. A **random sample** is one in which every member of a population has an equal chance of being selected.

Population

A **population** is a group of phenomena that have something in common. The term often refers to a group of people, as in the following examples:

- All registered voters in India.
- All members of the International Cricket Team.

Inferential statistics

It use measurements from the sample of subjects in the experiment to compare the treatment groups and make generalizations about the larger population of subjects.

Sampling

Sampling is the process of selecting observations (a sample) to provide an adequate description and robust inferences of the population.
Sampling can be broadly grouped into two :

- Random (Probability) Sampling
- Non Random Sampling.

Sampling Errors

- **Sampling Errors–**
  Errors caused by the act of taking a sample.
  Make sample results inaccurate.
- **Random Sampling Error**
  Errors caused by the chance in selecting a random sample.
- **Nonsampling Error**
  Errors not related to the act of selecting a sample from the population. They can even be present in a census.

Null Hypothesis

The null hypothesis for a 1-sample t-test is :
$H_0 : \mu = \mu_0$ where :

- μ = the population mean
- μ_0 = the hypothesized mean

You can choose any one of three alternative hypotheses :

$H_1 : \mu > \mu_0$	One-tailed test
$H_1 : \mu < \mu_0$	One-tailed test
$H_1 : \mu \neq \mu_0$	Two-tailed test

Two Sample t-test

Let the two independent samples be $x_1, x_2, x_3, ... x_{n1}$ and $y_1, y_2, y_3, ..., y_{n2}$ with means $\bar{x} = \frac{\Sigma x}{n_1}$ and $\bar{y} = \frac{\Sigma y}{n_2}$ from two normal population with means μ_1 and μ_2 and common variance σ^2 (unknown).

Let $s_1^2 = \frac{1}{n_1 - 1}\Sigma(x-\bar{x})^2$ and $s_2^2 = \frac{1}{n_2 - 1}\Sigma(y-\bar{y})^2$

Thus, standard error can be given by $s = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$

The t-test formula is given as : $t = \frac{\bar{x}-\bar{y}}{s}\sqrt{\frac{n_1 n_2}{n_1 + n_2}}$

Example

A healthcare consultant wants to compare the patient satisfaction ratings of two hospitals. The consultant collects ratings from 20 patients for each of the hospitals. The consultant performs a 2-sample t-test to determine whether there is a difference in the patient ratings between the hospitals.

One Sample t-test

The One-Sample t-Test procedure tests whether the mean of a single variable differs from a specified constant.
Example : A researcher might want to test whether the average IQ score for a group of students differs from 100.
Formula for One Sample t-test $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

where, t = the t statistic
$\bar{x}$ = the mean of the sample
μ = the comparison mean
s = the sample standard deviation
n = the sample size

Parameter & Statistics

Parameter: The statistical constants of the population like mean (μ), variance (σ^2) are referred as population parameters.
Statistic : Any statistical measure computed from sample is known as statistic.

Standard Error

Sl	Statistic	Standard Error
1.	Sample mean ($\bar{x}$)	$\sigma / \sqrt{n}$
2.	Observed sample proportion (p)	$\sqrt{PQ / n}$
3.	Sample standard deviation (s)	$\sqrt{\sigma^2 / 2n}$
4.	Sample variance (s^2)	$\sigma^2\sqrt{2 / n}$

where, $\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}}$

σ = population standard deviation
N = the size of the population
x_i = each value from the population
μ = population mean
P = Population Proportion
Q = 1-P

Trace the Mind Map
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Index numbers and Time based Data

Index Number

Index numbers are used as an indicator to indicate the changes in economic activity. They also provide frame work for decision making and to predict future events. There are three types of index numbers (i) Price index, (ii) quantity index and (iii) value index

Methods of Constructing Index Numbers

1. Unweighted (simple) Index Number
   - Simple aggregative method
   - Simple average of price relative method
2. Weighted Index Number
   - Weighted average of price relative method
   - Weighted aggregative method

