UNIT I: ALGEBRA

CHAPTER-1
QUADRATIC EQUATIONS

Revision Notes

Solutions of Quadratic Equations

- A quadratic equation in variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfy an equation are called the solutions or roots or zeroes of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- **A quadratic equation can be solved by the following algebraic methods :**
  (i) Splitting the middle term,
  (ii) Making perfect squares and
  (iii) Using quadratic formula.
- If $ax^2 + bx + c = 0$, where $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Method for splitting the middle term of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Form the product a and c *i.e.*, "ac"
  (ii) Find a pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such number, it can't be factorised).
  (iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x \pm b_2x$. Now factorize, by grouping the pairs of terms.
- Roots of the quadratic equation can be found by equating each linear factor to zero. Since, product of two numbers is zero, then either or both of them are zero.
- Any quadratic equation can be converted into the form $(x + a)^2 - b^2 = 0$ by adding and subtracting same terms. This method of finding the roots of quadratic equation is called the method of making the perfect square.
- Method of making the perfect square for quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

  (i) Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

  (ii) Multiplying and dividing the coefficient of x by 2

$$x^2 + 2x\frac{b}{2a}x + \frac{c}{a} = 0$$

  (iii) Adding and subtracting $\frac{b^2}{4a^2}$

$$x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2$$

If $(b^2 - 4ac) \geq 0$, then by taking square root :

$$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The Old-Babylonians (400 *BC*) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid's developed a geometrical approach for finding out roots, which are solutions of quadratic equations.
- In Vedic manuscripts, procedures are described for solving quadratic equations by geometric methods related to completing a square.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.
- Sridharacharya (C.E. 870-930) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.
- Golden ratio ϕ is the root of quadratic equation $x^2 - x - 1 = 0$.

Discriminant and Nature of Roots

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant D $= b^2 - 4ac$.
- Nature of roots of a quadratic equation :
  (i) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  (ii) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii) If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

Know the Terms

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$
- **Quadratic identities :**
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$

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CHAPTER-2
ARITHMETIC PROGRESSION

Revision Notes

To Find n^th Term of the Arithmetic Progression

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number *d* to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.

- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.,* $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots..$
- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, the common difference changes to negative of original sequence common difference.

Know the Formulae

- The general term of an A.P. is expressed as :

  $$a_n = a + (n - 1)d. \text{ .... from the starting.}$$

  where, a is the first term d is the common difference and n is the number of terms.
- The general term of an A.P. $l, l - d, l - 2d, \ldots\ldots.., a$ is given by :

  $$a_n = l + (n - 1)(- d) = l - (n - 1)d \text{ .... from the end.}$$

  where, l is the last term, d is the common difference and n is the number of terms.
- Sum of n terms of an A.P. is given by :

  $$S_n = \frac{n}{2}[2a + (n - 1)d]$$

  where, a is the first term, d is the common difference and n is the total number of terms.
- Sum of n terms of an A.P., when first and last term is given, is

  $$S_n = \frac{n}{2}[a + l]$$

  where, a is the first term and l is the last term.
- The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum of first $(n - 1)$ terms of it. *i.e.,*

  $$a_n = S_n - S_{n-1}.$$

Mnemonics

SAND

S : Means Sum of terms
A : Means first term
N : Means nth term of n term
D : Means common difference

Know the Terms

- A sequence is defined as an ordered list of numbers.
  The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.
- If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.
  Example : 2 + 4 + 6 + 8 + ... is a series.
- The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician Leonardo Fibonacci, when he was dealing with the problem of rabbit population.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by a constant, the resulting sequence is also an A.P.
- The selection of three terms in an A.P. are
  (i) $a - d, a, a + d$
  (ii) The selection of four terms in an A.P. are $a - 3d, a - d, a + d, a + 3d$.

- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.
- If the terms are selected at a regular interval, the given sequence is in A.P.
- If three consecutive number a, b and c are in A.P., the sum of first and third number is twice the middle number *i.e.*, $2b = a + c$.

Facts about the Common Difference

If common difference is:

(a) Positive, the A.P. is increasing. E.g. 2,4,6,8,…….

(b) Zero, the A.P. is constant. E.g. 5,5,5,5,5,……..

