UNIT I: NUMBER SYSTEMS

Chapter - 1 : Real Numbers

Revision Notes

Fundamental Theorem of Arithmetic

- **The Fundamental Theorem of Arithmetic**
  Every composite number can be expressed as a product of primes and this factorisation is unique, apart from the order in which the prime factors occur. Fundamental theorem of arithmetic is also called a **Unique Factorization Theorem.**

  Composite number = Product of prime numbers

  Or

  Any integer greater than 1 can either be a prime number or can be written as a unique product of prime numbers.
  e.g.,
  (i) $2 \times 11 = 22$ is the same as $11 \times 2 = 22$.
  (ii) 6 can be written as 2×3 or 3×2, where 2 and 3 are prime numbers.
  (iii) 15 can be written as 3×5 or 5×3, where 3 and 5 are prime numbers.
  The prime factorization of a natural number is unique, except to the order of its factors.
  e.g., 12 detained by multiplying the prime numbers 2, 2 and 3 together,

  $$12 = 2 \times 2 \times 3$$

  We would probably write it as $12 = 2^2 \times 3$
- By using Fundamental Theorem of Arithmetic, we shall find the HCF and LCM of given numbers (two or more). This method is also called **Prime Factorization Method.**
- **Prime Factorization Method to find HCF and LCM:**
  (i) Find all the prime factors of given numbers.
  (ii) HCF of two or more numbers = Product of the smallest power of each common prime factor, involved in the numbers.
  (iii) LCM of two or more numbers = Product of the greatest power of each prime factor, involved in the numbers.

Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

- **Rational Numbers:** A number in the form $\frac{p}{q}$, where p and q are co-prime numbers and $q \neq 0$, is known as rational number.
  For example: $2, -3, \frac{3}{7}, -\frac{2}{5}$, etc. are rational numbers.
- **Irrational Numbers:** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ are irrational numbers.
- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- **Terminating Decimals:** If decimal expansion of rational number $\frac{p}{q}$ comes to an end, then the decimal obtained from $\frac{p}{q}$ is called terminating decimal.

- **Non-terminating Repeating (or Recurring) Decimals:** The decimal expansion obtained from $\frac{p}{q}$ repeats periodically, then it is called non-terminating repeating (or recurring) decimal.
- Just divide the numerator by the denominator of a fraction. If you end up with a remainder of 0, you have a terminating decimal otherwise repeating or recurring decimal.
- The sum or difference of a rational and an irrational number is irrational.
- The product and quotient of a non-zero rational and an irrational number is irrational.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which terminates.
- Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form $\frac{p}{q}$, where p and q are co-primes and the prime factorization of q is of the form 2^m5^n, where m and n are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating.

Know the Formulae

For two positive integers a and b, we have

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

and

$$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

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UNIT II: ALGEBRA

Chapter - 2 : Polynomials

Revision Notes

Zeroes of a Polynomial and Coefficients of Quadratic Polynomials

- **Polynomial:** An algebraic expression in the form of $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers) is called a polynomial in one variable x of degree n.
- **Value of a Polynomial at a given point :** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$,
  is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial:** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.
  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-co-ordinates of the points, where the graph of $y = p(x)$ intersects the X-axis.
  (i) A linear polynomial has one and only one zero.
  (ii) A quadratic polynomial has at most two zeroes.
  (iii) A cubic polynomial has at most three zeroes.
  (iv) In general, a polynomial of degree n has at most n zeroes.

- **Graphs of Different types of Polynomials:**
  - **Linear Polynomial:** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.
  - **Quadratic Polynomial: (i)** Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects X-axis at a maximum of two distinct points.
    (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two distinct points.
  - **Cubic polynomial:** Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects X-axis at a maximum of three distinct points.
- **Relationship between the Zeroes and the Coefficients of a Polynomial :**

(i) Zero of a linear polynomial $= \dfrac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\dfrac{-b}{a}$

(ii) In a quadratic polynomial,

$$\text{Sum of zeroes of a quadratic polynomial} = \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes of a quadratic polynomial} = \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

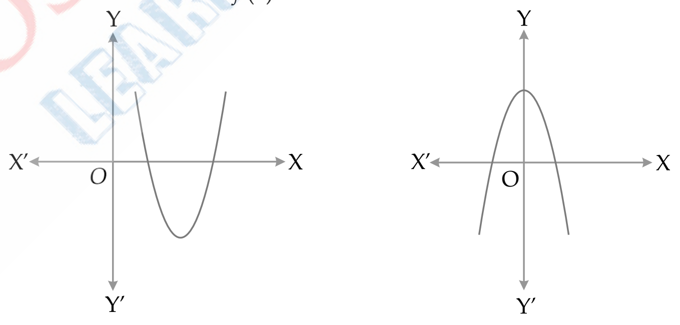
(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma = (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D.

