PART – I : ELECTROSTATICS

Chapter - 1 : Electric Charges and Fields

Revision Notes

Electric Field and Dipole

Electric Charge

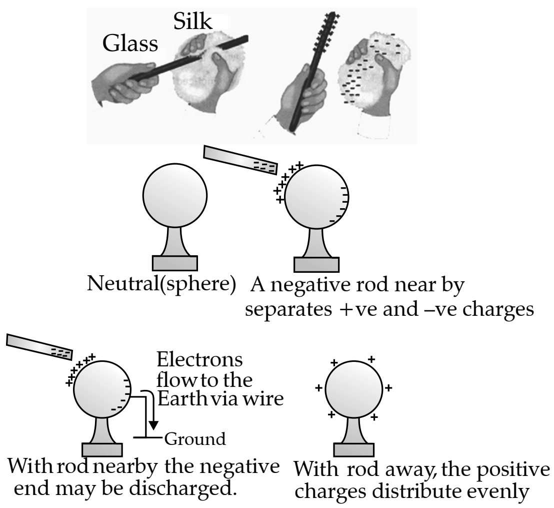
- Electric charge is the property of a matter due to which, it experiences a force when placed in an electromagnetic field.
- Point charge is an accumulation of the electric charges at a point, without spatial extent.
- Electrons are the smallest and lightest fundamental particles in an atom having negative charge as these are surrounded by invisible force known as electrostatic field.
- Protons are comparatively larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons or two protons will tend to repel each other as they carry like charges, negative and positive respectively.
- The electron and proton will get attracted towards each other due to their unlike charges.
- The charge present on the electron is equal and opposite to charge on the proton.

  Charge on a proton $= +1.6 \times 10^{-19}$ C

  and charge on an electron $= -1.6 \times 10^{-19}$ C

Electrostatic Charge

- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental physical quantity like length, mass and time.
- Charge on a body is expressed as $q = \pm ne$
- The magnitude of charge is independent of the speed of the particle.
- Based on the flow of charge across them, materials are classified as:
  - Conductors - Allow electric charge to flow freely, *e.g.* metals
  - Semi-conductors - Behave as the conductor or insulator depending on the number of free electrons and holes availability. e.g. silicon
  - Insulators - Do not allow electric charge to flow, *e.g.* rubber, wood, plastic, etc.
- **Net charge on a body is given by:**
  - Charging by friction - Charging insulators
  - Charging by conduction - Charging metals and other conductors
  - Charging by induction - Wireless charging



Charging by Rubbing

- On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged.
- If a plastic rod is rubbed with wool, it becomes negatively charged.

Charging by Induction

- Charging by induction means charging without contact.
- If a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and attracts positive charges on metal.

- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.
- When the rod is removed, the captive positive charge is distributed evenly.

Properties of Electric Charge

Addition of charges

- If a system contains three point charges q_1, q_2 and q_3, then the total charge of the system will be the algebraic addition of q_1, q_2 and q_3, *i.e.*, charges will add up.

$$q = q_1 + q_2 + q_3$$

Conservation of charges

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge can neither be created nor destroyed in the process, but only exists in positive-negative pairs.

Quantization of charges

- Electric charge is always quantized *i.e.*, electric charge is always an integral multiple of charge '*e*'.
- Net charge q_{net} of an object having N_e electrons, N_p protons and N_n neutrons is:
  $q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = \pm ne$
- Neutron (n): $m = 1.675 \times 10^{-27}$ kg; $q = 0$
- Proton (p): $m = 1.673 \times 10^{-27}$ kg; $q = +1.6 \times 10^{-19}$ C
- Electron (e): $m = 9.11 \times 10^{-31}$ kg; $q = -1.6 \times 10^{-19}$ C

Coulomb's Law

- The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as:

$$F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

  where,

  F = Force of attraction/repulsion between charges q_1 and q_2.

  q_1, q_2= Magnitudes of charge 1 and charge 2 respectively

  r = Distance between charges q_1, q_2

  k = Constant whose value depends on medium where charges are kept. $k = \dfrac{1}{4\pi\varepsilon}$

  As $\varepsilon = K'\varepsilon_0$, $k = \dfrac{1}{4\pi K'\varepsilon_0}$

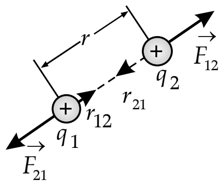
  ε_0 = Permittivity of vacuum = 8.854×10^{-12} F/m

  K' = Relative permittivity of medium or dielectric constant.

- For vacuum, relative permittivity, $K' = 1$,
- As $\varepsilon = K'\ \varepsilon_0$, therefore force of attraction/repulsion between two electric charges q_1, q_2 placed in vacuum and medium will be:

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} \text{ (vacuum) and } F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{q_1q_2}{r^2} \text{ (medium)}$$

- The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- Current is the rate $\dfrac{dq}{dt}$ at which charge moves past a point or through a region, $i = \dfrac{dq}{dt}$, hence 1 C=(1 A) × (1 s).
- The vector form of Coulomb force with $\hat{r}_{12}$ = unit vector from q_1 to q_2 is given as:

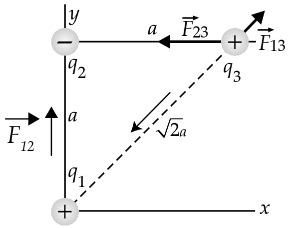


$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}\hat{r}_{12} \text{ and } \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21}$$

$\Rightarrow$
$$\vec{F}_{21} = -\vec{F}_{12}$$

Principle of Superposition

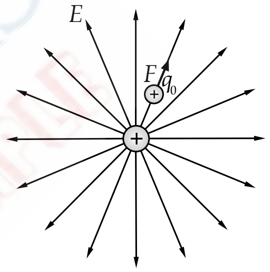
- The force on any charge due to other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.
- Force exerted by q_1 on $q_3 = \vec{F}_{13}$
- Force exerted by q_2 on $q_3 = \vec{F}_{23}$
- Net force exerted on q_3 is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$



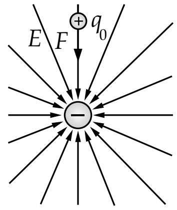
Electric field

- The space around a charge up to which its electric force can be experienced is called electric field.
- If a test charge q_0 is placed at a point where electric field is E, then force on the test charge is $F = q_0E$
- The electric field strength due to a point source charge 'q' at an observation point 'A' at a distance 'r' from the source charge is given by:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{r} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

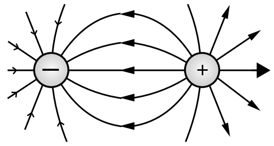


- The unit of electric field is N/C.
- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in an external field, the field in conductor is zero.
- In case of charged conductor, electric field is independent of the shape of conductor.



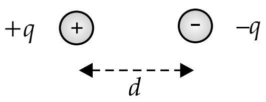
Electric field lines

- Electric field lines are imaginary lines which originates from the positive charge and towards negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge q_0 located around source charge.
- Electric field has same direction as force on positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge q_0.
- The electric field lines never intersect each other.
- Strength of electric field is encoded in density of field lines.



Electric Dipole

- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.



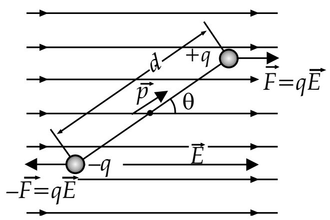
- The electric field exists due to a dipole.
- The force on a dipole in a uniform electric field is zero in both stable as well as unstable equilibrium.
- The potential energy of a dipole in an uniform electric field is minimum for a stable equilibrium and maximum for an unstable equilibrium.

Torque on a dipole

- In a dipole, when net force on dipole due to electric field is zero and center of mass of dipole remains fixed, the forces on charged ends produce net torque τ about its center of mass.

$$\tau = F\,d\sin\theta = qE\,d\sin\theta = pE\sin\theta$$
$$\vec{\tau} = \vec{p}\times\vec{E}$$

  - If $\theta = 0^\circ$ or 360°, dipole exists in stable equilibrium state.
  - If $\theta = 180^\circ$, dipole exists in an unstable equilibrium state.
- In uniform electric field, dipole experiences torque, net force on dipole is zero.
- In uniform electric field, dipole experiences a rotatory motion.



- In non-uniform electric field, dipole experiences torque and net force.
- In non-uniform electric field, dipole experiences rotatory and translatory motion.
  - The torque aligns dipole with electric field and it becomes zero.
  - The direction of torque is normal to the plane going inward.

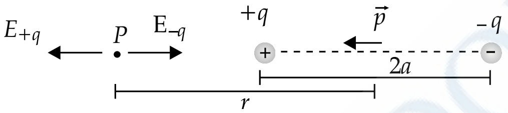
Electric Dipole Moment

- Dipole moment is a vector quantity whose unit is coulomb-metre (Cm).
- Dipole moment vector of electric dipole is $\vec{p} = \vec{q} \times 2a$ between pair of charges q, $-q$, along the line, separated by distance $2a$.

Electric field due to a dipole

- For point P at distance r from centre of dipole on charge q, for $r >> a$, total field at point P is

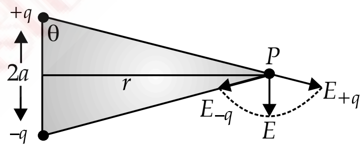
$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}$$



$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \quad (\text{if } a << r)$$

- For point P on the equatorial plane due to charges $+q$ and $-q$, electric field of dipole at a large distance,

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$



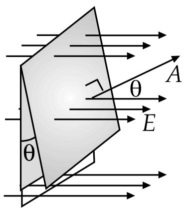
Gauss's Theorem and its Applications

Electric Flux

- Electric flux is proportional to algebraic number of electric field lines passing through the surface, outgoing lines with positive sign, incoming lines with negative sign.
- Due to arbitrary arrangement of electric field lines, electric flux can be quantify as $\phi_E = EA$
- If vector A is perpendicular to surface, magnitude of vector A parallel to electric field is $A\cos\theta$

  $A_{II} = A\cos\theta$

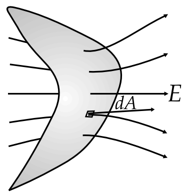
  $\phi_E = EA_{II} = EA\cos\theta$



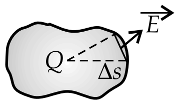
- In non-uniform electric field, the flux will be $\phi_E = \int E dA$

Continuous Charge Distribution

- It is a system in which the charge is uniformly distributed over the material. In this system, infinite number of charges are closely packed and have minor space among them. Unlike the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the material. There are three types of continuous charge distribution system.

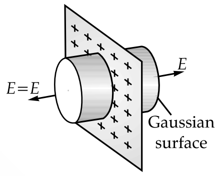


- For linear charge distribution (λ), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_x \frac{\lambda}{r^2} dx\hat{r}$ (Where, λ = linear charge density)
- For surface charge distribution (σ), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_s \frac{\sigma}{r^2} dS\hat{r}$ (Where, σ = surface charge density)
- For volume charge distribution (ρ), $\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_v \frac{\rho}{r^2} dV\hat{r}$ (Where, ρ = volume charge density)



Gauss theorem

- The net outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times to net charge enclosed by the surface.
- The electric field flux at all points on Gaussian surface is $\phi = E\oint dA = \frac{q}{\varepsilon_0}$
- If there is a positive flux, net positive charge is enclosed.
- If there is a negative flux, net negative charge is enclosed.
- If there is zero flux, no net charge is enclosed.
- The expression for electric field due to a point charge on Gaussian surface is $E = \frac{q}{4\pi\varepsilon_0 r^2}$
- In an insulating sheet, charge remains in the sheet, so electric field, $E = \frac{\sigma}{2\varepsilon_0}$
- Gauss theorem works in cases of cylindrical, spherical and rectangular symmetries.



- The field outside the wire points radially outward which depends on distance from wire, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$, where, λ is linear density of charge.
- Closed surface : It is a surface which divides the space inside and outside region, where one can't move from one region to another without crossing the surface.
- Gaussian surface : It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- Electrostatic Shielding : It is the phenomenon of protecting a certain region of space from external electric field.
- Dielectric : The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. *e.g.* Air, H_2 gas, glass, mica, paraffin wax, transformer oil, etc.

Mnemonics

Concept: Characteristics of Electric field lines

Mnemonics: **I**ndia **S**tarts **P**laying **N**ight **C**ricket **T**ournament Daily with **N**ew **I**nspiration.

How to remember all the 5 characteristics ?

Remember this sentence.

India Starts Playing Night Cricket Tournament Daily with New Inspiration.

- India → Imaginary line
- Starts → Start from
- Playing → Positive charge
- Night → Never forms
- Cricket → closed loops
- Tournament → Tangent at any point
- Daily → Direction of field
- New → Never
- Inspiration → Intersects

Know the Terms

- 1 coulomb: When two point charges placed at a distance of 1 m in vacuum, repel/attract each other with force of 9×10^9 N, the charge on each is known as 1 coulomb.
- Electric line of force: It is a curve drawn in such a way that the tangent at each point to curve gives the direction of the net field at that point.

Know the Formulae

- Coulomb's force: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$;

  where all alphabets have their usual meanings.