Formulae

1. Simple aggregative method

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

2. Simple average of price relative method

$$P_{01} = \frac{\sum \left(\frac{p_1}{p_0} \times 100\right)}{N}$$

3. Weighted aggregative method

$$P_{01} = \frac{\sum p_1 w}{\sum p_0 w} \times 100$$

4. Weighted average of price relative method

$$P_{01} = \frac{\sum \left(\frac{p_1}{p_2} \times 100\right) \times 100}{\sum w}$$

where, p_0 = price of base year,
p_1 = price of current year,
N = number of items,
w = weight related to item

Some other important formulae

- Laspeyre's price Index Number

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

- Passche's price Index Number

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

- Fisher's price Index Number

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$$

$$= \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}} \times 100$$

where p_0 = base year price
p_1 = current year price
q_0 = base year quantity
q_1 = current year quantity

Time Reversal Test

Index number satisfies the relationship

$$P_{01}^{F} \times P_{10}^{F} = 1$$

where P_{01}^{F} is Fisher's price index number

Time Series

Its componants:-
A time series is a set of observations taken at specified times, usually at equal time intervals

It has 4 components:-
(i) Secular (simple) Trend
(ii) Seasonal variations
(iii) cyclic variations
(iv) Irregular or random variations

Mathematical models for time series

Additive model:
$Y = T + S + C + I$
Multiplicative modes:-
$Y = T \times S \times C \times I$
where, Y = original value
T = Trend value
S = Seasonal variations
C = Cyclic variations
I = Irregular variations

Measurement of Trends

Method of moving averages

It helps to make forecast when all statistical information we take into account the last few data points, e.g: data for last three, four, five and six periods.
Three period moving average is defined as a value by adding numerical data for three periods and then dividing by 3.
Four period moving average is defined as a value by adding numerical data for four periods and then dividing by 4

Method of least square

The straight line trend in represented by
$Y = aX + b$
where, Y = Trend value to be computed
X = Unit of time (Independent variable)
a = constant to be calculated
b = constant to be calculated
Also, $a = \frac{\sum Y}{n}$ and $b = \frac{\sum xy}{\sum x^2}$
n = number of items

How to understand Mind Map?
First Level · Second Level · Third Level

Financial Mathematics

Nominal Rate of Return

The nominal rate of return is the total rate of return earned on investment before adjusting for any deductions or remiums.

$$\text{Nominal rate of return} = \frac{(\text{Ending value of investment} - \text{Begning value of investment})}{\text{Begning value of investment}} \times 100$$

e.g. Suppose Ram has invested ₹10,000 on 1st feb. 2020 on 31 Jan. 2021, calculate NROR, if investment grown to 20,000

Sol. $NROR = \frac{₹20,000 - ₹10,000}{₹10,000} \times 100$

$= 100\%$

EMI

An equated monthly installment (EMI) is a fixed payment amount mode by a borrower to a lender at a specified date of each calender month.

$$EMI = \frac{(1+i)^n}{(1+i)^n - 1} \times (P \times i)$$

where, i = monthly interest rate [annual rate /12/100]

n = number of installments

P = principal amount of loan

Perpetuity

A perpetuity is a type of annuity that lasts forever.

e.g.: In a real state sector when an owner purchases a property and then rents it out. This owner is entitled to an infinite stream of cash flow from the renter as long as property continues to exist.

It is of 2 types :

1. Flat perpetuity = $\frac{\text{Cash flow}}{\text{Interest rate or yield}}$
2. Growing perpetuity = $\frac{\text{Payment}}{\text{Interest rate - Growth rate}}$

Sinking Fund

A sinking fund is a fund containing money set aside or saved to pay off a debt or bond.

$$\text{Sinking fund A} = \frac{\left[1 + \left(\frac{r}{m}\right)^{n \times m}\right] - 1}{\left(\frac{r}{m}\right)} \times P$$

where P = periodic payments

r = annualized rate of interest

n = No. of year

m = No. of payments per year

Stocks and Shares

Shares: The capital of a company is divided into small units, which are commonly known as shares.

Stocks: The conversions of the fully paid up shares of a member into a single fund is known as stock.

Debenture

Debentures are some of the debt instruments which can be used by government, companies, organizations for the purpose of issuing loan.