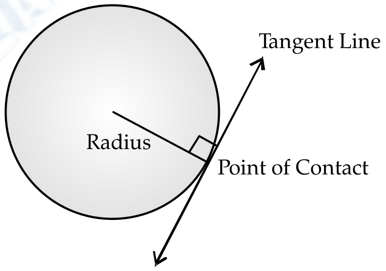
(c) Negative, the A.P. is decreasing. E.g. 24,21,18,15,……

UNIT II: GEOMETRY

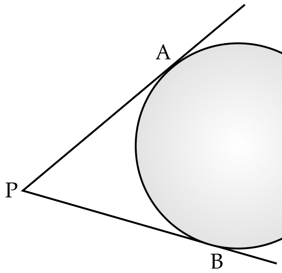
CHAPTER-3
CIRCLES

Revision Notes

- A tangent to a circle is a line that intersects the circle at one point only.
- The common point of the circle and the tangent is called the point of contact.
- The length of the segment of the tangent drawn from the external point P and the point of contact with the circle is called the length of the tangent.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point outside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.



- The lengths of the tangents drawn from an external point to a circle are equal.



In the figure, $PA = PB$.

Know the Terms

- The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish mathematician **Thomas Fincke** in 1583.
- The line containing the radius through the point of contact is also called the 'normal' to the circle at that point.
- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

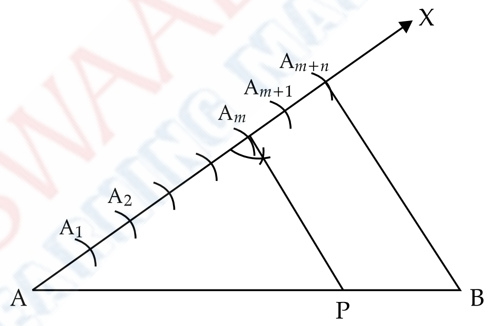
CHAPTER-4
CONSTRUCTIONS

Revision Notes

Division of Line Segment in a Given Ratio

- The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.
- **To divide a line segment internally**

  In a given ratio $m : n$, where both m and n are positive integers, we follow the following steps :



Step 1. Draw a line segment AB of given length by using a ruler.

Step 2. Draw any ray AX making an acute angle with AB.

Step 3. Along AX mark off $(m + n)$ points say $A_1, A_2, \ldots\ldots\ldots, A_m, A_{m+1}, \ldots\ldots\ldots, A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1}A_{m+n}$.

Step 4. Join BA_{m+n}.

Step 5. Through the point A_m, draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ *i.e.*, $\angle AA_mP = \angle AA_{m+n}B$

This line meets AB at point P.

The point P is the required point which divides AB internally in the ratio $m : n$.

Tangents to a Circle from a Point Outside it

- **To draw the tangent to a circle at a given point on it, when the centre of the circle is known.**

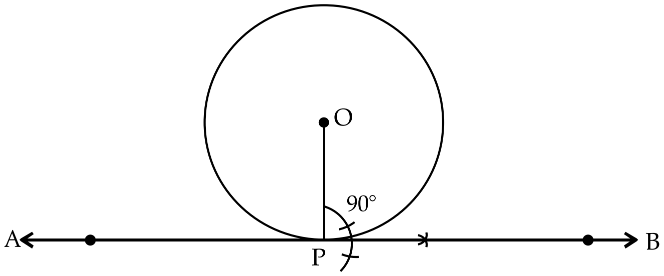
  Given : A circle with centre O and a point P on it.

  To construct : The tangent to the circle at the point P.

Steps of construction :

(i) Join *OP*.

(ii) Draw a line segment $AB \perp OP$ at the point *P*. Segment *APB* is the required tangent at *P*.



➢ **To draw the tangent to a circle at a given point on it, when the centre is not known.**

Given : *P* is a point on the circle.

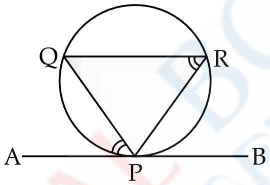
Construct : Draw a tangent at point *P*.

Steps of construction :

(i) Draw any chord *PQ* and join *P* and *Q* to a point *R*.

(ii) Draw $\angle QPA$ equal to $\angle PRQ$ on opposite side of chord *PQ*.

The line segment *BPA* is the tangent to the circle at *P*.



➢ **To draw the tangents to a circle from an external point when its centre is known.**

Given : A circle with centre *O* and a point *P* outside it.