- Electric field due to point charge q: $E = \frac{k|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

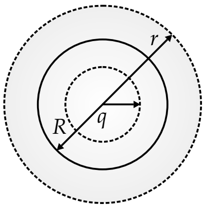
- Electric field due to a dipole at a point on the dipole axis: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$ $(r >>> a)$

- Electric field at a point on equatorial plane: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$ $(r >>> a)$

- Torque on an electric dipole placed in an electric field, $\tau = pE\sin\theta$

- Electric flux through an area A: $\phi = E.A = EA\cos\theta$

- Electric flux through a Gaussian surface: $\phi = \oint E.dS$



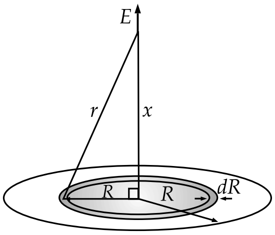
- Gauss's Law: $\phi = \frac{q_{enc}}{\varepsilon_0}$

- Electric Field due to an infinite line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$

  where, E = electric field [N/C], λ = charge per unit length [C/m]

  ε_0 = permittivity of free space = 8.85×10^{-12} [$C^2/N\,m^2$], r = distance (m), $k = 9 \times 10^9\ N\,m^2\,C^{-2}$

- Electric field due to a ring at a distance x is: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qx}{(r^2+x^2)^{3/2}}$



- When, $x >>> r$: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}$

- When $x <<< r$: $E = 0$

- Electric field due to a charged disc: $E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$

  where ,

  E = electric field [N/C] σ = charge per unit area [C/m^2]

  $\varepsilon_0 = 8.85 \times 10^{-12}$ [C^2/Nm^2] x = distance from charge [m]

  R = radius of the disc [m]

- Electric field due to a thin infinite sheet: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

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Chapter - 2 : Electrostatic Potential and Capacitance

Revision Notes

Electric Potential

Electric potential

- Electric potential is the amount of work done by an external force in moving a unit positive charge from infinity to a point in an electrostatic field without producing an acceleration.
- It is written as $V = \frac{W}{q}$

  where, W = work done in moving charge q through the field, q = charge being moved through the field.
- The SI units of electric potential are $\frac{\text{J}}{\text{C}}$, Volt, $\frac{\text{Nm}}{\text{C}}$.

Potential difference

- Electric potential difference is defined as the amount of work done in carrying a unit charge from one point to another in an electric field.

  Electric potential difference = $\frac{\text{Work}}{\text{Charge}} = \frac{\Delta PE}{\text{Charge}} = \frac{W}{q}$

  Between two points A and B, $W_{AB} = -V_{AB} \times q$

  where, $V_{AB} = V_B - V_A$ is potential difference between A and B.
- In a region of space having an electric field, the work done by electric field dW, when positive point charge q, is displaced by a distance ds, then,

$$dW = q\vec{E}.\vec{ds}$$

$$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B q\vec{E}.\vec{ds}}{q} = -\int_A^B \vec{E}.\vec{ds}$$

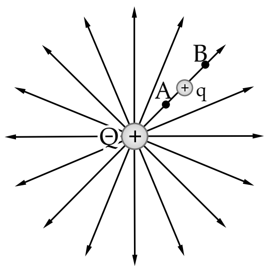
Electric potential due to point charge

- The electric potential by point charge q, at a distance r from the charge, can be written as,

$$V_E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

  where, ε_0 is permittivity of vacuum (free space).
- Electric potential is a scalar quantity.
- Dimension of Electric potential is $[\text{M L}^2\text{T}^{-3}\text{A}^{-1}]$.
- For a single point charge q, the potential difference between A and B is given by,

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}.\vec{ds} = -\int_A^B Eds\cos 0° = -E\int_A^B ds$$



  where, E is the field due to a point charge, $ds = dr$, so that,

$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0}\cdot\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

- If $r_B = \infty$, then $V_B = 0$ so,

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_A} = \frac{kq}{r_A}$$

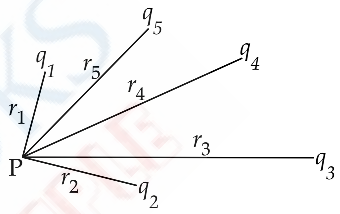
Dipole and system of charges

- Electric dipole consists of two equal but opposite electric charges which are separated by a distance.
- The net potential due to a dipole at any point on its equatorial line is always zero. So, work done in moving a charge on an equatorial line is always zero.
- Electric potential due to dipole at a point at distance r and making an angle θ with the dipole moment p is given by, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}.\hat{r}}{r^2} (r >> a)$
- Potential at a point due to system of charges is the sum of potentials due to individual charges.
- In a system of charges $q_1, q_2, q_3, \ldots q_n$ having positive vectors $r_1, r_2, r_3, \ldots r_n$ relative to point P, the potential at point P due to total charge configuration is algebraic sum of potentials due to individual charges, so,

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



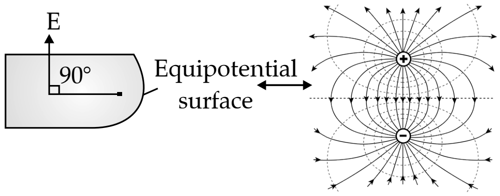
- It is known that in a uniformly charged spherical shell, here electric potential outside the shell is given as:

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad (r \geq R)$$

where, q is the total charge on shell and R is the shell radius.

Equi-potential surfaces

- Equi-potential surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface, hence the surface will have no electric field, so E will be at right angle to the surface.
- Work done in moving a charge over equi-potential surface is always zero.
- Electric field is always perpendicular to the equi-potential surface.
- Spacing among equi-potential surfaces allows to locate regions of strong and weak electric field.
- Equi-potential surfaces never intersect each other. If they intersect then the intersecting point of two equi-potential surfaces results in two values of electric potential at that point, which is impossible.

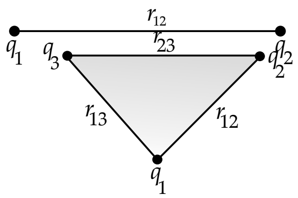


- Potential energy of a system of two charges,

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

- Potential energy of a system of three charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



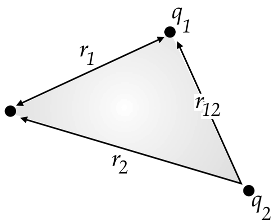
- Potential energy due to single charge in an external field:
Potential energy of a charge q at a distance r in an external field,

$$U = qV(\vec{r})$$

Here, $V(\vec{r})$ is the external potential at point r.

- Potential energy due to two charges in an external field,

$$U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$



- Potential energy of a dipole in an external field:

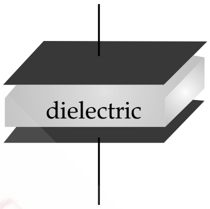
  When a dipole of charge $q_1 = +q$ and $q_2 = -q$ having separation '$2a$' is placed in an external field ($\vec{E}$).

  $U(\theta) = -pE\cos\theta$

  Here, $p = 2aq$ and θ is the angle between electric field and dipole.

Capacitance

Conductors and insulators



- Conductors are the materials through which charge can move freely. **Examples:** Metals, semi-metals as carbon, graphite, antimony and arsenic.
- Insulators are materials in which the electrical current does not not flow easily. Such materials cannot be grounded and do not easily transfer electrons. Examples: Plastics and glass.

Dielectrics

- These are the materials in which induced dipole moment is linearly proportional to the applied electric field.
- Electrical displacement or electrical flux density, $D = \varepsilon_r\varepsilon_0 E$.
  where, ε_r= Electrical relative permittivity, ε_0 = Electrical permittivity of free space and E is electric field.
- If a dielectric is kept in between the plates of capacitor, capacitance increases by factor 'κ' (kappa) known as dielectric constant, so $C = \kappa\varepsilon_0\dfrac{A}{d}$

  where, A = area of plates

  κ = dielectric constant of material is also called relative permittivity $\kappa = \varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$

Material	Dielectric Constant (κ)	Dielectric strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Pyrex Glass	5.6	14
Water	80	-

- In dielectric, polarisation and production of induced charge takes place when dielectric is kept in an external electric field.

Electric polarization

- Electric polarization P is the difference between electric fields D (induced) and E (imposed) in dielectric due to bound and free charges written as $P = \dfrac{D-E}{4\pi}$
- In term of electric susceptibility: $P = \chi_e E$
- In MKS: $P = \varepsilon_0\chi_e E$,
- The dielectric constant κ is always greater than 1 as $\chi_e > 0$

Capacitor

- A capacitor is a device which is used to store charge.
- Amount of charge 'Q' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate is connected to negative terminal.
- Electric charge stored between plates of capacitor is directly proportional to potential difference between its plates, *i.e.*,

  $$Q = CV$$

  where, C = Capacitance of capacitor, V= potential difference between the plates
- In capacitor, energy is stored in the form of electrical energy, in the space between the plates.

Capacitance

- Capacitance of a capacitor is ratio of magnitude of charge stored on the plate to potential difference between the plates, written as $C = \dfrac{Q}{\Delta V}$

  where, C = capacitance in farads (F), Q = charge in Coulombs (C), ΔV = electric potential difference in Volts (V),
- SI unit of capacitance is farad (F)
- $1\,F = \dfrac{1\,C}{1\,V} = 9 \times 10^{11}$ stat farad,

  Where, stat-farad is electrostatic unit of capacitance in C.G.S. system
- Capacitance of a conductor depends on size, shape, medium and other conductors in surrounding.
- Parallel plate capacitor with dielectric among its plates has capacitance which is given as:

$$C = \kappa\varepsilon_0 \frac{A}{d},$$

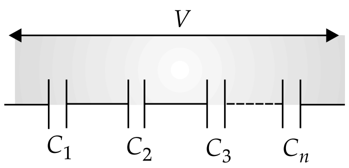
  where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
- Capacitor having capacitance of 1 Farad is too large for electronics applications, so components with lesser values of capacitance such as μ (micro), n (nano) and p (pico) are applied such as:

PREFIX	MULTIPLIER	
μ	10^{-6} (millionth)	$1\,\mu F = 10^{-6}$ F
n	10^{-9} (thousand-millionth)	$1\,nF = 10^{-9}$ F
p	10^{-12} (million-millionth)	$1\,pF = 10^{-12}$ F

Combination of capacitors in series and parallel

Capacitors in series

- (i) If a number of capacitors of capacitances C_1, C_2, C_3, ... C_n are connected in series, then their equivalent capacitance is given by:



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots\ldots\ldots \frac{1}{C_n}$$

- In series combination, the charge on each capacitor is same, but the potential difference on each capacitor depends on their respective capacitance, *i.e.*,

$$q_1 = q_2 = q_3 \ldots\ldots\ldots\ldots q_n = q$$

- If V_1, V_2, V_3, ..., V_n be the potential differences across the capacitors and V be the emf of the charging battery, then

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots + V_n$$

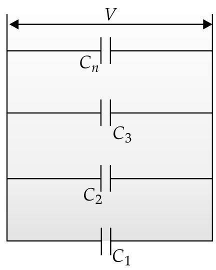
- As charge on each capacitor is same, therefore

$$q = V_1C_1 = V_2C_2 = V_3C_3 \ldots\ldots\ldots\ldots$$

  the potential difference is inversely proportional to the capacitance, *i.e.*,

$$V \propto \frac{1}{C}$$

- In series, potential difference across largest capacitance is minimum.
- The equivalent capacitance in series combination is less than the smallest capacitance in combination.



Capacitors in parallel

- (i) If a number of capacitors of capacitances C_1, C_2, C_3................C_n are connected in parallel, then their equivalent capacitance is given by,

$$C_p = C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots\ldots + C_n$$

- In parallel combination, the potential difference across each capacitor is same and equal to the emf of the charging battery, *i.e.*,

$$V_1 = V_2 = V_3 = \ldots\ldots\ldots\ldots = V_n = V$$

while the charge on different capacitors may be different.

- If $q_1, q_2, q_3, \ldots\ldots\ldots, q_n$ be the charges on the different capacitors, then

$$q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n = VC_p$$

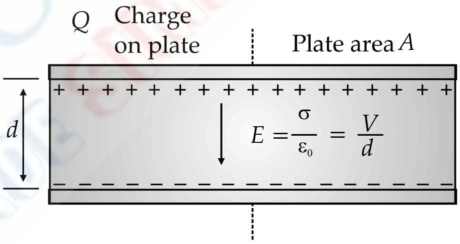
- As potential drop across each capacitor is same, so

$$\Rightarrow \qquad V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \ldots\ldots = \frac{q_n}{C_n}$$

- The charges on capacitors are directly proportional to capacitances, *i.e.*, $q \propto C$
- Parallel combination is useful when large capacitance with large charge gets accumulated on combination.
- Force of attraction between parallel plate capacitor will be $F = \frac{1}{2}\left[\frac{QV}{d}\right] = \frac{1}{2}QE$ where Q is charge on capacitor.