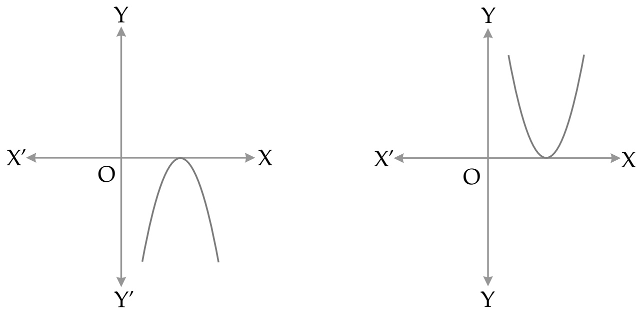
The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

Case I : If $D > 0$, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-co-ordinates of points of intersection with X-axis is known as 'zeroes' of $f(x)$.



$\therefore f(x)$ will have two zeroes and we can say that roots/zeroes of the two given polynomials are real and unequal.

Case II : If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.



$\therefore f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

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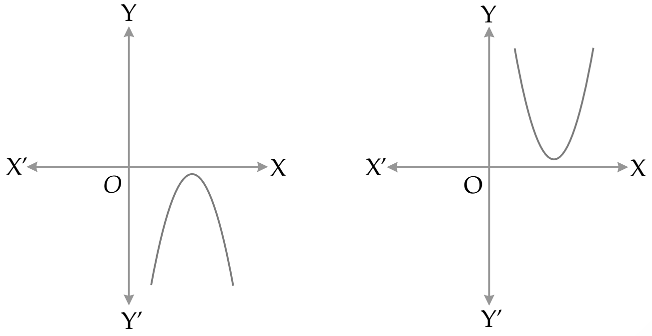
Case III: If D < 0, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.



$\therefore\ f(x)$ will not have any real zero.

Know the Formulae

Relationship between the zeroes and the coefficients of a Polynomial :

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b$, where $a \neq 0$	1	$k = -\frac{b}{a}$, *i.e.*, $k = \frac{-\text{Constant term}}{\text{Coefficient of } x}$
2.	Quadratic	$ax^2 + bx + c$, where $a \neq 0$	2	Sum of zeroes, $(\alpha + \beta) = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$ Product of zeroes, $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d$, where $a \neq 0$	3	Sum of zeroes, $(\alpha + \beta + \gamma) = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time, $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$ Product of zeroes, $(\alpha\beta\gamma) = \frac{-\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$

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Mnemonics

Concept: Formula $\rightarrow \alpha.\beta = \frac{c}{a}$

Amitabh **B**achchan went **C**anada **by** **a**eroplane.

Interpretation:

Amitabh's A $\Rightarrow$ Alpha (α)
Bachchan's B $\Rightarrow$ Beta (β)
Canada's C $\Rightarrow$ Constant (c)
By for Divide by and aeroplane's a $\Rightarrow$ Variable.

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Chapter - 3 : Pair of Linear Equations in two Variables

Revision Notes

Graphical Solution of Linear Equations in Two Variables

- **Linear equation in two variables:** An equation in the form of $ax + by + c = 0$, where a, b and c are real numbers and a and b are not zero, is called a linear equation in two variables x and y.

  General form of a pair of linear equations in two variables is:

  $$a_1x + b_1y + c_1 = 0$$

  and $$a_2x + b_2y + c_2 = 0,$$

  where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that

  $$a_1, b_1 \neq 0 \text{ and } a_2, b_2 \neq 0.$$

  e.g., $$3x - y + 7 = 0,$$

  and $$7x + y = 3$$

  are linear equations in two variables x and y.

- There are two methods of solving simultaneous linear equations in two variables:

  (i) Graphical method, and

  (ii) Algebraic methods.

  1. Graphical Method:

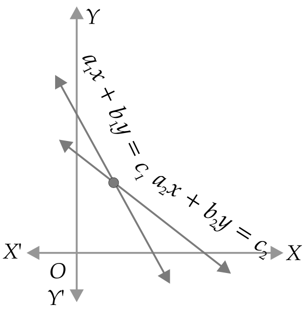
  (i) Express one variable (say y) in terms of the other variable x, $y = ax + b$, for the given equation.

  (ii) Take three values of independent variable x and find the corresponding values of dependent variable y, take integral values only.

  (iii) Plot these values on the graph paper in order to represent these equations.

  (iv) **If the lines intersect** at a distinct point, then point of intersection will be the unique solution for given equations. In this case, the pair of linear equations is **consistent.**

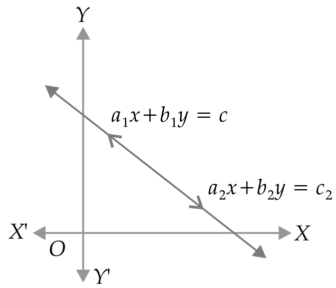
  If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.



