UNIT–I : NUMBER SYSTEMS

CHAPTER-1

REAL NUMBERS

Topic-1

Fundamental Theorem of Arithmetic

Concepts Covered • *Fundamental Theorem of Arithmetic:*

For any two positive integers a and b,

We have HCF (a, b) × LCM (a, b) = a × b

or HCF (a, b) = $\frac{a \times b}{LCM\ (a, b)}$ *or* LCM (a, b) = $\frac{a \times b}{HCF\ (a, b)}$

Revision Notes

- **The Fundamental Theorem of Arithmetic**

  Every composite number can be expressed as a product of primes and this factorisation is unique, apart from the order in which the prime factors occur. Fundamental theorem of arithmetic is also called a **Unique Factorization Theorem**.

  Composite number = Product of prime numbers

  Or

  Any integer greater than 1 can either be a **prime numbers** or can be written as a unique product of prime numbers. e.g.,

  (i) $2 \times 11 = 22$ is the same as $11 \times 2 = 22$.

  (ii) 6 can be written as 2×3 or 3×2, where 2 and 3 are **prime numbers**.

  (iii) 15 can be written as 3×5 or 5×3, where 3 and 5 are **prime numbers.**

  The prime factorization of a natural number is unique, except to the order of its factors.

  e.g., 12 detained by multiplying the **prime numbers** 2, 2 and 3 together,

  $$12 = 2 \times 2 \times 3$$

  We would probably write it as

  $$12 = 2^2 \times 3$$

- By using Fundamental Theorem of Arithmetic, we shall find the HCF and LCM of given numbers (two or more). This method is also called **Prime Factorization Method.**

- **Prime Factorization Method to find HCF and LCM:**

  (i) Find all the prime factors of given numbers.

  (ii) HCF of two or more numbers = Product of the smallest power of each common prime factor, involved in the numbers.

  (iii) LCM of two or more numbers = Product of the greatest power of each prime factor, involved in the numbers.

Key Words

Highest Common Factor (HCF): The HCF of two or more numbers is the highest number among all the common factors of the given numbers.

Prime Numbers: A number that can be divided exactly only by itself and 1.

Fundamental Facts

(1) The Euclidean algorithm is useful for reducing a common fraction to lowest terms.

For example: $\frac{714}{765} = \frac{51 \times 14}{51 \times 15} = \frac{14}{15}$.

(2) The concept of LCM is important to solve problem related to racetracks, traffic light etc.

(3) In Mathematics problem, where we pair two objects against each other, the LCM value is useful in optimizing the quantities of the given objects.

Example

Find the LCM of 40, 36 and 126 by applying the prime factorization method.

Step 1. Factorise each of the given positive integers such as:

$$40 = 2 \times 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3$$
and
$$126 = 2 \times 3 \times 3 \times 7$$

Step 2. Express them as a product of powers of primes in ascending order of magnitudes of primes:

$$40 = 2^3 \times 5,\ 36 = 2^2 \times 3^2 \text{ and } 126 = 2 \times 3^2 \times 7$$

Step 3. To find LCM, list all prime factors of 40, 36 and 126 with their greatest exponents as:

$$\therefore \quad \text{LCM} = 2^3 \times 3^2 \times 5 \times 7$$
$$= 8 \times 9 \times 5 \times 7$$
$$= 2520.$$

Mnemonics

Concept: Euclid's Division Lemma ($a = bq + r$)
Mnemonics: Alibaba's **b**est product **q**uotation is a**s**sent **r**eward.
Interpretation:
Alibaba's A = a
best B = b
quotation Q = q
assets A = addition of bq and r
reward R = r
Then $a = b \times q + r$.

Topic-2 Irrational Numbers

Concepts Covered • *Rational & Irrational Numbers.*

Revision Notes

- **Rational Numbers:** A number in the form $\frac{p}{q}$, where p and q are **co-prime numbers** and $q \neq 0$, is known as rational number.

  For example: $2, -3, \frac{3}{7}, -\frac{2}{5}$, etc. are rational numbers.
- **Irrational Numbers:** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ are irrational numbers.
- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- The sum or difference of a rational and an irrational number is irrational.
- The product and quotient of a non-zero rational and an irrational number is irrational.

Key Words

Co-Prime Numbers: Co-prime numbers are those numbers that have only one common factor. **For example:** 3 and 5, 11 and 13 etc.

Fundamental Facts

(1) The discovery of irrational numbers is usually attributed to Pythagoras, more specifically to the Pythagorean Hippasus of metapontum who produced a proof of the irrationality of $\sqrt{2}$.

(2) Irrational number are numbers that cannot be expressed as the ratio of two whole numbers.

Example

Show that $2\sqrt{3}+7$ is an irrational number.

Step 1. Let $2\sqrt{3}+7$ be a rational number. Since, a rational number can be expressed as $\frac{a}{b}$, where $b \neq 0$ and a & b are integers.

Step 2. Then

$$2\sqrt{3}+7 = \frac{a}{b}$$

or

$$2\sqrt{3} = \frac{a}{b}-7$$

or

$$\sqrt{3} = \frac{1}{2}\left(\frac{a}{b}-7\right)$$

Here, L.H.S. = $\sqrt{3}$ is an irrational.

But, R.H.S. = $\frac{1}{2}\left(\frac{a}{b}-7\right)$ is a rational.

So, it is not possible.

Step 3. Hence, our assumption that $2\sqrt{3}+7$ is a rational is incorrect.

Hence, $2\sqrt{3}+7$ is an irrational number.

UNIT–II : ALGEBRA

CHAPTER-2

POLYNOMIALS

Revision Notes

- **Polynomial:** An algebraic expression in the form of $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers) is called a polynomial in one variable x of degree n.
- **Value of a Polynomial at a given point:** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial:** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.

  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-coordinates of the points, where the graph of $y = p(x)$ intersects the X-axis.

  (i) A linear polynomial has one and only one zero.

  (ii) A quadratic polynomial has at most two zeroes.

  (iii) A cubic polynomial has at most three zeroes.

  (iv) In general, a polynomial of degree n has at most n zeroes.
- **Graphs of Different types of Polynomials:**
  - **Linear Polynomial:** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.
  - **Quadratic Polynomial: (i)** Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects X-axis at a maximum of two distinct points.

    (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two distinct points.
  - **Cubic polynomial:** Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects X-axis at a maximum of three distinct points.

- **Relationship between the Zeroes and the Coefficients of a Polynomial:**

(i) Zero of a linear **polynomial**

$$= \frac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\frac{-b}{a}$

(ii) In a quadratic polynomial,
Sum of zeroes of a quadratic polynomial

$$= \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes of a quadratic polynomial

$$= \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

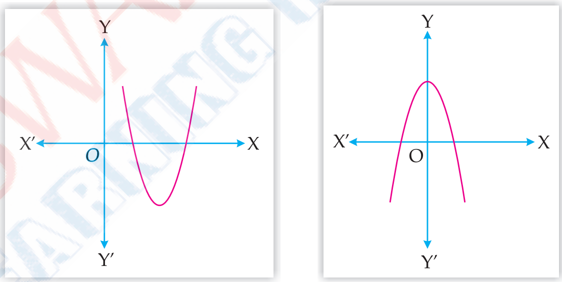
$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma = (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

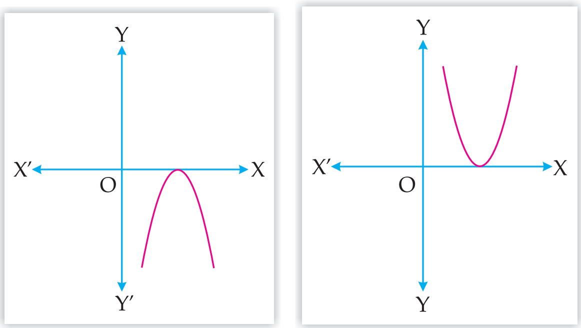
- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D. The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

Case I: If D > 0, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-coordinates of points of intersection with X-axis is known as 'zeroes' of $f(x)$.