Capacitance of parallel plate capacitor with and without dielectric medium between the plates

- Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium.
- The electric field between two large parallel plates is given as:



$$E = \frac{\sigma}{\varepsilon_0},$$

Where, σ = charge density and ε_0 = permittivity of free space

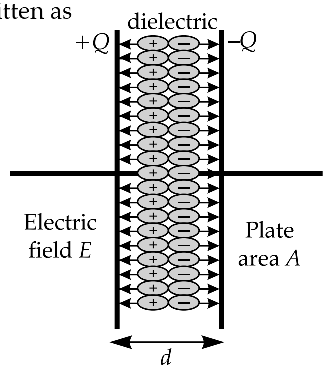
Surface charge density,

$$\sigma = \frac{Q}{A},$$

where, Q = charge on plate and A = plate area

- Capacitance of parallel-plate capacitor with area A separated by a distance d is written as

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$



- If a dielectric slab is placed in between the plates of a capacitor, then its capacitance will increase by certain amount.
- Capacitance of parallel plate capacitor depends on plate area A, distance d between the plates, medium between the plates (κ) and not on charge on the plates or potential difference between the plates.
- If we have number of dielectric slabs of same area as the plates of the capacitor and thicknesses $t_1, t_2, t_3, \ldots$. and dielectric constant $\kappa_1, \kappa_2, \kappa_3 \ldots\ldots$ between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \frac{t_3}{\kappa_3} + \ldots},$$

Where, $d = t_1 + t_2 + t_3 + \ldots\ldots$

- If slab of conductor of thickness t is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{\frac{t}{\kappa} + \frac{(d-t)}{1}} = \frac{\varepsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}}$$

$$C = \frac{\varepsilon_0 A}{d - t} \qquad (\because \kappa = \infty \text{ for a conductor})$$

➢ When the medium between the plates consists of slabs of same thickness but areas A_1, A_2, A_3,... and dielectric constants κ_1, κ_2, κ_3..., then capacitance is given by

$$C = \frac{\varepsilon_0(\kappa_1 A_1 + \kappa_2 A_2 + \kappa_3 A_3 \ldots.)}{d}$$

$$\therefore \qquad \kappa = \frac{C_m}{C_0} = \frac{\text{Capacitance in medium}}{\text{Capacitance in vacuum}}$$

➢ When space between the plates is partly filled with medium of thickness t and dielectric constant κ, then capacitance will be:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}$$

When there is no medium between the plates, then $\kappa = 1$, so $C_{\text{vacuum}} = \frac{\varepsilon_0 A}{d}$

➢ Capacitance of spherical conductor of radius R in a medium of dielectric constant κ is given by,

$$C = 4\pi\varepsilon_0\kappa R$$

Energy stored in capacitor

➢ In capacitor, energy gets stored when a work is done on moving a positive charge from negative conductor to positive conductor against the repulsive forces.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

➢ **Polar atom:** Atom in which positive and negative charges possess asymmetric charge distribution about its centre.
➢ **Polarisation:** The stretching of atoms of a dielectric slab under an applied electric field.
➢ **Dielectric strength:** The maximum value of electric field that can be applied to dielectric without its electric breakdown.
➢ **Dielectric:** It is an electrically insulated or non-conducting material considered for its electric susceptibility.
➢ Permittivity: It is a property of a dielectric medium that shows the forces which electric charges placed in medium exerts on each other.

OR

It is the measure of resistance that is encountered when forming an electric field in a particular medium. More specifically, permittivity describes the amount of charge needed to generate one unit of electric flux in a particular medium.

Mnemonics

Concept: Characteristics of equi-potential surface
Mnemonics: Exclusive peace and No war Noble India is super power
Interpretation: **Exclusive peace and:** **E**lectric field is **p**erpendicular to the surface **No war:** **N**o **W**ork is done on moving a charge on the surface **Noble India is:** **N**ever **I**ntersects **Super Power:** **S**ame **p**otential everywhere on the surface

Key Formulae

➢ Electric Potential, $V = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.

➢ Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$

➢ Electric potential due to a point charge q at a distance r away: $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

➢ Finding V from E: $V_f - V_i = -\int_i^f \vec{E}.d\vec{S}$

➢ Potential energy of two point charges in absence of external electric field: $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}}\right]$

➢ Potential energy of two point charges in presence of external electric field: $q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$

Note: All symbols have their usual meanings.
Concept: Characteristics of equi-potential surface.
Exclusive peace and No war Noble India is super power.
Interpretation:
Electric field is perpendicular to the surface.
No Work is done in moving a charge on the surface.
Never intersects.
Same potential everywhere on the surface.

➢ Capacitance, $C = \frac{Q}{V}$, measured in Farad; 1 F = 1 coulomb/volt

➢ Parallel plate capacitor:

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

➢ Cylindrical capacitor:

$$C = 2\pi\kappa\varepsilon_0 \frac{L}{\ln(b/a)}$$

where, L= length [m], b= radius of the outer conductor [m], a = radius of the inner conductor [m]

➢ Spherical capacitor:

$$C = 4\pi\kappa\,\varepsilon_0\left(\frac{ab}{b-a}\right)$$

where, b = radius of the outer conductor [m], a = radius of the inner conductor [m]

➢ Maximum charge on a capacitor:

$$Q = VC$$

➢ For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the **dielectric constant** κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ and Q is unchanged. In vacuum, κ = 1 and when dielectrics are used, replace ε_0 with $\kappa\,\varepsilon_0$.

➢ Electrical energy stored in a capacitor: [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

➢ Surface charge density or Charge per unit area: [C/m^2]

$$\sigma = \frac{q}{A}$$

➢ **Energy density:**
- Electric energy density is also called Electrostatic pressure.
- Electric force between plates of capacitor,

$$F = \frac{1}{2}\varepsilon_0 E^2 \cdot A$$

- Energy stored in terms of Energy density,

$$\frac{E}{A \times d} = \frac{1}{2}\varepsilon_0 E^2$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

where, U = energy per unit volume [J/m^3], ε_0= permittivity of free space, = 8.85 × 10^{-12} C^2/Nm^2, E = energy [J]

- **Capacitors in series:**

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

- **Capacitors in parallel:**

$$C_{eff} = C_1 + C_2 \ldots$$

❐❐

PART – II : CURRENT ELECTRICITY

Chapter - 3 : Current Electricity

Revision Notes

Electric Current, Resistance and Cells

Electric current

- Electric current is defined as the rate of flow of charge, across the cross section of conductor *i.e.*, $I = \frac{dq}{dt}$
- When charge flows at a constant rate, the corresponding electric current can be written as : $I = \frac{q}{t}$
- Conventional current in an external circuit flows from positive terminal to negative terminal.
- Free electrons flow from the negative terminal to the positive terminal in the external circuit.
- 1 ampere current = 6.25 × 10^{18} electrons flowing per second.
- Direct current is unidirectional flow of electric charge.

Flow of electric charges in metallic conductor

- When an electric field is applied to a metal at certain points, free electrons experience force and start moving
- Without external applied emf, free electrons will move randomly through metal from one point to other giving zero net current.
- Motion of conducting electrons in electric field is a combination of motion due to random collisions.
- Drift velocity, mobility and their relation with electric current
- Drift Velocity is an average velocity which is obtained by certain particle like electron due to the presence of electric field.
- Drift velocity is written as :

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

where, relaxation time, $\tau = \frac{\lambda}{v}$, here e = charge, m = mass, λ = mean free path

- When electric current is set up in a conductor, electrons drift through the conductor with velocity v_d, is given as

$$v_d = \frac{I}{neA} \text{ or } I = neAv_d$$

where, I = Electric current through conductor, n = Number of free electrons per unit volume, A = Area of cross-section, e = Charge of electron

- Drift velocity of electrons under ordinary conditions is of the order of 0.1 mm/s.
- Mobility is the drift velocity of an electron when applied electric field is unity.

Mobility, $$\mu = \frac{v_d}{E}$$

or, $$\mu = \frac{e\tau E / m}{E} = \frac{e\tau}{m}$$

Electrical resistivity and conductivity

- Resistivity is the specific resistance that is given by the conductor having unit length and unit area of cross-section.

$$\rho = \frac{m}{ne^2\tau}$$

- Conductivity is the reciprocal of resistivity shown as :

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

Ohm's law

- The flow of current through conductor is directly proportional to the potential difference established across the conductor, provided physical conditions remains constant.

or, $$I \propto V$$
or, $$I = GV$$
Here, $$G = \frac{1}{R}$$
or, $$I = \frac{1}{R}V$$
or, $$V = IR$$
where, R = resistance of conductor

Electrical resistance

- It is an obstacle that is shown by the body during the flow of current as :

$$R = \frac{V}{I} = \frac{m}{ne^2\tau}\frac{l}{A}$$

- The resistance of the conductor is given as : $R = \rho\frac{l}{A}$

where, $\rho = \frac{m}{ne^2\tau}$ is specific resistance or resistivity of the material of conductor.

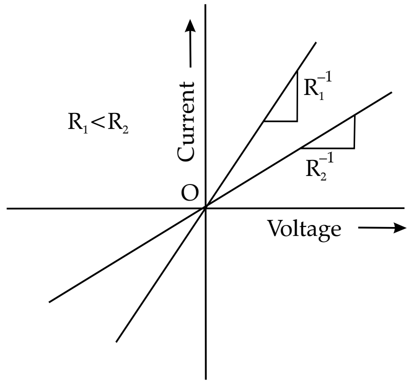
- In the series combination of resistances, the current is same throughout each resistor.
- In the parallel combination of resistances, the potential difference is same across each resistor.

***V-I* characteristics (linear and non-linear)**

- *V-I* characteristic curves show the relationship between the current flowing through an electronic device and applied voltage across its terminals.

Linear *V-I* Characteristics

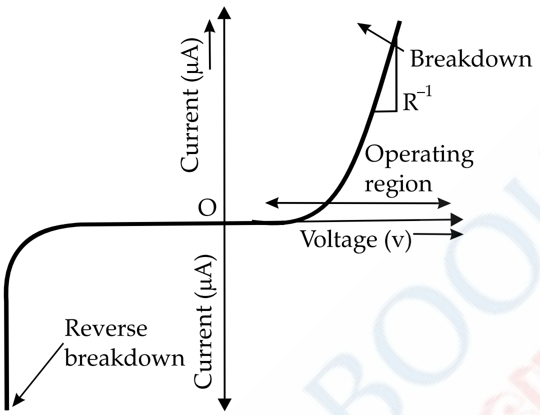
- A linear *V-I* curve has a constant slope and hence a constant resistance. Carbon resistors and metals obey the Ohm's law and have a constant resistance. This means that the *V-I* curve is a straight line passing through the origin.



- An electronic component may exhibit linear characteristic only in a particular region. For example, a resistance shows linear behaviour mostly in its operating region.

Non-linear *V-I* Characteristics

- A circuit component has a non-linear characteristic if the resistance is not constant throughout and is some function of voltage or current. The diode, for example, has varying resistance for different values of voltage.
- However, it has linear characteristic for a narrow operating region. Note that in the graph above, we can also see the maximum forward and reverse voltage in which the diode can be operated without causing breakdown and burning up of the diode.



Electrical energy and power

- Electrical energy is stored in the charged particles in an electric field.

$$E = V \times i \times t = i^2 \times R \times t = \frac{V^2}{R} \times t$$

where, E = Electrical energy, V = Potential difference, t = Time, i = Current, R = Resistance

- Power is the work done per unit time which is the rate of energy consumed in a circuit.

$$P = \frac{W}{t}$$

Since Voltage $V = \frac{W}{q}$,

So, $P = V\frac{q}{t} = VI$ $\left[\text{Here, } I = \frac{q}{t}\right]$

or $P = I^2R$ or $\frac{V^2}{R}$

The unit of power is J/s or W (Watt).

Temperature dependence of resistivity

- With small change in temperature, resistivity varies with temperature as :

$$\rho = \rho_0(1 + \alpha\,\Delta T)$$

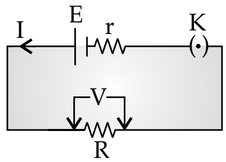
where, α = temperature coefficient of resistivity.

Internal resistance of cell

- Cell is a device that maintains the potential difference that is present between the two electrodes as a result of chemical reaction.
- Internal resistance is the resistance of electrolyte that is present in a battery which resists the flow of current when connected to a circuit.
- Emf E is the potential difference between the electrodes of cell, when no current flows through it.

Potential difference and emf of a cell

- **The emf and terminal potential difference of a cell :** Let emf of a cell be E and its internal resistance, r. If an external resistance R be connected across the cell through a key, then IR = V = potential difference across the external resistance R. This is equal to the terminal potential difference across the cell.



$$E = V + Ir$$

$$\Rightarrow \qquad I = \frac{E - V}{r}$$

$$\text{So} \qquad V = E - Ir$$

$$\therefore \qquad V < E. \qquad \text{(if there is flow of current)}$$

When current is drawn from a cell, its terminal potential difference is less than the emf.

Combination of cells in series and parallel

➢ **(i) Series combination of cells :** This combination is used when an external resistance (R) of the circuit is much larger as compared to the internal resistance (r) of the cell. *i.e.,*

$$R >> r$$

Let n cells, each of emf E and internal resistance r are connected in series across an external resistance R, then the current in the circuit will be

$$I_S = \frac{nE}{R + nr}$$

(ii) Parallel combination of cells : This combination is used when the external resistance R is much smaller as compared to the internal resistance (r) of the cell, *i.e.,*

$$R << r$$

When m cells are connected in parallel across a resistance R, then current through the resistance is given by

$$I_P = \frac{E}{R + r/m} = \frac{mE}{mR + r}$$

If m cells of emfs E_1, E_2, E_3,.....E_m and of internal resistances r_1, r_2, r_3,.....r_m are connected in parallel across an external resistance R, then the current through the external resistance is given by

$$I_P = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} ..... + \frac{E_m}{r_m}}{R + \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_m}\right)}$$

Kirchhoff's Laws, Wheatstone Bridge and their Applications

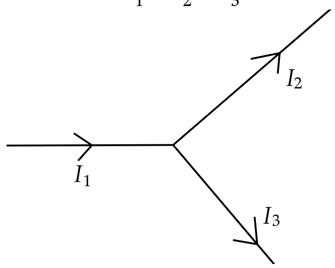
Kirchhoff's Laws

➢ Kirchhoff's Laws tell us about the relationship between voltages and currents in circuits.

First Law

➢ Kirchhoff's first law is also known as junction law which states that for a given junction or node in a circuit, sum of the currents entering in a junction will be equal to sum of currents leaving that junction.

$$I_1 = I_2 + I_3$$



OR

➢ The algebraic sum of all currents meeting at a junction in a closed circuit is zero. *i.e.,* $\Sigma I = 0$

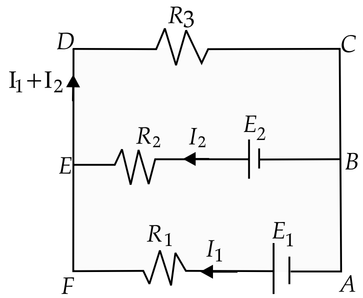
➢ This is called the law of conservation of charge.