Intersecting Lines

  (v) If **the lines** representing the linear equations **coincides**, then system of equations has infinitely many solutions. In this case, the pair of linear equations is **consistent and dependent.**

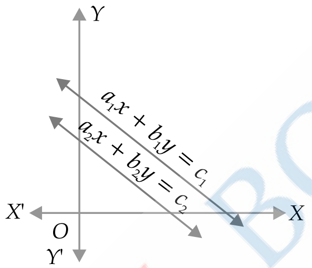
  If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.



Coincident Lines

(vi) If **the lines** representing the pair of linear equations **are parallel**, then the system of equations has no solution and is called **inconsistent**.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.



Parallel Lines

➢ **Possibilities of solutions and Inconsistency:**

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Conditions for solvability
$x - 2y = 0$ $3x - 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{-4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution or Unique solution	System is consistent
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

Algebraic Methods to Solve Pair of Linear Equations

➢ **Algebraic Method:** We can solve the linear equations algebraically by **substitution method, elimination method** and **cross-multiplication method**.

1. Substitution Method:

(i) Find the value of one variable (say y) in terms of the other variable *i.e.*, x from either of the equations.

(ii) Substitute this value of y in other equation and reduce it to an equation in one variable.

(iii) Solve the equation so obtained and find the value of x.

(iv) Put this value of x in one of the equations to get the value of variable y.

2. Elimination Method:

(i) Multiply given equations with suitable constants, make either the x-coefficients or the y-coefficients of the two equations equal.

(ii) Subtract or add one equation from the other to get an equation in one variable.

(iii) Solve the equation so obtained to get the value of the variable.

(iv) Put this value in any one of the equation to get the value of the second variable.

Note:

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.*, it is inconsistent.

- **Equations reducible to a pair of Linear Equations in two variables:** Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

Steps to be followed for solving word problem

S. No.	Problem type	Steps to be followed
1.	Age Problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then, 'a' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after 'b' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them.
2.	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
3.	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4.	Problems based on Distance, Speed and Time	$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ or Distance = Speed × Time and $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. Then, the speed of boat in downstream = $(x + y)$ km/h and speed of boat in upstream = $(x - y)$ km/h.
5.	Problems based on commercial Mathematics	For solving specific questions based on commercial mathematics, ● To the fare of 1 full ticket may be taken as ₹ x and the reservation charges may be taken as ₹ y, so that one full fare = $x + y$ and one half fare $= \frac{x}{2} + y$. ● To solve the questions of profit and loss, take the cost price of 1st article as ₹ x and that of 2nd article as ₹ y. ● To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at some other rate of interest as per given in question.

6.	Problems based on Geometry and Mensuration	• Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle. • In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary.

Know the Formulae

➢ If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a pair of linear equations in two variables x and y such that:

(i) System has unique solution

if $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

(ii) System has infinite number of solutions

if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(iii) System has no solution

if $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Mnemonics

Algebra Methods

1. Concept:

System has unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

For unique feature Audi **A**$_1$ and **A**$_2$ are not same as BMW **B**$_1$ and **B**$_2$

Interpretation:

$A_1 = a_1$
$A_2 = a_2$
$B_1 = b_1$
$B_2 = b_2$

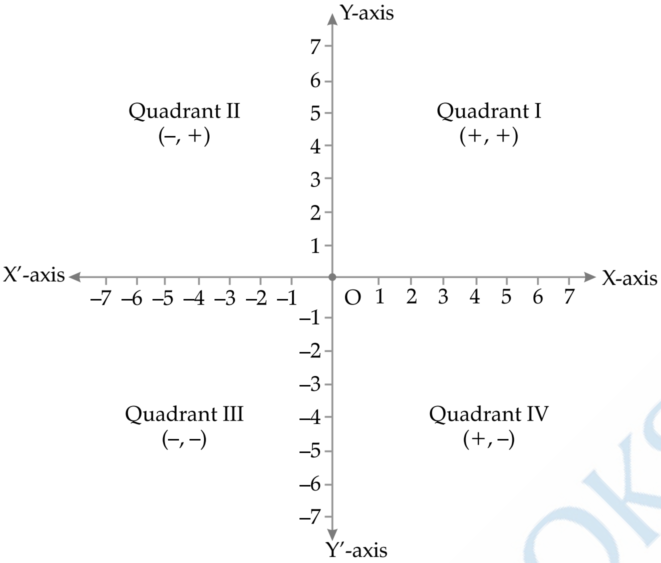
UNIT III: CO-ORDINATE GEOMETRY

Chapter - 4 : Lines (in two-dimensions)

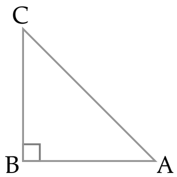
Revision Notes

Distance between two points and Section formula

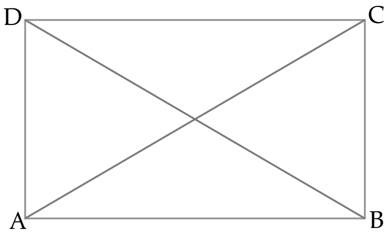
➢ Two perpendicular number lines intersecting at origin are called co-ordinate axes. The horizontal line is the X-axis (denoted by X′OX) and the vertical line is the Y-axis (denoted by Y′OY).



