UNIT I: ALGEBRA

CHAPTER-1
QUADRATIC EQUATIONS

Revision Notes

Solutions of Quadratic Equations

- A quadratic equation in variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfy the equation are called the solutions or roots or zeros of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- A quadratic equation can be solved by the following algebraic methods:
  (i) By factorization (splitting the middle term),
  (ii) Making perfect squares and
  (iii) Using quadratic formula.
- If $ax^2 + bx + c = 0$, where $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Method for factorization of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Find the product of a and c *i.e.*, "ac"
  (ii) Find a pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such number, it can't be factorized)
  (iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x \pm b_2x$. Now factorize, by grouping the pairs of terms.
- Roots of the quadratic equation can be found by equating each linear factor to zero. Since, product of two numbers is zero, then either or both of them are zero.
- **Solution of a Quadratic equation using Quadratic formula**
  The roots of the quadratic $ax^2 + bx + c = 0$; $a \neq 0$ can be find using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  The above result is known as **quadratic formula** or **Sridharacharya Formula**.
  Here, $b^2 - 4ac \geq 0$, for real roots.
  History backgrand
- The Old-Babylonians (400 *BC*) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid developed a geometrical approach for finding out roots, which are solutions of quadratic equations.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.
- Sridharacharya (C.E. 1025) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber Embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.

Discriminant and Nature of Roots

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant $D = b^2 - 4ac$.

- **Nature of roots of a quadratic equation:**
  (i) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  (ii) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii) If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

Mnemonics

Concept: To Find the roots of quadratic equation, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

A Negative Boy could not decided if he did or didn't want to go to a Radical party. The Boy was Square so he missed out on 4 Awesome Chicks.
This was all over by 2 a.m.

Interpretation:

A negative Boy = $(-b)$

he did or didn't want to go = $(+/-)$

To a Radical party = $(\sqrt{})$

Boy was Square = (b^2)

Missed out = $(-)$

4 Awesome = $4a$

Chicks = c

All over = Divided by

2 a.m. = $2a$

Know the Formulae

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$
- **Quadratic identities:**
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$
- Discriminant, $D = b^2 - 4ac$.

CHAPTER-2
ARITHMETIC PROGRESSION

Revision Notes

To Find n^{th} Term of the Arithmetic Progression

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding or subtracting a fixed number d to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.
- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of k.
- The standard form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots..$
- If an A.P. $a, a + d, a + 2d, \ldots\ldots..., l$ is reversed to $l, l - d, l - 2d, \ldots\ldots..., a$, then common difference changes to negative of original sequence common difference.

Sum of n Terms of an Arithmetic Progression

- Sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where, a is the first term, d is the common difference and n is the total number of terms.

- Sum of n terms of an A.P. when first and last term is given.

$$S_n = \frac{n}{2}[a + l]$$

where, a is the first term and l is the last term.

- The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum to first $(n - 1)$ terms of it.
i.e., $a_n = S_n - S_{n-1}$.

Mnemonics

Concept: n^{th} Term of Arithmetic Progression $n = a + (n - 1)d$.

Nokia Offers Additional Programmer in English To Attract Positive New One Buyer Daily

Interpretation:

Nokia's 'N' is n^{th} term.
Offer's 'O' is **of**
Additional's 'A' is Arithmetic
Programmer's 'P' is Progression
In's 'I' is **is.**
English's 'E' is **Equal**
To's 'T' is **To**
Attract's 'A' is a
Positive's 'P' is **+**
New's 'N' is n
One buyer is **– 1**
Daily's 'D' is d

Know the Formulae

- The general (n^{th}) term of an A.P. is expressed as:

$$a_n = a + (n-1)d. \text{ .... from the starting.}$$

where, a is the first term and d is the common difference.

- The general (n^{th}) term of an A.P. $l, l-d, l-2d, \ldots\ldots, a$ is given by:

$$a_n = l + (n-1)(-d) = l - (n-1)d \text{ .... from the end.}$$

where, l is the last term, d is the common difference and n is the number of terms.