Bond Valuation

Bond valuation is a process of calculating its fair price.

$$\text{Present value of Bond} = \sum_{t=1}^{N} \frac{C}{(1+r)^t} + \frac{P}{(1+r)^N}$$

where c = periodic coupon payment

r = Market interest rate

P = Per value

N = Years of maturity

Depreciation and Linear Method

Depreciation is an accounting practice of spreading the cost of an asset purchased across its useful lifetime.

Linear Method for Calculating Depreciation:

Annual Depreciation Expense=

(Asset cost – Residual value)/Useful life of the asset

e.g.: Seema wants to buy a mobile phone for ₹ 5000. Based on the post experience she estimates a 3 year life span and a salvage value of ₹ 200, then

$$\text{Annual depreciation expenses} = \frac{₹(5000 - 200)}{3}$$

$= ₹1600$

Compounded Annual Growth Rate

CAGR indicates the average rate of growth when Ne reinvest the return over a number of years.

$$CAGR = \left(\frac{\text{End value}}{\text{begining value}}\right)^{\frac{1}{n}} - 1$$

where n = investment period

e.g:

	Date	Value
Begining value	3-01-2010	2,000/-
End value	3-07-2017	5,000/-

$$CAGR = \left(\frac{5000}{2000}\right)^{\frac{1}{7}} - 1 = 1.1398 - 1$$

$= 0.1398$

$= 13.98\%$

Effective Interest Rate

Effective Interest rate , $E = (1+i)^n - 1$

where i = rate of interest/nominal or actual rate of interest e.g. UCO bank offers nominal rate of interest 12% on its certificate of deposit to its client. The client initially invested ₹ 1,000 and agreed to have the interest rate compounded monthly for a full year.

$$\text{Effective rate} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 12.683\%$$

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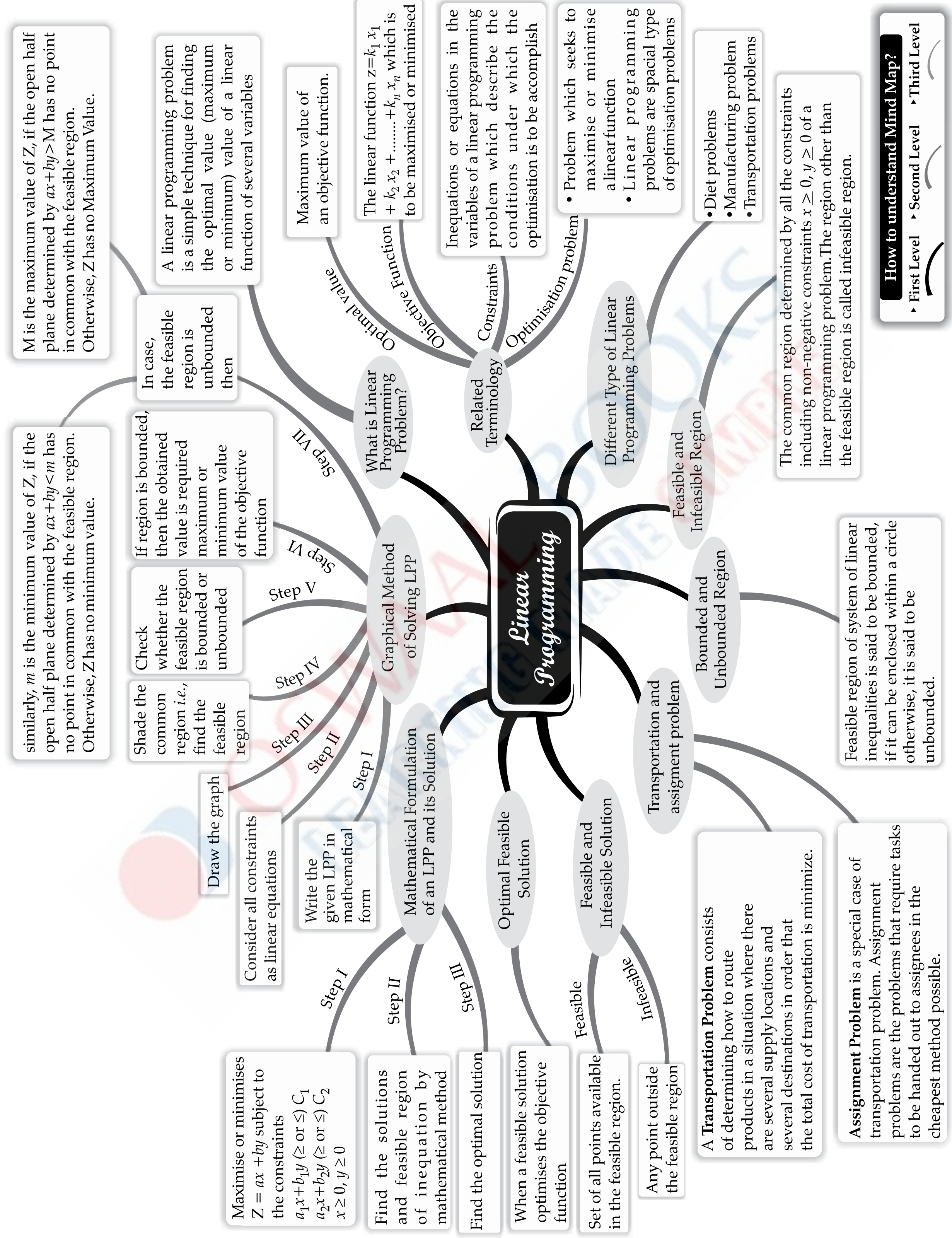

