SOLVED PAPER	CBSE - 2020 Class-X Delhi/Outside Delhi Sets	Mathematics (Standard)

Time : 3 hrs. **Max. Marks : 80**

General Instructions :

(i) *This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory.*

(ii) *Section A : Q. No. 1 to 20 question of one mark each.*

(iii) *Section B : Q. No. 21 to 26 comprises of 6 question of two mark each.*

(iv) *Section C : Q. No. 27 to 34 comprises of 8 questions of three marks each.*

(v) *Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.*

(vi) *There is no overall choice in the question paper. However, an internal choice has been provided in 2 question of one mark each. 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choices in such questions.*

(vii) *In addition to this, separate instructions are given with each section and question, wherever necessary.*

(viii) *Use of calculators is not permitted.*

Delhi Set-I **Code No. 30/1/1**

SECTION - A

Q. No. 1 to 10 are multiple choice type questions of 1 mark each. Select the correct option.

1. If one zero of the quadratic polynomial $x^2 + 3x + k$ *is* 2, then the value of k is

(a) 10 **(b)** –10

(c) –7 **(d)** –2

2. The total number of factors of prime number is

(a) 1 **(b)** 0

(c) 2 **(d)** 3

3. The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is

(a) $x^2 + 5x + 6$ **(b)** $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ **(d)** $-x^2 + 5x + 6$

4. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution is,

(a) -2 **(b)** $\neq 2$

(c) 3 **(d)** 2

5. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 **(b)** 12, 420

(c) 3, 420 **(d)** 420, 3

6. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P., is

(a) 6 **(b)** -6

(c) 18 **(d)** -18

7. The first term of A.P. is p and the common difference is q, then its 10th term is

(a) $q + 9p$ **(b)** $p - 9q$

(c) $p + 9q$ **(d)** $2p + 9q$

8. The distance between the points $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$, is

(a) $a^2 + b^2$ **(b)** $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ **(d)** $\sqrt{a^2 - b^2}$

9. If the point P(k, 0) divides the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ in the ratio 1 : 2, then the value of k is

(a) 1 **(b)** 2

(c) -2 **(d)** -1

10. The value of p, for which the points $A(3, 1)$, $B(5, p)$ and $C(7, -5)$ are collinear, is

(a) -2 **(b)** 2

(c) -1 **(d)** 1

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 marks.

11. In Fig. 1, ΔABC is circumscribing a circle, the length of BC is_______ cm.

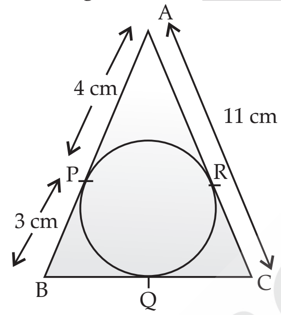

Fig.1

12. Given $\Delta\, ABC \sim \Delta\, PQR$, if $\dfrac{AB}{PQ} = \dfrac{1}{3}$, then $\dfrac{ar(\Delta\, ABC)}{ar(\Delta\, PQR)} =$ ________.

13. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is _______.

14. $\dfrac{\cos 80°}{\sin 10°} + \cos 59° \text{ cosec } 31° =$ ———.

15. The value of $\left(\sin^2\theta + \dfrac{1}{1+\tan^2\theta}\right) =$ ———.

OR

The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$ ———.

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

16. The ratio of the length of a vertical rod and the length its shadow is $1 : \sqrt{3}$. Find the angle of elevation of the sunat that moment ?

17. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes ?

18. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant.

19. A die is thrown once. What is the probability of getting a number less than 3 ?

OR

If the probability of wining a game is 0.07, what is the probability of losing it ?

20. If the mean of the first n natural number is 15, then find n.

SECTION - B

Q. Nos. 21 to 26 carry 2 mark each.

21. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P. .

22. In Fig. 2, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\dfrac{BE}{EC} = \dfrac{BC}{CP}$

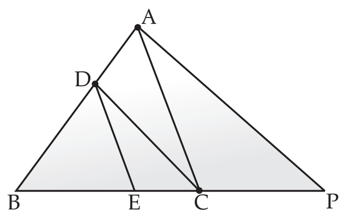

Fig.2

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OR

In Fig. 3, two tangents *TP* and *TQ* are drawn to circle with centre *O* from an external point *T*. Prove that $\angle PTQ = 2\angle OPQ$.

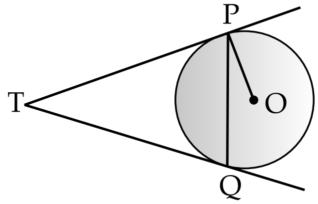

Fig.3

23. The rod *AC* of TV disc antenna is fixed at right angles to wall *AB* and a rod *CD* is supporting the disc as shown in Fig. 4. If *AC*=1.5 m long and *CD* = 3 m, find (i) tan θ (ii) sec θ + cosec θ.

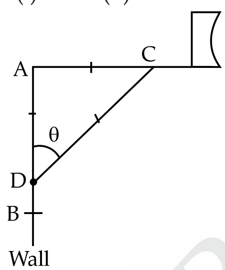

Fig.4

24. A piece of wire 22 cm long is bent into the form an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$

25. If a number x is chosen at random from the number – 3, – 2, – 1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

26. Find the mean the following distribution :

Class :	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency :	5	10	10	7	8

OR

Find the mode of the following date :

Class :	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency :	6	8	10	12	6	5	3

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

27. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$.

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

28. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

If 4 is zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

29. In a flight of 600 km, an aircraft was slowed due to bad wether. Its average speed for the trip was reduce by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

30. Find the area of triangle *PQR* formed by the points *P*(– 5, 7), *Q*(– 4, –5) and *R*(4, 5).

OR

If the point *C*(– 1, 2) divides internally the line segment joining *A*(2, 5) and *B*(*x*, *y*) in the ratio 3 : 4, find the coordinates of *B*.

31. In Fig. 5, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, Prove that ΔBAC is an isosceles triangle.

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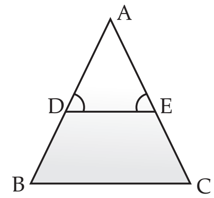

Fig.5

32. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first is a right angle.

33. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

34. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-point of its height and parallel to its. Compare the volume of the two parts.

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

35. Show that the square of any positive integer cannot be of the from $(5q + 2)$ or $(5q + 3)$ for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3.

36. The sum of four consecutive number in A.P. is 32 and the ratio of the product of the first and last and term to the product of two middle terms is 7: 15. Find the numbers.

OR

Solve : $1 + 4 + 7 + 10 + ... + x = 287$

37. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

38. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

39. A bucket in the from of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$

40. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive

OR

The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	**Frequency**
0 – 100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Delhi Set-II — Code No. 30/1/2

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

14. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 2\cos 60° =$ ______.

15. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ______.

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

20. A die thrown once. What is the probability of getting an even prime number ?

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. Find the sum of first 20 terms of the following A.P. :
1, 4, 7, 10, ...

26. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

33. Prove that the parallelogram circumscribing a circle is a rhombus.

34. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

39. A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre. (Use $\pi = 3.14$)

40. Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the first triangle.

Delhi Set-III — Code No. 30/1/3

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

14. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is _________ m.

15. $\frac{2\cos 67°}{\sin 23°} - \frac{\tan 40°}{\cot 50°} - \cos 0° =$ __________.

Q. No. 16 to 20 are short answer type questions of 1 mark each.

20. A pair of dice is thrown once. What is the probability of getting a doublet ?

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

26. The sum of the first 7 terms of an A.P. is 63 and that of its next 7 terms is 161. Find the A.P. .

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

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33. In given Fig. 5, two circles touch each other at the point *C*. Prove that the common tangent to the circles at *C*, bisects the common tangent at *P* and *Q*.

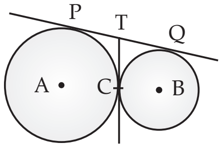

Fig.5

34. Prove that : $\dfrac{\cot\theta + \text{cosec}\theta - 1}{\cot\theta - \text{cosec}\theta + 1} = \dfrac{1 + \cot\theta}{\sin\theta}$

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

39. Draw a ΔABC with $BC = 7$ cm, $\angle B = 45°$ and $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

40. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

❑❑

Outside Delhi Set-I — Code No. 30/2/1

SECTION - A

Question numbers 1 to 10 are Multiple Choice Questions of 1 mark each. Select the correct option.

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 **(b)** 4
(c) 5 **(d)** 2

2. Euclid's division Lemma states that for two positive integers *a* and *b*, there exists unique integer *q* and *r* satisfying $a = bq + r$, and

(a) $0 < r < b$ **(b)** $0 < r \le b$
(c) $0 \le r < b$ **(d)** $0 \le r \le b$

3. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

(a) $m, m + 3$ **(b)** $-m, m + 3$
(c) $m, -(m + 3)$ **(d)** $-m, -(m + 3)$

4. The value of *k* for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is

(a) $-\frac{14}{3}$ **(B)** $\frac{2}{5}$
(c) 5 **(d)** 10

5. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) ± 0.2 **(b)** ± 0.02
(c) 0.4 **(d)** 2

6. The common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, ,$ ... is

(a) 1 **(B)** $\frac{1}{p}$
(c) -1 **(d)** $-\frac{1}{p}$

7. The n^{th} term of the A.P. $a, 3a, 5a, \ldots$ is

(a) na **(b)** $(2n - 1)a$
(c) $(2n + 1)a$ **(d)** $2na$

8 The point *P* on *x*-axis equidistant from the points $A(-1, 0)$ and $B(5, 0)$ is

(a) (2, 0) **(b)** (0, 2)
(c) (3, 0) **(d)** (2, 2)

9. The co-ordinates of the point which is reflection of point (–3, 5) in x-axis are

(a) (3, 5) **(b)** (3, –5)

(c) (–3, –5) **(d)** (–3, 5)

10. If the point $P(6, 2)$ divides the line segment joining $A(6, 5)$ and $B(4, y)$ in the ratio 3 : 1, then the value of y is

(a) 4 **(b)** 3

(c) 2 **(d)** 1

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

11. In fig. 1, $MN \parallel BC$ and $AM : MB = 1 : 2$, then

$\frac{ar(\Delta AMN)}{ar(\Delta ABC)}$ = ...

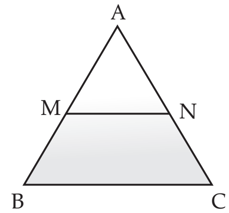

Fig. 1

12. In given Fig. 2, the length PB = ... cm.

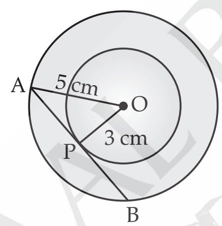

Fig. 2

13. In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B$ = ...

OR

Two triangles are similar if their corresponding sides are ....

14. The value of (tan 1° tan 2° ...... tan 89°) is equal to ....

15. In fig. 3, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ....

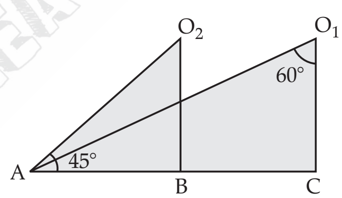

Fig. 3

Q. Nos. 16 to 20 are Short Answer Type Questions of 1 mark each.

16. If $\sin A + \sin^2 A = 1$, then find the value of the expression $(\cos^2 A + \cos^4 A)$.

17. In fig. 4 is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$

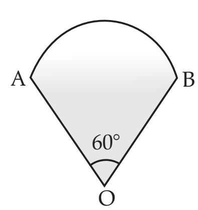

Fig. 4

18. If a number x is chosen at random from the numbers –3, –2, –1. 0, 1, 2, 3, then find the probability of $x^2 < 4$.

OR

What is the probability that a randomly taken leap year has 52 Sundays ?

19. Find the class-marks of the classes 10–25 and 35–66.

20. A die is thrown once. What is the probability of getting a prime number.

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

21. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$\mathbf{2x + 3, 3x^2 + 7x + 2, 4x^3 + 3x^2 + 2, x^3 + \sqrt{3x} + 7}$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

Answer the following question :

(i) How many of the above ten, are not polynomials ?

(ii) How man of th above ten, are quadratic polynomials ?

22. In fig. 5, ABC and DBC are two triangles on the same base BC. If AD intersects B at O, show that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

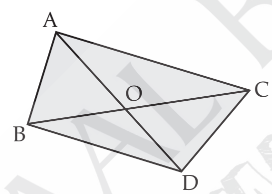

Fig. 5

OR

In fig. 6, if $AD \perp BC$, then prove that $AB^2 + CD^2 = BD^2 + AC^2$.