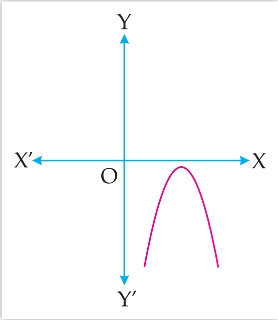
$\therefore$ $f(x)$ will have two zeroes and we can say that roots/zeroes of the two given polynomials are real and unequal.

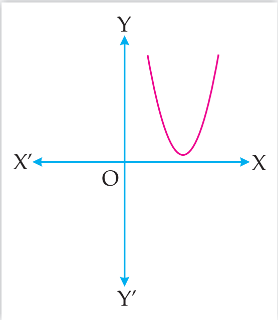
Case II: If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.



$\therefore$ $f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

Case III: If D < 0, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.





$\therefore$ $f(x)$ will not have any real zero.

Key Words

Distinct: The term distinct number is used to refer to a number in a set that is not equal to another number.

Graph: A diagram that represents the variation of a variable in comparison with that of one or more other variables.

Polynomial: An expression of more than two algebraic terms, especially the sum of several terms that contain different powers of the same variable(s).

Key Formulae

Relationship between the zeroes and the coefficients of a Polynomial:

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b$, where $a \neq 0$	1	$k = -\dfrac{b}{a}$, *i.e.*, $k = \dfrac{-\text{Constant term}}{\text{Coefficient of } x}$
2.	Quadratic	$ax^2 + bx + c$, where $a \neq 0$	2	Sum of zeroes, $(\alpha + \beta) = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{b}{a}$ Product of zeroes, $(\alpha\beta) = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d$, where $a \neq 0$	3	Sum of zeroes, $(\alpha + \beta + \gamma) = \dfrac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\dfrac{b}{a}$ Product of sum of zeroes taken two at a time, $(\alpha\beta + \beta\gamma + \gamma\alpha) = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{c}{a}$ Product of zeroes, $(\alpha\beta\gamma) = \dfrac{-\text{Constant term}}{\text{Coefficient of } x^3} = -\dfrac{d}{a}$

Fundamental Facts

(1) Polynomial are also an essential tool in describing and predicting traffic patterns so appropriate traffic control measures, such as traffic lights, can be implemented.

(2) Economists use polynomials to model economic growth patterns, and medical researchers used them to describe the behaviour of bacterial colonies.

Mnemonics

Concept: $\alpha.\beta = \dfrac{c}{a}$

Mnemonics: Amitabh **B**achchan went **C**anada **by a**eroplane.

Interpretation:

Amitabh's A $\Rightarrow$ **Alpha** (a)

Bachchan's B $\Rightarrow$ **Beta** (b)

Canada's C $\Rightarrow$ **Constant (c)**

By for Divide by and aeroplane's a $\Rightarrow$ **Variable.**

Example 1

If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, then find a. Also find its zeroes.

Solution: Let $p(x) = 2x^2 + 2ax + 5x + 10$

Step 1. If $(x + a)$ is a factor of $p(x)$, then $-a$ is a zero of $p(x)$.

$\therefore \quad p(-a) = 0$

Step 2. Putting $x = -a$ in $p(x)$, we get $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow \quad 2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow \quad 5a = 10$

$\Rightarrow \quad a = 2.$

Step 3. Putting the value of $a = 2$ in $p(x)$,

$$p(x) = 2x^2 + 4x + 5x + 10$$
$$= 2x(x + 2) + 5(x + 2)$$
$$= (x + 2)(2x + 5)$$

Step 4. For finding zero, we get, $p(x) = 0$

$\therefore \quad (x + 2)(2x + 5) = 0$

Either, $\quad x + 2 = 0 \Rightarrow x = -2$

or, $\quad 2x + 5 = 0 \Rightarrow x = -\dfrac{5}{2}.$

Hence, -2 and $-\dfrac{5}{2}$ are zeroes of $p(x)$.

CHAPTER-3

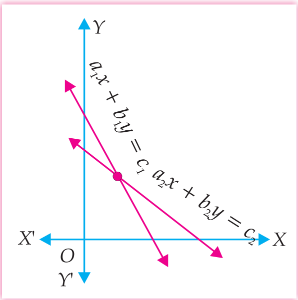
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Topic-1 Graphical Solution of Linear Equations in two variables

Concepts Covered • *To Solve the equations by graphical method.*
• *Possibilities of solutions and consistency/inconsistency.*
• *Conditions of unique solution/infinite number of solutions/ no Solutions.*

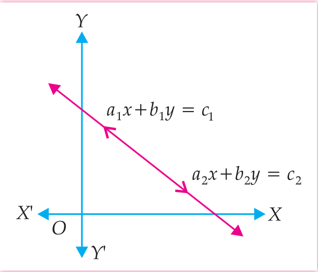
Revision Notes

- **Linear equation in two variables:** An equation in the form of $ax + by + c = 0$, where a, b and c are real numbers and a and b are not zero, is called a linear equation in two **variables** x and y.
  General form of a pair of linear equations in two **variables** is:
  $$a_1x + b_1y + c_1 = 0$$
  and $$a_2x + b_2y + c_2 = 0,$$
  where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that
  $$a_1, b_1 \neq 0 \text{ and } a_2, b_2 \neq 0.$$
  e.g., $3x - y + 7 = 0$,
  and $7x + y = 3$
  are linear equations in two **variables** x and y.
- There are two methods of solving simultaneous linear **equations** in two variables:
  (i) Graphical method, and
  (ii) Algebraic methods.
  1. Graphical Method:
  (i) Express one variable (say y) in terms of the other variable x, $y = ax + b$, for the given **equation**.
  (ii) Take at least two values of independent variable x and find the corresponding values of dependent variable y, take integral values only.
  (iii) Plot these values on the graph paper in order to represent these **equation**.
  (iv) If the lines intersect at a distinct point, then point of intersection will be the unique solution for given equations. In this case, the pair of linear equations is **consistent.**
  If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique **solution.**



Intersecting Lines

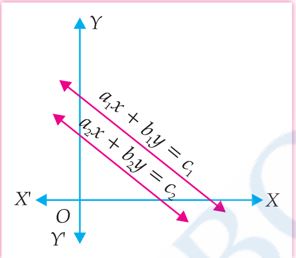
  (v) If **the lines** representing the linear equations **coincides**, then system of equations has infinitely many **solutions**. In this case, the pair of linear equations is **consistent and dependent.**
  If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many **solutions.**



Coincident Lines

(vi) If **the lines** representing the pair of linear equations **are parallel**, then the system of equations has no solution and is called **inconsistent**.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.



Parallel Lines

► **Possibilities of solutions and Inconsistency:**

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Conditions for solvability
$x - 2y = 0$ $3x - 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{-4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution or Unique solution	System is consistent
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

Key Words

Variable: A quantity that may assume any one of a set of values.

Equation: An equation is a formula that expresses the equality of two expressions by connecting them with equal sign.

Solution: Process of solving a problem.

Fundamental Facts

(1) A linear equation in two variable is represented geometrically by a line whose points make up the collection of solutions of the equation.

(2) To graph a linear equations in two variables by plotting the points.

(3) In case of two variables, each solution may be interpreted as the cartesian coordinates of a point of the Euclidean plane.

Key Formulae

If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a pair of linear equations in two variables x and y such that:

(i) System has unique solution

if $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

(ii) System has infinite number of solutions

if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(iii) System has no solution

if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Fundamental Facts

Concept: Algebra Methods

System has unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Mnemonics: For unique feature **Audi** $\mathbf{A_1}$ and $\mathbf{A_2}$ are not same as BMW $\mathbf{B_1}$ and $\mathbf{B_2}$

Interpretations:

$A_1 = a_1$

$A_2 = a_2$

$B_1 = b_1$

$B_2 = b_2$

Topic-2 Algebraic Methods to Solve Pair of Linear Equations

Concept Covered • *Solve the linear equation algebraically by Substitution and Elimination method.*
• *To solving word problems.*

Revision Notes

- **Algebraic Method:** We can solve the linear equations algebraically by **substitution method and elimination method.**

1. **Substitution Method:**
   (i) Find the value of one variable (say y) in terms of the other variable *i.e.*, x from either of the equations.
   (ii) Substitute this value of y in other equation and reduce it to an equation in one variable.
   (iii) Solve the equation so obtained and find the value of x.
   (iv) Put this value of x in one of the equations to get the value of variable y.
2. **Elimination Method:**
   (i) Multiply given equations with suitable constants, make either the x-coefficients or the y-coefficients of the two equations equal.
   (ii) Subtract or add one equation from the other to get an equation in one variable.
   (iii) Solve the equation so obtained to get the value of the variable.
   (iv) Put this value in any one of the equation to get the value of the second variable.