Second Law

➢ Kirchhoff's second law is also known as loop law which shows that around any closed loop in a circuit, sum of the potential differences across all elements will be zero.

i.e., $\Sigma V = 0$ or $\Sigma V = \Sigma IR$

➢ This is called the law of conservation of energy.

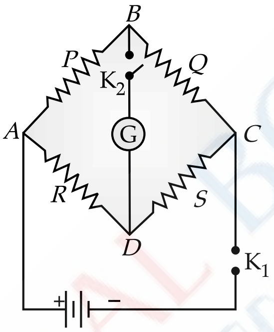


For example : Applying Junction law in loop *AFEBA*,

$$E_1 - E_2 = I_1R_1 - I_2R_2$$

Wheatstone Bridge

➢ It is a circuit having four resistances *P*, *Q*, *R* and *S*, a galvanometer and a battery connected as shown.



Wheatstone Bridge

➢ **Conductance :** The reciprocal of resistance with unit as siemens, "S."
➢ **Node :** An end point to any branch of a network or a junction common to two or more branches.
➢ **Permittivity :** The ability of a material to store electrical potential energy under the influence of an electric field measured by the ratio of the capacitance of a capacitor with the material as dielectric to its capacitance with vacuum as dielectric.
➢ **Galvanometer :** An instrument for detecting and measuring small electric currents.

Metre Bridge, Potentiometer and their Applications

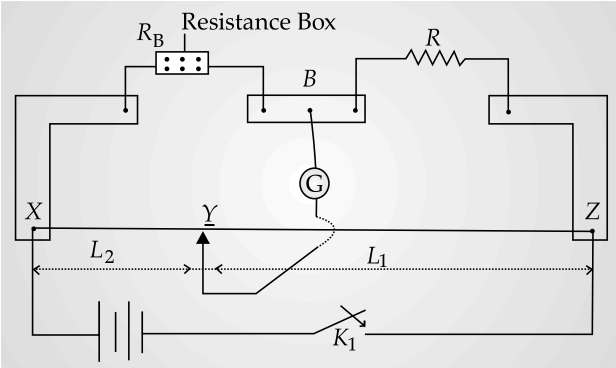
Metre Bridge

➢ It is an instrument which is used to find the unknown resistance of a coil or a material connected in a circuit.
➢ It is also known as slide wire bridge which is an instrument that works on the principle of Wheatstone bridge.
➢ Metre bridge has two metallic strips which act as holders for the wire that are made of metals like copper.
➢ **In metre bridge :**

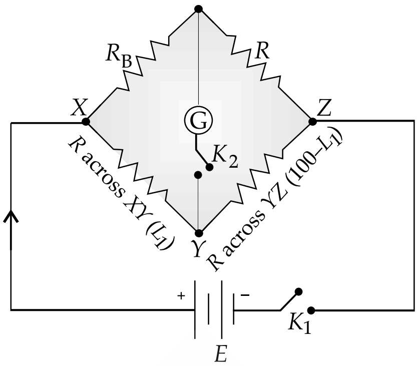
- Resistance box R_B and unknown resistance *R* are connected across the two gaps of metallic strips.
- One end of galvanometer is connected to the middle lead of metallic strip placed between L shaped strips while other end is connected to a jockey.
- Jockey which is a metal wire having one end as knife edge is used for sliding on the bridge wire.

Measurement from the Metre bridge :

➢ At negative terminal of galvanometer, there appears zero deflection that makes jockey to connect to negative point on the wire.
➢ The distance from point *X* to *Y* is taken as L_1 cm while the distance from point *Y* to point *Z* is taken as L_2 cm which can be $(100 - L_1)$ cm.
➢ Metre bridge can be drawn similar to Wheatstone bridge as :



Metre bridge



Wheatstone bridge

From the above arrangement :

$$\frac{R_B}{\text{Resistance across } XY} = \frac{R}{\text{Resistance across } YZ}$$

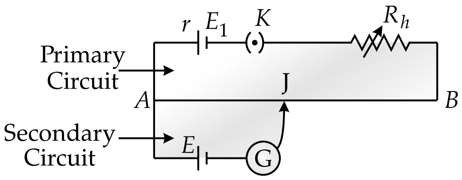
Now, $$\frac{R_B}{\frac{\rho L_1}{A}} = \frac{R}{\frac{\rho L_2}{A}} \qquad \left[\text{As}, R = \frac{\rho L}{A}\right]$$

Further, $$\frac{R_B}{\frac{\rho L_1}{A}} = \frac{R}{\frac{\rho(100 - L_1)}{A}} \qquad [\because L_2 = 100 - L_1]$$

Hence, $$\frac{R_B}{L_1} = \frac{R}{100 - L_1}$$

Potentiometer

- Potentiometer is a device which measures the emf of a particular cell and helps in comparing the emfs of different cells.
- Potentiometer depends on deflection method where zero deflection results in non drawn of current from the cell or circuit.
- It serves as an ideal instrument of infinite resistance for measuring the potential difference.
- Potentiometer comprises of long resistive wire AB of length L (about 6 m to 10 m long) made up of manganin or constantan.
- In this, a battery of known voltage E and internal resistance r forms the primary circuit.
- In the potentiometer circuit, one terminal of other cell is connected at one end of main circuit while other terminal is connected at any point on the resistive wire through galvanometer G which forms the secondary circuit.



where, J = Jockey, K = Key, R_h = Variable resistance which controls the current through the wire AB

In the circuit :

- Specific resistance (ρ) of wire is high while its temperature coefficient of resistance is low.
- At point A, all high potential points of primary and secondary circuits are connected together, while all low potential points are connected to point B or jockey.
- Value of known potential difference is more than the value of unknown potential difference that is to be measured.
- The current in primary circuit should remain constant and jockey should not slide with the wire.

Principle of potentiometer

- Potentiometers are displacement sensors that produce electrical output in proportion to the mechanical displacement.
- It can be used to measure the internal resistance and emf of a cell which cannot be measured by voltmeter.

- The basic principle of potentiometer is that the potential drop along any length of the wire is directly proportional to its length. So, when a constant current flows through a wire of uniform cross-section and composition then,

$$V \propto l.$$

- When there is zero potential difference between two points, there will be no flow of electric current.
- **Applications of potentiometer :** In measuring potential difference and comparing emf of cells in measuring potential difference.
- In a potentiometer, auxiliary circuit comprises of battery of emf E connected across terminals A and B with rheostat R_h, resistance box and key K in series where resistance R_1 is connected to terminal A and jockey J through galvanometer with cell E_1 and key K_1 in series, then if key K_1 is closed, current will flow through resistance R_1 where a potential difference is developed.
- If J is the position of jockey on potentiometer wire which gets adjusted in such a way that galvanometer shows no deflection, then AJ will be the balancing length l on potentiometer wire.
- Here, the galvanometer will show no deflection as potential is same if key K is potential gradient of potentiometer wire, then potential difference across resistance R_1 will result as :

$$V = Kl$$

- If r is the resistance of potentiometer of length L, then current through potentiometer will be :

$$I = \frac{E}{R+r}$$

- Potential drop across potentiometer wire will be :

$$Ir \text{ or } \left(\frac{E}{R+r}\right)\times r$$

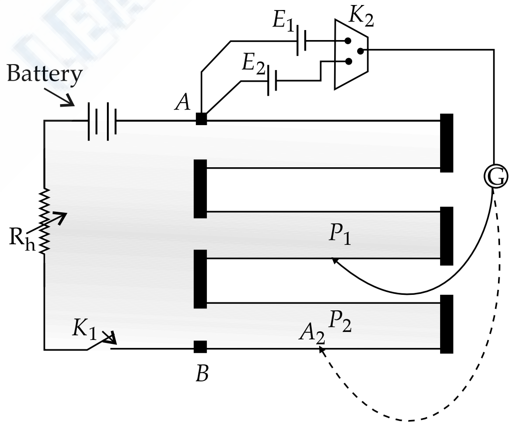
- Now potential gradient of potentiometer wire which is potential per unit length is :

$$K = \left(\frac{E}{R+r}\right)\times\frac{r}{L}$$

$$\therefore \quad V = \left(\frac{E}{R+r}\right)\times\frac{rl}{L}$$

Application of potentiometer comparing emf of cells

- If a positive terminal of the cell of emf E_1 is connected to terminal A while negative terminal is connected to jockey by galvanometer, then by closing the key, jockey will move along the wire AB and null point is obtained where galvanometer shows no deflection.



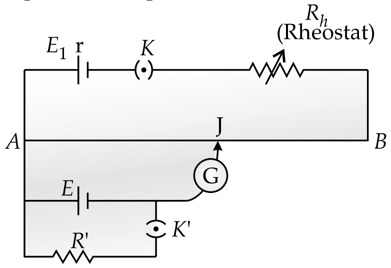
- When the length of wire AP as l_1 is measured, then potential difference across it, will balance the emf E_1, So $E_1 = Kl_1$, where K is potential gradient of the wire.
- When cell of emf E_1 is disconnected while cell of emf E_2 is connected, then $E_2 = Kl_2$.
- On comparing and dividing, we get an expression :

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

- By knowing the values of l_1 and l_2, the emf of two cells can be compared.

Applications of potentiometer in measurement of internal resistance of cell

(i) Initially, in secondary circuit, key K' remains open and the balancing length (l_1) is obtained. Since, cell E is in open circuit so its emf balances on length l_1, *i.e.* $E = Kl_1$ ...(a)



(ii) Now key K' is closed, so cell E comes in closed circuit. If the process is repeated again, then the potential difference V balances on length l_2, *i.e.*, $V = Kl_2$ ...(b)

(iii) By using formula, internal resistance, $r = \left(\frac{E}{V} - 1\right).R'$

$$r = \left(\frac{l_1 - l_2}{l_2}\right).R' \quad \text{[Using eqns. (a) and (b)]}$$

Know the Terms

- **Conductors :** These are materials, which develop electric currents in them, when an electric field is applied to them
- **Conventional current :** The current that flows from a point at higher (positive) potential to a point at lower (negative) potential.
- **Relaxation time :** The short time for which a free electron accelerates before it undergoes a collision with positive ion in the conductor.
- **Conductance :** It is reciprocal of the resistance of a conductor. *i.e.*,

$$G = \frac{1}{R}$$

Unit : ohm^{-1} (Ω^{-1})/siemen (S)/mho.

- **Conductivity :** It is the reciprocal of the resistivity of the material of a conductor *i.e.*,

$$\sigma = \frac{1}{\rho}$$

- **Superconductivity :** The phenomenon due to which a substance loses all signs of its resistance, when cooled to its critical temperature.
- **Temperature coefficient of resistance :** It is defined as the measure of change in electrical resistance of any substance per degree of temperature change.

Know the Formulae

- **Electric current,** $I = \frac{q}{t}$
- **Drift velocity v_d with electric field,** $v_d = \frac{-e\vec{E}\tau}{m}$
- **Current I with drift velocity v_d,** $I = neAv_d$
- **Mobility of charge,** $\mu = \frac{v_d}{E} = \frac{q\tau}{m}$
- **Mobility and drift velocity,** $v_d = \mu_e E$
- **Current and mobility,** $I = neA\,\mu_e E$

- **Resistance, potential difference and current,** $R = \frac{V}{I}$
- **Resistance R with specific resistivity,** $R = \rho\frac{l}{A}$
- **Resistivity with electrons ,** $\rho = \frac{m}{ne^2\tau}$
- **Current density,** $\vec{J} = \frac{I}{\vec{A}}$
- **Conductance,** $G = \frac{1}{R}$
- **Conductivity,** $\sigma = \frac{1}{\rho}$
- **Microscopic form of Ohm's law,** $\vec{J} = \sigma\vec{E}$
- **Temperature coefficient of resistance,** $\alpha = \frac{R_t - R_0}{R_0 \times (t_t - t_0)}$
- **In a cell, emf and internal resistance,** $I = \frac{E}{R+r}$
- ***n* cells of emf *E* in series,** $\mathbf{E_{emf}} = nE$
- **Resistance of *n* cells in series, (where *R* is external resistance)** $nr + R$
- **Current in circuit with *n* cells in series,** $I = \frac{nE}{R+nr}$
- ***n* cells in parallel, then** **emf** = E
- **Resistance of *n* cells in parallel,** $R + \frac{r}{n}$
- **Internal resistance of a cell,** $r = \left(\frac{E-V}{V}\right) \times R$
- **Power of a circuit,** $P = VI = I^2R = \frac{V^2}{R}$
- **Energy consumed,** $E = IVt$
- Kirchhoff's Law (Junction law), $\Sigma I = 0$

  Kirchhoff's Law (Loop law), $\Sigma V = 0$

  Wheatstone Bridge, $\Delta V_{BD} = 0$ or $\frac{P}{Q} = \frac{R}{S}$

Note : All symbols have their usual meanings.

- Potential gradient (*K*) : $K = \frac{V}{L} = \frac{iR}{L} = \left(\frac{E}{R + R_h + r}\right) \times \frac{R}{L}$
- Internal resistance of a cell : $r = \left(\frac{E}{V} - 1\right) \times R = \left(\frac{l_1 - l_2}{l_2}\right) \times R$
- Comparison of emf's of two cells $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

❑❑

PART – III : MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

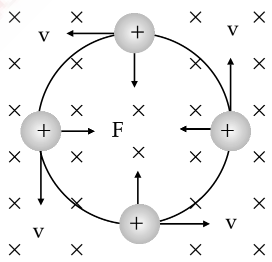
Chapter - 4 : Moving Charges and Magnetism

Revision Notes

Magnetic Field & Cyclotron

Concept of Magnetic field

- Magnetic field is a region around a magnet where force of magnetism acts which affects other magnets and magnetic materials.
- Magnetic field also known as B-field can be pictorially represented by magnetic field lines.
- Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.
- **Lorentz Force:** When a charge q moving with velocity v enters a region where both magnetic fields and electric fields exist, both fields exert a force on it.