Know the Terms

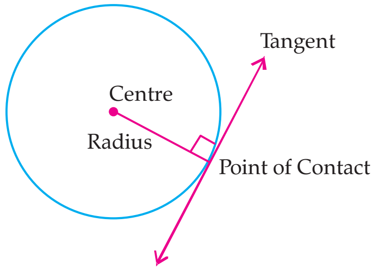
- A sequence is defined as an ordered list of numbers.
  The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.
- If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.
  Example: 2 + 4 + 6 + 8 + ... is a series.
- The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician *Leonasalo Fibonacci*, when he was dealing with the problem of rabbit population.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by a constant, the resulting sequence is also an A.P.
- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.
- If the terms are selected at a regular interval, the given sequence is in A.P.
- If three consecutive numbers a, b and c are in A.P., the sum two numbers is twice the middle number *i.e.*, $2b = a + c$.

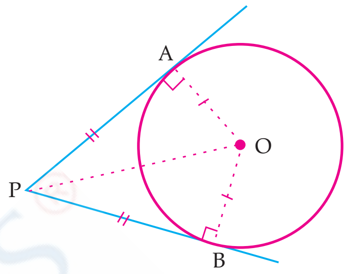
UNIT II: GEOMETRY

CHAPTER-3
CIRCLES

Revision Notes

- **Tangent:** A tangent to a circle is a line that intersects the circle at one point only.
- The common point of the circle and the tangent is called the point of contact.
- **Secant:** Two common points (A and B) between line PQ and circle.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.





- There are exactly two tangents to a circle through a point outside the circle.
- The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent.
- The lengths of the tangents drawn from an external point to a circle are equal.

  In the figure,

$$PA = PB.$$

Know the Facts

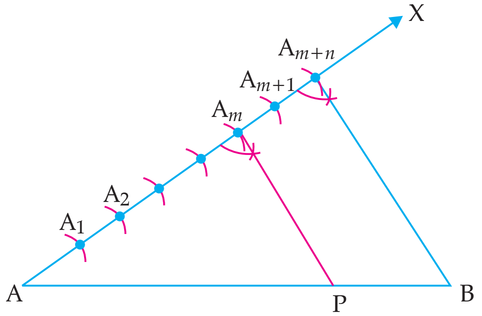
- The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish mathematician **Thomas Fincke** in 1583.
- The line perpendicular to the tangent and passing through the point of contact, is known as the normal.
- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

CHAPTER-4
CONSTRUCTIONS

Revision Notes

Division of a Line Segment in a Given Ratio

- **To divide a line segment internally in a given ratio $m : n$, where both m and n are positive integers.**
  - 1^{st} **Method:** we follow the following steps:



Step 1. Draw a line segment AB of given length by using a ruler.
Step 2. Draw any ray AX making an acute angle with AB.

Step 3. Along AX mark off $(m + n)$ points $A_1, A_2, \ldots\ldots, A_m, A_{m+1}, \ldots\ldots, A_{m+n}$, such that

$$AA_1 = A_1A_2 = A_{m+n-1}\,A_{m+n}.$$

Step 4. Join BA_{m+n}.

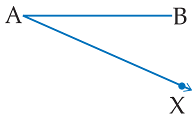
Step 5. Through the point A_m draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at A_m.
i.e., $\angle AA_mP = \angle AA_{m+n}B$.

This line meets AB at point P.

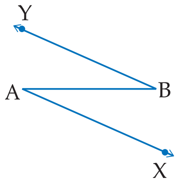
The point P is the required point which divides AB internally in the ratio $m : n$.

- **2nd Method:**

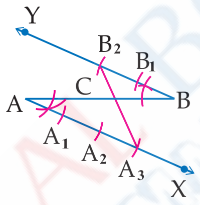
1. Draw an ray AX making an acute angle with line segment AB.



2. Draw ray $BY \parallel AX$



3. Locate A_1, A_2, A_3 $(m = 3)$ on AX and B_1, B_2 $(n = 2)$ at equal distances. Join A_3B_2, intersecting AB at C, $AC : BC = 3 : 2$.