Note:

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.*, it is inconsistent.

Steps to be followed for solving word problems

S. No.	Problem type	Steps to be followed
1.	Age Problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then, 'a' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after 'b' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them.
2.	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
3.	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4.	Problems based on Distance, Speed and Time	$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ or $\text{Distance} = \text{Speed} \times \text{Time}$ and $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. Then, the speed of boat in downstream $= (x + y)$ km/h and speed of boat in upstream $= (x - y)$ km/h.
5.	Problems based on commercial Mathematics	For solving specific questions based on commercial mathematics, • To the fare of 1 full ticket may be taken as ₹ x and the reservation charges may be taken as ₹ y, so that one full fare $= x + y$ and one half fare $= \frac{x}{2} + y$. • To solve the questions of profit and loss, take the cost price of 1st article as ₹ x and that of 2nd article as ₹ y. • To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at some other rate of interest as per given in question.
6.	Problems based on Geometry and Mensuration	• Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle. • In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary.

Fundamental Facts

(1) Algebraic expressions are the mathematical equations consisting of variables, constants, terms and coefficients.

(2) An equation is a statement indicating that two algebraic expressions are equal.

CHAPTER-4

QUADRATIC EQUATIONS

Revision Notes

Solution of Quadratic Equations

- A quadratic **equation** in variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfy the equation are called the solutions or roots or zeros of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- A quadratic equation can be solved by the following algebraic methods:
  (i) By factorization (splitting the middle term),
  (ii) Making perfect squares and
  (iii) Using quadratic formula.
- If $ax^2 + bx + c = 0$, where $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the **quadratic** equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Method for factorization of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Find the product of a and c *i.e.*, "ac"
  (ii) Find a pair of numbers b_1 and b_2 whose product is "ac"and whose sum is "b" (if you can't find such number, it can't be factorized).
  (iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x \pm b_2x$. Now factorize, by grouping the pairs of terms.
- **Roots** of the quadratic equation can be found by equating each linear factor to zero. Since, product of two numbers is zero, then either or both of them are zero.

Key Words

Equation: The process of equating one thing with another.
Quadratic: The term quadratic describes something that pertains to squares, to the operation of squaring, to terms of second degree etc.
Roots: A solution to an equation, usually expressed as a number or an algebraic.

- Solution of a Quadratic equation using Quadratic formula
  The roots of the quadratic equation $ax^2 + bx + c = 0$; $a \neq 0$ can be find using the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The above result is known as **quadratic formula** or **Sridharacharya Formula**.
Here, $b^2 - 4ac \geq 0$, for real roots.

- The Old-Babylonians (400 BC) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid developed a geometrical approach for finding out roots, which are solutions of quadratic equations.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.
- Sridharacharya (C.E. 1025) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber Embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.

Discriminant and Nature of Roots

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant $D = b^2 - 4ac$.
- **Nature of roots of a quadratic equation:**
  (i) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real **roots**.
  (ii) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii) If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

Fundamental Facts

(1) Quadratic equations are second order polynomials. This means that the highest power of the variable is two.
(2) Many physical and mathematical problems are in the form of quadratic equations.
(3) An equation in the form of $ax^2 + bx + c = 0$ is called as a quadratic equation. It has two roots. Both of them may be real, equal or imaginary.

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$
- Quadratic identities:
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$
- Discriminant $D = b^2 - 4ac$.

Example 1

Solve $6x^2 - 23x - 35 = 0$ by quadratic formula.
Step 1. Given: $6x^2 - 23x - 35 = 0$
Comparing with $ax^2 + bx + c = 0$, we get $a = 6, b = -23$ and $c = -35$
Step 2. Take formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Step 3. Putting the values of a, b, and c.

$$x = \frac{(-23) \pm \sqrt{(-23)^2 - 4\times 6\times(-35)}}{2\times 6}$$

$$= \frac{23 \pm \sqrt{529+840}}{12}$$

$$= \frac{23 \pm \sqrt{1369}}{12}$$

$$= \frac{23 \pm 37}{12}$$

Step 4. Now, required value of x,

$$x = \frac{23+37}{12}, \frac{23-37}{12}$$

$$= \frac{60}{12}, -\frac{14}{12}$$

$$= 5, -\frac{7}{6}.$$

Example 2

Find the value of k for which the equation $4x^2 + kx + 25 = 0$ has equal roots.
Step 1. Given: $4x^2 + kx + 25 = 0$
Comparing above equation with $ax^2 + bx + c = 0$, $a = 4$, $b = k$ and $c = 25$

Step 2. Condition for equal roots is $D = 0$.

i.e., $b^2 - 4ac = 0$

Step 3. Substituting the values of *a, b* and *c* in above condition.

$$(k)^2 - 4 \times 4 \times 25 = 0$$
$$\Rightarrow \quad k^2 - 400 = 0$$
$$\Rightarrow \quad (k)^2 - (20)^2 = 0$$
$$\Rightarrow \quad (k + 20)(k - 20) = 0$$
$$\Rightarrow \quad k = 20 \text{ or } -20.$$

Mnemonics

Concept: To Find the roots of quadratic equation, $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Mnemonics: A negative **B**oy could not decided if he did or didn't want to go to a **Radical** party. The Boy was **Square** so he **missed out** on 4 **A**wesome **C**hicks.

This was all over by **2 a.m.**

Interpretation:

A negative Boy = $(-b)$

he did or didn't want to go = (+/–)

to a Radical party = $(\sqrt{\quad})$

Boy was Square = (b^2)

missed out = (–)

4 Awesome = 4a

Chicks = c

all over = Divided by

2 a.m. = 2a

CHAPTER-5

ARITHMETIC PROGRESSIONS

Topic-1 To Find n^{th} Term of the Arithmetic Progression

Concepts Covered
- *Define first term, common difference.*
- *Define finite and infinite A.P.*
- *Formula for finding n^{th} term of an A.P.*

Revision Notes

- An arithmetic progression is a **sequence** of numbers in which each **term** is obtained by adding or subtracting a fixed number d to the preceding **term**, except the first **term**.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the **sequence** of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.
- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.,* $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots\ldots$

- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, the common difference changes to negative of original **sequence** common difference.

Key Formulae

- The general (n^{th}) term of an A.P. is expressed as:

$$a_n = a + (n - 1)d. \text{ .... from the starting.}$$

where, a is the first term and d is the common difference.

- The general (n^{th}) term of an A.P. $l, l - d, l - 2d, \ldots\ldots, a$ is given by:

$a_n = l + (n - 1)(- d) = l - (n - 1)d$ .... from the end.

where, l is the last term, d is the common difference and n is the number of terms.

Key Words

Term: A term is a word or expression used with a particular meaning.

Sequence: A sequence is an enumerated collection of objects in which repetitions are allowed and order matters.

Fundamental Facts

(1) An A.P. or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

(2) The 7^{th} Century Indian Mathematician and astronomer Brahmagupta is the father of Arithmetic.

(3) If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.

(4) If each term of an A.P. is multiplied or divided by a non-zero constant, the resulting sequence is also an A.P.

(5) If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.

(6) If the terms are selected at a regular interval, the given sequence is in A.P.

(7) If three consecutive numbers a, b and c are in A.P., the sum two numbers is twice the middle number *i.e.*, $2b = a + c$.

(8) A sequence is defined as an ordered list of numbers.
The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.

(9) If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.
Example: 2 + 4 + 6 + 8 + ... is a series.