Lorentz Force, $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

where, $\vec{F}$ = magnetic force, $\vec{q}$ = charge, $\vec{v}$ = velocity, $\vec{B}$ = magnetic field, $\vec{E}$ = electric field, $q\vec{E}$ = electric force on the charge, $q(\vec{v} \times \vec{B})$ = magnetic force on the charge

- SI unit of magnetic field is Tesla, while smaller magnetic fields are measured in terms of Gauss.

$$1 \text{ Tesla} = 10^4 \text{ G}$$

- When a test charge q_0 enters a magnetic field $\vec{B}$ directed along negative z-axis with a velocity $\vec{v}$ making an angle θ with the z-axis, then,

$$\vec{F}_m = q_0(\vec{v} \times \vec{B}) = q_0 vB \sin\theta \, \hat{n}$$

Characteristics of motion of particle in magnetic field

- Velocity and kinetic energy of particle do not change, as force is always perpendicular to velocity.
- Direction of velocity will continuously change, if $\theta = 0$.
  when $\theta = 0$, no force will act on the particle, hence there will be no change in velocity.
- When $\theta = 90°$, test charge describes a circle of radius $\frac{mv}{q_0 B}$,
  where, m is mass of the particle; larger the momentum, bigger the circle described.
- In case of θ being any other angle than 0° and 90°, test charge will show circular path of radius $\frac{mv\sin\theta}{q_0 B}$, which moves along the direction of magnetic field with speed of $v\cos\theta$.
- Momentum along the direction of magnetic field will remain same.
- Angular speed of test charge $\frac{q_0 B}{m}$ is independent of initial speed of particle.
- Centripetal force on test charge $q_0 v B\sin\theta$ is independent of the mass of particle.
- When the particle enters the magnetic field with the same momentum, then radius of path will be,

$$r = \frac{mv}{q_0 B}$$

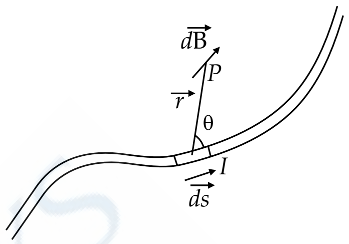
where, $$r \propto \frac{1}{q_0}$$

Oersted's experiment

Oersted observed that:

- When there is no current, compass needle below a wire shows no deflection.
- When the flow of current is in single direction, then the compass needle deflects in a particular direction.
- When the flow of current is reversed, deflection in compass needle occurs in the opposite direction.
- From an experiment, it is concluded that an electrical current produces a magnetic field which surrounds the wire.

Biot-Savart's law



- The magnetic field due to a current element at a nearby point is given by:

$$\vec{dB} = \left[\frac{\mu_0}{4\pi}\right] I \frac{\vec{ds} \times \vec{r}}{r^3}$$

where,

$\vec{dB}$ = Magnetic field produced by current element

$\vec{ds}$ = Vector length of small section of wire in direction of current

$\vec{r}$ = Positional vector from section of wire to where magnetic field is measured

I = Current in the wire

θ = Angle between $\vec{ds}$ and $\vec{r}$

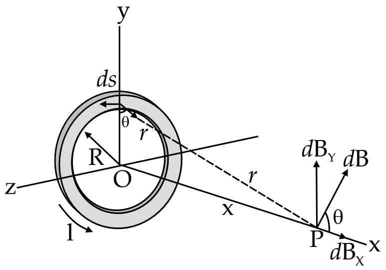
μ_0 = Permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am

The magnitude of magnetic field, $|\vec{dB}| = \left(\frac{\mu_0}{4\pi}\right)\frac{Idl\sin\theta}{r^2}$

Applications of Biot-Savart's Law

- Magnetic field at a point in circular loop will be:

$$\vec{B} = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}\vec{r}$$ **[Here, $r^2 = R^2 + x^2$]**



- Magnetic field at the centre of the coil

$$\vec{B} = \frac{\mu_0 N I}{2R}\vec{r}, \qquad [x = 0]$$

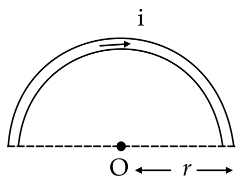
- Magnetic field at very large distance from the centre: $B = \frac{2\mu_0 N i A}{4\pi x^3}$

[Here, $R^2 << r^2$ or, $R^2 + x^2 \approx x^2$]

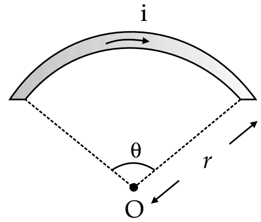
where, A = Area of circular loop
$= \pi R^2$

- Magnetic field due to current carrying circular arc with centre O will be:

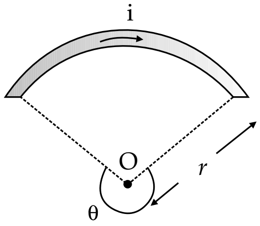
(i) $B = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{r} = \frac{\mu_0 i}{4r}$



(ii) $B = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{r}$

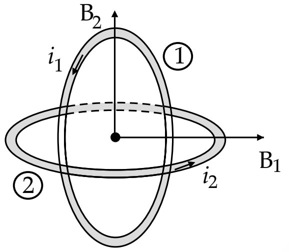


(iii) $B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta)i}{r}$



➢ Magnetic field at common centre of non-coplanar and concentric coils, where both coils are perpendicular to each other will be:

$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$



Ampere's Circuital Law and its Applications

➢ Ampere's circuital law states that the line integral of magnetic field around a closed path is μ_0 times of total current enclosed by the path, $\oint B.dI = \mu_0 I$

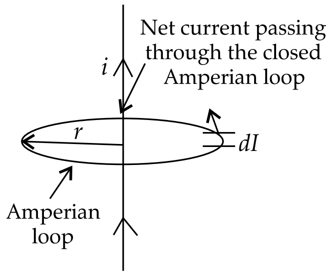
where,

B = Magnetic field

dl = Infinitesimal segment of the path

μ_0 = Magnetic permeability of free space

I = Enclosed electric current by the path



➢ Magnetic field at a point will not depend on the shape of Amperian loop and will remain same at every point on the loop.

Forces between two parallel currents

➢ Two parallel wires separated by distance r having currents I_1 and I_2 where magnetic field strength at second wire due to current flowing in first wire is given as:

$B = \frac{\mu_0 I_1}{2\pi r}$

• In this, the field is orientated at right-angles to second wire where force per unit length on the second wire will be:

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

• Magnetic field-strength at first wire due to the current flowing in second wire will be:

$B = \frac{\mu_0 I_2}{2\pi r}$

➢ One ampere is the magnitude of current which, when flowing in each parallel wire one metre apart, results in a force between the wires as 2×10^{-7} N per metre of length.

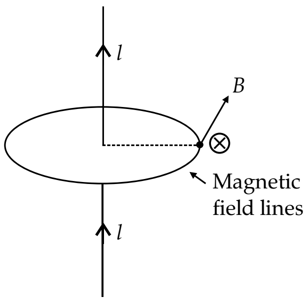
Applications of Ampere's law to infinitely long straight wire, straight and toroidal solenoids:

(i) Magnetic Field due to long straight wire

➢ Ampere's law describes the magnitude of magnetic field of a straight wire as:

$$B = \frac{\mu_0 I}{2\pi r}$$

where,



- Field B is tangential to a circle of radius r centered on the wire.
- Magnetic field B and path length L will remain parallel where magnetic field travels.

(ii) Magnetic Field due to Solenoid

- **Solenoid:** An electromagnet that generates a controlled magnetic field.
- Solenoid is a tightly wound helical coil of wire whose diameter is small compared to its length.
- Magnetic field generated in the centre, or core of a current carrying solenoid is uniform and is directed along the axis of solenoid.
- Magnetic field due to a straight solenoid:
  - at any point in the solenoid, $B = \mu_0 nI$
  - at the ends of solenoid, $B_{end} = \dfrac{\mu_0 nI}{2}$

  where, n = number of turns per unit length, I = current in the coil.

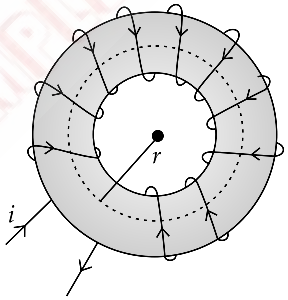
(iii) Magnetic Field due to Toroid

- **Toroid:** It is an electronic component made of hollow circular ring wound with number of turns of copper wire.
- The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound.
- In a toroid with n turns per unit length with mean radius r, where current i is flowing through it, the magnetic field experienced by the toroid with total number of turns N will be:

$$\oint \vec{B}.\vec{dl} = \mu_0 i \quad \Rightarrow B \times 2\pi r = \mu_0 Ni$$

where, r = average radius

Here,
$$B = \frac{\mu_0 Ni}{2\pi r} = \mu_0 ni$$
$$\left(\text{here}, n = \frac{N}{2\pi r}\right)$$



- At any point, empty space surrounded by toroid and outside the toroid, magnetic field B will be zero as net current is zero.

Torque and Galvanometer

Torque experienced by a current loop in uniform magnetic field

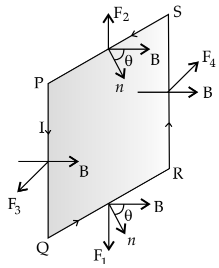
- In a rectangular loop of length l, breadth b with current I flowing through it in a uniform magnetic field of induction B where angle θ is between the normal and in direction of magnetic field, then the torque experienced will be:

$$\tau = nBIA \sin\theta$$

where, n = number of turns in the coil

$\therefore$ $nIA = m$

Further, $\tau = mB \sin\theta$

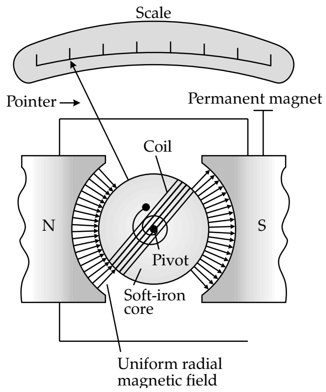


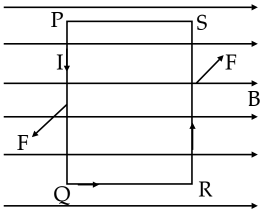
- Torque will be maximum when the coil is parallel to magnetic field and will be zero when coil is perpendicular to magnetic field.
- In vector notation, torque $\vec{\tau}$ experienced will be $\vec{\tau} = \vec{m} \times \vec{B}$

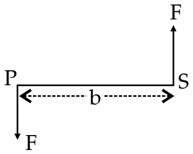
Moving coil galvanometer

- It is an instrument used for detection and measurement of small electric currents.
- In this, when a current carrying coil is suspended in uniform magnetic field, it experiences a torque which rotates the coil.
- The force experienced by each side of the galvanometer will be $F = BIl$ which are opposite in direction.
- Opposite and equal forces form the couple which generates deflecting torque on the coil having number of turns n is given as:

$$\tau = F \times b$$
$$= nBIl \times b$$
$$= nBIA$$







- In moving coil galvanometer, current in the coil will be directly proportional to the angle of the deflection of the coil, $I \propto \theta$

  i.e., where, θ is the angle of deflection.

  Current sensitivity of galvanometer

- Current sensitivity of galvanometer is the deflection produced when unit current passes through the galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

$$I = \frac{C}{nBA}\theta$$

Current Sensitivity, $$\frac{\theta}{I} = \frac{nBA}{C}$$

- Voltage sensitivity of galvanometer is the deflection per unit voltage given as

Voltage Sensitivity, $$\frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG}$$

  where, G = galvanometer resistance, C = torsional constant.

- Increase in sensitivity of moving coil galvanometer depends on:

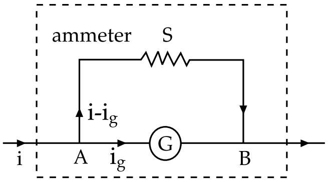
  (i) number of turns n (ii) magnetic field B (iii) area of coil A and (iv) torsional constant.

Conversion of galvanometer into ammeter

- Galvanometer can be converted into ammeter by connecting a low resistance known as shunt in parallel with the galvanometer coil.
- If I_g being the maximum current with full scale deflection passes through galvanometer, then current through shunt resistance will be $i_s = (i - i_g)$

  where, G = Galvanometer resistance, S = Shunt resistance and i = Current in circuit

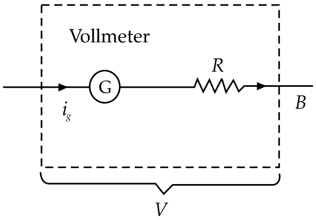
- Now, effective resistance of ammeter will be:



$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

$$R_a = \frac{GS}{G+S}$$

Conversion of galvanometer into voltmeter



- Voltmeter measures the potential difference between the two ends of a current carrying conductor.
- Galvanometer can be converted to voltmeter by connecting high resistance in series with galvanometer coil.
- As resistance R is connected in series with galvanometer, current through the galvanometer will be,

$$i_g = \frac{V}{R+G} \text{ or, } R = \frac{V}{i_g} - G$$

- Effective resistance of voltmeter is $R_v = G + R$,

  where, R_v is very large making the voltmeter to connect in parallel since it can draw less current from the circuit.

Mnemonics

Path of a charged particle in a uniform magnetic field depending on the angle between the magnetic field and the velocity of the particle:

Mnemonics : Circle ninety (90°) angle, go straight if it zero (0°), go for helical, all other angle magnet field is zero.