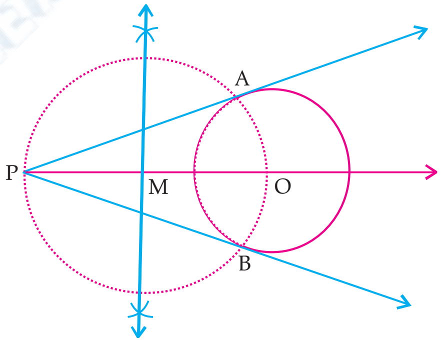


Tangents to a Circle from a Point Outside it

➤ **To draw the tangent to a circle from an external point when its centre is known.**

Given: A circle with centre O and a point P outside it.

To construct: The tangents to the circle from P.



Steps of construction:

(i) Join OP and bisect it. Let M be the mid-point of OP.

(ii) Taking M as centre and MO as radius, draw a circle to intersect $C(O, r)$ in two points, say A and B.

(iii) Join PA and PB. These are the required tangents from P to the circle.

➤ **To draw tangents to a circle from a point outside it when its centre is not known.**

Given: P is a point outside the circle.

To construct: To draw tangents to the circle from the point P.

Steps of construction:

(i) Draw a circle of given radius.

(ii) Through P draw a secant PAB to meet the circle at A and B.

(iii) Produce AP to C such that $PC = PA$. Bisect CB at Q.

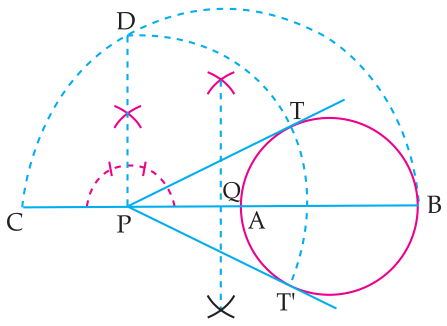
(iv) With CB as diameter and centre as Q, draw a semi-circle.

(v) Draw $PD \perp CB$, to meet semi-circle at the point D.

(vi) Taking P as centre and PD as radius draw an arc to interest the circle at T and T'.

(vii) Join P to T and T'.

Hence, PT and PT' are the required tangents.

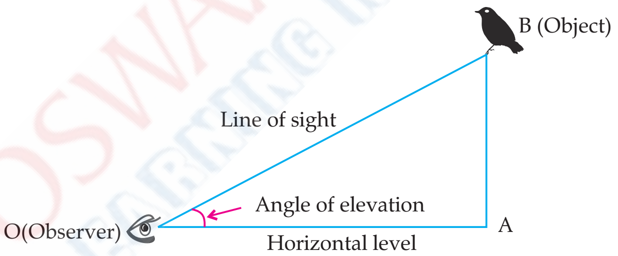


UNIT III: Trigonometry

CHAPTER-5
HEIGHTS AND DISTANCES

Revision Notes

- The line of sight is the line drawn from the eye of an observer to the point on the object viewed by the observer.
- The angle of elevation of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at a point on the object.
- Line of sight, angles and altitude (height).

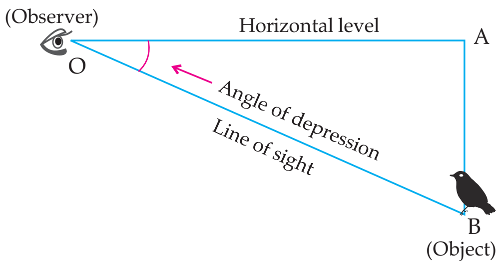


(i) $\angle AOB$ is the angle of elevation.

(ii) By height AB, means object is at point B from the point A located at the ground.

(iii) AO is the distance of the observer from the point A.

- The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at a point on the object.



- The height of object above the water surface is equal to the depth of its image below the water surface.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

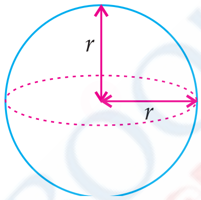
UNIT IV: MENSURATION

CHAPTER-6
SURFACE AREAS AND VOLUMES

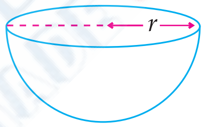
Revision Notes

Surface Areas and Volumes

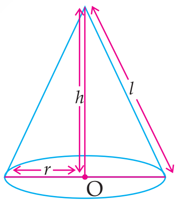
- A sphere is a perfectly round geometrical object in three-dimensional space .