(10) The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician *Leonasalo Fibonacci*, when he was dealing with the problem of rabbit population.

(11) If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.

Mnemonics

Concept: n^{th} Term of Arithmetic Progression $n = a + (n - 1)d$.

Mnemonics: **N**okia **O**ffers **A**dditional **P**rogrammers **in** English **T**o **A**ttract **P**ositive **N**ew **O**ne **B**uyer **D**aily

Interpretation:

Nokia's 'N' is n^{th} term.
Offer's 'O' is **of**
Additional's 'A' is Arithmetic Programmer's 'P' is Progression
In's 'I' is **is**.
English's 'E' is **Equal**
To's 'T' is **To**
Attract's 'A' is a
Positive's 'P' is +
New's 'N' is n
One buyer is **–1**
Daily's 'D' is d

Example

Which term of the A.P. 6, 13, 20, 27,....... is 98 more than its 24^{th} term ?

Step 1. The given A.P. is 6, 13, 20, 27,.............

Here, first term, $a = 6$

common difference, $d = 13 - 6 = 20 - 13 = 7$

Step 2. According to question: $a_n = a_{24} + 98$

$\Rightarrow$ $a + (n + 1)d = a + (24 - 1)\, d + 98$

$\Rightarrow$ $7(n - 1) = 23 \times 7 + 98$

$\Rightarrow$ $n - 1 = 23 + 14$

$\Rightarrow$ $n = 38$

Hence, 38^{th} term is the required term.

Topic-2

Sum of *n* Terms of an Arithmetic Progression

Concepts Covered • *Understand the formula t find the sum of n terms of A.P.*

• *Students will be able to recall some patterns which occur in their daily life.*

Revision Notes

- Sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

where, a is the first term, d is the common difference and n is the total number of terms.

- Sum of n terms of an A.P. when first and last term is given.

$$S_n = \frac{n}{2}[a + l]$$

where, a is the first term and l is the last term.

- The n^{th} term of an A.P. is the difference of the sum of first n terms and the sum to first $(n - 1)$ terms of it. *i.e.*, $a_n = S_n - S_{n-1}$.

Fundamental Facts

(1) To find the sum of n terms of an A.P., we use a formula first founded by Johann carl friedrich Gauss in the 19^{th} century.

(2) A.P. can be applied in real life by analysing a certain pattern, for example, A.P. is used in straight line depreciation.

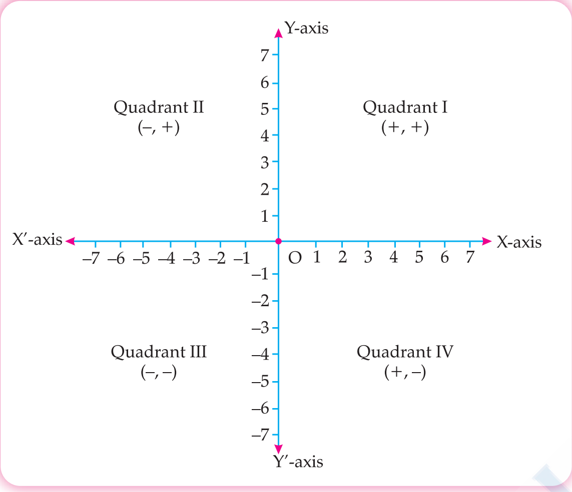
UNIT–III : CO-ORDINATE GEOMETRY

CHAPTER-6

LINES (IN TWO-DIMENSIONS)

Revision Notes

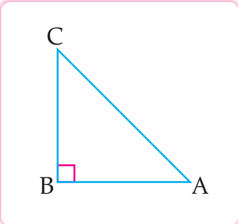
- Two perpendicular number lines intersecting at origin are called co-ordinate axes. The horizontal line is the X-**axis** (denoted by X'OX) and the vertical line is the Y-axis (denoted by Y'OY).



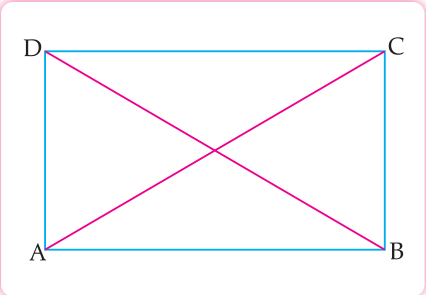
Key Words

Axis: A line, used as a reference to determine position, symmetry and rotation.
Point: To indicate the presence or position of.
Plane: A level or flat surface.

- The point of intersection of X-*axis* and Y-*axis* is called origin and denoted by O.
- Cartesian **plane** is a plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate **plane** or XY-**plane**.
- The X-co-ordinate of a **point** is its perpendicular distance from Y-**axis**.
- The y-co-ordinate of a **point** is its perpendicular distance from X-**axis**.
- The point where the X-axis and the Y-axis intersect has co-ordinate **point** (0, 0).
- The abscissa of a point is the X-coordinate of the point.
- The ordinate of a point is the Y-coordinate of the point.
- If the abscissa of a point is x and the ordinate of the point is y, then (x, y) is called the co-ordinates of the point.
- The axes divide the Cartesian plane into four parts called the quadrants (one fourth part), numbered I, II, III and IV anti-clockwise from OX.
- The co-ordinates of a point on the X-axis are of the form $(x, 0)$ and that of the point on Y-axis are $(0, y)$.
- Sign of co-ordinates depicts the quadrant in which it lies. The co-ordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.
- Three points A, B and C are collinear if the distances AB, BC and CA are such that the sum of two distances is equal to the third.
- Three points A, B and C are the vertices of an equilateral triangle if $AB = BC = CA$.
- The points A, B and C are the vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$.
- Three points A, B and C are the vertices of a right triangle, if $AB^2 + BC^2 = CA^2$.



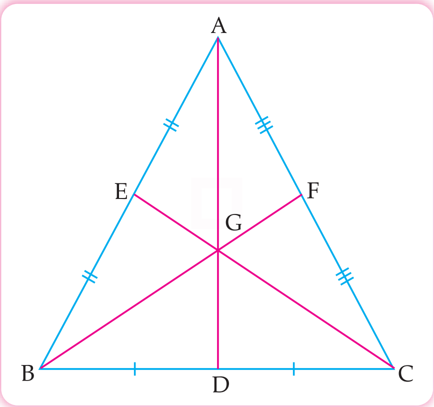
- For the given four points A, B, C and D:



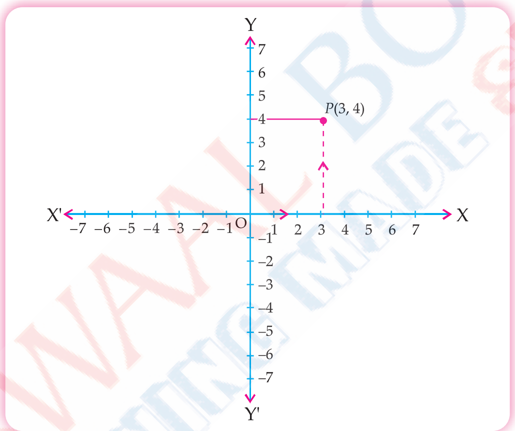
1. If $AB = BC = CD = DA$; $AC = BD$, then ABCD is a square.
2. If $AB = BC = CD = DA$; $AC \neq BD$, then ABCD is a rhombus.
3. If $AB = CD$, $BC = DA$; $AC = BD$, then ABCD is a rectangle.
4. If $AB = CD$, $BC = DA$; $AC \neq BD$, then ABCD is a parallelogram.

- Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.

- Diagonals of rhombus and square bisect each other at right angle.
- Centroid is the point of intersection of the three medians of a triangle. In the figure, G is the centroid of a triangle ABC.



- Centroid divides each median of a triangle in a ratio of 2 : 1 from vertex to base of the side.
- If $x \neq y$, then $(x, y) \neq (y, x)$ and if $(x, y) = (y, x)$, then $x = y$.
- To plot a point P(3, 4) in the cartesian plane.
  (i) A distance of 3 units along X-axis.
  (ii) A distance of 4 units along Y-axis.