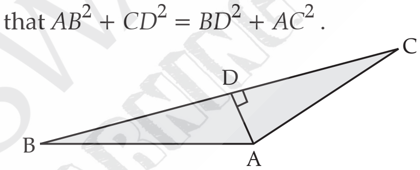

Fig. 6

23. Prove that $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = \mathbf{cosec}\ \alpha$

OR

Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

24. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

25. 'A child has a die whose six faces show the letters as shown below :

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A, (ii) D ?

26. compute the mode for the following frequency distribution :

Size of items (in cm)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28
Frequency	5	7	9	17	12	10	6

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SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

27. If $2x + y = 23$ and $4x - y = 19$, find the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$.

OR

Solve for x : $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$, $x \neq -4, 7$.

28. Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the let term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

OR

Solve the equation :

$1 + 4 + 7 + 10 + ... + x = 287$.

29. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

30. If the mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$ and $x + y - 10 = 0$, find the value of k.

OR

Find the area of triangle ABC with $A(1, -4)$ and the mid-points of sides through A being $(2, -1)$ and $(0, -1)$.

31. In Fig. 7, if $\Delta ABC - DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.

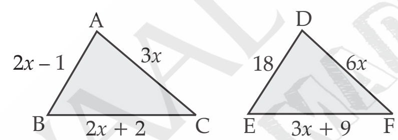

Fig. 7

32. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that

$AQ = \frac{1}{2}(BC + CA + AB)$

33. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$.

34. The area of a circular play ground is 22176 cm^2. Find the cost of fencing this ground at the rate of 50 per metre.

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

35. Prove that $\sqrt{5}$ is an irrational number.

36. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately ?

37. Draw a circle of radius 2 cm with centre O and take a point P outside the circle such that $OP = 6.5$ cm. From P, draw two tangents to the circle.

OR

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

38. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

39. Find the area of the shaded region in fig. 8, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.

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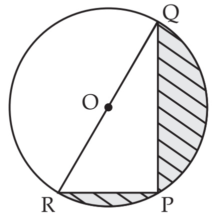

Fig. 8

OR

Find the curved surface area of the frustum of a cone, the diameters of whose circular ends are 20 m and 6 m and its height is 24 m.

40. The mean of the following frequency distribution is 18. The frequency f in the class interval 19 – 21 is missing. Determine f.

Class interval	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

OR

The following table gives production yield per hectare of wheat of 100 farms of a village :

Production yield	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70
No. of farms	4	6	16	20	30	24

Change the distribution to a 'more than' type distribution and draw its ogive.

Outside Delhi Set-II — Code No. 30/2/2

Note : All other Questions are from Set I

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

15. The value of $\sin 23° \cos 67° + \cos 23° \sin 67°$ is ....

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

19. If $\tan A = \cot B$, then find the value of $(A + B)$.

20. Find the class marks of the classes 15 – 35 and 45 – 60.

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. A child has a die whose six faces show the letters as shown below :

A	A	B	C	C	C

The die is thrown once. What is the probability of getting (i) A, (ii) C ?

26. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. If in an A.P., the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m + n)$ terms is $-(m + n)$.

OR

Find the sum of all 11 terms of an A.P. whose middle term is 30.

33. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

34. If $1 + \sin^2\theta = 3 \sin\theta \cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

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SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

39. Draw two tangents to a circle of radius 4 cm, which are inclined to each other at an angle of 60°.

OR

Construct a triangle *ABC* with sides 3 cm, 4 cm and 5 cm. Now, construct another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔABC.

40. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

Outside Delhi Set-III — Code No. 30/2/3

Note : Except these all other questions are from Set I & Set II

SECTION - A

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

15. The value of sin 32° cos 58° + cos 32° sin 58° is ....

OR

The value of $\frac{\tan 35°}{\cot 55°}+\frac{\cot 78°}{\tan 12°}$ is ....

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

19. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take π = 3.14)

20. Find the class marks of the classes 20- 50 and 35- 60.

SECTION - B

Q. Nos. 21 to 26 carry 2 marks each.

25. Find the mode of the following frequency distribution :

Class	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	3	8	9	10	3	2

26. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

SECTION - C

Q. Nos. 27 to 34 carry 3 marks each.

32. Which term of the A.P. 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

OR

Find the middle term of the A.P. 7, 13, 19, ..., 247.

33. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm standing water is required ?

34. Show that :

$$\frac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}=1$$

SECTION - D

Q. Nos. 35 to 40 carry 4 marks each.

39. Draw a circle of radius 3.5 cm. From a point *P*, 6 cm from its centre, draw two tangents to the circle.

OR

Construct a ΔABC with *AB* = 6 cm, *BC* = 5 cm and ∠B = 60°. Now construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.

40 A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone is 7 cm and height of cone is 3.5 cm,

find the volume of the solid. $\left(\text{Take}\ \pi=\frac{22}{7}\right)$

❑❑

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SOLUTIONS

Delhi Set-I — Code No. 30/1/1

SECTION - A

1. Correct option : (b)

Let $p(x) = x^2 + 3x + k$

$\because$ 2 is a zero of $p(x)$, then

$$p(2) = 0$$
$$\therefore \quad (2)^2 + 3(2) + k = 0$$
$$\Rightarrow \quad 4 + 6 + k = 0$$
$$\Rightarrow \quad 10 + k = 0$$
$$\Rightarrow \quad k = -10$$

2. Correct option : (c)

Explanation : We have only two factors (1 and number itself) of any prime number.

3. Correct option : (a)

Explanation : Let α and β be the zeroes of the quadratic polynomial, then

$$\alpha + \beta = -5$$

and $\alpha\beta = 6$

So, required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6$$

4. Correct option : (d)

Explanation : Given equations :

$$x + y - 4 = 0$$

and $2x + ky - 3 = 0$

Here, $\dfrac{a_1}{a_1} = \dfrac{1}{2}, \dfrac{b_1}{b_2} = \dfrac{1}{k}$ and $\dfrac{c_1}{c_2} = \dfrac{-4}{-3} = \dfrac{4}{3}$

$\because$ System has no solution

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$
$$\Rightarrow \quad k = 2 \text{ or } k \neq \frac{3}{4}$$

5. Correct option : (c)

Explanation :

$$12 = 2 \times 2 \times 3$$
$$21 = 3 \times 7$$

and $15 = 3 \times 5$

$$\therefore \quad \text{HCF} = 3$$

and $\text{LCM} = 2 \times 2 \times 3 \times 5 \times 7 = 420$

6. Correct option : (a)

Explanation : $2x$, $(x + 10)$ and $(3x + 2)$ are in A.P.

$$\therefore \quad (x + 10) - 2x = (3x + 2) - (x + 10)$$
$$\Rightarrow \quad -x + 10 = 2x - 8$$
$$\Rightarrow \quad -x - 2x = -8 - 10$$
$$\Rightarrow \quad -3x = -18$$
$$\Rightarrow \quad x = 6$$

7. Correct option : (c)

Explanation : $a = p$ and $d = q$ (given)

$$\therefore \quad 10^{th} \text{ term} = a + (10 - 1)d$$
$$= p + 9q$$

8. Correct option : (c)

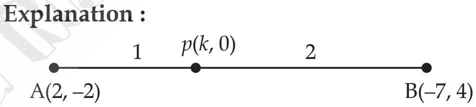
Explanation : Here, $x_1 = a\cos\theta + b\sin\theta$, $y_1 = 0$

and $x_2 = 0$, $y_2 = a\sin\theta - b\cos\theta$

$$\therefore \quad \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2}$$
$$= \sqrt{(-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}$$
$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$
$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$
$$= \sqrt{a^2 \times 1 + b^2 \times 1} = \sqrt{a^2 + b^2}$$

9. Correct option : (d)

Explanation :



$$\therefore \quad k = \frac{1(-7) + 2(2)}{1 + 2}$$
$$\left[\because x = \frac{mx_2 + nx_1}{m + n}\right]$$
$$\Rightarrow \quad k = \frac{-7 + 4}{3}$$
$$\Rightarrow \quad 3k = -3$$
$$\Rightarrow \quad k = -1$$

10. Correct option : (a)

Explanation : Here, $x_1 = 3$, $x_2 = 5$, $x_3 = 7$ and $y_1 = 1$, $y_2 = p$, $y_3 = -5$

If points are collinear, then area of triangle = 0

$$\therefore \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$
$$\Rightarrow \quad \frac{1}{2}[3(p + 5) + 5(-5 - 1) + 7(1 - p)] = 0$$
$$\Rightarrow \quad \frac{1}{2}[3p + 15 - 30 + 7 - 7p] = 0$$
$$\Rightarrow \quad -4p - 8 = 0$$
$$\Rightarrow \quad -4p = 8$$
$$\Rightarrow \quad p = -2$$

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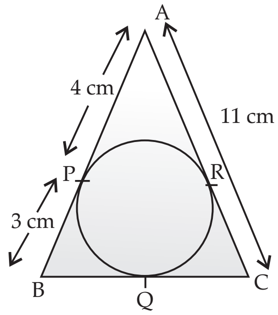
11. 10

Explanation : $\because$ AP and AR are tangents to the circle from external point A.

$\therefore$ $AP = AR$ *i.e.*, $AR = 4$ cm



Similarly, PB and BQ are tangents.

$\therefore$ $BP = BQ$ *i.e.*, $BQ = 3$ cm

Now, $CR = AC - AR = 11 - 4 = 7$cm

Similarly, CR and CQ are tangents.

$\therefore$ $CR = CQ$ *i.e.*, $CQ = 7$ cm

Now, $BC = BQ + CQ = 3 + 7 = 10$ cm.

Hence, the length of BC is 10 cm.

12. 1 : 9

Explanation : Since, $\Delta ABC \sim \Delta PQR$, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{1}{3}\right)^2$$

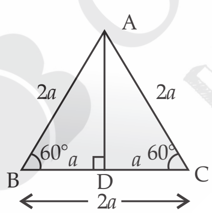
[$\therefore$ By using property of similar triangle]

$$= \frac{1}{9}$$

13. $a\sqrt{3}$

Explanation : ABC is an equilateral triangle in which $AD \perp BC$.

From Δ ABC,



$$AB^2 = (AD)^2 + (BD)^2$$

(By using Pythogoras Theorem)

$$\Rightarrow (2a)^2 = (AD)^2 + (a)^2$$

$$\Rightarrow 4a^2 - a^2 = (AD)^2$$

$$\Rightarrow (AD)^2 = 3a^2$$

$$\Rightarrow AD = a\sqrt{3}$$

Hence, the length of attitude is $a\sqrt{3}$

14. 2

Explanation : $\dfrac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \text{ cosec } 31^\circ$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos(90^\circ - 31^\circ) \times \frac{1}{\sin 31^\circ}$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \sin 31^\circ \times \frac{1}{\sin 31^\circ}$$

$$= 1 + 1 = 2$$

15. 1

Explanation : $\sin^2\theta + \dfrac{1}{1+\tan^2\theta} = \sin^2\theta + \dfrac{1}{\sec^2\theta}$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

OR

1

Explanation : $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$$= \sec^2\theta(1 - \sin\theta)(1 + \sin\theta)$$

$$= \sec^2\theta(1 - \sin^2\theta)$$

$[\because (a - b)(a + b) = a^2 - b^2]$

$$= \sec^2\theta \times \cos^2\theta$$

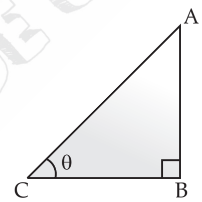
$$= \frac{1}{\cos^2\theta} \times \cos^2\theta$$

$$= 1$$

16. Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$.

In Δ ABC,



$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \qquad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}} \text{(given)}\right]$$

$$\Rightarrow \tan\theta = \tan 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

Here the angle of elevation of the sun is 30°.

17. Let h_1 and h_2 be height and r_1, r_2 be radii of two cones, then ratio of their volumes

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

Given : $\dfrac{h_1}{h_2} = \dfrac{1}{3}$ and $\dfrac{r_1}{r_2} = \dfrac{3}{1}$

$$= \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right) = \left(\frac{3}{1}\right)^2\left(\frac{1}{3}\right) = \frac{3}{1}$$

Hence, ratio of their volumes is 3 : 1.