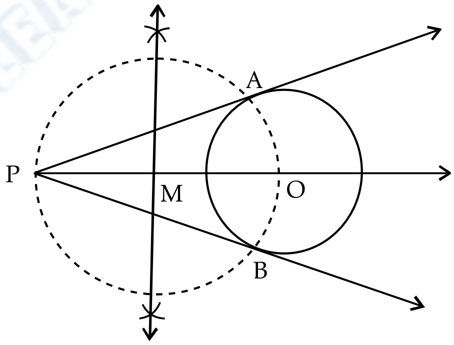
To construct : The tangents to the circle from point *P*.

Steps of construction :

(i) Join *OP* and bisect it. Let *M* be the mid-point of *OP*.

(ii) Taking *M* as centre and *MO* as radius, draw a circle to intersect *C* (*O*, *r*) in two points, say *A* and *B*.

(iii) Join *PA* and *PB*. These are the required tangents from *P* to the circle.



➢ **To draw tangents to a circle from a point outside it, when its centre is not known**

Given : P is a point outside the circle.

To construct : To draw tangents to the circle from the point P.

Steps of construction :

(i) Draw a circle of given radius.

(ii) Through *P* draw a secant *PAB* to meet the circle at *A* and *B*.

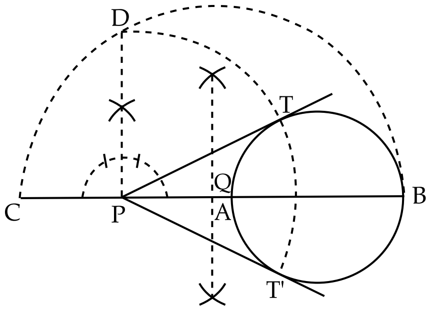
(iii) Produce *AP* to *C* such that *PC* = *PA*. Bisect *CB* at *Q*.

(iv) With *CB* as diameter and centre as *Q*, draw a semi-circle.

(v) Draw $PD \perp CB$, to meet semi-circle at the point *D*.

(vi) Taking *P* as centre and *PD* as radius draw an arc to interest the circle at *T* and *T'*.

(vii) Join P to T and P to T'.
Hence, PT and PT' are the required tangents.

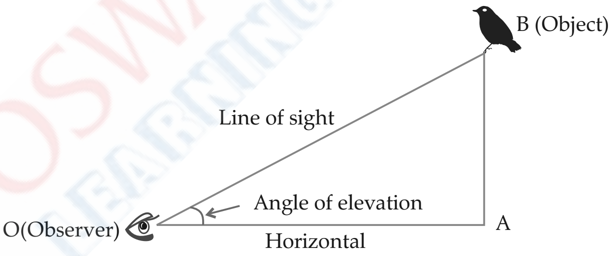


UNIT III: TRIGONOMETRY

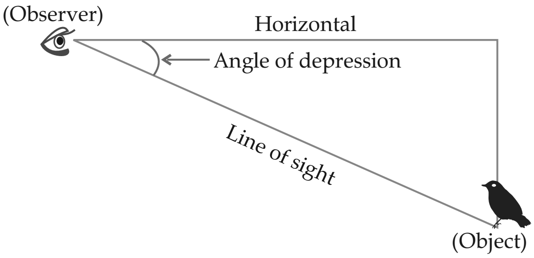
CHAPTER-5
SOME APPLICATIONS OF TRIGONOMETRY
(HEIGHTS AND DISTANCES)

Revision Notes

- The line of sight is the line drawn from the eye of an observer to the point of the object viewed by the observer.
- The angle of elevation of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at a point on the object.



- The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at a point on the object.



- The height of object above the water surface is equal to the depth of its image below the water surface.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

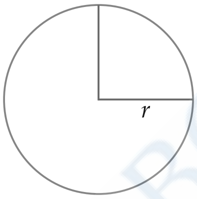
UNIT IV: MENSURATION

CHAPTER-6
SURFACE AREAS AND VOLUMES

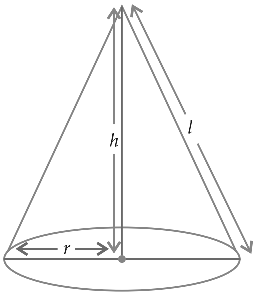
Revision Notes

Surface Areas and Volumes (Simple-Case)

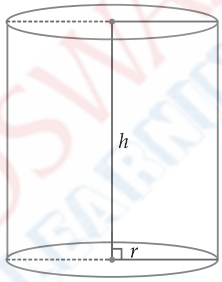
- A sphere is a perfectly round geometrical object in three-dimensional space that is the surface of a completely round ball.