Key Formulae

- The distance between two points *i.e.*, P(x_1, y_1) and Q(x_2, y_2) is

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

- The distance of a point P(x, y) from origin is $\left|\sqrt{x^2 + y^2}\right|$
- Co-ordinates of point (x, y) which divides the line segment by joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally are

$$x = \left(\frac{mx_2 + nx_1}{m + n}\right)$$

and $$y = \left(\frac{my_2 + ny_1}{m + n}\right)$$

- Co-ordinates of mid-point of the line segment by joining the points (x_1, y_1) and (x_2, y_2) are

$$x = \left(\frac{x_2 + x_1}{2}\right)$$

and $$y = \left(\frac{y_2 + y_1}{2}\right)$$

Fundamental Facts

(1) Co-ordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.
(2) Cartesian plane was discovered by ***Rene Descartes***.
(3) The other name of co-ordinate geometry is Analytical Geometry.
(4) Co-ordinate Geometry acts as a bridge between the Algebra and Geometry.
(5) Medians of a triangle are concurrent. The point of concurrency is called the centroid.
(6) Trisection of a line segment means dividing it into 3 equal parts, so 2 points are required.
(7) Centroid of a triangle divides its median in the ratio of 2 : 1.

UNIT–IV : GEOMETRY

CHAPTER-7

TRIANGLES

Revision Notes

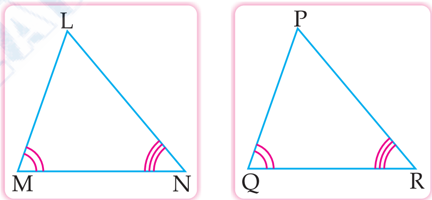
- A triangle is one of the basic shapes of geometry. It is a polygon with 3 sides and 3 vertices/corners.
- Two figures are said to be congruent if they have the same shape and the same size.
- Those figures which have the same shape but not necessarily the same size are called **similar figures.**

  Hence, we can say that **all congruent figures are similar but all similar figures are not congruent.**
- **Similarity of Triangles:** Two triangles are similar, if:

  (i) their **corresponding** sides are proportional.

  (ii) their **corresponding** angles are equal.

  If ΔABC and ΔDEF are **similar**, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.
- **Criteria for Similarity of Triangles:**



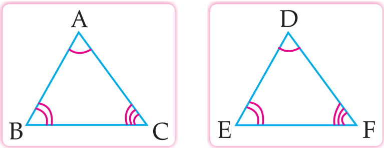
In ΔLMN and ΔPQR, if

(a) $\angle L = \angle P, \angle M = \angle Q$ and $\angle N = \angle R$

(b) $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$,

then $\Delta LMN \sim \Delta PQR$.

(i) **AAA-Criterion:** In two triangles, if corresponding angles are equal, then the triangles are similar and hence their **corresponding** sides are in the same ratio.



If ΔABC and ΔDEF are similar

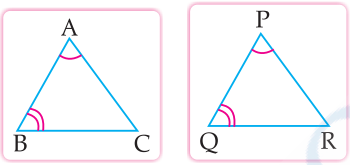
$\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$.

Then, $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

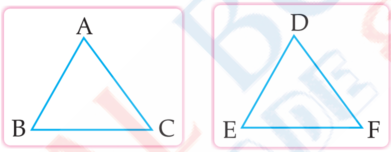
Remark: If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

AA-Criterion: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

As we know that the sum of all angles in a triangle is 180° so if two angles in ΔABC and ΔPQR are same *i.e.*, $\angle A = \angle P, \angle B = \angle Q$.



(ii) SSS-Criterion: In two triangles if the sides of one triangle are proportional to the sides of another triangle, then the two triangles are similar and hence corresponding angles are equal.



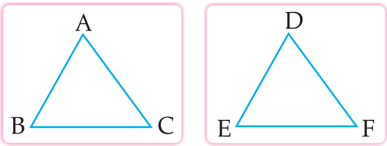
If $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$\therefore$ $\Delta ABC \sim \Delta DEF$

then $\angle A = \angle D, \angle B = \angle E$

and $\angle C = \angle F$

(iii) SAS-Criterion: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.



If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

Key Words

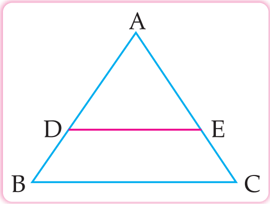
Similar: A thing similar to another.

Corresponding: Having in the same relationship.

Parallel: It means that two lines that never intersect.

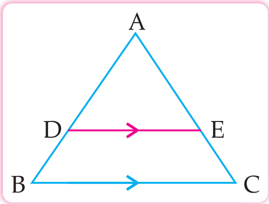
Some theorems based on similarity of triangles:

(i) If a line is drawn **parallel** to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. It is known as **'Basic Proportionality Theorem'** or **'Thales Theorem'.**



In ΔABC, let $DE \parallel BC$, then

(a) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **(b)** $\dfrac{AB}{DB} = \dfrac{AC}{EC}$ **(c)** $\dfrac{AD}{AB} = \dfrac{AE}{AC}$.



(ii) If a line divides any two sides of a triangle in the same ratio, then the line is **parallel** to the third side. It is the '**Converse of Basic Proportionality Theorem**'.

If $\dfrac{AD}{DB} = \dfrac{AE}{EC}$,

then $DE \parallel BC$

Fundamental Facts

(1) The use of similar triangles has made possible the measurements of heights and distances.

(2) Thales of Miletus was the great mathematician who found the similar triangles.

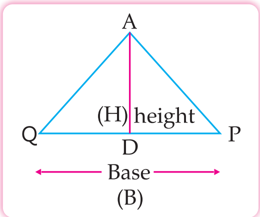
Mnemonics

Concept: Area of triangle $= \dfrac{1}{2} \times$ Base $\times$ height

Mnemonics: Audi is the product of **h**alf of **B**MW and **H**onda

Interpretations:

A $\Rightarrow$ **Area**
B $\Rightarrow$ **Base**
H $\Rightarrow$ **Height**



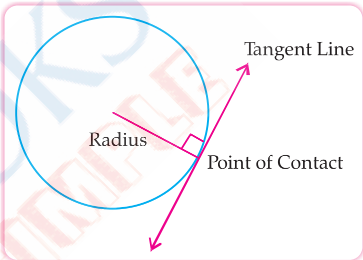
CHAPTER-8

CIRCLES

Concepts Covered
- *Tangent line secant*
- *Property/theorems of tangents to a circle*

Revision Notes

- Tangent: A **tangent** to a **circle** is a line that intersects the circle at one point only.
- The common point of the **circle** and the tangent is called the point of **contact**.



- Secant: Two common points (A and B) between line PQ and circle.
- A **tangent** to a circle is a special case of the secant when the two end points of the corresponding chord are coincide.
- There is no **tangent** to a circle passing through a point lying inside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of **contact**.

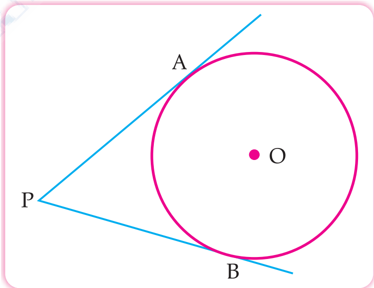
Key Words

Circle: A circle is a round shaped figure has no corners or edges.
Tangent: A line, curve or surface touching but not intersecting another.
Contact: To touch or make touch physical.

- There are exactly two tangents to a circle through a point outside the circle.
- The length of the segment of the tangent from the external point P and the point of **contact** with the circle is called the length of the tangent.
- The lengths of the tangents drawn from an external point to a circle are equal.

In the figure,

$$PA = PB.$$



Fundamental Facts

(1) The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish mathematician **Thomas Fincke** in 1583.

(2) The line perpendicular to the tangent and passing through the point of contact, is known as the normal.