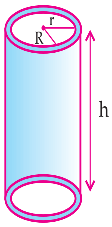
- A hemisphere is half of a sphere.



- A cone is a three dimensional geometric shape tapers smoothly from a flat base to a point called the apex or vertex.



- A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.

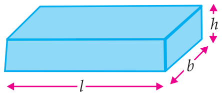


Conversion of One Type of Metallic Solid into Another

- While converting one metallic object into another, the volume will remain same by assuming no wastage of metal.
- Total surface area always be different from the original.
- Total surface area of the solid formed by the combination of solids is the sum of the curved surface areas of each individual solid.
- The solids having the same curved surface do not necessarily have the same volume.

Know the Formulae

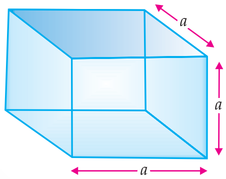
- **Cuboid:**



$$\text{Lateral surface area or area of four walls} = 2(l + b)h$$
$$\text{Total surface area} = 2(lb + bh + hl)$$
$$\text{Volume} = l \times b \times h$$
$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$

Here, l is length, b is breadth and h is height of the cuboid.

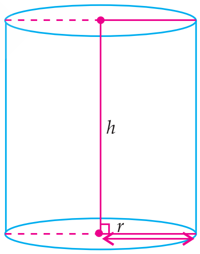
- **Cube:**



$$\text{Lateral surface area or area of four walls} = 4 \times a^2$$
$$\text{Total surface area} = 6 \times a^2$$
$$\text{Volume} = a^3$$
$$\text{Diagonal of a cube} = \sqrt{3} \times a$$

Here, a is edge of cube.

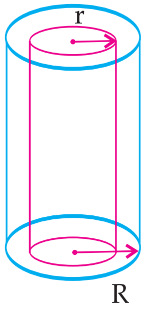
- **Right Circular Cylinder:**



$$\text{Area of base or top face} = \pi r^2$$
$$\text{Area of curved surface or curved surface area} = \text{perimeter of the base} \times \text{height}$$
$$= 2\pi rh$$
$$\text{Total surface area (including both ends)} = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$
$$\text{Volume} = (\text{Area of the base} \times \text{height}) = \pi r^2 h$$

Here, r is the radius of base and h is the height of the right circular cylinder.

- **Right Circular Hollow Cylinder:**



Total surface area = (External surface area + Internal surface area) + (Area of brim)

$$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$$
$$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$$
$$= [2\pi(R + r)(h + R - r)]$$

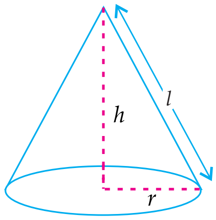
$$\text{Curved surface area} = (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$$

Volume of the material used = (External volume) – (Internal volume)

$$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$$

Here, R and r are the external and internal radii and h is the height of the right circular hollow cylinder.

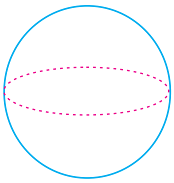
- **Right Circular Cone:**



$$\text{Slant height, } l = \sqrt{h^2 + r^2}$$
$$\text{Area of curved surface} = \pi rl$$
$$= \pi r\sqrt{h^2 + r^2}$$
$$\text{Total surface area} = \text{Area of curved surface} + \text{Area of base}$$
$$= \pi rl + \pi r^2$$
$$= \pi r(l + r)$$
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Here, r, h and l are the radius, vertical height and slant height respectively of the right circular cone.

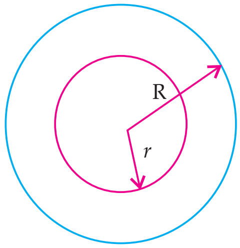
- **Sphere:**



$$\text{Surface area} = 4\pi r^2$$
$$\text{Volume} = \frac{4}{3}\pi r^3$$

Here, r is the radius of the sphere.

- **Spherical Shell:**

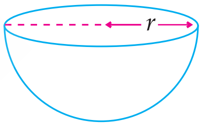


$$\text{Surface area (outer)} = 4\pi R^2$$

$$\begin{aligned}\text{Volume of material used} &= \text{(External volume - Internal Volume)}\\ &= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3\\ &= \frac{4}{3}\pi(R^3 - r^3)\end{aligned}$$

Here, R and r are the external and internal radii of the spherical shell.