Circle circle ninety (90°) angle ⇨	Path is a circle if angle between magnetic field and velocity of charged particle is 90°.
Go **straight** if it zero **(0°)** ⇨	Path is a straight line if angle between magnetic field and velocity of charged particles is 90°
Go for **helical**, all **other angle** ⇨	Path is helix if any other angle between magnetic field and velocity of charged particle.
Magnet field is hero.	

Key Formulae

- Lorentz force, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- In uniform magnetic field B, frequency of circular motion of charged particle,

$$f = \frac{qB}{2\pi m}$$

and $$KE_m = \frac{q^2 r^2 B^2}{2m}$$

- Biot-Savart's law, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(\vec{dl} \times \vec{r})}{r^3}$
- Magnetic field at a point due to circular loop, $\vec{B} = \frac{\mu_0}{2} \cdot I \frac{R^2}{(R^2 + x^2)^{3/2}}$
- Ampere's circuital law: $\oint \vec{B} . \vec{dl} = \mu_0 I$
- Magnetic field at the surface of a solid cylinder: $B = \frac{\mu_0 I}{2\pi R}$
- Magnetic field inside the solenoid: $B = \mu_0 n I$
- Magnetic field in a toroid with mean radius r: $r = \frac{\mu_0 N i}{2\pi r}$
- Force between two parallel wires, $F = \frac{\mu_0}{4\pi} \times \frac{2 i_1 i_2}{a} \times l$
- Force between two moving charge particle, $F_m = \frac{\mu_0}{4\pi} \times \frac{q_1 q_2 v_1 v_2}{r^2}$
- $\tau_{max} = NBiA$
- Current Sensitivity $= \frac{\theta}{I} = \frac{nAB}{C}$
- Voltage Sensitivity $= \frac{\theta}{V} = \frac{nAB}{CG}$

Chapter - 5 : Magnetism and Matter

Revision Notes

Magnetic Dipole

Current loop as a magnetic dipole and its magnetic dipole moment

- Magnetic dipole is a small magnet of microscopic dimensions similar to flow of electric charge around a loop.
- Magnetic dipole moment is the strength of magnetic dipole that measures dipole's ability to align itself with external magnetic field.
- Magnetic dipole moment, known as magnetic moment, is the maximum amount of torque generated by magnetic force on dipole which appears per unit value of surrounding magnetic field in vacuum.
- Magnetic field produced at large distance r from the centre of circular loop along its axis will be

$$B = \frac{2\mu_0 IA}{4\pi r^3}$$

where, I = Current in the loop, A = Area

- Magnetic moment of current loop is the product of current and loop area,

$$M = I \times A$$

- A current loop may experience a torque in a constant magnetic field,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

Magnetic dipole moment of revolving electron

- For an electron of charge e revolving around a nucleus of charge Ze at an orbit of radius r, with velocity v and magnetic moment μ_1, the orbital magnetic moment will be

$$\mu_1 = -\frac{em_e vr}{2m_e}$$

But angular momentum of electron,

$$L = m_e vr$$

$$\therefore \qquad \mu_l = -\frac{e}{2m_e}L$$

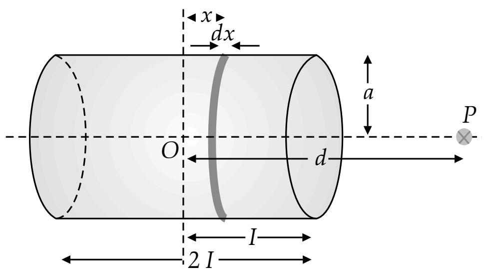
Here (–) sign shows that angular momentum's direction is opposite to the magnetic moment's direction.

Magnetic field of a Solenoid, Earth's Magnetism & Magnetic properties of Materials

Bar magnet as an equivalent solenoid

- If a solenoid of length $2l$, radius a with current I having n number of turns per unit length, then the magnetic moment of solenoid,

$$M\ (= NIA),\ B = \frac{\mu_0 2M}{4\pi d^3}$$



- Magnetic moment of a bar magnet is equal to magnetic moment of an equivalent solenoid that produces same magnetic field.

Gauss' Law for Magnetic Fields

- Gauss' Law for magnetism applies to the magnetic flux through a closed surface.
- It shows that no magnetic monopoles exist and total flux through closed surface will be zero.
- The Gauss's law for magnetic fields in integral form is given by

$$\phi = \int \vec{B}.d\vec{A} = 0$$

Earth's Magnetism

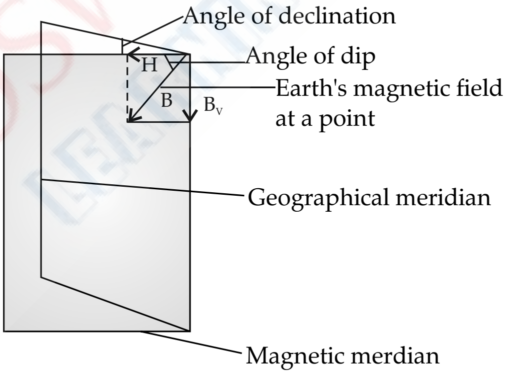
- Earth shows magnetic properties. This is evident from the following facts:
  - A freely suspended needle stays in north - south direction.
  - Availability of neutral points. At neutral points, magnetic field due to suspended magnet is equal and opposite to the horizontal component of Earth's magnetic field.
- The source of Earth's magnetism is still undefined, though certain theories have good scientific justifications like ions revolving with Earth.

Characteristics of Earth's Magnetism

- Earth's south pole and north pole are defined by Sun's direction. These are known as geographical north and south poles. Magnetic north and south poles are the points where the magnetic needle becomes perpendicular to earth's surface. Hence, there are two systems of directions.
- Due to two systems of directions, we can draw two meridians. (Plane joining geographic North and South pole is geographic meridian and plane joining magnetic North and South pole is magnetic meridian)

Elements of earth's magnetic field

- **Angle of Declination:** At any place on Earth, the acute angle between magnetic meridian and the geographical meridian is called the angle of declination.
- **Angle of Dip:** The angle of dip at any place is the angle between Earth's magnetic field intensity B with horizontal in the magnetic meridian at that place.



- **Horizontal Component of Earth's Magnetic field:** The horizontal component of Earth's magnetic field H is in the horizontal direction in the magnetic meridian.

$$B_H \text{ or } H = B\cos\theta$$
$$B_V = B\sin\theta$$

Where θ is angle of Dip, $\tan\theta = \dfrac{B_V}{B_H}$

We find the earth's magnetic field B at any place by measuring its horizontal component. Hence,

$$B = \frac{H}{\cos\theta} \text{ and } B_V = H\tan\theta$$

- There is variation in magnetic field between place to place depending upon angle of Dip, angle of declination and horizontal component of Earth, Hence, these are known as elements of Earth's magnetic field.

Mnemonics

Concept: Four characteristics of magnetic field lines:

Mnemonics: **I l**ove **n**ew **st**ories **T**ina found new **C**ookies.

Interpretation:

(i) **I**maginary Lines

(ii) **E**xtended North to South pole

(iii) **T**angent gives (magnetic) field direction

(iv) **N**ever Cross each other

Know the Formulae

- Magnetic field due to short dipole at distance *'d'* on axial line:

$$B_{\text{axial}} = \frac{\mu_0 2M}{4\pi d^3}$$

- Magnetic field due to short dipole at distance *'d'* on equatorial line:

$$B_{\text{equi}} = \frac{\mu_0 M}{4\pi d^3}$$

PART – IV : ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

Chapter - 6 : Electromagnetic Induction

Revision Notes

Electric Field and Dipole

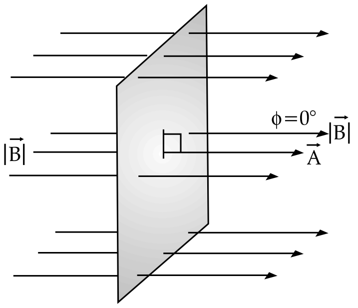
(a) Concept Notes

Electromagnetic induction

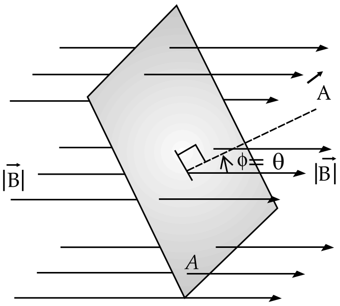
- Electromagnetic induction is the process of generating the electric current with a changing magnetic field.
- It takes place whenever a magnetic field is changing or electric conductors move relative to one another when they are in fluctuating magnetic field.
- The current produced by electromagnetic induction is more when the magnet or coil moves faster. When magnet or coil moves back and forth repeatedly, then alternating current is produced.

Magnetic flux:

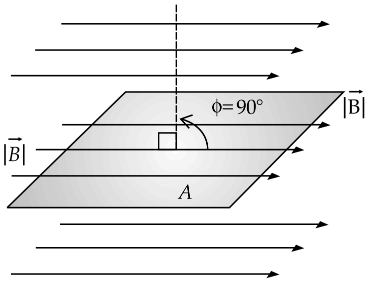
- Magnetic flux through an enclosed area is the number of magnetic field lines cutting through a surface area *A*, defined by unit area vector.
- The unit of magnetic flux is weber, where, 1 Wb = 1 T/m^2.
- Magnetic flux (ϕ_B) is related to number of field lines passing through a given area.
- If magnetic field is changing, the changing magnetic flux will be $\phi_B = NBA \cos\theta$, where θ is the angle between magnetic field and normal to the plane.



$\vec{B}$ parallel to A (ϕ = 0°) magnetic flux ϕ_B = BA.



$\vec{B}$ at an angle ϕ with the perpendicular to A : magnetic flux $\phi_B = BA\cos\theta$



$\vec{B}$ perpendicular to A (ϕ=90°) : magnetic flux $\phi_B = 0$.

Magnetic flux density

- The change in magnetic flux per unit change in area is called magnetic flux density.
- Magnetic flux is given by: $d\phi = \vec{B}\cdot d\vec{A}$

  For $\vec{B}$ parallel to $d\vec{A}$, we have

  $$d\phi = B(dA)\cos 0° = B(dA)$$

  Therefore, $$B = \frac{d\phi}{dA} \quad ....(i)$$

  i.e., **magnetic induction** is equal to the magnetic flux density. In other words, the magnetic field may be measured in terms of magnetic flux density. From equation (i), we find:

  Unit of $$B = \frac{\text{Unit of } d\phi}{\text{Unit of } dA}$$

  Or, $$T = \frac{\text{Wb}}{\text{m}^2}$$

  i.e., Tesla = weber per square metre.

Faraday's Laws of Electromagnetic Induction

- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law.
- **Faraday's First Law** of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field, an emf is induced which is known as induced emf and if the conductor circuit is closed, current is also induced which is called alternating current.
- **Faraday's Second Law** of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is the product of number of turns in the coil and flux associated with the coil.

  $$\varepsilon = -\frac{d\phi_B}{dt}$$

  ϕ_B is magnetic flux through the circuit and is represented as $\phi_B = \int \vec{B}.d\vec{A}$

  With N loops of similar area in a circuit and ϕ_B being the flux through each loop, emf is induced in every loop. Writing the formula for Faraday's law as

  $$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

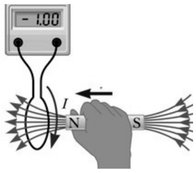
  where, ε = Induced emf [V], N = Number of turns in the coil
  Δϕ = Change in the magnetic flux [Wb], Δt = Change in time [s]
  The negative sign indicates that ε opposes its cause.
- If there is no change in magnetic flux, no emf is induced.

Induced emf and current

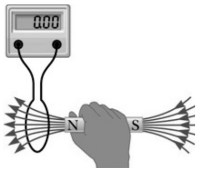
- A changing magnetic flux induces an electric field which induces a current in the circuit.
- A wire moving in the field induces a current which acts same as current provided by a battery.
- Changing magnetic flux and induced electric field are related to induced emf as per Faraday's law.
- The induced EMF in a conductor moving is related to the magnetic field as $E = B.l.v\sin\theta$

Induced current

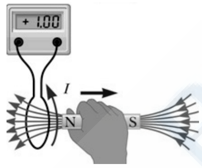
- When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.
- When a magnet is moved towards a loop of wire connected to an ammeter, ammeter shows current induced in the loop.



- When a magnet is held stationary, there will be no induced current in the loop, even though the magnet is inside the loop.



- When a magnet is moved away from the loop, the ammeter shows opposite current induced in the loop.



Motional emf

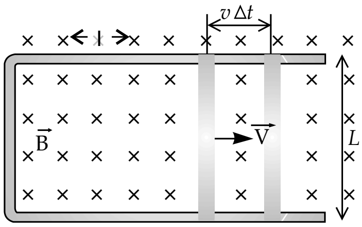
- The relationship between an induced emf ε in a wire or a conductor moving at a constant speed v through a magnetic field B is given by:

$$\phi_B = Blx$$

$$\varepsilon = \frac{-d\phi_B}{dt} = \frac{-d}{dt}(Blx)$$

$$= -Bl\frac{dx}{dt}$$

$$= Blv \qquad \left(\frac{dx}{dt} = -v\right)$$



- An induced emf from Faraday's law is generated from a motional emf that opposes the change in flux.
- Magnetic and electric forces on charges in a rod moving perpendicular to magnetic field is given as:

At equilibrium

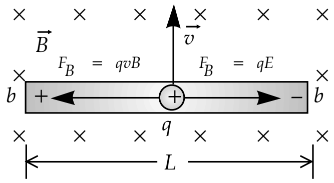
$$F_E = F_B$$

$$qE = qvB$$

$$E = vB$$

$$\frac{V}{l} = vB \qquad \left[\text{Here, } E = \frac{V}{l}\right]$$

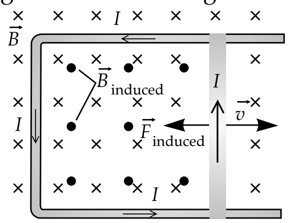
$$V = Bvl$$



Lenz's law

- Lenz's law is used to determine the direction of induced magnetic fields, currents and *emfs*.
- The direction of an induced emf always opposes the change in magnetic flux which causes the emf.
- It explains the negative sign in Faraday's flux rule, $\varepsilon = -\frac{d\phi_B}{dt}$ showing that the polarity of induced emf tends to produce a current that opposes the cause *i.e.* change in magnetic flux.