- The point of intersection of X-axis and Y-axis is called origin and denoted by O.
- Cartesian plane is a plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate plane or XY-plane.
- The x-co-ordinate of a point is its perpendicular distance from Y-axis.
- The y-co-ordinate of a point is its perpendicular distance from X-axis.
- The point where the X-axis and the Y-axis intersect has co-ordinate point (0, 0).
- The abscissa of a point is the x-co-ordinate of the point.
- The ordinate of a point is the y-co-ordinate of the point.
- If the abscissa of a point is x and the ordinate of the point is y, then (x, y) is called the co-ordinates of the point.
- The axes divide the Cartesian plane into four parts called the quadrants (one fourth part), numbered I, II, III and IV anti-clockwise from OX.
- The co-ordinates of a point on the X-axis are of the form $(x, 0)$ and that of the point on Y-axis are $(0, y)$.
- Sign of co-ordinates depicts the quadrant in which it lies. The co-ordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.
- Three points A, B and C are collinear if the distances AB, BC and CA are such that the sum of two distances is equal to the third.
- Three points A, B and C are the vertices of an equilateral triangle if $AB = BC = CA$.
- The points A, B and C are the vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$.
- Three points A, B and C are the vertices of a right triangle, if $AB^2 + BC^2 = CA^2$.



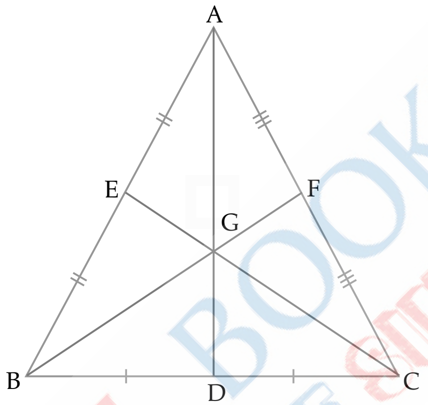
- For the given four points A, B, C and D :



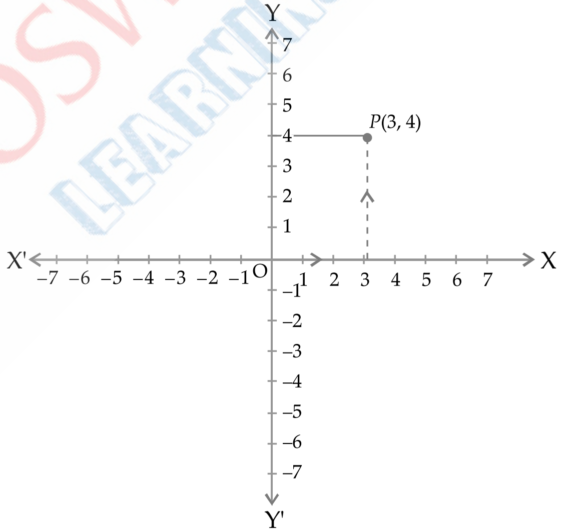
1. If $AB = BC = CD = DA$; $AC = BD$, then ABCD is a square.

2. If $AB = BC = CD = DA$; $AC \neq BD$, then ABCD is a rhombus.
3. If $AB = CD$, $BC = DA$; $AC = BD$, then ABCD is a rectangle.
4. If $AB = CD$, $BC = DA$; $AC \neq BD$, then ABCD is a parallelogram.

- Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.
- Diagonals of rhombus and square bisect each other at right angle.
- All given points are collinear, if the area of the obtained polygon is zero.
- Three given points are collinear, if the area of triangle is zero.
- Centroid is the point of intersection of the three medians of a triangle. In the figure, G is the centroid of a triangle ABC.



- Centroid divides each median of a triangle in a ratio of 2 : 1 from vertex to base of the side.
- If $x \neq y$, then $(x, y) \neq (y, x)$ and if $(x, y) = (y, x)$, then $x = y$.
- To plot a point P(3, 4) in the cartesian plane.
  (i) A distance of 3 units along X-axis.
  (ii) A distance of 4 units along Y-axis.