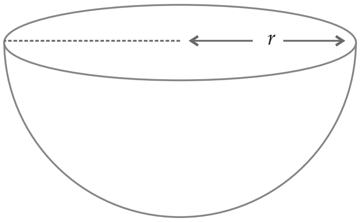
- A Cone is a three dimensional geometric shape that tapers smoothly from a flat base to a point called the apex or vertex.



- A right circular cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.



- A hemisphere is half of a sphere.



Problems Involving converting one type of metallic solid into another

- While converting one metallic object into another, the volume will remain same by assuming no wastage of metal.
- Total surface area of the solid formed is always different from the original solid.
- Total surface area of the solid formed by the combination of solids remains as the sum of the curved surface areas of each of the individual parts.
- The solids having the same curved surface do not necessarily have the same volume.

Table for the converting of the solids :

Name of solids	Volume	Total surface Area	Lateral surface Area
Cube	$V = a^3$	$TSA = 6a^2$	$LSA = 4a^2$
Cuboid	$V = l \times b \times h$	$TSA = 2\,(lb + bh + hl)$	$LSA = 2h\,(l + b)$
Cylinder	$V = \pi r^2 h$	$TSA = 2\pi r\,(h + r)$	$CSA = 2\pi rh$
Hollow cylinder ($R > r$)	$V = \pi(R^2 - r^2)h$	$TSA = 2\pi\,(R + r)\,(h + R - r)$	$CSA = 2\pi\,(R + r)h$
Cone	$V = \frac{1}{3}\pi r^2 h$	$TSA = \pi r\,(l + r)$	$CSA = \pi rl$
Sphere	$V = \frac{4}{3}\pi r^3$	$TSA = 4\pi r^2$	$CSA = 4\pi r^2$
Hemisphere	$V = \frac{2}{3}\pi r^3$	$TSA = 3\pi r^2$	$CSA = 2\pi r^2$

Mnemonics

Curved Surface area = 2πrh			
Interpretation		IS FAMOUS FOR	is equal to
CHAT	Curved	TWO	2
SHOP	Surface	PIZZA	π (pie)
IN AGRA	Area	ROLLED	r (Radius)
		HAVING CHEESE	h (Height)

Know the Formulae

➢ **Cuboid :**

Lateral surface area or area of four walls $= 2(l + b)h$

Total surface area $= 2\,(lb + bh + hl)$

Volume $= l \times b \times h$

Diagonal $= \sqrt{l^2 + b^2 + h^2}$

Here, l = length, b = breadth and h = height

➢ **Cube :**

Lateral surface area or area of four walls $= 4 \times (\text{edge})^2$

Total surface area $= 6 \times (\text{edge})^2$

Volume $= (\text{edge})^3$

Diagonal of a cube $= \sqrt{3} \times \text{edge}.$

➢ **Right circular cylinder :**

Area of base or top face $= \pi r^2$

Area of curved surface or curved surface area $=$ perimeter of the base $\times$ height $= 2\pi rh$

Total surface area (including both ends) $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Volume $=$ (Area of the base $\times$ height)$= \pi r^2 h$

Here, r is the radius of base and h is the height.

- **Right circular hollow cylinder :**

Total surface area = (External surface area + internal surface area) + (Area of ends)

$$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$$
$$= [2\pi h(R + r) + 2\pi(R^2 - r^2)] = [2\pi h(R + r) + 2\pi(R + r)(R - r)]$$
$$= [2\pi(R + r)(h + R - r)]$$

Curved surface area $= (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$

Volume of the material used = (External volume) – (Internal volume)

$$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$$

Here, R and r are the external and internal radii and h is the height of the hollow cylinder.

- **Right circular cone :**

$$\text{Slant height, } l = \sqrt{h^2 + r^2}$$
$$\text{Area of curved surface} = \pi rl = \pi r\sqrt{h^2 + r^2}$$
$$\text{Total surface area} = \text{Area of curved surface} + \text{Area of base}$$
$$= \pi rl + \pi r^2 = \pi r(l + r)$$
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Here, r, h and l are the radius, vertical height and slant height respectively of the cone.