18. In the English language, there are 26 alphabets. Consonants are 21.

$\therefore$ The probability of choosing a consonant $= \dfrac{21}{26}$

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19. Total possible outcomes $= 6$

$\therefore$ P(number less than 3) $= \frac{2}{6} = \frac{1}{3}$

OR

P(winning the game) $= 0.07$

P(losing the game) $= 1 - 0.07$

$= 0.93$

20. Given : 1, 2, 3, 4, ... to n terms.

$\therefore$ The sum of first n natural numbers $= \frac{n(n+1)}{2}$

So, mean $= \frac{n(n+1)}{2 \times n}$

$\Rightarrow \quad \frac{n+1}{2} = 15$

$\Rightarrow \quad n + 1 = 30$

$\Rightarrow \quad n = 29$

SECTION - B

21. Given : $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$

Common difference, $d_1 = (a^2 + b^2) - (a - b)^2$

$= (a^2 + b^2) - (a^2 + b^2 - 2ab)$

$= a^2 + b^2 - a^2 - b^2 + 2ab$

$= 2ab$

and $d_2 = (a + b)^2 - (a^2 + b^2)$

$= a^2 + b^2 + 2ab - a^2 - b^2$

$= 2ab$

Since, $d_1 = d_2$

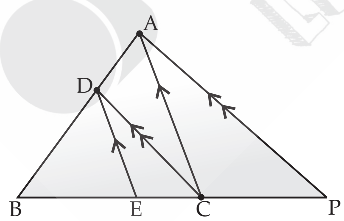
Thus, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in *A.P.*

Hence Proved

22. In ΔABP,

$DC \parallel AP$ (Given)

$\therefore \quad \frac{BD}{DA} = \frac{BC}{CP}$ (From *BPT*) ....(i)



In ΔABC,

$DE \parallel AC$ (Given)

$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC}$ (From *BPT*) ....(ii)

From equations (i) and (ii), we have

$\frac{BE}{EC} = \frac{BC}{CP}$ **Hence Proved**

OR

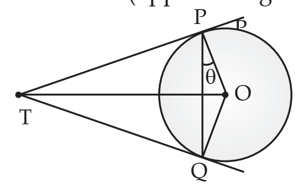
Let $\angle OPQ$ be θ, then

$\angle TPQ = 90° - \theta$

Since, $TP = TQ$

$\therefore \quad \angle TQP = 90° - \theta$

(opposite angles of equal sides)



Now, $\angle TPQ + \angle TQP + \angle PTQ = 180°$

(Angle sum property of a Triangle)

$\Rightarrow \quad 90° - \theta + 90° - \theta + \angle PTQ = 180°$

$\Rightarrow \quad \angle PTQ = 180° - 180° + 2\theta$

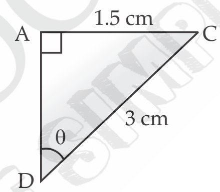
$\Rightarrow \quad \angle PTQ = 2\theta$

Hence, $\angle PTQ = 2\angle OPQ$

Hence Proved.

23. Given, $AC = 1.5$ m

and $CD = 3$ m



In right angle triangle *CAD*,

$AD^2 + AC^2 = DC^2$ (Using Pythagoras theorem)

$\Rightarrow \quad AD^2 + (1.5)^2 = (3)^2$

$\Rightarrow \quad AD^2 = 9 - 2.25 = 6.75$

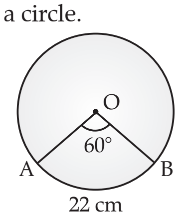
$\Rightarrow \quad AD = \sqrt{6.75} = 2.6$m **(Approx)**

(i) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$

(ii) $\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC}$

$= \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$

24. *AB* is an arc of a circle.



i.e., $AB = 22$ cm

and $\theta = 60°$

$\therefore$ Length of an arc $= \frac{2\pi r\theta}{360°}$

$\Rightarrow \quad 22 = \frac{2 \times 22 \times r \times 60°}{7 \times 360°}$

$\Rightarrow \quad 22 = \frac{22 \times r}{21}$

$\Rightarrow \quad 22 \times r = 22 \times 21$

$\Rightarrow \quad r = 21$

Hence, The radius of the circle (r) is 21 cm.

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25. Total number of outcomes = 7

Favourable outcomes = 5(–2, –1, 0, 1, 2)

$$\therefore \quad P(x^2 \leq 4) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{5}{7}$$

26.

Class	Frequency (f)	Mid-Value (x)	$f \times x$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
	$\Sigma f = 40$		$\Sigma fx = 326$

$$\therefore \quad \text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{326}{40} = 8.15$$

OR

Modal class = 60 – 80

$$\therefore \quad \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Hence, $l = 60, f_1 = 12, f_0 = 10, f_2 = 6$ and $h = 20$

$$\text{Mode} = 60 + \frac{12-10}{2\times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{2 \times 20}{24 - 16}$$

$$= 60 + \frac{40}{8} = 60 + 5$$

$$= 65$$

SECTION - C

27. Let α and β be zero of the given polynomial $ax^2 + bx + c$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$s = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

$$p = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$$

The required polynomial, $x^2 - sx + p$ or $cx^2 + bx + a$

OR

$$\begin{array}{r|l} & x - 2 \\ \hline -x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\ & -x^3 + x^2 - x \\ & (+)\ (-)\ (+) \\ \hline & 2x^2 - 2x + 5 \\ & 2x^2 - 2x + 2 \\ & (-)\ (+)\ (-) \\ \hline & 3 \end{array}$$

Thus, quotient = $x - 2$ and remainder = 3

Then, $f(x) = g(x) \times q(x) + r(x)$

$= (-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

Hence Verified.

28. Given, $2y - x = 8$

$\Rightarrow \quad x = 2y - 8$

y	0	4	5
$x = 2y - 8$	– 8	0	2

$5y - x = 14$

$\Rightarrow \quad x = 5y - 14$

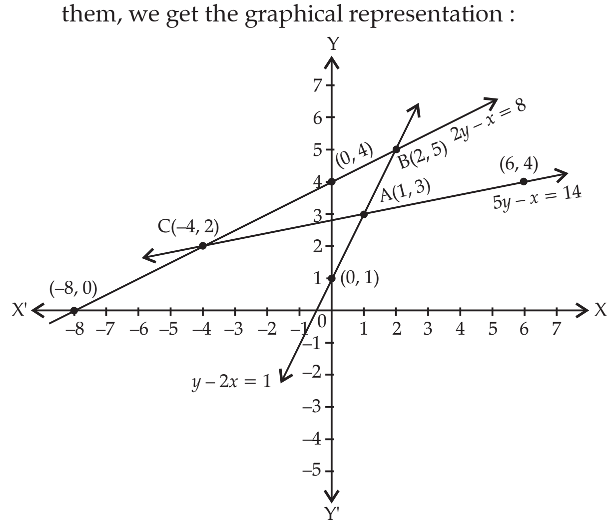
y	3	4	2
$x = 5y - 14$	1	6	– 4

and $y - 2x = 1$

$\Rightarrow \quad y = 1 + 2x$

x	0	1	2
$y = 1 + 2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation :



Hence, the coordinates of the vertices of a triangle ABC are $A(1, 3)$, $B(2, 5)$ and $C(-4, 2)$.

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OR

$x^3 - 3x - 10x + 24$

Let α, β and γ be the zeroes of given polynomial

$\therefore \quad \alpha + \beta + \gamma = 3$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = -10$ ...(ii)

$\alpha\beta\gamma = -24$ ...(iii)

Given : $\alpha = 4$ from eqn. (i) $\beta + \gamma = -1$

from eqn (ii) $\beta\gamma = -6$

$(\beta - \gamma)^2 = (\beta + \gamma)^2 - 4\beta\gamma$

$= (-1)^2 - 4(-6)$

$= 25$

$\therefore \quad \beta - \gamma = \pm 5$

$\beta - \gamma = 5$ ...(iv)

$\beta + \gamma = -1$

$2\beta = 4 \Rightarrow \beta = 2$

and $\gamma = -3$

Hence zeroes are –3, 2 and 4.

29. Let original speed of flight be x km/hr, then according to question,

$$\frac{600}{x-200} - \frac{600}{x} = 30 \text{ minutes}$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad 600\left[\frac{1}{x-200} - \frac{1}{x}\right] = \frac{30}{60}$$

$$\Rightarrow \quad \frac{x - x + 200}{x(x-200)} = \frac{1}{2 \times 600}$$

$$\Rightarrow \quad \frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$$\Rightarrow \quad x^2 - 200x = 240000$$

$$\Rightarrow \quad x^2 - 200x - 240000 = 0$$

Here, $a = 1$, $b = -200$ and $c = -240000$

$$\therefore \quad x = \frac{200 \pm \sqrt{40000 + 960000}}{2 \times 1}$$

$$= \frac{200 \pm \sqrt{1000000}}{2}$$

$$= \frac{200 \pm 1000}{2}$$

$$= \frac{200 + 1000}{2}, \frac{200 - 1000}{2}$$

$$= 600, -400$$

Since, speed cannot be negative, therefore

original speed = 600 km/hr.

and original distance = 600 km

$$\therefore \quad \text{Time} = \frac{\text{original distance}}{\text{original speed}}$$

$$= \frac{600 \text{ km}}{600 \text{ km/hr}} = 1 \text{ hr}$$

Hence, the original duration of flight is 1 hr.

30. Here $x_1 = -5$, $x_2 = -4$, $x_3 = 4$ and $y_1 = 7$, $y_2 = -5$, $y_3 = 5$

$$\therefore \text{ Area of } \Delta PQR = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-5(-5-5) - 4(5-7) + 4(7+5)]$$

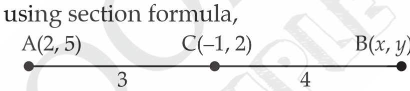
$$= \frac{1}{2}[-5(-10) - 4(-2) + 4(12)]$$

$$= \frac{1}{2}[50 + 8 + 48]$$

$$= \frac{1}{2} \times 106 = 53 \text{ sq units.}$$

OR

By using section formula,



$$-1 = \frac{mx_2 + nx_1}{m+n} = \frac{3 \times x + 4 \times 2}{3+4} = \frac{3x+8}{7}$$

$$\Rightarrow \quad 3x + 8 = -7$$

$$\Rightarrow \quad 3x = -15$$

$$\Rightarrow \quad x = -5$$

$$\text{and} \quad 2 = \frac{my_2 + ny_1}{m+n} = \frac{3 \times y + 4 \times 5}{3+4} = \frac{3y+20}{7}$$

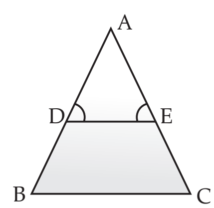
$$\Rightarrow \quad 3y + 20 = 14$$

$$\Rightarrow \quad 3y = 14 - 20 = -6$$

$$\Rightarrow \quad y = -2$$

Hence, the coordinates of $B(x, y)$ is (–5, –2).

31. Given : $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



To prove : ΔBAC is an isosceles triangle.

Proof : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

$\therefore \quad \angle ADE = \angle ABC$ (Corresponding angles)

and $\quad \angle AED = \angle ACB$ (Corresponding angles)

$\because \quad \angle ADE = \angle AED$ (Given)

$\therefore \quad \angle ABC = \angle ACB$

So, BAC is an isosceles triangle. **Hence Proved.**

32. **Given :** $AC^2 = AB^2 + BC^2$

To prove : Angle opposite to the first side is a right angle.

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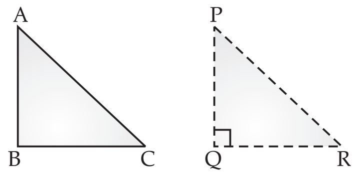


Construction : Draw ΔPQR, where $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.

Proof : In ΔPQR,

$PR^2 = PQ^2 + QR^2$ (By using Pythagoreas Theorem)

$\because$ $AB = PQ, BC = QR$ [From construction]

$\therefore$ $PR^2 = AB^2 + BC^2$

Now, $AC^2 = AB^2 + BC^2$ (given)

$\therefore$ $AC^2 = PR^2$

$\Rightarrow$ $AC = PR$

In ΔABC and ΔPQR,

$AB = PQ$ (By construction)

$BC = QR$ (By construction)

and $AC = PR$ (Proved above)

$\therefore$ $\Delta ABC \cong \Delta PQR$ (By SSS congruency rule)

So, $\angle B = \angle Q$ (By CPCT)

But $\angle Q = 90°$ (by construction)

Hence, $\angle B = =90°$ **Hence Proved.**

33. Given, $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow$ $1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow$ $2\sin\theta\cos\theta = 3 - 1 = 2$

$\Rightarrow$ $\sin\theta\cos\theta = 1$ ...(i)

Now taking LHS,

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

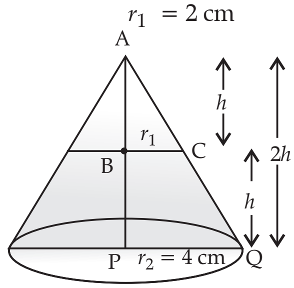
$$= \frac{1}{1} = 1 \text{ [Form eq. (i)]}$$

Hence, $\tan\theta + \cot\theta = 1$ **Hence Proved.**

34. $\Delta ABC \sim \Delta APQ$

$\therefore$ $\frac{h}{2h} = \frac{r_1}{4}$

$\Rightarrow$ $r_1 = 2$ cm



Volume of smaller cone $= \frac{1}{3}\pi r_1^2 h = \frac{1}{3}\pi(2)^2 h$

$= \frac{1}{3} \times 4\pi h$

Volume of frustum $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3}\pi \times h(2^2 + 4^2 + 2 \times 4)$

$= \frac{1}{3}\pi h \times 28$

$\therefore$ Required ratio $= \dfrac{\frac{1}{3} \times 4\pi h}{\frac{1}{3} \times 28\pi h}$

$= \frac{4}{28} = \frac{1}{7}$

SECTION - D

35. Let n be any positive integer.

By Euclid's division lemma,

$n = 5p + r$, where $0 \le r < 5$

Then, $n = 5p, 5p + 1, 5p + 2, 5p + 3$ or $5p + 4$, where $p \in N$.