- **Hemisphere:**



$$\begin{aligned}\text{Area of curved surface} &= 2\pi r^2\\ \text{Total surface area} &= \text{Area of curved surface + Area of base}\\ &= 2\pi r^2 + \pi r^2\\ &= 3\pi r^2\\ \text{Volume} &= \frac{2}{3}\pi r^3\end{aligned}$$

Here, r is the radius of the hemisphere.

Know the Terms

- The platonic solids also called the regular solids or regular polyhedra. Five such solids are : dodecahedron, icosahedron, octahedron and tetrahedron.
- Greek mathematician Plato equated tetrahedron with the 'element' fire, the cube with earth, the icosahedron with water, the octahedron with air and dodecahedron with the stuff of which the constellations and heavens were made.
- The stone of platonic solids are kept in Ashmolean Museum in Oxford.
- The tomb of Archimedes carried a sculpture consisting of a sphere and cylinder circumscribing it.

UNIT V: STATISTICS AND PROBABILITY

CHAPTER-7
STATISTICS

Revision Notes

Mean, Median and Mode

- Statistics deals with the collection, presentation and analysis of numerical data.
- Three measures of central tendency are :

  (i) Mean, **(ii)** Median and **(iii)** Mode
- **Mean:** In statistics mean stands for the arithmetic mean of the given items.

i.e., $$\text{Mean} = \frac{\text{Sum of given items}}{\text{No. of items}}$$

- **Median:** It is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or descending order of their magnitudes.

  It divides the arranged series in two equal parts *i.e.,* 50% of the observations lie below the median and the remaining are above the median.

- **Mode:** Mode is the observation which occurred maximum times. In ungrouped data, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the **modal class**. The mode of the data is a value inside the modal class.

Cumulative Frequency Graph

- **Cumulative Frequency Distribution:**

  (i) Cumulative frequency of a particular value of the variable (or class) is the sum (total) of all the frequencies up to that value (or class).

  (ii) There are two types of cumulative frequency distributions:

  (a) Cumulative frequency distribution of less than type.

  (b) Cumulative frequency distribution of more than type.

  For example:

Class interval (marks)	Frequency (No. of Students)	Less than type		More than type	
		Marks Out of 50	***c.f.***	**Marks Out of 50**	***c.f.***
0 – 10	2	Less than 10	2 = 2	More than 0	60 – 0 = 60
10 – 20	10	Less than 20	2 + 10 = 12	More than 10	60 – 2 = 58
20 – 30	25	Less than 30	12 + 25 = 37	More than 20	58 – 10 = 48
30 – 40	20	Less than 40	37 + 20 = 57	More than 30	48 – 25 = 23
40 – 50	3	Less than 50	57 + 3 = 60	More than 40	23 – 20 = 3

Know the Formulae

- **Mean:**

  (a) For Raw Data:

  If n observations $x_1, x_2, ..., x_n$ are given, then their arithmetic mean is given by :

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  (b) For Ungrouped Data:

  If there are n distinct observations $x_1, x_2,..., x_n$ of variable x with frequencies $f_1, f_2,..., f_n$ respectively, then the arithmetic mean is given by:

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

(c) For Grouped Data:

(i) To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point.

(ii) Direct Method:

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.

(iii) Assumed Mean Method or Short-cut Method:

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

where a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i.

- **Median of Grouped Data:**

Let $n = f_1 + f_2 + f_3 + ... + f_n$. First of all find $\frac{n}{2}$ and then the class in which $\frac{n}{2}$ lies. This class is known as the **median class.** Median of the given distribution lies in this class.

Median of the grouped data can be calculated using the formula:

$$\text{Median } (M_e) = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

where l = lower limit of median class, f = frequency of median class, n = number of observations, $c.f.$ = cumulative frequency of the class preceding the median class, h = class-size or width of the class-interval.

- **Mode of Grouped Data:**

Mode of the grouped data can be calculated by using the formula:

$$\text{Mode (M)} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where l = lower limit of the modal class, h = width or size of the class-interval, f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class, f_2 = frequency of the class succeeding the modal class.