Know the Formulae

- The distance between two points *i.e.*, P(x_1, y_1) and Q(x_2, y_2) is

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

- The distance of a point P(x, y) from origin is $\sqrt{x^2+y^2}$
- Co-ordinates of point (x, y) which divides the line segment by joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally are

$$x = \left(\frac{mx_2+nx_1}{m+n}\right) \text{ and } y = \left(\frac{my_2+ny_1}{m+n}\right)$$

- Co-ordinates of mid-point of the line segment by joining the points (x_1, y_1) and (x_2, y_2) are

$$x = \left(\frac{x_2+x_1}{2}\right) \text{ and } y = \left(\frac{y_2+y_1}{2}\right)$$

Know the Facts

- Co-ordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.
- Cartesian plane was discovered by ***Rene Descartes***.
- The other name of co-ordinate geometry is Analytical Geometry.
- Co-ordinate Geometry acts as a bridge between the Algebra and Geometry.
- Medians of a triangle are concurrent. The point of concurrency is called the centroid.
- Trisection of a line segment means dividing it into 3 equal parts, so 2 points are required.
- Centroid of a triangle divides its median in the ratio of 2 : 1.

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UNIT IV: GEOMETRY

Chapter - 5 : Triangles

Revision Notes

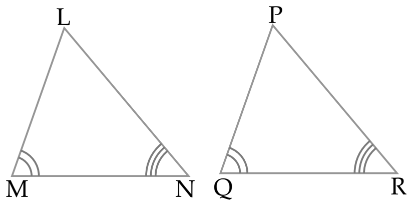
- A triangle is one of the basic shapes of geometry. It is a polygon with 3 sides and 3 vertices/corners.
- Two figures are said to be congruent if they have the same shape and the same size.
- Those figures which have the same shape but not necessarily the same size are called **similar figures.**

  Hence, we can say that **all congruent figures are similar but all similar figures are not congruent.**
- **Similarity of Triangles:** Two triangles are similar, if:

  (i) their corresponding sides are proportional.

  (ii) their corresponding angles are equal.

  If ΔABC and ΔDEF are similar, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.
- **Criteria for Similarity of Triangles:**

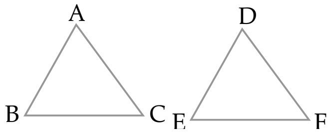


In ΔLMN and ΔPQR, if

(a) $\angle L = \angle P$, $\angle M = \angle Q$ and $\angle N = \angle R$

,

(b) $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$



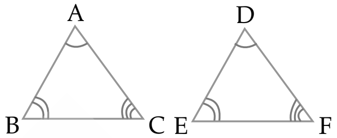
then $\Delta LMN \sim \Delta PQR$.

(i) AAA-Criterion: In two triangles, if corresponding angles are equal, then the triangles are similar and hence their corresponding sides are in the same ratio.

If ΔABC and ΔDEF are similar

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Then,

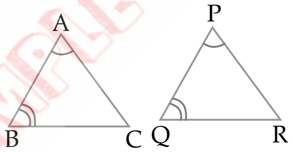


$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Remark: If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

AA-Criterion: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

As we know that the sum of all angles in a triangle is 180° so if two angles in ΔABC and ΔPQR are same *i.e.*, $\angle A = \angle P$, $\angle B = \angle Q$.



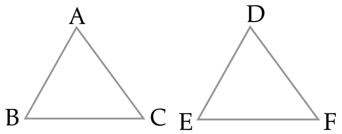
(ii) SSS-Criterion: In two triangles if the sides of one triangle are proportional to the sides of another triangle, then the two triangles are similar and hence corresponding angles are equal.

If $\quad \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

$\therefore \quad \Delta ABC \sim \Delta DEF$

then $\quad \angle A = \angle D, \angle B = \angle E$

and $\quad \angle C = \angle F$

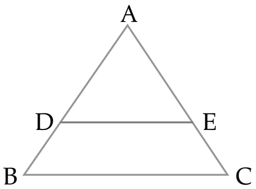


(iii) SAS-Criterion: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

If $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

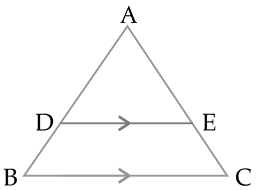
Some theorems based on similarity of triangles:

(i) If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. It is known as **'Basic Proportionality Theorem'** or **'Thales Theorem'.**



In ΔABC, let DE || BC, then

(a) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **(b)** $\dfrac{AB}{DB} = \dfrac{AC}{EC}$ **(c)** $\dfrac{AD}{AB} = \dfrac{AE}{AC}$.



(ii) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the **'Converse of Basic Proportionality Theorem'.**

If $\dfrac{AD}{DB} = \dfrac{AE}{EC}$,

then $DE || BC$

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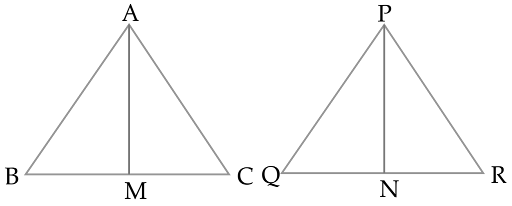
(iii) If two triangles are similar, then the ratio of areas of these triangles is equal to the ratio of squares of their corresponding sides.