Now $n^2 = (5p)^2 = 25p^2 = 5(5p^2) \Rightarrow 5q$

(where q is any integer)

$n^2 = (5p + 1)^2 = 25p^2 + 1 + 10p$

$= 5(5p^2 + 2p) + 1 = 5q + 1$

and $n^2 = (5p + 2)^2 = 25p^2 + 20p + 4$

$= 5(5p^2 + 4p) + 4 = 5q + 4$

Similarly, $n^2 = (5p + 3)^2 = 5q + 4$

and $n^2 = (5p + 4)^2 = 5q + 1$

Thus, square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$.

OR

Let n be any positive integer.

$\therefore$ $n = 3q + r$, where $r = 0, 1, 2$

Putting $r = 0$,

$n = 3q + 0 = 3q$, which is divisible by 3.

Putting $r = 1$,

$n = 3q + 1$, which is not divisible by 3.

Putting $r = 2$,

$n = 3q + 2$, which is not divisible by 3.

Hence, one of every three consecutive positive integers is divisible by 3. **Hence Proved.**

36. Let the four consecutive terms of A.P. be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

By given conditions,

$a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow$ $4a = 32 \Rightarrow a = 8$

and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow$ $\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$

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$$\Rightarrow \quad \frac{(8)^2 - 9d^2}{(8)^2 - d^2} = \frac{7}{15}$$

$$\Rightarrow \quad \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$$

$$\Rightarrow \quad 960 - 135d^2 = 448 - 7d^2$$

$$\Rightarrow \quad 7d^2 - 135d^2 = 448 - 960$$

$$\Rightarrow \quad -128d^2 = -512$$

$$\Rightarrow \quad d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

OR

Given, $a = 1$ and $d = 3$.

Let number of terms in the series be n,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad \frac{n}{2}[2 \times 1 + (n-1)3] = 287$$

$$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 287$$

$$\Rightarrow \quad \frac{n}{2}[3n - 1] = 287$$

$$\Rightarrow \quad 3n^2 - n = 574$$

$$\Rightarrow \quad 3n^2 - n - 574 = 0$$

$$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow \quad 3n(n-14) + 41(n-14) = 0$$

$$\Rightarrow \quad (n-14)(3n+41) = 0$$

i.e., $n = 14$ or $n = -\frac{41}{3}$, it is not possible

Thus, the 14th term is x,

$$a + (n-1)d = x$$

$$\Rightarrow \quad x = 1 + (14-1)3$$

Hence, $\quad x = 40$

37. Steps of construction :

(i) Draw a line segment $AB = 7$ cm.

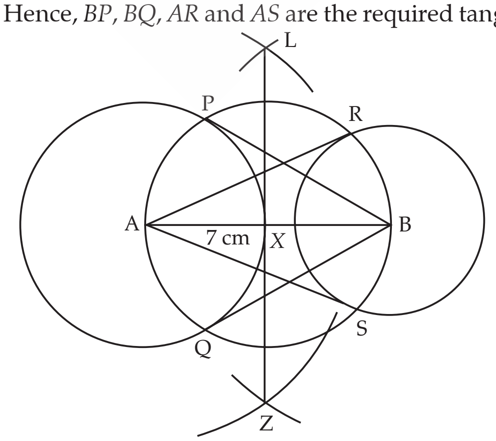
(ii) With A as centre and radius 3 cm draw a circle.

(iii) With B as centre and radius 2 cm draw another circle.

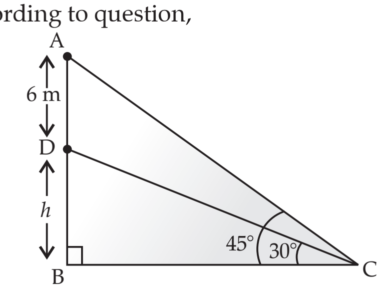
(iv) Taking AB as diameter draw another circle, which intersects first two circles at P and Q, R and S.

(v) Join B to P, B to Q, A to R and A to S.

Hence, BP, BQ, AR and AS are the required tangents.



38. According to question,



AD is a flagstaff and BD is a tower.

In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{h+6}{BC}$$

$$\Rightarrow \quad BC = h + 6 \quad ...(i)$$

In ΔDBC,

$$\tan 30° = \frac{DB}{BC} \quad \text{[from (i)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{h+6}$$

$$\Rightarrow \quad h\sqrt{3} = h + 6$$

$$\Rightarrow \quad h\sqrt{3} - h = 6$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 6$$

$$\Rightarrow \quad h = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{6(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad h = 3(\sqrt{3}+1) = 3(1.73 + 1)$$

$$\Rightarrow \quad h = 3 \times 2.73$$

$$\Rightarrow \quad h = 8.19 \text{ m.}$$

39. Height of frustum, $h = 30$ cm,

$R_1 = 20$ cm and $R_2 = 10$ cm

$$\text{Capacity of the bucket} = \frac{1}{3}\pi h[R_1^2 + R_2^2 + R_1R_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 30[(20)^2 + (10)^2 + 20 \times 10]$$

$$= \frac{220}{7}[400 + 100 + 200]$$

$$= \frac{220}{7} \times 700$$

$$= 22000 \text{ cm}^3$$

$$= 22000 \times \frac{1}{1000} \text{ litre}$$

$$[\because 1 \text{ litre} = 1000 \text{ cm}^3]$$

$$= 22 \text{ litre.}$$

Total cost of milk which can completely **fill the bucket = ₹ 40 × 22 = ₹ 880.**

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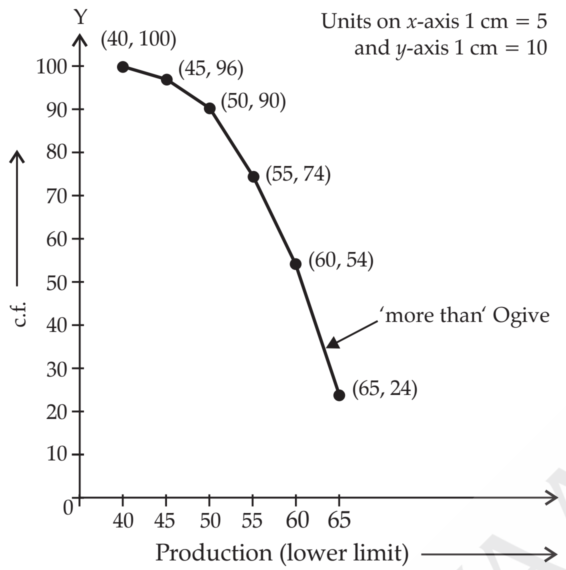
40.

Production yield/hectare	*C.f.*
more than 40	100
more than 45	96
more than 50	90
more than 55	74
more than 60	54
more than 65	24



OR

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	***N* = 100**	

Also, $76 + x + y = 100$

$\Rightarrow \quad x + y = 100 - 76 = 24$ ...(i)

Given, Median = 525, which lies between class 500 – 600.

$\Rightarrow$ Median class = 500 – 600

Now, $\quad \text{Median} = l + \dfrac{\frac{n}{2} - c.f.}{f} \times h$

$$\Rightarrow \quad 525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$$\Rightarrow \quad 25 = (50 - 36 - x)\, 5$$

$$\Rightarrow \quad 14 - x = \frac{25}{5} = 5$$

$$\Rightarrow \quad x = 14 - 5 = 9$$

Putting the value of x is eq. (i), we get

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$.

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Delhi Set-II — Code No. 30/1/2

SECTION - A

14. 1

Explanation : $\left(\dfrac{\sin 35°}{\cos 55°}\right)^2 + \left(\dfrac{\cos 43°}{\sin 47°}\right) - 2 \cos 60°$

$$= \left[\frac{\sin(90° - 55°)}{\cos 55°}\right]^2 + \left[\frac{\cos(90° - 47°)}{\sin 47°}\right] - 2 \cos 60°$$

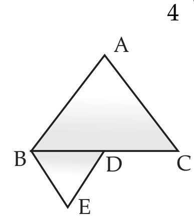
$$= \left(\frac{\cos 55°}{\cos 55°}\right)^2 + \left(\frac{\sin 47°}{\sin 47°}\right)^2 - 2 \cos 60°$$

$$= (1)^2 + (1)^2 - 2 \times \frac{1}{2}$$

$$= 1 + 1 - 1 = 1$$

15. 4 : 1

Explanation : $\dfrac{ar(\Delta ABC)}{ar(\Delta BDE)} = \dfrac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2}$



$$= \frac{(BC)^2}{\left(\frac{(BC)}{2}\right)^2}$$

$$= \frac{4BC^2}{BC^2} = \frac{4}{1}$$

$= 4 : 1$

20. Total possible outcomes $= 6$

Favourable outcomes $= \{2\}$ *i.e.*, 1

$$P\text{ (getting an even prime number)} = \frac{1}{6}$$

SECTION - B

25. Given A.P. : 1, 4, ,7, 10,, ...

Here, $a = 1, d = 4 - 1 = 3$ and $n = 20$

$$\therefore \quad S_{20} = \frac{n}{2}[2a + (n-1)d]$$

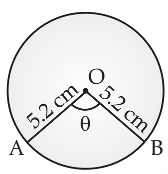
$$= \frac{20}{2}[2 \times 1 + (20-1)3]$$

$$= 10(2 + 57)$$

$$= 10 \times 59 = 590$$

26. Perimeter of the sector $= 2r + \frac{2\pi r\theta}{360°}$

$$\Rightarrow \quad 16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360°}$$



$$\Rightarrow \quad \frac{2\pi \times 5.2 \times \theta}{360°} = 6$$

$$\Rightarrow \quad \theta = \frac{6 \times 360°}{2\pi \times 5.2}$$

Now, area of sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{6 \times 360°}{2\pi \times 5.2 \times 360°} \times \pi \times (5.2)^2$$

$= 15.6$ sq. units.

SECTION - C

32. Let the speed of the train $= x$ km/hr

Total distance covered by the train = 480 km

$\therefore$ Time taken cover the distance 480 km $= \frac{480}{x}$ hr

If the speed has increased 8 km/hr, *i.e.*, $(x + 8)$ km/hr

Then, time taken to cover the distance 480 km $= \frac{480}{x-8}$ hr.

According to question,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow \quad 480\left[\frac{x - x + 8}{x(x-8)}\right] = 3$$

$$\Rightarrow \quad \frac{8}{x^2 - 8x} = \frac{3}{480} = \frac{1}{160}$$

$$\Rightarrow \quad x^2 - 8x - 1280 = 0$$

Compare with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -8$ and $c = -1280$

$$\therefore \quad x = \frac{8 \pm \sqrt{64 + 4 \times 1280}}{2 \times 1} = \frac{8 \pm \sqrt{5184}}{2}$$

$$= \frac{8 \pm 72}{2} = \frac{8 + 72}{2}, \frac{8 - 72}{2}$$

$$= \frac{80}{2}, \frac{-64}{2} = 40, -32$$

Since, negative speed cannot be possible.

Hence, the original speed of the train = 40 km/hr.

33. Let $ABCD$ be the parallelogram.

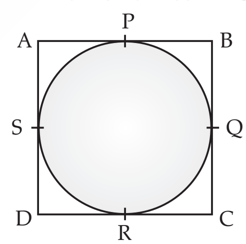
$\therefore \quad AB = CD$ and $AD = BC$ ...(i)

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$.

Adding the above equations.

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$



$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow \quad AB + CD = AD + BC$$

From eq. (i),

$$2AB = 2AD$$

or, $\quad AB = AD$

Hence, $ABCD$ is a rhombus. **Hence Proved.**

34. L.H.S.

$= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta]$
$- 3(\sin^4\theta + \cos^4\theta) + 1$

$[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1 \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= -1 + 1$

$= 0 =$ R.H.S.