- As per conservation of energy, induced emf opposes its cause, making mechanical work to continue with the process which gets converted into electrical energy.
- Slide wire containing induced current, magnetic field and magnetic force:



Electric Generators and Back Emf

- Electric generator rotates a coil in a magnetic field inducing an emf which is given as a function of time

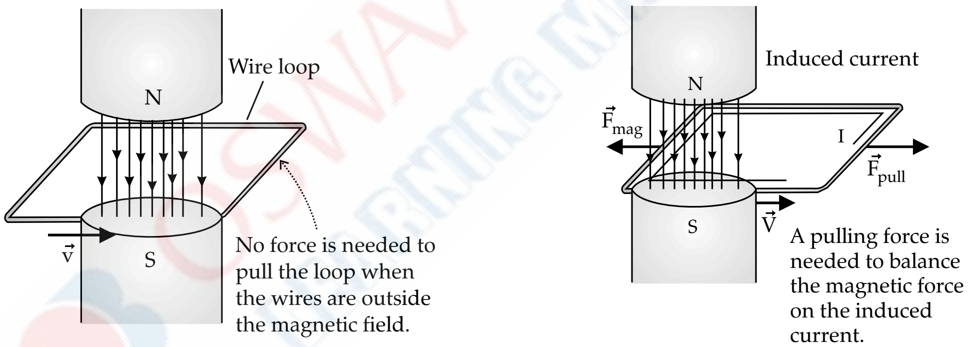
$$\varepsilon = NBA\,\omega \sin(\omega t).$$

where, A = Area of N-turn coil rotated at constant angular velocity ω in uniform magnetic field $\vec{B}$.
- The peak emf of a generator is, $\varepsilon_0 = NBA\omega$
- Any rotating coil produces an induced emf. In motors, it is known as back emf as it opposes the emf input to the motor.

Eddy Currents, Self and Mutual Induction

Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called as magnetic damping.
- Eddy currents give rise to magnetic fields that oppose any external change in the magnetic field.
- Eddy currents are induced electric currents that flow in a circular path:



- Eddy currents flowing in a material will generate their own secondary magnetic field that opposes the coil's primary magnetic field.

Mutual Induction

- The production of induced emf in a circuit, when the current in the neighbouring circuit changes is called **mutual induction.**

  When the circuit of the primary coil is closed or opened, deflection is produced in the galvanometer of the secondary coil. This is due to the mutual induction.
- The mutual induction between two coils depends on the following factors:
  - The number of turns of primary and secondary coils.
  - The shape, size or geometry of the two coils. *i.e.*, the area of cross-section and the length of the coils.

Coefficient of mutual induction:

- Suppose, the instantaneous current in the primary coil is I. Let the magnetic flux linked with the secondary coil be ϕ. It is found that the magnetic flux is proportional to the current. *i.e.*,

$$\phi \propto I \quad \text{or} \quad \phi = MI \qquad \text{...(i)}$$

where, M is the constant of proportionality. It is called coefficient of mutual induction.

The induced emf ε in the secondary coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \qquad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law *i.e.*, the induced emf in the secondary coil opposes the variation of current in the primary coil.

Taking magnitude of induced emf the equation (ii), we find

$$M = \frac{\varepsilon}{(dI/dt)}$$

Therefore, Unit of $M = \frac{\text{V}}{\text{As}^{-1}} = \text{VA}^{-1}\text{s}$

If n_1, n_2 be the number of turns per unit length in primary and secondary coils per unit length and r be their radius, then coefficient of mutual inductance is given as

$$M = \mu_0 n_1 n_2 \pi r^2 l$$

Self-Induction:

➢ The production of induced emf in a circuit, when the current in the same circuit changes is known as **self-induction**.

Suppose the instantaneous current in the circuit is I and if the magnetic flux linked with the solenoid is ϕ, then it is found that:

$$\phi \propto \text{I} \text{ or } \phi = LI \quad \text{...(i)}$$

where, L is the constant of proportionality. It is called **coefficient of self-induction.**

The induced emf ε in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law *i.e.*, the induced emf opposes the variation of current in the coil.

Taking the magnitude of the induced emf from the equation (ii), we find:

$$L = \varepsilon/(dI/dt) \quad \text{...(iii)}$$

Then, the coefficient of self-induction is the ratio of induced emf in the circuit to the rate of change of the current in the circuit.

Unit of L: The unit of self-induction is also called henry (symbol H).

From equation (ii), we find that if $dI/dt = 1\ \text{As}^{-1}$ and $\varepsilon = 1$ V,

then $L = 1\ \text{H} \Rightarrow 1\ \text{VA}^{-1}\text{s}$

➢ If a rod of length l moves perpendicular to a magnetic field B with a velocity v, then the induced emf produced across it, is given by

$$\varepsilon = vBl$$

In general, we have, $\varepsilon = Blv \sin\theta$

➢ If a metallic rod of length l rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across its ends is given by

$$\varepsilon = \frac{B\omega l^2}{2} = \frac{B2\pi f l^2}{2} = BAf$$

Here, ω = angular velocity of rotation, $A = \pi l^2$ = area of circle and f = frequency of rotation.

➢ Inductance in the electrical circuit is equivalent to the inertia (mass) in mechanics.

➢ When a bar magnet is dropped into a coil, the electromagnetic induction in the coil opposes its motion, so the magnet falls with acceleration less than that due to gravity.

➢ The inductance of a coil depends on the following factors:

- area of cross-section,
- number of turns
- permeability of the core.

➢ Unit of induction, $H = \frac{\text{Wb}}{\text{A}} = \frac{\text{Vs}}{\text{A}} = \Omega.\text{s}$

➢ The self inductance of a circular coil is given by:

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu_0}{4\pi}.\frac{(2\pi NI)}{rI} \times AN \qquad \left[\because B = \frac{\mu_0}{4\pi}.\frac{2\pi NI}{r}\right]$$

$$L = \frac{\mu_0 N^2}{2r} A = \frac{\mu_0 N^2}{2r} \times \pi r^2$$

or $$L = \frac{\mu_0 N^2 \pi r}{2}$$

- The self inductance of a solenoid of length l is given by

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left(\frac{\mu_0 NI}{l}\right)\frac{AN}{I} \qquad \left[\because B = \frac{\mu_0 NI}{l}\right]$$

or $$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al = \mu_0 n^2 V \qquad \left[\because n = \frac{N}{l}\right]$$

Here, $n = N/l$ = Number of turns per unit length and $V = Al$ = Volume of the solenoid.

- If two coils of inductance L_1 and L_2 are coupled together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

where, k is called the coupling constant.

- The value of k lies between 0 and 1.

For perfectly coupled coils, $k = 1$, it means that the magnetic flux of primary coil is completely linked with the secondary coil.

- Eddy currents do not cause sparking.
- If a current I is set up in a coil of inductance L, then the magnetic field energy stored in it is given by

$$U_m = \frac{1}{2}LI^2$$

Mnemonics

Concept: Induced emf in a conductor moving in a magnetic field:
Mnemonics: **I e**at **L**oaf and **B**oiled **V**egetables
Interpretation: **I: I**nduce **e**at: **e**mf **L**oaf and: **L**ength of Conductor **B**oiled: **B** (magnetic field) **V**egetables: **V** (Velocity)

Know the Terms

- **Electric generator:** Device for converting mechanical work into electrical energy that induces an emf by rotating a coil in magnetic field
- **Induced electric field:** Field generated due to changing magnetic flux with time
- **Induced emf:** A short-lived voltage generated by a conductor or coil, moving in a magnetic field
- **Magnetic damping:** A process in which energy of motion is converted in to heat by way of electric eddy currents induced in a coil that passes between the poles of a magnet
- **Magnetic flux:** The number of magnetic field lines measured through a given area
- **Motional *emf:*** Voltage produced by the movement of conducting wire or a conductor in a magnetic field
- **Peak *emf*:** The maximum *emf* produced by a generator
- **Back emf:** The emf generated by a running motor due to coil that turns in a magnetic field which opposes the voltage that powers the motor
- **Inductor:** A device used to store electrical energy in the form of magnetic field when electric current flows
- EMF produced by an electric generator: $\varepsilon = NBA\omega \sin(\omega t)$

Know the Formulae

- Magnetic flux: $\phi_m = \int \vec{B}.\vec{dA}$
- Faraday's law: $\varepsilon = -N\frac{d\phi_m}{dt}$
- Motional induced emf: $\varepsilon = Blv$
- Motional emf around a circuit: $\varepsilon = \oint E.dl = -\frac{d\phi_m}{dt}$
- EMF produced by an electric generator $\varepsilon = NBAsin\ \omega t$
- For Self Induction $\varepsilon = \frac{d\phi}{dt} = -L\frac{dI}{dt}$
- For Mutual Induction $\varepsilon = \frac{d\phi}{dt} = -M\frac{dI}{dt}$
- The inductance in series is given by $L_s = L_1 + L_2 + L_3 + ...$
- The inductance in parallel is given by $\frac{1}{L_p} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+....$
- Mutual Inductance of two coils is given by

$$M= \frac{n_0 n_r N_P N_S A_P}{I_P} = \frac{n_0 n_r N_P N_S A_S}{I_P}$$

where, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$).
μ_r is the relative permeability of the soft iron core.
N_S is number of turns in secondary coil.
N_P is number of turns in primary coil.
A_P is the cross-sectional area of primary coil in m^2.
A_S is the cross-sectional area of secondary will in m^2.
I is the coil current.

- For *A.C.* Generator $\varepsilon = \varepsilon_0 \sin\omega t$ or $\varepsilon = \varepsilon_0 \sin 2\pi\nu t$

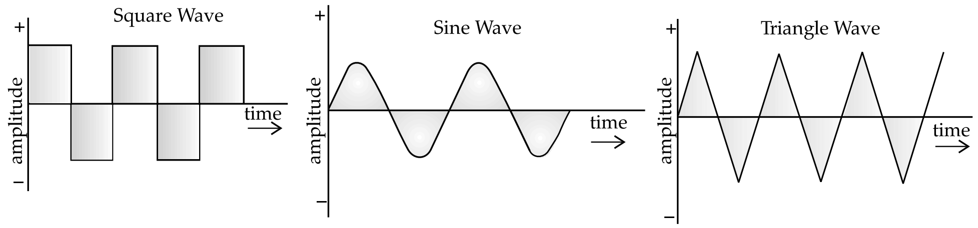
□□

Chapter - 7 : Alternating Current

Revision Notes

Alternating Current

- Alternating current
- Alternating current changes continuously in magnitude and periodically in direction.
- It is represented by sine curve or cosine curve as $I = I_0 \sin wt$ or $I = I_0 \cos wt$ where, I_0 is peak value of current and I is instantaneous value of current.
- Frequency of an alternating current supply f, is defined as the number of cycles completed per second. It is measured in Hertz (Hz). In India, the frequency is 50 Hz.
- The time period T, of an alternating supply, is time taken to complete one cycle.
- The behaviour of ohmic resistance R in ac circuit is the same as in dc circuit.
- Alternating current can be produced by using a device called as an alternator.
- AC waveforms are:



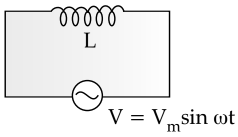
- Peak and ***rms*** value of alternating current/voltage:
- Root mean square or *rms* is the root mean square of voltage or current in an *ac* circuit for one complete cycle denoted by V_{rms} or I_{rms}.
- *Rms* value is the standard way of measuring alternating current and voltage as it gives the *dc* equivalent values.
- *Rms* value of *ac* is also called effective value or virtual value of *ac* represented as I_{rms}, I_{eff} or I_v shown as

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- *Rms* voltage value is the square root of averages of the squares of instantaneous voltages in a time varying waveform.

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

- AC voltage applied to pure inductive circuits:



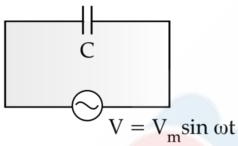
$$V = V_m \sin \omega t$$

$$i = i_m\left(\sin \omega t - \frac{\pi}{2}\right)$$ [which shows current lags the voltage by $\frac{\pi}{2}$]

$$\text{Average } P_L = \frac{i_m V_m}{2}[\sin (2\omega t)] = 0$$ [Since average of sin $2\omega t$ over a complete cycle is zero]

Thus the average power supplied to an inductor over one complete cycle is zero.

AC applied to pure capacitive circuit:



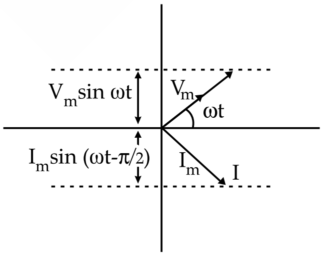
$$V = V_m \sin \omega t$$

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$ [which shows current leads the voltage by $\frac{\pi}{2}$]

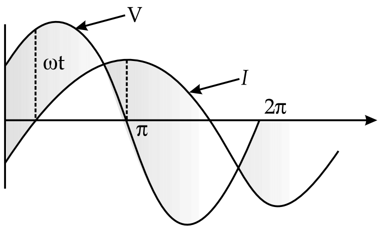
$$\text{Average } P_C = \frac{I_m V_m}{2}\sin(2\omega t) = 0$$ [Since average of sin $2\omega t$ over a complete cycle is zero]

Thus the average power supplied to an capacitor over one complete cycle is zero.