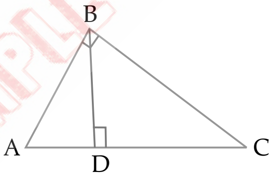
Let $\Delta ABC \sim \Delta PQR$,

then $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2 = \left(\dfrac{AM}{PN}\right)^2$



➢ **Theorems Based on Right Angled Triangles:**

(i) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.



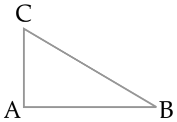
In right ΔABC, B ⊥ AC,

then $\Delta ADB \sim \Delta ABC$

$\Delta BDC \sim \Delta ABC$

and $\Delta ADB \sim \Delta BDC$

(ii) In a right angled triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides. It is known as **Pythagoras Theorem.**



In right ΔABC,

$$BC^2 = AB^2 + AC^2.$$

Some Important Notes:

➢ In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.

➢ Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the median of the triangle.

➢ Three times of the square of any side of an equilateral triangle is equal to four times the square of the altitude.

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Mnemonics

Area of Right angled Triangle

Audi is the product of half of BMW and Honda

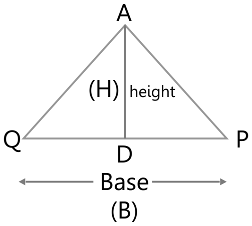
Concept: Area of triangle = $\frac{1}{2}$ **× Base × height**

Interpretation:

A = Area

B = Base

H = Height



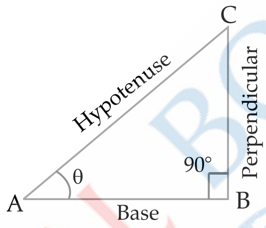
UNIT V: TRIGONOMETRY

Chapter - 6 : Introduction to Trigonometry & Trigonometric Identities

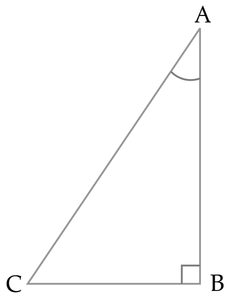
Revision Notes

Trigonometric Ratios and Complementary Angles

- In fig., a right triangle ABC right angled at B is given and $\angle BAC = \theta$ is an acute angle. Here side AB which is adjacent to $\angle$A is base, side BC opposite to $\angle$A is perpendicular and the side AC is hypotenuse which is opposite to the right angle B.



Trigonometric Identities



- An equation is called an identity if it is true for all values of the variable(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

In $\Delta\, ABC$, right-angled at B, By Pythagoras Theorem,

$$AB^2 + BC^2 = AC^2 \quad \text{...(i)}$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \quad \text{...(ii)}$$

This is true for all values of A such that $0° \le A \le 90°$. So, this is a trigonometric identity. Now divide eqn.(i) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \quad \text{...(iii)}$$

Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, $\tan A$ and $\sec A$ are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \le A < 90°$.

Again dividing eqn. (i) by BC^2.

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$$\frac{AB^2}{BC^2}+\frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2+\left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \text{cosec}^2 A \qquad \text{...(iv)}$$

Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all value of A such that $0° < A \leq 90°$.

Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, *i.e.*, if any one of the ratios is known, we can determine the values of other trigonometric ratios.

Know the Formulae

The trigonometric ratios of $\angle A$ in right triangle ABC are defined as

$$\text{sine of } \angle A = \sin\theta = \frac{\text{Perpendicular or opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

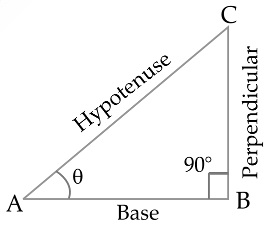
$$\text{cosine of } \angle A = \cos\theta = \frac{\text{Base or adjecent side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan\theta = \frac{\text{Perpendicular or opposite side}}{\text{Base adjacent side}} = \frac{BC}{AB}$$

$$\text{cotangent of } \angle A = \cot\theta = \frac{\text{Base or adjacent side}}{\text{Perpendicular or oppsite side}} = \frac{AB}{BC} = \frac{1}{\tan\theta}$$

$$\text{secant of } \angle A = \sec\theta = \frac{\text{Hypotenuse}}{\text{Base or adjacent side}} = \frac{AC}{AB} = \frac{1}{\cos\theta}$$

$$\text{cosecant of } \angle A = \text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular or opposite side}} = \frac{AC}{BC} = \frac{1}{\sin\theta}$$



It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

Also, $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.