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SECTION - D

39. Height of a frustum of a cone, $h = 16$ cm $R_1 = 8$ cm and $R_2 = 20$ cm

Quantity of milk in a bucket

$$= \frac{1}{3}\pi h\,[R_1^2 + R_2^2 + R_1R_2]$$

$$= \frac{1}{3} \times 3.14 \times 16\,[(8)^2 + (20)^2 + 8 \times 20]$$

$$= \frac{3.14 \times 16}{3}\,[64 + 400 + 160]$$

$$= \frac{3.14 \times 16 \times 624}{3}$$

$$= 3.14 \times 16 \times 208 \text{ cm}^3$$

$$= 10449.92 \times \frac{1}{1000} \text{ litre}$$

$$= 10.45 \text{ litre}$$

Total cost of milk, which can completely fill the bucket = ₹ 40 × 10.45 litre

= ₹ 418

40. Steps of construction :

1. Draw a line segment $BC = 5$ cm.

2. With B as centre and radius, $AB = 4$ cm, draw an arc.

3. With C as centre and radius, $AC = 6$ cm, draw another arc, intersecting the arc drawn in step 2 at the point A.

4. Join AB and AC to obtain ΔABC.

5. Below BC, make an acute angle $\angle CBX$.

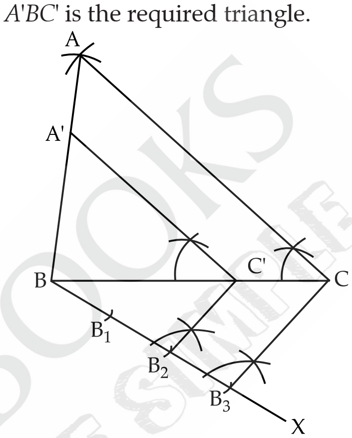
6. Along BX mark off three points B_1, B_2, B_3 such that $BB_1 = B_1B_2 = B_2B_3$.

7. Join B_3C.

8. From B_2, draw $B_2C' \parallel B_3C$.

9. From C', draw $C'A' \parallel CA$, meeting BA at the point A'.

Then $A'BC'$ is the required triangle.

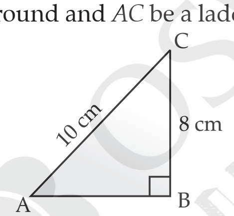


Delhi Set-III — Code No. 30/1/3

SECTION - A

14. 6 m

Explanation : Let BC be the height of the window above the ground and AC be a ladder.



Here, $BC = 8$ cm and $AC = 10$ cm

∴ In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

(By using pythogoras Theorem)

$$\Rightarrow \quad (10)^2 = AB^2 + (8)^2$$

$$\Rightarrow \quad AB^2 = 100 - 64 = 36$$

$$\Rightarrow \quad AB = 6 \text{ m}$$

15. 0

Explanation : $\dfrac{2\cos 67°}{\sin 23°} - \dfrac{\tan 40°}{\cot 50°} - \cos 0°$

$$= \frac{2\cos(90° - 23°)}{\sin 23°} - \frac{\tan(90° - 50°)}{\cot 50°} - \cos 0°$$

$$= \frac{2\sin 23°}{\sin 23°} - \frac{\cot 50°}{\cot 50°} - \cos 0°$$

$$= 2 - 1 - 1 \qquad [\because \cos 0° = 1]$$

$$= 0$$

20. Total possible outcomes $= 6 \times 6 = 36$

Favourable outcomes $= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

i.e.,

$$\therefore \quad P(\text{getting doublet}) = \frac{6}{36} = \frac{1}{6}$$

SECTION - B

25. ∵ Angle subtended in 1 minutes = 6°

∴ Angle subtended in 35 minutes = 6° × 35 = 210°

Area of the face of the clock by the minute hand

= Area of the sector

$$= \frac{\pi r^2 \theta}{360°}$$

$$= \frac{22}{7} \times \frac{12 \times 12 \times 210°}{360°}$$

$$= \frac{665280}{2520} = 264 \text{ cm}^2$$

26. Since, $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

Given, $S_7 = 63$

So, $S_7 = \dfrac{7}{2}[2a + 6d] = 63$

or, $2a + 6d = 18$ ...(i)

Now, sum of 14 terms is :

$$S_{14} = S_{\text{first 7 terms}} + S_{\text{next 7 terms}}$$

$$= 63 + 161 = 224$$

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$\therefore \quad \frac{14}{2}[2a + 13d] = 224$

$\Rightarrow \quad 2a + 13d = 32 \quad ...(ii)$

On subtracting (i) from (ii), we get

$(2a + 13d) - (2a + 6d) = 32 - 18$

$\Rightarrow \quad 7d = 14$

$\Rightarrow \quad d = 2$

Putting the value of d in (i), we get

$a = 3$

Hence, the $A. P.$ will be : 3, 5, 7, 9, ...

SECTION - C

32. Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

$\therefore$ Relative Speed of boat in upstream $= (x - y)$ km/hr and Relative speed of boat in downstream $= (x + y)$ km/hr

According to question, $\frac{20}{x+y} = 2$

$\Rightarrow \quad x + y = 10 \quad ...(i)$

and $\frac{4}{x-y} = 2$

$\Rightarrow \quad x - y = 2 \quad ...(ii)$

On adding eq. (i) and (ii), we get

$2x = 12$

$\Rightarrow \quad x = 6$

Putting the value of x is eq. (i),

$6 + y = 10$

$\Rightarrow \quad y = 10 - 6 = 4$

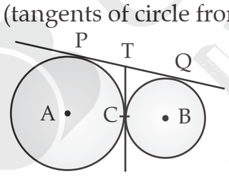
Speed of a boat in still water = 6 km/hr

and speed of the stream = 4 km/hr.

33. Since, $PT = TC$ (tangents of circle)

and $QT = TC$

(tangents of circle from extended point)



So, $PT = QT$

Now $PQ = PT + TQ$

$\Rightarrow \quad PQ = PT + PT$

$\Rightarrow \quad PQ = 2PT$

$\Rightarrow \quad \frac{1}{2}PQ = PT$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

34. L.H.S. $= \frac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

= R.H.S. **Hence Proved**

SECTION - D

39. In ΔABC,

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 105° + 45° + \angle C = 180°$

$\Rightarrow \quad \angle C = 30°$

Steps of Constructions :

(i) Draw BC = 7 cm.

(ii) Construct $\angle CBY = 45°$ and $\angle BCZ = 30°$.

(iii) Rays BY and CZ intersect at A.

(iv) ΔABC is given.

(v) From B, draw a ray BX below BC making acute angle with BC.

(vi) Along it mark 4 points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

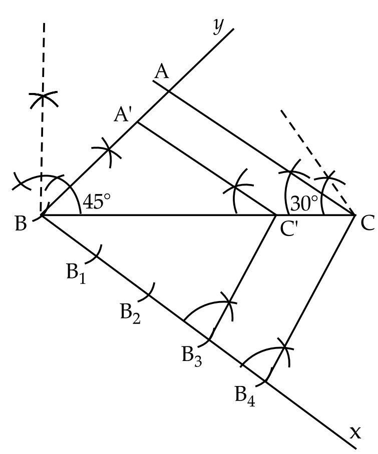
(vii) Join B_4C. Make $\angle BB_4C$ at B_3 such that the ray intersects BC at C'.

$\therefore \quad \angle BB_4C = \angle BB_3C'$.

So, $\quad B_4C \parallel B_3C'$.

(viii) From C', make $\angle BC'A' = \angle BCA$ so that $C'A' \parallel CA$.

Thus, $A'BC'$ is the required triangle.



40. Let AB be a building, then $AB = 7$ cm

CD be the height of tower, so

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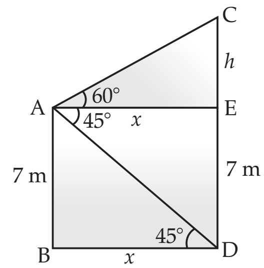


$$CD = (7 + h)\text{m}$$
$$BD = AE = x \text{ m}$$

In ΔABD,

$$\tan 45° = \frac{AB}{BD}$$
$$\Rightarrow \quad 1 = \frac{7}{x}$$
$$\Rightarrow \quad x = 7 \text{ cm}$$

In ΔCEA,

$$\tan 60° = \frac{CE}{AE}$$
$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$
$$\Rightarrow \quad h = x\sqrt{3}$$

putting the value of x, we get

$$h = 7\sqrt{3}$$

Now, $CD = CE + ED$

$= (7 + 7\sqrt{3})$ m

Hence, height of tower $= 7(1 + \sqrt{3})$ m

$= 7(1 + 1.732)$ m

$= 7 \times 2.732$ m

$= 19.124$ m

$= 19.12$ m (Approx)

❑❑

Outside Delhi Set-I — Code No. 30/2/1

SECTION - A

1. **Correct option :** (b)

   Explanation :

   Prime factors of 196 $= 2^2 \times 7^2$

2	196
2	98
7	49
7	7
	1

   $\therefore$ The sum of exponents of prime factor

   $= 2 + 2 = 4.$

2. **Correct option :** (c)

   Explanation : For given positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$ where $0 \le r < b$.

3. **Correct option :** (b)

   Explanation : Given, $x^2 - 3x - m(m + 3)$

   putting $x = -m$, we get

   $= (-m)^2 - 3(-m) - m(m + 3)$

   $= m^2 + 3m - m^2 - 3m = 0,$

   putting $x = m + 3$, we get

   $= (m + 3)^2 - 3(m + 3) - m(m + 3)$

   $= (m + 3)[m + 3 - 3 - m]$

   $= (m + 3)[0] = 0.$

   Hence, $-m$ and $m + 3$ are the zeroes of given polynomial.

4. **Correct option :** (d)

   Explanation :

   $x + 2y - 3 = 0$

   and $5x + ky + 7 = 0$

   If system is inconsistent, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

   Taking first two orders, we have

$$\frac{1}{5} = \frac{2}{k} \Rightarrow k = 10.$$

5. **Correct option :** (a)

   Explanation :

$$x^2 - 0.04 = 0$$
$$\Rightarrow \quad x^2 = 0.04$$
$$\Rightarrow \quad x = \pm\sqrt{0.04}$$
$$\Rightarrow \quad x = \pm 0.2.$$

6. **Correct option :** (c)

   Explanation :

   Given A.P. $= \frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$

   $\therefore$ Common difference $= \frac{1-p}{p} - \frac{1}{p}$

   $= \frac{1-p-1}{p} = \frac{-p}{p}$

   $= -1.$

7. **Correct option :** (b)

   Explanation :

   Given A.P. $= a, 3a, 5a, \ldots$

   Here first term, $a = a$ and $d = 3a - a = 2a$

   $\therefore$ n^{th} term $= a + (n-1)d$

   $= a + (n-1)2a$

   $= a + 2na - 2a$

   $= 2na - a = (2n - 1)a.$

8. **Correct option :** (a)

   Explanation : Let the position of the point P on x-axis be $(x, 0)$, then

$$PA^2 = PB^2$$

$\Rightarrow \quad (x+1)^2 + (0)^2 = (5-x)^2 + (0)^2$

$\Rightarrow \quad x^2 + 2x + 1 = 25 + x^2 - 10x$

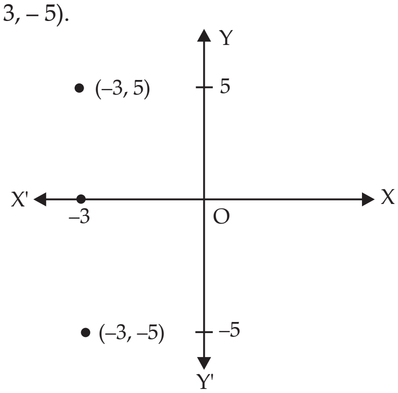
$\Rightarrow \quad 2x + 10x = 25 - 1$

$\Rightarrow \quad 12x = 24 \Rightarrow x = 2$

Hence, the point $P(x, 0)$ is $(2, 0)$.

9. Correct option : (c)

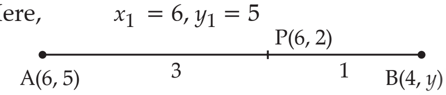
Explanation : By using the graph of coordinate plane, we have the reflection of point $(-3, 5)$ is x-axis is $(-3, -5)$.