(3) In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

UNIT–V : TRIGONOMETRY

CHAPTER-9

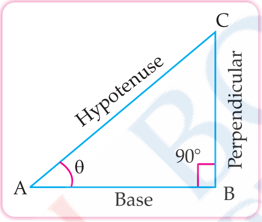
TRIGONOMETRY & TRIGONOMETRIC IDENTITIES

Topic-1 Trigonometric Ratios and Its Values

Concepts Covered • *Six trigonometric ratios with their sides of a right angled triangle.* • *Values of trigonometric ratios between 0° to 90°.*

Revision Notes

- In fig., a right triangle ABC right angled at B is given and $\angle BAC = \theta$ is an acute angle. Here side AB which is adjacent to ∠A is base, side BC opposite to ∠A is perpendicular and the side AC is hypotenuse which is opposite to the right angle B.



Key Words

Hypotenuse: The longest side of a right angled triangle.
Base: Adjacent side of an angle is a base of a triangle.
Perpendicular: Opposite side of an angle in the right triangle.

Key Formulae

The trigonometric ratios of ∠A in right triangle ABC are defined as

$$\text{sine of } \angle A = \sin\theta = \frac{\textbf{Perpendicular} \text{ or opposite side}}{\textbf{Hypotenuse}} = \frac{BC}{AC}$$

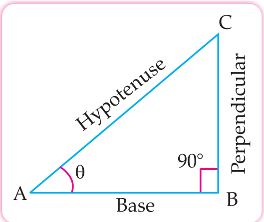
$$\text{cosine of } \angle A = \cos\theta = \frac{\textbf{Base} \text{ or adjecent side}}{\textbf{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan\theta = \frac{\textbf{Perpendicular} \text{ or opposite side}}{\text{Base adjacent side}} = \frac{BC}{AB}$$

$$\text{cotangent of } \angle A = \cot\theta = \frac{\textbf{Base} \text{ or adjacent side}}{\textbf{Perpendicular} \text{ or oppsite side}} = \frac{AB}{BC} = \frac{1}{\tan\theta}$$

$$\text{secant of } \angle A = \sec\theta = \frac{\textbf{Hypotenuse}}{\textbf{Base} \text{ or adjacent side}} = \frac{AC}{AB} = \frac{1}{\cos\theta}$$

$$\text{cosecant of } \angle A = \operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular or opposite side}} = \frac{AC}{BC} = \frac{1}{\sin\theta}$$



It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

Also, $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined (∞)
cot A	Not defined (∞)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined (∞)
cosec A	Not defined (∞)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Fundamental Facts

(1) The concept of trigonometry is completely based on right angles.
(2) The three basic functions in trigonometry are sine, cosine and tangent.
(3) Trigonometry, as the name might suggest, is all about triangles.

Mnemonics

1. The relation of Trigonometric Ratios

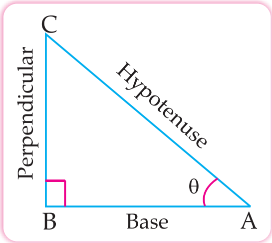
Mnemonics:

In right angled ΔABC, we have

$$\sin\theta = \frac{BC}{AC},\ \cos\theta = \frac{BA}{AC},\ \tan\theta = \frac{BC}{AB},$$

$$\cot\theta = \frac{AB}{BC},\ \sec\theta = \frac{AC}{BA},\ \text{cosec}\,\theta = \frac{AC}{BC}$$

sin	cos	tan
Pandit	Badri	Prasad
Har	Har	Bhole
cosec	sec	cot



Interpretation:

Here,

$$\sin\theta = \frac{\textbf{P}\text{andit}}{\textbf{H}\text{ar}} = \frac{P}{H} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\textbf{B}\text{adri}}{\textbf{H}\text{ar}} = \frac{B}{H} = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BA}{AC}$$

$$\tan\theta = \frac{\textbf{P}\text{rasad}}{\textbf{B}\text{hole}} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot\theta = \frac{\textbf{B}\text{hole}}{\textbf{P}\text{rasad}} = \frac{B}{P} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

$$\sec\theta = \frac{\textbf{H}\text{ar}}{\textbf{B}\text{adri}} = \frac{H}{B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BA}$$

$$\text{cosec}\,\theta = \frac{\textbf{H}\text{ar}}{\textbf{P}\text{andit}} = \frac{H}{P} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}.$$

2. Trigonometric Ratios

Mnemonics: We learn these ratios in following ways:

(i) "**S**ome **p**eople **h**ave" $\sin\theta = \frac{P}{H}$

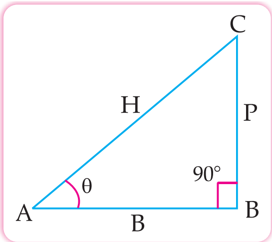
(ii) "**C**urly **B**rown **H**air" $\cos\theta = \frac{B}{H}$

(iii) "**T**hrough **p**roper **B**rushing" $\tan\theta = \frac{P}{B}$.

(i) $\sin\theta = \frac{BC}{AC} = \frac{P}{H}$

Interpretation:

Some	**P**eople	**H**ave
↓	↓	↓
sin θ	Perpendicular	Hypotenuse



(ii) $\cos\theta = \frac{AB}{AC} = \frac{B}{H}$

Interpretation:

Curly	**B**rown	**H**air
↓	↓	↓
cos θ	Base	Hypotenuse

(iii) $\tan\theta = \dfrac{BC}{AB} = \dfrac{P}{B}$

Interpretation:

Through → $\tan\theta$

Proper → Perpendicular

Brushing → Base

Topic-2 Trigonometric Identities

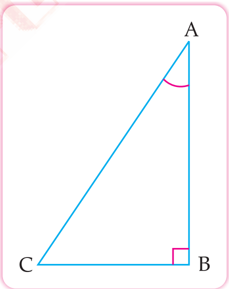
Concepts Covered • ***Three important identities are:***

(i) $\sin^2\theta + \cos^2\theta = 1$, *(ii)* $1 + \tan^2\theta = \sec^2\theta$, *(iii)* $1 + \cot^2\theta = \text{cosec}^2\theta$.

Revision Notes

- An equation is called an identity if it is true for all values of the variable(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

In ΔABC, right-angled at B, By Pythagoras Theorem,



$$AB^2 + BC^2 = AC^2 \quad ...(i)$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \quad ...(ii)$$

This is true for all values of A such that $0° \le A \le 90°$. So, this is a trigonometric identity. Now divide eqn.(i) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \quad ...(iii)$$

Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, $\tan A$ and $\sec A$ are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \le A < 90°$.

Again dividing eqn. (i) by BC^2.

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \text{cosec}^2 A \quad ...(iv)$$

Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all value of A such that $0° < A \le 90°$.

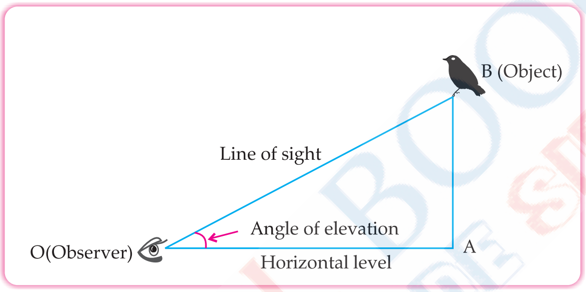
Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, *i.e.*, if any one of the ratios is known, we can determine the values of other trigonometric ratios.

CHAPTER-10
HEIGHTS AND DISTANCES

(ANGLE OF ELEVATION, ANGLE OF DEPRESSION)

Revision Notes

- The line of sight is the line drawn from the eye of an observer to the point on the object viewed by the observer.
- The angle of elevation of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at a point on the object.
- Line of sight, angles and altitude (height).

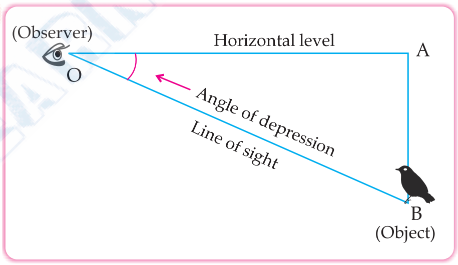


(i) $\angle AOB$ is the angle of elevation.

(ii) By height AB, means object is at point B from the point A located at the ground.

(iii) AO is the distance of the observer from the point A.