Linear Programming
What is Linear Programming Problem?
A linear programming problem is a simple technique for finding the optimal value (maximum or minimum) value of a linear function of several variables
Related Terminology
Optimal value
Maximum value of an objective function.
Objective Function
The linear function $z = k_1 x_1 + k_2 x_2 + \ldots + k_n x_n$ which is to be maximised or minimised
Constraints
Inequations or equations in the variables of a linear programming problem which describe the conditions under which the optimisation is to be accomplish
Optimisation problem
• Problem which seeks to maximise or minimise a linear function
• Linear programming problems are spacial type of optimisation problems
Different Type of Linear Programming Problems
•Diet problems
•Manufacturing problem
•Transportation problems
Feasible and Infeasible Region
The common region determined by all the constraints including non-negative constraints $x \geq 0, y \geq 0$ of a linear programming problem. The region other than the feasible region is called infeasible region.
Bounded and Unbounded Region
Feasible region of system of linear inequalities is said to be bounded, if it can be enclosed within a circle otherwise, it is said to be unbounded.
Transportation and assigment problem
A **Transportation Problem** consists of determining how to route products in a situation where there are several supply locations and several destinations in order that the total cost of transportation is minimize.
Assignment Problem is a special case of transportation problem. Assignment problems are the problems that require tasks to be handed out to assignees in the cheapest method possible.
Feasible and Infeasible Solution
Feasible
Set of all points available in the feasible region.
Infeasible
Any point outside the feasible region
Optimal Feasible Solution
When a feasible solution optimises the objective function
Mathematical Formulation of an LPP and its Solution
Step I
Maximise or minimises $Z = ax + by$ subject to the constraints
$a_1x + b_1y\ (\geq \text{ or } \leq)\ C_1$
$a_2x + b_2y\ (\geq \text{ or } \leq)\ C_2$
$x \geq 0, y \geq 0$
Step II
Find the solutions and feasible region of inequation by mathematical method
Step III
Find the optimal solution
Graphical Method of Solving LPP
Step I
Write the given LPP in mathematical form
Step II
Consider all constraints as linear equations
Step III
Draw the graph
Step IV
Shade the common region i.e., find the feasible region
Step V
Check whether the feasible region is bounded or unbounded
Step VI
If region is bounded, then the obtained value is required maximum or minimum value of the objective function
Step VII
In case, the feasible region is unbounded then
M is the maximum value of Z, if the open half plane determined by $ax+by>M$ has no point in common with the feasible region. Otherwise, Z has no Maximum Value.
similarly, m is the minimum value of Z, if the open half plane determined by $ax+by<m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.
How to understand Mind Map?
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Second Level
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