10. Correct option : (d)

Explanation :

Here, $x_1 = 6, y_1 = 5$



and $x_2 = 4, y_2 = y$

Then $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$

$\therefore \quad 2 = \dfrac{3 \times y + 1 \times 5}{3+1} = \dfrac{3y+5}{4}$

$\Rightarrow \quad 3y + 5 = 8$

$\Rightarrow \quad 3y = 8 - 5 = 3$

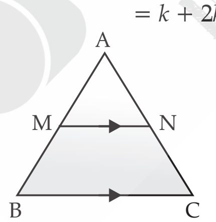
$\Rightarrow \quad y = 1.$

11. Here $AM = k, MB = 2k.$

and $MN \parallel BC$

$\therefore \quad AB = AM + MB$

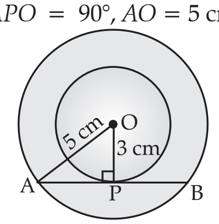
$= k + 2k = 3k$



$\therefore \quad \dfrac{\text{ar}(\Delta AMN)}{\text{ar}(\Delta ABC)} = \dfrac{AM^2}{AB^2} = \dfrac{k^2}{9k^2} = \dfrac{1}{9}.$

12. Since AB is a tangent at P and OP is radius.

$\therefore \quad \angle APO = 90°, AO = 5$ cm and $OC = 3$ cm



In right angled ΔOPA,

$AP^2 = AO^2 - OP^2$

(By pythagoras theorem)

$AP^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

$\Rightarrow \quad AP = 4$ cm

$\because$ Perpendicular from centre to chord bisect the chord

$\Rightarrow \quad AP = BP = 4$ cm.

13. Given that AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

It can be observed that

$AB^2 = 108$ cm, $AC^2 = 144$ cm and $BC^2 = 36$ cm

Now $AB^2 + BC^2 = 108 + 36 = 144$ cm and $AC^2 = 144$ cm

i.e., $AB^2 + BC^2 = AC^2$, which is satistys Pythagoras theorem

So, $\angle B = 90°$.

OR

Two triangles are similar if their corresponding sides are in the same ratio.

14. tan 1° tan 2° tan 3° .... tan 89°

= (tan 1° tan 89°) (tan 2° tan 88°) (tan 3° tan 87°) ... (tan 45° tan 45°)

= [tan 1° tan (90° – 1)] [tan 2° tan (90° – 2)] [tan 3° tan (90° – 3)] .... [tan 45° tan (90° – 45°)]

= (tan 1° cot 1°) (tan 2° cot 2°) (tan 3° cot 3°) .... [tan 45° cot 45°]

$= \left(\tan 1° \times \dfrac{1}{\tan 1°}\right)\left(\tan 2° \times \dfrac{1}{\tan 2°}\right)\left(\tan 3° \times \dfrac{1}{\tan 3°}\right) \ldots \left[\tan 45° \times \dfrac{1}{\tan 45°}\right]$

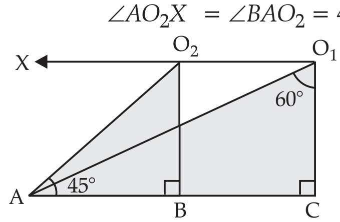
$= 1 \times 1 \times 1 \ldots\ldots\ldots\ldots 1 \times 1$

$= 1.$

15. Draw $AC \parallel O_1X$

$\therefore \quad \angle AO_1X = 90° - 60° = 30°$

and $\angle AO_2X = \angle BAO_2 = 45°.$



16. Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \quad \sin A = 1 - \sin^2 A = \cos^2 A$

On squaring both sides, we get

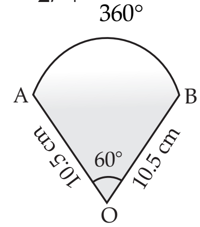
$\sin^2 A = \cos^4 A$

$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \quad \cos^2 A + \cos^4 A = 1.$

17. Perimeter of the sector

$= 2r + \dfrac{2\pi r\theta}{360°}$



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$$= 2 \times 10.5 + \frac{2 \times 22 \times 10.5 \times 60°}{7 \times 360°}$$

$$= 21 + 11$$

$$= 32 \text{ cm.}$$

18.

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9

Total possible outcomes = 7

Favourable outcomes = $x^2 < 4$ *i.e.*, $x = -1, 0, 1$ = 3

$$P(x^2 < 4) = \frac{3}{7}$$

OR

Number of days in a leap year = 366

$$\text{Number of weeks} = \frac{366}{7} = 52.28$$

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on remaining 2 days

The possible pairing of days are

Sunday — Monday

Monday — Tuesday

Tuesday — Wednesday

Wednesday — Thursday

Thursday — Friday

Friday — Saturday

Saturday — Sunday

There are total 7 pairs and out of 7 pairs, only 2 pairs have Sunday. The remaining 5 pairs does not include Sunday.

Therefore, the probability of only 52 Sundays in a Leap year is $\frac{5}{7}$

19. Class mark of 10 – 25 $= \frac{10+25}{2} = \frac{35}{2} = 17.5$

and class mark of 35 – 55 $= \frac{35+55}{2} = \frac{90}{2} = 45.$

20. Total possible outcomes = 6

Favourable outcomes = {2, 3, 5} *i.e.*, 3

$\therefore$ Probability $= \frac{3}{6} = \frac{1}{2}.$

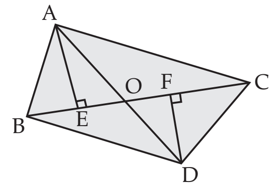
SECTION - B

21. (i) $x^3 + \sqrt{3x} + 7$, $2x^2 + 3 - \frac{5}{x}$ and $x + \frac{1}{x}$ are not polynomials.

(ii) $3x^2 + 7x + 2$ is only one quadratic polynomial.

22. To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{AO}{DO}$

Construction : Draw $AE \perp BC$ and $DF \perp BC$.



Proof : In ΔAOE and ΔDOF,

$\angle AOE = \angle DOF$ (vertically opposite angles)

$\angle AEO \sim \angle DFO$ (Each 90°)

$\Delta AOE \sim \Delta DOF$ (By AA similarity)

$$\frac{AO}{DO} = \frac{AE}{DF} \quad \text{...(i)}$$

Now, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF}$$

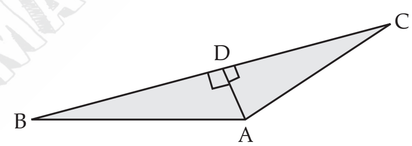
Hence, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{AO}{DO} \quad \text{[From eq. (i)]}$$

Proved

OR

In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad \text{...(i)}$$



In right ΔADB,

$$AB^2 = AD^2 + BD^2 \quad \text{...(ii)}$$

Subtracting eq. (i) from eq. (ii),

$$AB^2 - AC^2 = BD^2 - CD^2$$

Hence $$AB^2 + CD^2 = AC^2 + BD^2$$ **Proved**

23. $$\text{L.H.S} = 1 + \frac{\cot^2\alpha}{1+\text{cosse } \alpha}$$

$$= 1 + \frac{\text{cosec}^2\alpha - 1}{1+\text{cosse } \alpha}$$

$$= 1 + \frac{(1+\text{cosec}\,\alpha)(\text{cosec}\,\alpha - 1)}{1+\text{cosec } \alpha}$$

$$= 1 + \text{cosec } \alpha - 1$$

$$= \text{cosec } \alpha = \text{R.H.S.}$$

OR

$$\text{L.H.S.} = \tan^4\theta + \tan^2\theta$$

$$= \tan^2\theta\,(1 + \tan^2\theta)$$

$$= \tan^2\theta \times \sec^2\theta$$

$$= (\sec^2\theta - 1)\sec^2\theta$$

$$= \sec^4\theta - \sec^2\theta = \text{R.H.S.}$$

24. Given,

Volume of a right circular cylinder = $25\frac{1}{7}$ cm

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i.e., $\pi r^2 h = \frac{176}{7}$

where height, h = radius r, then

$\Rightarrow \quad \frac{22}{7} \times h^2 \times h = \frac{176}{7}$

$\Rightarrow \quad h^3 = \frac{176}{22} = 8 = 2^3.$

Hence, height of the cylinder = 2 cm.

25. Total possible outcomes, $n(S) = 6$

(i) Let E_1 = getting event letter A, then

$n(E_1) = 2$

$\therefore \quad \text{Probability} = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(ii) Let E_2 = getting event letter D, then

$n(E_2) = 1$

$\therefore \quad \text{Probability} = \frac{n(E_2)}{n(S)} = \frac{1}{6}$

26. Here, Modal class = 12 – 16

$\therefore l = 12, f_1 = 17, f_0 = 9, f_2 = 12$ and $h = 4$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 12 + \left(\frac{17-9}{2\times 17 - 9 - 12}\right) \times 4$$

$$= 12 + \frac{8 \times 4}{13}$$

$$= 12 + 2.46 = 14.46.$$

SECTION - C

27. Given,

$2x + y = 23$ ...(i)

and $4x - y = 19$ ...(ii)

On adding eq. (i) and (ii), we get

$6x = 42 \Rightarrow x = 7$

Putting the value of x in eq. (i), we get

$14 + y = 23$

$\Rightarrow \quad y = 23 - 14 = 9$

Hence, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14$

$= 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = \frac{-5}{7}$

OR

Given, $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$

$\Rightarrow \quad \frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$

$\Rightarrow \quad \frac{3}{x^2+4x+7x+28} = \frac{11}{30}$

$\Rightarrow \quad \frac{3}{x^2+11x+28} = \frac{11}{30}$

$\Rightarrow \quad 11x^2 + 121x + 308 = 90$

$\Rightarrow \quad 11x^2 + 121x + 218 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 11, b = 121$ and $c = 218$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-121 \pm \sqrt{14641 - 9592}}{22}$$

$$\Rightarrow \quad x = \frac{-121 \pm \sqrt{5049}}{22}$$

$$= \frac{-121 \pm 71.06}{22}$$

$$\Rightarrow \quad x = \frac{-121 + 71.06}{22}, \frac{-121 - 71.06}{22}$$

$$\Rightarrow \quad x = \frac{-49.94}{22}, \frac{-192.06}{22}$$

$$\Rightarrow \quad x = -2.27, -8.73.$$

28. Given, first term, $A = a$

and second term = b

$\Rightarrow$ common difference, $d = b - a$

Last term, $l = c$

$\Rightarrow \quad A + (n-1)\,d = c$

$\Rightarrow \quad a + (n-1)\,d = c$

$a + (n-1)(b-a) = c$

$\Rightarrow \quad (b-a)(n-1) = c - a$

$\Rightarrow \quad n - 1 = \frac{c-a}{b-a}$

$\Rightarrow \quad n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$

$\Rightarrow \quad n = \frac{b+c-2a}{b-a}$

Now $\quad \text{sum} = \frac{n}{2}\,[\text{A} + l]$

$$= \frac{(b+c-2a)}{2(b-a)}\,[a + c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$

Hence Proved.

OR

Given, $a = 1$ and $d = 4 - 1 = 3$

Let number of terms is the series be n, the

$$S_n = \frac{n}{2}\,[2a + (n-1)\,d]$$

$$\Rightarrow \quad \frac{n}{2}\,[2 \times 1 + (n-1)\,3] = 287$$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 287$

$\Rightarrow \quad 3n^2 - n - 574 = 0$

$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow \quad 3n(n - 14) + 41(n - 14) = 0$

$\Rightarrow \quad (n - 14)(3n + 41) = 0$

Either $n = 14$ or $n = -\frac{41}{3}$, it is not possible

Thus 14^{th} thus is x

$\therefore \quad a + (n - 1)d = x$

$\Rightarrow \quad x = 1 + 13 \times 3 = 40.$

29. Please see the solution of Question No. 29 of Delhi Set - I

30. Here, $\frac{3+k}{2} = x$

and $y = \frac{4+6}{2} = \frac{10}{2} = 5$

Given, $x + y - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} + 5 - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} = 5$

$\Rightarrow \quad 3 + k = 10$

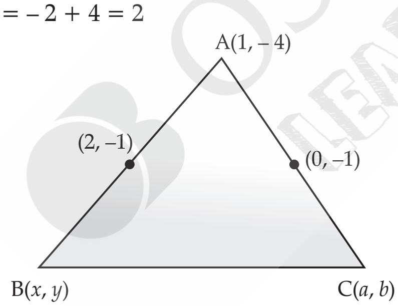
$\Rightarrow \quad k = 10 - 3 = 7.$

OR

Let the coordinates of the points B and C be (x, y) and (a, b), then $\frac{x+1}{2} = 2$

$\Rightarrow x = 4 - 1 = 3$ and $\frac{y-4}{2} = -1$

$\Rightarrow y = -2 + 4 = 2$



Similarly, $\frac{a+1}{2} = 0 \Rightarrow a = -1$

and $\frac{b-4}{2} = -1 \Rightarrow b = -2 + 4 = 2$

So, the coordinates of B and C are (3, 2) and (– 1, 2)

Here, $x_1 = 1, y_1 = -4, x_2 = 3, y_2 = 2$ and $x_3 = -1$, $y_3 = 2$

$\therefore$ Area of ΔABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2 - 2) + 3(2 + 4) - 1(-4 - 2)]$

$= \frac{1}{2}[0 + 18 + 6]$

$= \frac{1}{2} \times 24$

= 12 square units.