- The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at a point on the object.



- The height of object above the water surface is equal to the depth of its image below the water surface.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

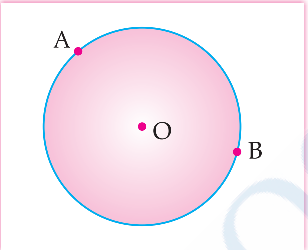
UNIT–VI : MENSURATION

CHAPTER-11

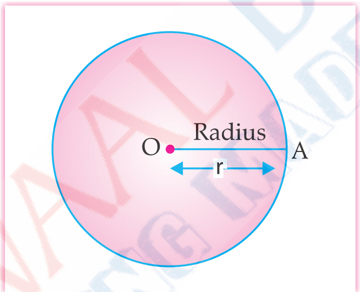
AREAS RELATED TO CIRCLES

Revision Notes

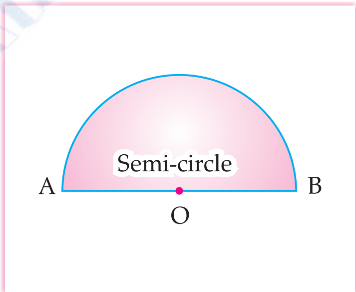
- A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

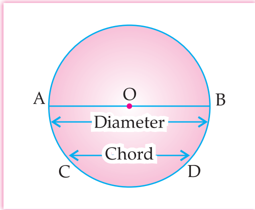


- A line segment joining the centre of the circle to a point on the circumference of the circle is called its radius.

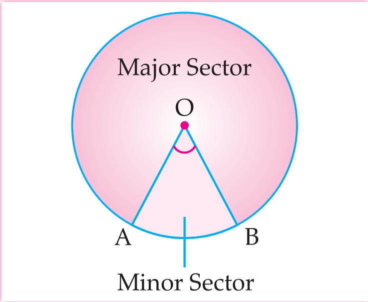


- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle. Here AB is a diameter, which is a longest chord.
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.





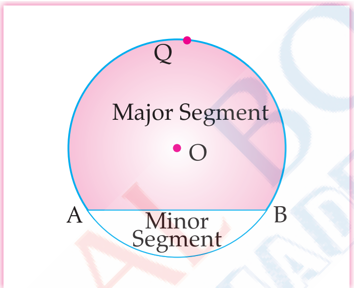
- A part of a **circumference** of circle is called an arc.
- An **arc** of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.
- An **arc** of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.
- The region bounded by an **arc** of a circle and two radii at its end points is called a **sector**.



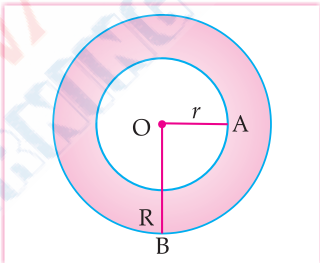
Key Words

Circumference: carrying around means the perimeter of circle.
Arc: Any smooth curve joining two points.
Sector: A part of a circle made of the arc of the circle along with its two radii.

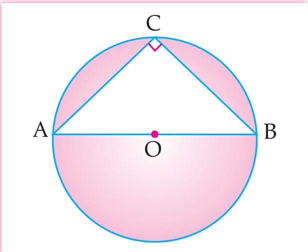
- A chord divides the interior of a circle into two parts, each called a segment.



- Circles having the same centre but different radii are called concentric circles.



- Two circles (or arcs) are said to be congruent if on placing one over the other cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.



- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Key Formulae

1. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is diameter and r is the radius of the circle.
2. Area of a circle = πr^2.
3. Area of a semi-circle = $\frac{1}{2}\pi r^2$.
4. Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)r$
5. Area of a ring or an annulus = $\pi(R + r)(R - r)$. where R is the outer radius and r is the inner radius.
6. Length of arc, $l = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$, where θ is the angle subtended at centre by the arc.
7. Area of a sector = $\frac{\pi r^2\theta}{360°}$ or area of sector = $\frac{1}{2}(l \times r)$, where l is the length of arc.
8. Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$.
9. Area of major segment = Area of the circle – Area of minor segment
   = πr^2 – Area of minor segment.
10. If a chord subtends a right angle at the centre, then area of the corresponding segment= $\left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$
11. If a chord subtends an angle of 60° at the centre, then area of the corresponding segment= $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$.
12. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
13. Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.
14. Perimeter of a sector = $\frac{\pi r\theta}{180°} + 2r$.

Fundamental Facts

- An Indian mathematician Srinivas Ramanujan worked out the identity using the value of π correct to million places of decimals.
- The Indian mathematician Aryabhatta gave the value of π as $\frac{62832}{20000}$
- "How I made a greater discovery" this mnemonic help us in getting the value of π = 3.14159 ....
- Give it under separate reading with explanation how to use

	CAN	I	HAVE	A	SMALL	CONTAINER	OF	COFFEE
	↓	↓	↓	↓	↓	↓	↓	↓
No. of Letters →	3	1	4	1	5	9	2	6

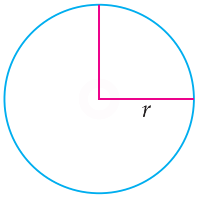
- Archimedes calculated the area of a circle by approximating it to a square.
- Area of sector of a circle depends on two parameters-radius and central angle.

CHAPTER-12

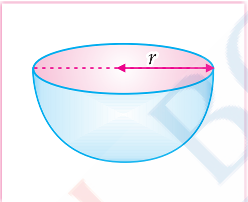
SURFACE AREAS AND VOLUMES

Revision Notes

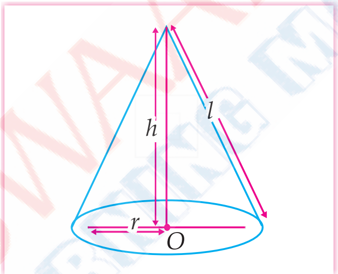
- A sphere is a perfectly round geometrical object in three-dimensional space.



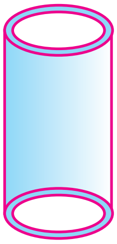
- A hemisphere is half of a sphere.



- A cone is a three dimensional geometric shape tapers smoothly from a flat base to a point called the apex or vertex.



- A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.



Key Words

Surface area: The amount of space covering the outside of a three dimensional shape.

Volume: The amount of space occupied by a three dimensional object.

Materials: A substance or mixture of substances that constitutes an object.

Key Formulae

- **Cuboid:**

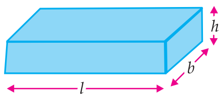
Lateral **surface area** or area of four walls

$= 2(l + b)h$

Total surface area $= 2(lb + bh + hl)$

Volume $= l \times b \times h$

Diagonal $= \sqrt{l^2 + b^2 + h^2}$



Here, l is length, b is breadth and h is height of the cuboid.

- **Cube:**

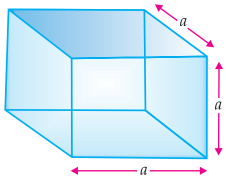
Lateral **surface area** or area of four walls

$= 4 \times a^2$

Total surface area $= 6 \times a^2$

Volume $= a^3$

Diagonal of a cube $= \sqrt{3} \times a$



Here, a is edge of cube.

- **Right Circular Cylinder:**

Area of base or top face $= \pi r^2$

Area of curved surface or curved surface area

$=$ perimeter of the base $\times$ height

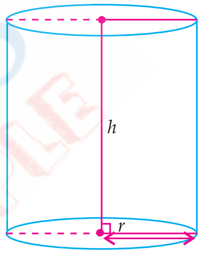
$= 2\pi rh$

Total **surface area** (including both ends)

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Volume $=$ (Area of the base $\times$ height) $= \pi r^2 h$

Here, r is the radius of base and h is the height of the right circular cylinder.