- **Sphere :**

$$\text{Surface area} = 4\pi r^2$$
$$\text{Volume} = \frac{4}{3}\pi r^3$$

Here, r is the radius of the sphere.

- **Spherical shell :**

$$\text{Surface area (outer)} = 4\pi R^2$$
$$\text{Volume of material} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(R^3 - r^3)$$

Here, R and r are the external and internal radii of the spherical shell.

- **Hemisphere :**

$$\text{Area of curved surface} = 2\pi r^2$$

Total surface area = Area of curved surface + Area of base

$$= 2\pi r^2 + \pi r^2$$
$$= 3\pi r^2$$
$$\text{Volume} = \frac{2}{3}\pi r^3$$

Here, r is the radius of the hemisphere.

Know the Terms

- The platonic solids also called the regular solids or regular polyhedra. 5 such solids are : cube, dodecahedron, icosahedron, octahedron and tetrahedron.
- Greek mathematician Plato equated tetrahedron with the 'element' fire, the cube with earth, the icosahedron with water, the octahedron with air and dodecahedron with the stuff of which the constellations and heavens were made.
- The stone of platonic solids are kept in Ashmolean Museum in Oxford.
- The tomb of Archimedes carried a sculpture consisting of a sphere and cylinder circumscribing it.

UNIT V: STATISTICS AND PROBABILITY

CHAPTER-7
STATISTICS

Revision Notes

Mean, Median and Mode

- Statistics deals with the collection, presentation and analysis of numerical data.
- Three measures of central tendency are :

  (i) Mean, **(ii)** Median **(iii)** Mode
- **Mean :**

  (a) For Raw Data :

  If n observations $x_1, x_2, ..., x_n$ are given, then their arithmetic mean is given by :

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  (b) For Ungrouped Data :

  If there are n distinct observations $x_1, x_2,..., x_n$ of variable x with frequencies $f_1, f_2,..., f_n$ respectively, then the arithmetic mean is given by :

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

  (c) For Grouped Data :

  To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point.

  (i) Direct Method :

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i},$$

  where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.

  (ii) Assumed Mean Method or Short-cut Method :

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$$

- **Median :**

  (a) For Ungrouped Data :

  If n is odd, Median $= \left(\frac{n+1}{2}\right)^{th}$ term

  If n is even, Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$

  (b) For Grouped Data :

  Let $n = f_1 + f_2 + f_3 + ... + f_n$. First of all find $\frac{n}{2}$ and then the class in which $\frac{n}{2}$ lies. This class is known as the **median class.** Median of the given distribution lies in this class.

Median of the grouped data can be calculated using the formula :

$$\text{Median } (M_e) = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h,$$

where, l = lower limit of median class, f = frequency of median class, n = number of observations, $c.f.$ = cumulative frequency of the class preceding the median class, h = class-size or width of the class-interval.

➢ **Mode of Grouped Data :**

(i) Mode is the observation which occurred maximum times. In ungrouped data, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the **modal class**. The mode of the data is a value inside the modal class.

(ii) Mode of the grouped data can be calculated by using the formula :

$$\text{Mode } (M) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$

Where, l = lower limit of the modal class, h = width or size of the class-interval, f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class, f_2 = frequency of the class succeeding the modal class.

Note :

(a) If the series has only one mode, then it is known as **Unimodal.**

(b) If the series has two modes, then it is known as **Bimodal.**

(c) If the series has three modes, then it is known as **Trimodal.**

(d) Mode may or may not be defined for a given series.

➢ **Empirical Relation Between Mean, Median and Mode :**

(i) Mode $= 3$ Median $- 2$ Mean

(ii) Median $= \frac{1}{3}$ Mode $+ \frac{2}{3}$ Mean

(iii) Mean $= \frac{3}{2}$ Median $- \frac{1}{2}$ Mode

Note : For calculating the mode and median for grouped data, it should be ensured that the class-intervals are continuous before applying the formula. Same condition also apply for construction of an ogive. Further, in case of ogives, the scale may not be the same on both the axes.

Mnemonics

SANDS (3m = m + 2AM)

(1) Concept :

Empirical Relation :

3 Medium = Mode + 2 mean

Interpretation

M = Median, m = mode, AM = Arithmetic Mean

Meerut has 3 times more GDP

Median

as compared with one times of Mumbai

Mode

and 2 times of Mehandipur GDP.

2 Mean