31. Given, $\Delta ABC \sim \Delta DEF$

Then according to question,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$\Rightarrow \quad \frac{2x-1}{2x+2} = \frac{18}{3x+9}$

$\Rightarrow \quad (2x - 1)(3x + 9) = 18(2x + 2)$

$\Rightarrow \quad (2x - 1)(x + 3) = 6(2x + 2)$

$\Rightarrow \quad 2x^2 - x + 6x - 3 = 12x + 12$

$\Rightarrow \quad 2x^2 + 5x - 12x - 15 = 0$

$\Rightarrow \quad 2x^2 - 7x - 15 = 0$

$\Rightarrow \quad 2x^2 - 10x + 3x - 15 = 0$

$\Rightarrow \quad 2x(x - 5) + 3(x - 5) = 0$

$\Rightarrow \quad (x - 5)(2x + 3) = 0$

Either $x = 5$ or $x = \frac{-3}{2}$, it is not possible

So, $x = 5$

Then in ΔABC, we have

$AB = 2x - 1 = 2 \times 5 - 1 = 9$

$BC = 2x + 2 = 2 \times 5 + 2 = 12$

$AC = 3x = 3 \times 5 = 15$

and in ΔDEF, we have

$DE = 18$

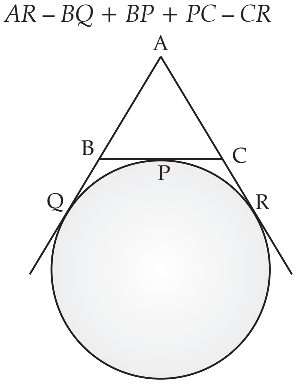
$EF = 3x + 9 = 3 \times 5 + 9 = 24$

$DE = 6x = 6 \times 5 = 30.$

32. $BC + CA + AB$

$= (BP + PC) + (AR - CR) + (AQ - BQ)$

$= AQ + AR - BQ + BP + PC - CR$



$\because$ From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

From the point C, $CP = CR$

$\therefore$ Perimeter of ΔABC, *i.e.*,

$$AB + BC + CA = 2AQ - BQ + BQ + CR - CR]$$

$$\Rightarrow \quad 2AQ = AB + BC + CA$$

$$\Rightarrow \quad AQ = \frac{1}{2}(BC + CA + AB)$$

Hence proved.

33. Given, $\sin\theta + \cos\theta = \sqrt{2}$

On squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2} \quad \text{...(i)}$$

Now taking L.H.S.,

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\ \sin\theta}$$

$$= \frac{1}{\sin\theta\ \cos\theta} = \frac{1}{1/2}$$

[From eq. (i)]

$$= 2 = R.H.S.$$

34. Area of a circular play ground = 22176 cm^2

i.e., $\pi r^2 = 22176\ cm^2$

$$\Rightarrow \quad r^2 = 22176 \times \frac{7}{22} = 7056$$

$$\Rightarrow \quad r = 84 \text{ cm} = 0.84 \text{ m}$$

Cost of fencing this ground = ₹ 50 $\times 2\pi r$

$$= ₹\ 50 \times 2 \times \frac{22}{7} \times 0.84$$

= ₹ 264.

SECTION - D

35. Let $\sqrt{5}$ be a rational number.

$$\therefore \quad \sqrt{5} = \frac{p}{q},$$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get

$$5 = \frac{p^2}{q^2}$$

or $\quad p^2 = 5q^2$

$\because p^2$ is divisible by 5

$\therefore p$ is divisible by 5

Let $p = 5r$ for some positive integer r,

$$p^2 = 25r^2$$

$$\therefore \quad 5q^2 = 25r^2$$

or $\quad q^2 = 5r^2$

$\because q^2$ is divisible by 5

$\therefore q$ is divisible by 5.

Here p and q are divisible by 5, which contradicts the fact that p and q are coprimes.

Hence, our assumption is false

$\therefore \sqrt{5}$ is an irrational number.

36. Let time taken to fill the pool by the larger diameter pipe = x hr

and time taken to fill the pool by the smaller diameter pipe = y hr

According to question,

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \quad \text{...(i)}$$

and
$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad \text{...(ii)}$$

Multiplying by 9 in eq. (i) and subtracting from eq. (ii), we get

$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2}$$

$$\frac{9}{x} + \frac{9}{y} = \frac{9}{12}$$

$$(-)\quad(-)\qquad(-)$$

$$\frac{-5}{x} = \frac{1}{2} - \frac{9}{12} = \frac{-3}{12}$$

$$\Rightarrow \quad 3x = 12 \times 5$$

$$\Rightarrow \quad x = 20$$

Putting the value of x in eq. (i), we get

$$\frac{1}{20} + \frac{1}{y} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60}$$

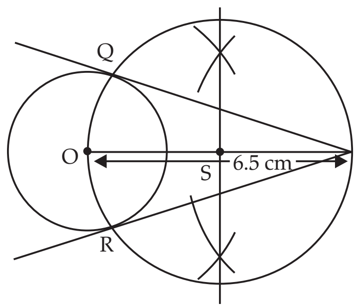
$$\Rightarrow \quad \frac{1}{y} = \frac{2}{60} = \frac{1}{30}$$

$$\Rightarrow \quad y = 30.$$

Hence, time taken to fill the fool by the larger and smaller diameter pipe respectively 20 hrs and 30 hrs.

37. Steps of construction :

(i) Draw a line segment OP = 6.5 cm.



(ii) Taking O as centre and radius 2 cm, draw a circle.

(iii) Taking OP as diameter draw another circle which intersects the first circle at Q and R.

(iv) Join P to Q and P to R.

Hence PQ and PR are two tangents.

OR

Steps of construction :

(i) Draw a line segment $BC = 5$ cm.

(ii) With B and C as centres and radii 7 cm and 6 cm draw two arcs, which intersects at A.

(iii) Join BA and CA to obtain ΔABC.

(iv) Below BC make an acute angle CBX.

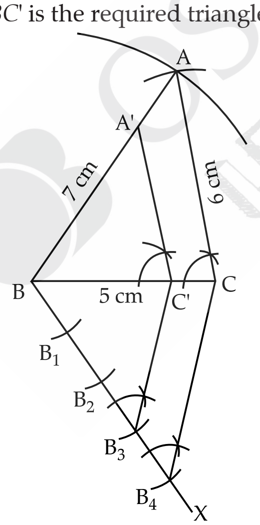
(v) Mark B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vi) Join B_4 to C.

(vii) Draw a line segment B_3C, || B_4C to meet BC at C.

(viii) Draw line segment $C'A'$ || CA to meet AB at A'

Hence, A' BC' is the required triangle.



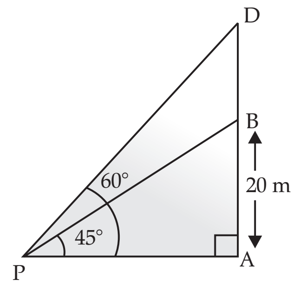
38. Let the height of the tower be BD

In ΔPAB,

$$\tan 45° = \frac{AB}{AP}$$

$$\Rightarrow \quad 1 = \frac{20}{AP}$$

$$\Rightarrow \quad AP = 20\text{m}$$



In ΔPAD,

$$\tan 60° = \frac{AD}{AP} = \frac{20 + BD}{20}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20 + BD}{20}$$

$$\Rightarrow \quad 20 + BD = 20\sqrt{3}$$

$$\Rightarrow \quad BD = 20\sqrt{3} - 20 = 20\,(\sqrt{3} - 1)$$

$$= 20\,(1.732 - 1) = 20 \times 0.732$$

$$= 14.64 \text{ cm.}$$

39. Given, $PQ = 24$ cm, $PR = 7$ cm

We know that the angle in the semicircle is right angle.

Here, $\angle RPQ = 90°$

In ΔRPQ,

$$RQ^2 = PR^2 + PQ^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$$

$$\therefore \quad RQ = 25 \text{ cm}$$

$$\therefore \text{ Area of } \Delta RPQ = \frac{1}{2} \times RP \times PQ$$

$$= \frac{1}{2} \times 7 \times 24 = 84 \text{ cm}^2$$

$$\text{and area of semi-circle} = \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2$$

$$= \frac{11 \times 625}{7 \times 4} = \frac{6875}{28} \text{ cm}$$

Now, area of shaded region

$$= \text{area of sem-circle} - \text{area of } \Delta RPQ$$

$$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28} = 161.54 \text{ cm}^2.$$

OR

Here, $h = 24$ m

$$r_1 = \frac{20}{2} = 10 \text{ m}$$

$$r_2 = \frac{6}{2} = 3 \text{ m}$$

$$\text{slant height } (l) = \sqrt{\left((r_1 - r_2)^2 + h^2\right)}$$

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$$= \sqrt{\left((10-3)^2+(24)^2\right)}$$

$$= \sqrt{625}$$

$$= 25 \text{ m}$$

$$\text{CSA} = \pi(r_1 + r_2)l$$

$$= \frac{22}{7}(10 + 3)25$$

$$= \frac{22}{7} \times 13 \times 25$$

$$= \frac{7150}{7} = 1021.42 \text{ m}^2$$

40.

Class	Class mark (x)	Frequency (f)	fx
11 – 13	12	3	36
13 – 15	14	6	84
15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	$20f$
21 – 23	22	5	110
23 – 25	24	4	96
		$\Sigma f = 40 + f$	$\Sigma fx = 704 + 20f$

For x $\quad \Sigma f = 40 + f$

$$\Sigma fx = 704 + 20f$$

$$\text{Mean} = 18 = \frac{704 \times 20f}{40 + f}$$

$$\Rightarrow \quad 720 + 18f = 704 + 20f$$

$$\Rightarrow \quad f = 8.$$

OR

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	$N = 100$	

Also, $\quad 76 + x + y = 100$

$$\Rightarrow \quad x + y = 100 - 76 = 24 \quad \text{...(i)}$$

Given, Median = 525, which lies between class 500 – 600.

$\Rightarrow$ Median class = 500 – 600

Now, $$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$$\Rightarrow \quad 525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$$\Rightarrow \quad 25 = (50 - 36 - x)\,5$$

$$\Rightarrow \quad 14 - x = \frac{25}{5} = 5$$

$$\Rightarrow \quad x = 14 - 5 = 9$$

Putting the value of x in eq. (i), we get

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$.

Outside Delhi Set-II — Code No. 30/2/2

SECTION - A

15. $\sin 23° \cos 67° + \cos 23° \sin 67°$

$= \sin 23° \cos(90° - 23°) + \cos 23° \sin(90° - 23°)$

$= \sin 23° \sin 23° + \cos 23° \cos 23°$

$= \sin^2 23° + \cos^2 23°$

$= 1.$ $\quad [\sin^2 A + \cos^2 A = 1]$

19. $\because \quad \tan A = \cot B$

$\Rightarrow \quad \tan A = \tan(90° - B)$

$[\because \tan(90° - \theta) \cot\theta]$

$\Rightarrow \quad A = 90° - B$

$\Rightarrow \quad A + B = 90°.$

20. class mark of 15 – 35 $= \frac{15+35}{2} = \frac{50}{2} = 25$

and class mark of 45 – 60 $= \frac{45+60}{2} = \frac{105}{2} = 52.5.$

SECTION - B

25. Total possible outcomes $n(s) = 6$

(i) Let E_1 = getting event letter A, then $n(E_1) = 2$

$$\therefore \quad \text{Probability} = \frac{n(E_1)}{n(s)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Let E_2 = Getting event letter C, then $n(E_2) = 3$

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$\therefore$ Probability $= \dfrac{n(E_2)}{n(s)} = \dfrac{3}{6} = \dfrac{1}{2}$.

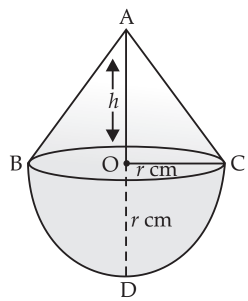
26. Let ABC be a cone, which is mounted on a hemisphere.