- **Right Circular Hollow Cylinder:**

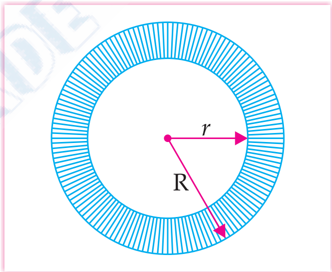
Total surface area $=$ (External surface area $+$ Internal surface area) $+$ (Area of brim)

$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$

$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$

$= [2\pi(R + r)(h + R - r)]$

Curved surface area $= (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$



Volume of the **material** used

$=$ (External volume) $-$ (Internal volume)

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

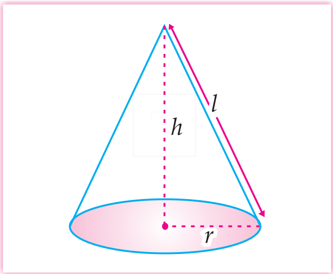
Here, R and r are the external and internal radii and h is the height of the right circular hollow cylinder.

- **Right Circular Cone:**

Slant height, $l = \sqrt{h^2 + r^2}$

Area of curved surface $= \pi rl$

$= \pi r\sqrt{h^2 + r^2}$



Total surface area $=$ Area of curved surface $+$ Area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

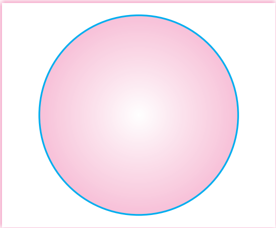
Volume $= \dfrac{1}{3}\pi r^2 h$

Here, r, h and l are the radius, vertical height and slant height respectively of the right circular cone.

- **Sphere:**

Surface area $= 4\pi r^2$

Volume $= \dfrac{4}{3}\pi r^3$



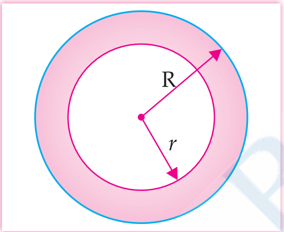
Here, r is the radius of the sphere.

- **Spherical Shell:**

Surface area (outer) = $4\pi R^2$

Volume of **material** $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(R^3 - r^3)$



Here, R and r are the external and internal radii of the spherical shell.

- **Hemisphere:**

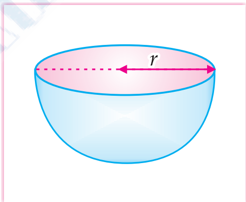
Area of curved surface = $2\pi r^2$

Total surface area = Area of curved surface + Area of base

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$



Here, r is the radius of the hemisphere.

Fundamental Facts

- The platonic solids also called the regular solids or regular polyhedra. Five such solids are : dodecahedron, icosahedron, octahedron and tetrahedron.
- Greek mathematician Plato equated tetrahedron with the 'element' fire, the cube with earth, the icosahedron with water, the octahedron with air and dodecahedron with the stuff of which the constellations and heavens were made.
- The stone of platonic solids are kept in Ashmolean Museum in Oxford.
- The tomb of Archimedes carried a sculpture consisting of a sphere and cylinder circumscribing it.

UNIT–VII : STATISTICS AND PROBABILITY

CHAPTER-13

STATISTICS

Revision Notes

- Statistics deals with the collection, presentation and analysis of numerical data.
- Three measures of central tendency are:
  (i) Mean, **(ii)** Median and **(iii)** Mode
- **Mean:** In statistics mean stands for the arithmetic mean of the given items.

  i.e., Mean $= \dfrac{\text{Sum of given items}}{\text{No. of items}}$
- **Median:** It is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or descending order of their magnitudes.

  It divides the arranged series in two equal parts *i.e.*, 50% of the observations lie below the median and the remaining are above the median.
- **Mode:** Mode is the observation which occurred maximum times. In ungrouped data, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the **modal class**. The mode of the data is a value inside the modal class.

Key Formulae

- **Mean:**

  (a) **For Raw Data:**

  If n observations $x_1, x_2, ..., x_n$ are given, then their arithmetic mean is given by :

  $$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  (b) **For Ungrouped Data:**

  If there are n distinct observations $x_1, x_2,..., x_n$ of variable x with frequencies $f_1, f_2,..., f_n$ respectively, then the arithmetic mean is given by:

  $$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

  (c) **For Grouped Data:**

  (i) To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point.

  (ii) Direct Method:

  $$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

  where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.

  (iii) Assumed Mean Method or Short-cut Method:

  $$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

  where a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i.

  (iv) Step-Deviation Method:

  $$\text{Mean } (\bar{x}) = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

where a is the assumed mean, h is the class size and $u_i = \frac{x_i - a}{h}$

- **Median of Grouped Data:**

  Let $n = f_1 + f_2 + f_3 + ... + f_n$. First of all find $\frac{n}{2}$ and then the class in which $\frac{n}{2}$ lies. This class is known as the **median class.** Median of the given distribution lies in this class.

  Median of the grouped data can be calculated using the formula:

$$\text{Median } (M_e) = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

  where l = lower limit of median class, f = frequency of median class, n = number of observations, $c.f.$ = cumulative frequency of the class preceding the median class, h = class-size or width of the class-interval.

- **Mode of Grouped Data:**

  Mode of the grouped data can be calculated by using the formula:

$$\text{Mode (M)} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

  where l = lower limit of the modal class, h = width or size of the class-interval, f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class, f_2 = frequency of the class succeeding the modal class.

- **Empirical relation between mean, median and mode:**

(i) Mode = 3 median – 2 mean

(ii) Median = $\frac{1}{3}$ mode + $\frac{2}{3}$ mean

(iii) Mean = $\frac{3}{2}$ median – $\frac{1}{2}$ mode

Fundamental Facts

- In statistics, that single value is called the central tendency and mean, median and mode are all ways to describe it.
- The mean is the average of a data set.
- The mode is the most common number in a data set.
- The median is the middle of the set of numbers.

CHAPTER-14

PROBABILITY

Revision Notes

- Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence.
- A random **experiment** is an experiment or a process for which the outcome cannot be predicted with certainty. *e.g.*, **(i)** tossing a coin, **(ii)** throwing a dice, **(iii)** selecting a card and **(iv)** selecting an object etc.
- Outcome associated with an experiment is called an event. *e.g.*, **(i)** Getting a head on tossing a coin, **(ii)** getting a face card when a card is drawn from a pack of 52 cards.
- The events whose probability is one are called sure/certain **events**.
- The **events** whose probability is zero are called impossible **events**.

- An event with only one possible outcome is called an elementary event.
- In a given **experiment,** if two or more events are equally likely to occur or have equal probabilities, then they are called equally likely events.
- Probability of an event always lies between 0 and 1.
- Probability can never be negative and more than one.
- A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of an ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are spades, hearts, diamonds and clubs.
- King, queen and jack are face cards.
- The sum of the probabilities of all elementary events of an experiment is 1.
- Two events A and B are said to be complementary to of each other if the sum of their probabilities is 1.
- Probability of an event E, denoted as P(E), is given by:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total possible number of outcomes}}$$

- For an event E, $P(\overline{E}) = 1 - P(E)$, where the event $\overline{E}$ representing 'not E' is the complement of the event E.
- For A and B two possible outcomes of an event,

  (i) If $P(A) > P(B)$, then event A is more likely to occur than event B.

  (ii) If $P(A) = P(B)$, then events A and B are equally likely to occur.

Key Words

Events: The set of outcomes from an experiment is known as events.
Experiment: Something that can be repeated that has a set of possible results.

Know the Facts

- The experimental or empirical probability of an event is based on what has actually happened while the theoretical probability of the event attempts to predict what will happen on the basis of certain assumptions.
- As the number of trials in an experiment go on increasing, we may expect the experimental and theoretical probabilities to be nearly the same.
- When we speak of a coin, we assume it to be '**fair** ' *i.e.*, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being '**unbiased**'.
- In the case of experiment we assume that the experiments have equally likely outcomes.
- A deck of playing cards consists of 4 suits : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Clubs and spades are of black colour, while hearts and diamonds are of red colour.

Fundamental Facts

- By the phrase '**random toss**', we mean that the coin is allowed to fall freely without any **bias** or **interference**.
- The first book on probability '**The Book on Games of Chance**' was written by Italian mathematician **J. Cardan**.
- The classical definition of probability was given by Pierre Simon Laplace.