$\angle$A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined (∞)
cot A	Not defined (∞)	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined (∞)
cosec A	Not defined (∞)	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Mnemonics

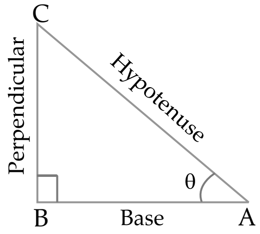
Concept

The relation of Trigonometric Ratios

In right angled ΔABC, we have

$$\sin\theta = \frac{BC}{AC},\ \cos\theta = \frac{BA}{AC},\ \tan\theta = \frac{BC}{AB},$$

$$\cot\theta = \frac{AB}{BC},\ \sec\theta = \frac{AC}{BA},\ \operatorname{cosec}\theta = \frac{AC}{BC}$$



$$\frac{\boxed{\text{sin}}\ \text{Pandit} \quad \boxed{\text{cos}}\ \text{Badri} \quad \boxed{\text{tan}}\ \text{Prasad}}{\text{Har}\ \boxed{\text{cosec}} \quad \text{Har}\ \boxed{\text{sec}} \quad \text{Bhole}\ \boxed{\text{cot}}}$$

Interpretation:

Here,

$$\sin\theta = \frac{\text{Pandit}}{\text{Har}} = \frac{P}{H} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{Badri}}{\text{Har}} = \frac{B}{H} = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BA}{AC}$$

$$\tan\theta = \frac{\text{Prasad}}{\text{Bhole}} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot\theta = \frac{\text{Bhole}}{\text{Prasad}} = \frac{B}{P} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

$$\sec\theta = \frac{\text{Har}}{\text{Badri}} = \frac{H}{B} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BA}$$

$$\operatorname{cosec}\theta = \frac{\text{Har}}{\text{Pandit}} = \frac{H}{P} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}.$$

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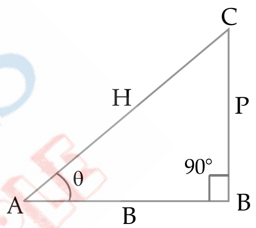
Trigonometric Ratios

Hints: We learn these ratios in following ways:

"**S**ome **p**eople **h**ave" $\sin\theta = \frac{P}{H}$

"**C**urly **B**rown **H**air" $\cos\theta = \frac{B}{H}$

"**T**hrough **p**roper **B**rushing" $\tan\theta = \frac{P}{B}$.



(i) $\sin\theta = \frac{BC}{AC} = \frac{P}{H}$

Interpretation:

Some	**P**eople	**H**ave
↓	↓	↓
$\sin\theta$	Perpendicular	Hypotenuse

(ii) Cos $\theta = \frac{AB}{AC} = \frac{B}{H}$

Interpretation:

Curly	**B**rown	**H**air
↓	↓	↓
$\cos\theta$	Base	Hypotenuse

(ii) $\tan\theta = \frac{BC}{AB} = \frac{P}{B}$

Interpretation:

Through	**P**roper	**B**rushing
↓	↓	↓
$\tan\theta$	Perpendicular	Base

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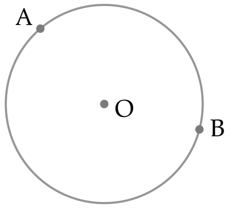
UNIT VI: MENSURATION

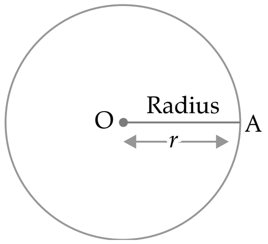
Chapter - 7 : Areas Related to Circles

Revision Notes

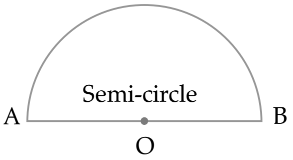
➢ A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

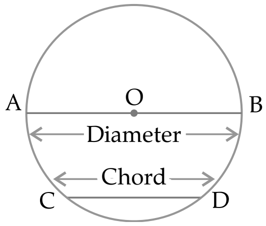


➢ A line segment joining the centre of the circle to a point on the circumference of the circle is called its radius.

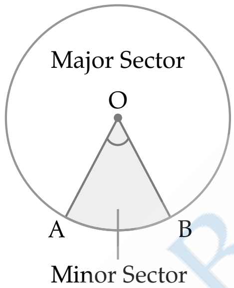


- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle. Here AB is a a diameter, which is a longest chord.
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.

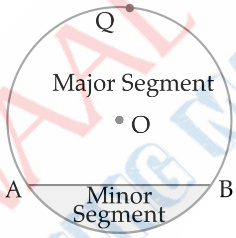




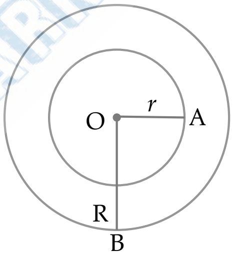
- A part of a circumference of circle is called an arc.
- An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.
- An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.
- The region bounded by an arc of a circle and two radii at its end points is called a sector.