Given : $OC = OD = r$ cm

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2)$$

$$= 2\pi r^2$$



Slant height of a cone,

$$l = \sqrt{r^2 + h^2}$$

So, curved surface area of a cone $= \pi rl$

$$= \pi r\sqrt{h^2 + r^2}$$

i.e., $2\pi r^2 = \pi r\sqrt{h^2 + r^2}$ (given)

$\Rightarrow$ $2r = \sqrt{h^2 + r^2}$

on squaring both of the sides, we get

$$4r^2 = h^2 + r^2$$

$\Rightarrow$ $4r^2 - r^2 = h^2$

$\Rightarrow$ $3r^2 = h^2$

$\Rightarrow$ $\dfrac{r^2}{h^2} = \dfrac{1}{3} \Rightarrow \dfrac{r}{h} = \dfrac{1}{\sqrt{3}}$

Hence, the ratio of the radius and the height

$$= 1 : \sqrt{3}.$$

SECTION - C

32. Let 1st term of series be a and common difference be d, then

$$S_m = n \text{ (given)}$$

$\Rightarrow$ $\dfrac{m}{2}[2a + (m-1)d] = n$

$\Rightarrow$ $m[2a + (m-1)d] = 2n$ ...(i)

and $S_n = m$ (given)

$\Rightarrow$ $\dfrac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow$ $n[2a + (n-1)d] = 2m$ ...(ii)

On subtracting,

$$2(n-m) = 2a(m-n) + d[m^2 - n^2 - (m-n)]$$

$\Rightarrow$ $2(n-m) = 2a(m-n) + d[(m-n)]$
$[-(m-n) - (m-n)]$

$\Rightarrow$ $2(n-m) = (m-n)[2a + d(m+n-1)]$

$\Rightarrow$ $-2 = 2a + d(m+n-1)$

Now, $S_{m+n} = \dfrac{m+n}{2}[2a + (m+n-1)d]$

$$= \frac{m+n}{2}(-2)$$

$= -(m+n)$ **Hence proved.**

OR

In an $A.P.$ with 11 terms, the middle term is $\left(\dfrac{11+1}{2}\right)$

$= 6^{th}$ term.

Now $t_6 = a + 5d = 30$

Thus, $S_{11} = \dfrac{11}{2}[2a + 10d]$

$= 11(a + 5d)$

$= 11 \times 30$

$= 330.$

33. Total distance of a journey = 600 km

Let speed of fast train be x km/hr, then speed of slow train $= (x - 10)$ km/hr

According to questions,

$$\frac{600}{x-10} - \frac{600}{x} = 3$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$\Rightarrow$ $600\left[\dfrac{x - x + 10}{(x-10)x}\right] = 3$

$\Rightarrow$ $\dfrac{6000}{x^2 - 10x} = 3$

$\Rightarrow$ $x^2 - 10x - 2000 = 0$

$\Rightarrow$ $x^2 - 50x + 40x - 2000 = 0$

$\Rightarrow$ $x(x - 50) + 40(x - 50) = 0$

$\Rightarrow$ $(x - 50)(x + 40) = 0$

Either $x = 50$ or $x = -40$

[$\because$ speed can not be possible is negative]

So, the speed of fast train = 50 km/hr, and the speed of slow train = 50 – 10 = 40 km,

34. Given, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

On dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$\Rightarrow$ $\text{cosec}^2\theta + 1 = 3\cot\theta$

$\Rightarrow$ $1 + \cot^2\theta + 1 = 3\cot\theta$

$\Rightarrow$ $\cot^2\theta - 3\cot\theta + 2 = 0$

$\Rightarrow$ $\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$

$\Rightarrow$ $\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$

$\Rightarrow$ $(\cot\theta - 2)(\cot\theta - 1) = 0$

$\Rightarrow$ $\cot\theta = 1$ or 2

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$$\tan\theta = 1 \text{ or } \frac{1}{2}.$$

Hence proved

SECTION - D

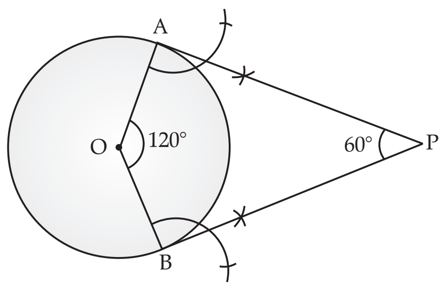
39. Steps of construction :

(i) Draw a circle of radius 4 cm with O as centre.

(ii) Draw two radii OA and OB inclined to each other at an angle of 120°.

(iii) Draw $AP \perp OA$ at A and $BP \perp OB$ at B. Which meet at P.

(iv) PA and PB are the required tangents inclined to each other an angle of 60°.



OR

Steps of construction :

(i) Draw a line segment BC = 5 cm.

(ii) With B as a centre and radius 3 cm, draw an arc.

(iii) With C as a centre and radius 4 cm, draw another arc meeting the previous arc at the point A.

(iv) Join AB and AC to obtain ΔABC.

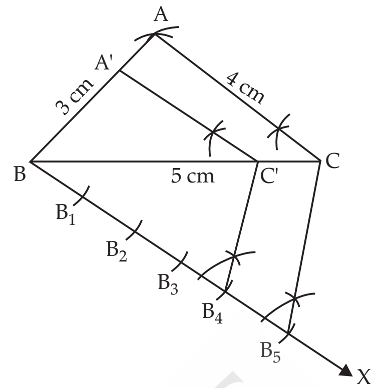
(v) Below BC make an acute angle $\angle CBX$.

(vi) Along BC mark off five points B_1, B_2, B_3, B_4, B_5 such as $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

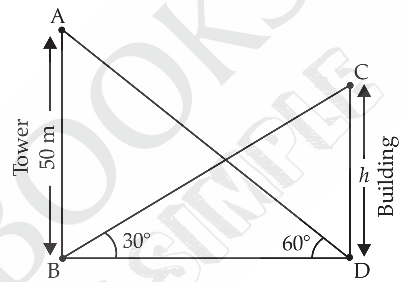
(vii) Join B_5C.

(viii) From B_4 draw $B_4C' \parallel B_5C$.

(ix) From C', draw $CA' \parallel CA$ meeting BA at the point A'. Hence, $A'BC'$ is the required triangle.



40. According to question,
In ΔABD



$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{50}{BD}$$

$$\Rightarrow BD = \frac{50}{\sqrt{3}}$$

Now in ΔBDC,

$$\tan 30° = \frac{CD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$\Rightarrow 3h = 50$$

$$\Rightarrow h = \frac{50}{3} = 16.67$$

Hence, the height of the building is 16.67 m.

Outside Delhi Set-III — Code No. 30/2/3

SECTION - A

15. $\sin 32° \cos 58° + \cos 32° \sin 58°$

$= \sin 32° \cos(90° - 32°) + \cos 32° \sin(90° - 32°)$

$= \sin 32° \sin 32° + \cos 32° \cos 32°$

$= \sin^2 32° + \cos^2 32°$

$= 1.$ $[\sin^2\theta + \cos^2\theta = 1]$

OR

$$\frac{\tan 35°}{\cot 55°} + \frac{\cot 78°}{\tan 12°}$$

$$= \frac{\tan(90° - 55)}{\cot 55°} + \frac{\cot(90° - 12°)}{\tan 12°}$$

$$= \frac{\cot 55°}{\cot 55°} + \frac{\tan 12°}{\tan 12°}$$

$$= 1 + 1 = 2.$$

19. Given, radius (r) = 6 cm

central angle (θ) = 30°

$$\text{Area of the sector} = \frac{\pi r^2\theta}{360°}$$

$$= \frac{3.14 \times 6 \times 6 \times 30°}{360°}$$

$$= 9.42 \text{ cm}^2.$$

20. Class mark of 20 – 50 = $\frac{20+50}{2} = \frac{70}{2} = 35$ and

class mark of 35 – 60 = $\frac{35+60}{2} = \frac{95}{2} = 47.5$.

SECTION - B

25. Here, modal class = 30 – 35

$\therefore l = 30, f_0 = 9, f_1 = 10, f_2 = 3$ and $h = 5$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10-9}{2\times10-9-3}\right) \times 5$$

$$= 30 + \frac{5}{8} = 30 + 0.625$$

$$= 30.625$$

26. Given, height (h) = 14 cm

and Base radius (r) = 6 cm

Volume of the remaining solid = Volume of a right circular cylinder – Volume of a right circular cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3}\times\frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^3.$$

SECTION - C

32. Here, $a = 20$

and $d = \frac{77}{4} - 20 = -\frac{3}{4}$

Let $t_n <, 0$

$\because t_n = a + (n-1)d$

$\therefore 20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow 80 - 3n + 3 < 0$

$\Rightarrow 83 - 3n < 0$

$\Rightarrow n > \frac{83}{3} \Rightarrow n > 27.6$

$\Rightarrow n = 28$

Hence, the first negative term is 28.

OR

In this A.P., $a = 7, d = 13 - 7 = 6$

and $t_n = 247$

$\because t_n = a + (n-1)d$

$\therefore 247 = 7 + (n-1)6$

$\Rightarrow 6(n-1) = 240$

$\Rightarrow n - 1 = 40$

$\Rightarrow n = 41$

Hence, the middle term $= \frac{n+1}{2}$

$= \frac{41+1}{2} = \frac{42}{2}$

$= 21.$

33. Speed of water in canal = 10 km/hr

In 30 min = $\frac{30}{60} = \frac{1}{2}$ hr

$\therefore$ Length of water $= 10 \times \frac{1}{2} = 5$ km

$= 5000$ m.

Volume of water is canal is 30 min = Volume of water for irrigated

$\Rightarrow 6 \times 1.5 \times 5000 = \frac{8}{100} \times l \times b$

$\Rightarrow l \times b = \frac{6\times1.5\times5000\times100}{8}$

$= \frac{4500000}{8} = 562500 \text{ m}^2$

Hence, area irrigated is 30 min is 562500 m^2.

34. L.H.S.

$$\frac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$$

$$= \frac{\cos^2(45°+\theta)+\sin^2(90°-45°+\theta)}{\tan(60°+\theta)\cot(90°-30°+\theta)}$$

$$= \frac{\cos^2(45°+\theta)+\sin^2(45°+\theta)}{\tan(60°+\theta)\cot(60°+\theta)}$$

[$\because \cos^2\theta + \sin^2\theta = 1$ and $\tan\theta = \frac{1}{\cot\theta}$]

$$= \frac{1}{1}$$

$= 1 =$ R.H.S.

SECTION - D

39. Steps of construction :

(i) Draw a line segment OP = 6 cm.

(ii) From the point O, draw a circle of radius = 3.5 cm.

(iii) Draw a perpendicular bisector of OP. Let M be the mid point of OP.

(iv) Taking M as centre and OM as radius draw a circle.

(v) This circle intersects the given circle at Q and R.

(vi) Join PQ and PR, which are tangents to the circles.

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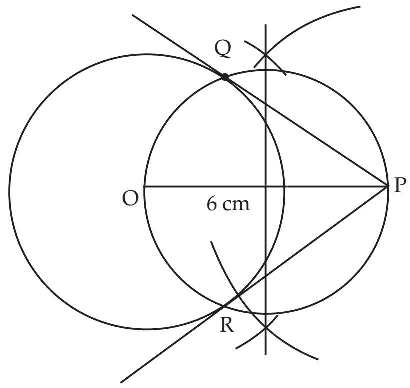


OR

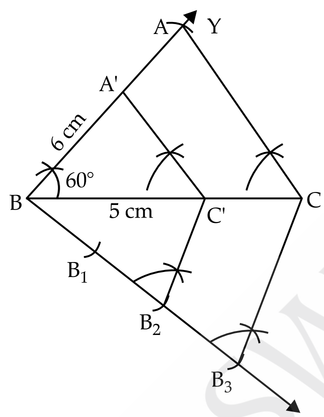
Steps of Construction :

(i) Draw a line segment $BC = 5$ cm.

(ii) At point B, draw a line By making an angle of 60°.

(iii) With B as centre mark an arc A of length 6 cm.

(iv) Join CA.



(v) Draw a ray BX making an acute angle with BC.

(vi) Locate three points B_1, B_2, B_3 on the line segment BX at equal distance.

(vii) Join B_3C. Draw a parallel line through B_2 to B_3C intersecting line segment BC at C'.

(viii) Through C' draw a line parallel to AC intersecting line segment AB at A

Hence, $\Delta A'BC'$ is the required triangle.

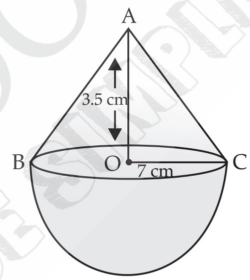
40. Here, radius $(r) = 7$ cm

and height of a cone $= 3.5$ cm

$\therefore$ Volume of the soild

= Volume of hemisphere + volume of a cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$



$$= \frac{2}{3}\times\frac{22}{7}\times(7)^3 + \frac{1}{3}\times\frac{22}{7}\times(7)^2\times 3.5$$

$$= \frac{1}{3}[2156 + 539]$$

$$= \frac{1}{3}\times 2695$$

$$= 898.33 \text{ cm}^3.$$

❑❑