- **Phasor-diagram:** A phasor diagram represents sinusoidal *ac* current and sinusoidal voltage in a circuit along with the phase difference between current and voltage. The length of phasor is proportional to the instantaneous values of V and I and the maximum length is proportional to V_0 and I_0.



Phasor diagram of purely Inductive circuit



Graphical representation of V **and** i **versus** ωt.

- Reactance and Impedance
- When an *ac* current is passed through a resistance, a voltage drop is produced which is in phase with the current and is measured in ohms (Ω).
- Reactance is the inertia against the motion of electrons where an alternating current after passing through it produces a voltage drop which is 90° out of phase with the current.

- Reactance is shown by "X" and is measured in ohms (Ω).
- Reactance is of two types: inductive and capacitive.
- Inductive reactance is linked with varying magnetic field that surrounds a wire or a coil carrying a current.
- Inductive reactance (X_L) is the resistance offered by an inductor and is given by $X_L = \omega L = 2\pi f L$
- Through a pure inductor, alternating current lags behind the alternating *emf* by phase angle of 90^o.
- Capacitive reactance is linked with changing electric field between two conducting surfaces separated from each other by an insulating medium.
- Capacitive reactance (X_C) is the resistance offered by a capacitor and is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

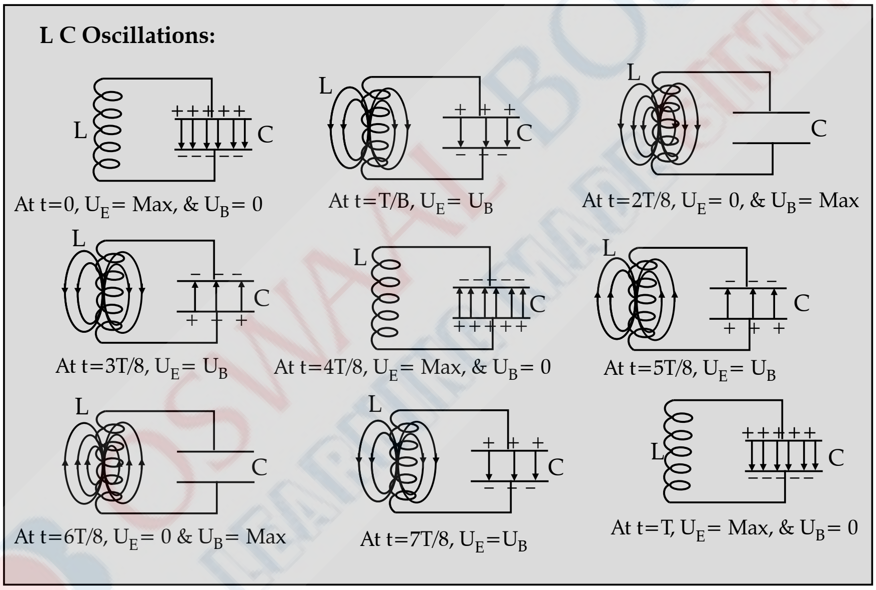
- Through a pure capacitor, alternating current leads the alternating *emf* by a phase angle of 90^o.

‹Impedance is the comprehensive expression of all forms of opposition to electron flow, including resistance and reactance, where an alternating current after passing through it produces a voltage drop between 0^o and 90^o which will be out of phase with current given as,

$$Z = \sqrt{R^2 + X^2}$$

where, Z = Impedance of circuit, R = Resistance, X = Reactance

- ***LC*** Oscillations (qualitative treatment only)



- *LC* circuit comprises of inductor and capacitor connected in series where energy from the cell is given to capacitor which keeps on oscillating between inductor and capacitor.
- When *ac* voltage is applied to the capacitor, it keeps on charging and discharging continuously.
- When capacitor is fully charged, it starts discharging and charge gets transferred to the inductor which is connected to capacitor.
- Due to change in current, there is change in magnetic flux of the inductor in the circuit, which induces an *emf* in the inductor.
- The *emf* is given by $e = -L\frac{dI}{dt}$ which opposes the growth of the current.
- When capacitor gets completely discharged, all the energy stored in it, gets stored in the inductor as a result of which, inductor starts charging the capacitor and energy stored in the capacitor starts increasing.
- As there is no current in the circuit, energy in the inductor is zero, so total energy of *LC* circuit will be

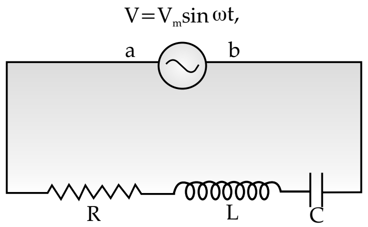
$$U_E = \frac{1}{2}.\frac{q^2}{C}$$

- Band Width: It is the range of angular frequencies over which the average power is greater than ½ the maximum value of average power.
- **Impedance:** In an *ac,* the impedance is analogous to resistance in a *dc* circuit that measures the combined effect of resistance, capacitive reactance and inductive reactance.

LCR Series Circuit

***LCR* series circuit**

- In an LCR series circuit with resistor, inductor and capacitor, the expression for the instantaneous potential difference between the terminals a and b is given as



- The potential difference in this will be equal to the sum of the magnitudes of potential differences across R, L and C elements as

$$V = V_m \sin \omega t = RI + L\frac{dI}{dt} + \frac{1}{C}q$$

  where, q is the charge on capacitor.

- The steady state situation will be

$$i = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \text{ and } i_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}}$$

where, $$\phi = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$$

- From the equation, steady-state current varies sinusoidal with time, so steady-state current can be written as $I = I_m \sin (wt - f)$
- In an LCR circuit:

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_m}{\sqrt{R^2 + X^2}} = \frac{V_m}{Z}$$

Here, Z = Impedance of the circuit, X = Reactance of the circuit, X_L and X_C = Inductive and Capacitive reactance.

- For steady-state currents, maximum current I_m is related to maximum potential difference V_m by

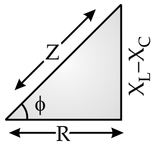
$$I_m = \frac{V_m}{Z}$$

- Total effective resistance of LCR circuit is called Impedance (Z) of the circuit given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle by which alternating voltage leads the alternating current in LCR circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$



- In an LCR circuit, impedance triangle is a right-angled triangle in which base is ohmic resistance R, perpendicular is reactance ($X_L - X_C$) and hypotenuse is impedance (Z)
- When a condenser of capacity C charged to certain potential is connected to inductor L, energy stored in C oscillates between L and C where frequency of energy oscillations is given by

$$X_L = X_C \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

- In LCR circuit, if there is no loss of energy, then total energy in L and C at every instant will remain constant.
- Sign for phase difference (f) between I and E for a series LCR circuit:

ϕ is positive, when $X_L > X_C$.
ϕ is negative, when $X_L < X_C$.
ϕ is zero, when $X_L = X_C$.
$\phi = \pi/2$, when $\omega = \square$.
$\phi = -\pi/2$, when $\omega = 0$.

Resonance

- Circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits maximum current, such circuit is called as series resonant circuit.
- The necessary condition for resonance in LCR series circuit is: $V_C = V_L$

$X_L = X_C$ which gives $\omega^2 = \frac{1}{LC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$

- In this, frequency of ac fed to circuit will be equal to natural frequency of energy oscillations in the circuit under conditions,

$$Z = R$$
$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$$

- The sharpness of tuning at resonance is measured by Q factor or quality factor of the circuit given as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- At series LCR resonance or acceptor circuit, current is maximum.

$$I_{\max} = \frac{E}{R}$$

Power in AC circuits

- When the current is out of phase with the voltage, the power indicated by the product of the applied voltage and the total current gives apparent power.
- If the instantaneous values of the voltage and current in an ac circuit are given by

$$E = E_0 \sin \omega t$$
$$i = i_0 \sin (\omega t - \phi)$$

where ϕ is the phase difference between voltage and the current. Then, the instantaneous power

$$P_{in} = E \times i = E_0 i_0 \sin \omega t . \sin (\omega t - \phi)$$

or average power $$P_{avg} = \frac{1}{2} E_0 i_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = V_{rms} \times I_{rms} \times \cos\phi$$

where, $\cos\phi$ is known as power factor.

- Power factor ($\cos\phi$) is important in power systems as it shows how closely the effective power equals the apparent power which is given as:

$$\cos\phi = \frac{\text{Effective power}}{\text{Apparent power}}$$

- The value of power factor varies from 0 to 1.

- The instantaneous rate at which energy is supplied to an electrical device by ac circuit is

$$P = VI$$

- Average power in LCR where, $X_L = X_C$ over a complete cycle in a non-inductive circuit or pure resistive circuit is given as $P = V_0 I_0$ or $I_0^2 R$

AC Generator and Transformer

***AC* generator**

- An alternator is an electrical machine which converts mechanical energy into alternating electrical energy.
- Alternator or a synchronous generator has a stator and rotor.
- It is similar to the basic working principle of a dc generator.
- It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced emf given as:

$$e = e_0 \sin \omega t = NBA\omega \sin \omega t$$

Transformer

- Transformer is an electrical device used for changing the alternating voltages.

  It is based on the phenomenon of mutual induction.
- The main use of transformer is in transmission of ac over long distances at extremely high voltages which reduces the energy losses in transmission.
- It comprises of two sets of coils which are insulated from each other and are wound on soft-iron core.
- In this, one of the coil is called as primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- Transformer Ratio:

$$E_s = \left(\frac{N_s}{N_p}\right) E_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right) I_p$$

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$ is defined as the transformer ratio.

The value of turns ratio of a transformer $\frac{N_p}{N_s} = \frac{V_p}{V_s} = n$

- Step-up transformer: If secondary coil has more number of turns than primary ($N_s > N_p$), voltage gets stepped up ($V_s > V_p$).

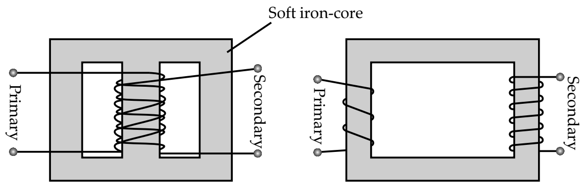
  In this, there is less current in secondary as compared to primary ($\frac{N_s}{N_p} > 1$ and $I_s < I_p$).

  The value of transformer ratio $k > 1$
- Step-down transformer: In this, the secondary coil has less number of turns than primary ($N_s < N_p$). In this, $V_s < V_p$ and $I_s > I_p$ as voltage gets stepped down or reduced with increase in current.

  In this, value of transformer ratio $k < 1$
- The main use of transformers is in stepping up voltage for power transmission.
- Electric power can be transmitted efficiently at high voltages than at low voltages due to less (I^2R) heat loss in a high voltage / low current transmission.
- Efficiency of transformer:

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\eta = \frac{E_s I_s}{E_p I_p}$$



- Inspite of heavy power losses, the efficiency in a transformer is usually above 90%.
- An ideal transformer is 100% efficient as it delivers all energy it receives.
- Real transformer is not 100% efficient and at full load, its efficiency lies between 94% to 96%.
- A transformer operating with constant voltage and frequency with very high capacity, efficiency results as 98%.
- Energy losses in transformers:
  1. Flux Leakage
  2. Resistance of windings
  3. Eddy currents
  4. Hysteresis

Mnemonics

Concept: In pure inductive and capacitive circuit

Mnemonics: **C**hocolate **C**ookies are **V**ery **I**nteresting !

Interpretation:
Chocolate: **C**urrent leads
Cookies are: in **C**apacitive circuit
Very: **V**oltage leads
Interesting !: in **I**nductive circuit

Key Formulae

- *rms* value for current $I_{rms} = \dfrac{I_0}{\sqrt{2}}$
- *rms* value for voltage $V_{rms} = \dfrac{V_0}{\sqrt{2}}$
- Power $P = V_{rms}I_{rms}$
- In a purely inductive circuit if, $V = V_m \sin \omega t$

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right), \quad \text{where } i_m = \frac{V_m}{X_L} \text{ and } X_L = \omega L$$

$$(P_{avg})_L = 0$$

- In a purely capacitive circuit if, $V = V_m \sin \omega t$

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{where, } i_m = \frac{V_m}{X_C} \text{ and } X_C = \frac{1}{\omega C}$$

- Average Power $= \dfrac{1}{2}V_0 I_0 \cos\phi = V_{rms}I_{rms}\cos\phi$ (where, $\cos\phi = \dfrac{R}{Z}$ is power factor)
- $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- Induced emf $= e = -L\dfrac{dI}{dt}$
- Energy in *LC* circuit, $U_E = \dfrac{1}{2}\dfrac{q^2}{C}$

- Impedance for a series *LCR* circuit,

$$Z = \sqrt{R^2 + X^2} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}.$$

- Average power, $P = \frac{E_0 I_0}{2} \cos\phi = V_{rms} I_{rms} \cos\phi$

- Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

For transformer:

- $$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

- $$V_s = \left(\frac{N_s}{N_p}\right)V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right)I_p$$

- The value of transformer ratio is greater than 1 for step up transformer and less than 1 for step-down transformer.

- $$\eta = \frac{E_s I_s}{E_p I_p}$$

- $$\%\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100\%$$

For generator:

- $e = e_0 \sin \omega t = NBA\omega \sin \omega t$
- $I = \frac{e}{r} = \frac{NBA\omega \sin \omega t}{R}$

❑❑