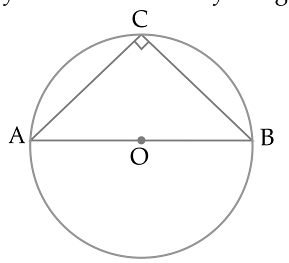
- A chord divides the interior of a circle into two parts, each called a segment.



- Circles having the same centre but different radii are called concentric circles.



- Two circles (or arcs) are said to be congruent if on placing one over the other cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.



- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Know the Formulae

1. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is diameter and r is the radius of the circle.
2. Area of a circle = πr^2.
3. Area of a semi-circle = $\frac{1}{2}\pi r^2$.
4. Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)r$
5. Area of a ring or an annulus = $\pi(R + r)(R - r)$. where R is the outer radius and r is the inner radius.
6. Length of arc, $l = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$, where θ is the angle subtended at centre by the arc.
7. Area of a sector = $\frac{\pi r^2\theta}{360°}$

   or area of sector = $\frac{1}{2}(l \times r)$, where l is the length of arc.
8. Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$.
9. Area of major segment = Area of the circle – Area of minor segment

   = πr^2 – Area of minor segment.
10. If a chord subtends a right angle at the centre, then

    area of the corresponding segment = $\left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$
11. If a chord subtends an angle of 60° at the centre, then

    area of the corresponding segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$.
12. If a chord subtends an angle of 120° at the centre, then

    area of the corresponding segment = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$.
13. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
14. Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.
15. Perimeter of a sector = $\frac{\pi r\theta}{180°} + 2r$.

Know the Facts

- An Indian mathematician Srinivas Ramanujan worked out the identity using the value of π correct to million places of decimals.
- The Indian mathematician Aryabhatta gave the value of π as $\frac{62832}{20000}$
- "How I made a greater discovery" this mnemonic help us in getting the value of π = 3.14159 ....
- Give it under separate reading with explanation how to use

	CAN	I	HAVE	A	SMALL	CONTAINER	OF	COFFEE
	↓	↓	↓	↓	↓	↓	↓	↓
No. of Letters →	3	1	4	1	5	9	2	6

- Archimedes calculated the area of a circle by approximating it to a square.
- Area of sector of a circle depends on two parameters-radius and central angle.

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UNIT VII: STATISTICS & PROBABILITY

Chapter - 8 : Probability

Revision Notes

- Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence.
- A random experiment is an experiment or a process for which the outcome cannot be predicted with certainty. *e.g.*,
  (i) tossing a coin, **(ii)** throwing a dice, **(iii)** selecting a card and **(iv)** selecting an object etc.
- Outcome associated with an experiment is called an event. *e.g.*, **(i)** Getting a head on tossing a coin, **(ii)** getting a face card when a card is drawn from a pack of 52 cards.
- The events whose probability is one are called sure/certain events.
- The events whose probability is zero are called impossible events.
- An event with only one possible outcome is called an elementary event.
- In a given experiment, if two or more events are equally likely to occur or have equal probabilities, then they are called equally likely events.
- Probability of an event always lies between 0 and 1.
- Probability can never be negative and more than one.
- A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of an ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are spades, hearts, diamonds and clubs.
- King, queen and jack are face cards.
- The sum of the probabilities of all elementary events of an experiment is 1.
- Two events A and B are said to be complementary to of each other if the sum of their probabilities is 1.
- Probability of an event E, denoted as P(E), is given by:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total possible number of outcomes}}$$

- For an event E, $P(\overline{E}) = 1 - P(E)$, where the event $\overline{E}$ representing 'not E' is the complement of the event E.
- For A and B two possible outcomes of an event,
  (i) If $P(A) > P(B)$, then event A is more likely to occur than event B.
  (ii) If $P(A) = P(B)$, then events A and B are equally likely to occur.

Know the Facts

- The experimental or empirical probability of an event is based on what has actually happened while the theoretical probability of the event attempts to predict what will happen on the basis of certain assumptions.
- As the number of trials in an experiment go on increasing, we may expect the experimental and theoretical probabilities to be nearly the same.
- When we speak of a coin, we assume it to be '**fair**' *i.e.*, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being '**unbiased**'.
- By the phrase '**random toss**', we mean that the coin is allowed to fall freely without any **bias** or **interference**.

- In the case of experiment we assume that the experiments have equally likely outcomes.
- A deck of playing cards consists of 4 suits : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Clubs and spades are of black colour, while hearts and diamonds are of red colour.
- The first book on probability '**The Book on Games of Chance**' was written by Italian mathematician **J. Cardan**.
- The classical definition of probability was given by Pierre Simon Laplace.

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