MATHEMATICS

Toppers' Answers-2018

Time : 3 Hours **Max. Marks : 80**

General Instructions :

(i) ***All*** *questions in both the sections are compulsory.*

(ii) *This question paper consists of* ***30*** *question divided into four sections – A, B, C and D.*

(iii) *Section A contains* ***6*** *questions* ***1*** *mark each. Section B contains* ***6*** *question of* ***2*** *marks each. Section C contains* ***10*** *questions of* ***3*** *marks each. Section D contains* ***8*** *questions of* ***4*** *marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in* ***four*** *question of 3 marks each and* ***3*** *questions of* ***4*** *marks each. You have to attempt only* ***one*** *of the alternatives in all such questions.*

(v) *Use of calculator is* ***not*** *permitted.*

Delhi Set **Code No. 30/1**

SECTION - A

1. 1) $x^2 - 2kx - 6 = 0$. let α be other root.

Product $= \frac{c}{a} = \frac{-6}{1} = -6$.

$3 \times \alpha = -6$

$\alpha = -2$.

Sum $= \frac{-b}{a} = \frac{-(-2k)}{1} = 2k$.

$\Rightarrow 3 + (-2) = 2k$

$1 = 2k,\ k = \frac{1}{2}$.

$\boxed{\text{Value of } k \text{ is } 1/2}$

2. 2) Smallest prime = 2

Smallest composite = 4

HCF $(2, 4) = 2$.

$\boxed{\text{The HCF of the smallest prime and smallest composite is 2.}}$

3. 3) Distance between (x, y) and $(0, 0)$.

$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$.

$\boxed{\text{The distance is } \sqrt{x^2+y^2}.}$

4. 4) $d = -4,\ a_7 = 4$.

$t_n = a + (n-1)d$.

$t_7 = a + (7-1)(-4)$

$4 = a + 6 \times (-4)$

$a = 24 + 4$

$a = 28$.

$\boxed{\text{The first term is 28.}}$

5. 5) $\cos^2 67° - \sin^2 23°$

~~$= \cos^2 67° - \cos^2 67°$~~

$(\cos 67 + \sin 23)(\cos 67 - \sin 23)$

1

$= (\cos 67 + \cos 67)(\cos 67 - \cos 67)$ $[\cos\theta = \sin(90-\theta)]$.

$= 0$.

$\boxed{\text{The value is } 0}$.

6. 6) $\frac{AB}{PQ} = \frac{1}{3}$. $\frac{ar\,\Delta ABC}{ar\,\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{1}{3^2} = \frac{1}{9}$.

1

$\boxed{\text{Ratio of areas is } \frac{1}{9}}$.

SECTION - B

7. 7) Given, $\sqrt{2}$ is irrational

2

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. So it is in form $\frac{a}{b}$. ~~[a, b ∈ R, b ≠ 0]~~ $[a, b \in Z, b \neq 0, HCF(a, b) = 1]$

$\Rightarrow 5+3\sqrt{2} = \frac{a}{b}$

$3\sqrt{2} = \frac{a}{b} - 5$.

$3\sqrt{2} = \frac{a-5b}{b}$

$\sqrt{2} = \frac{a-5b}{3b}$.

This shows that $\sqrt{2}$ is ~~irrational~~ rational ($a-5b$ and $3b$ are integers).

But we know that $\sqrt{2}$ is irrational.

This contradicts our assumption that $5+\sqrt{2}\times 3$ is rational.

$\Rightarrow 5+3\sqrt{2}$ is irrational, hence proved.

8. 8) Given, rectangle ABCD.

2

$\Rightarrow$ opposite sides are equal.

hence, $x+y = 30 \rightarrow (1)$

$x-y = 14 \rightarrow (2)$.

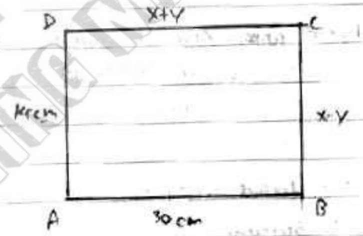
$(1)+(2)$, $2x = 44$

$x = 22$.

Substituting in (1), $22+y = 30$

$y = 8$.

$\Rightarrow \boxed{x=22, y=8}$



9. 9) Sum of first 8 multiples of 3:

2

Forms an AP, $a=3$, $d=3$, $n=8$.

Sum $= S_n = \frac{n}{2}[2a+(n-1)d]$

$S_8 = \frac{8}{2}[2\times3+(8-1)3] = 4[6+21] = 4\times27 = 108$.

$\boxed{\text{The sum of the first 8 multiples of 3 is } 108}$

10. 10) Points $A(2, 3)$, $B(6, -3)$ divided by $P(4, m)$.

Let the ratio be $k:1$.

2

By sec. section formula,

$P(4, m) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$(4, m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$

$\Rightarrow \frac{6k+2}{k+1} = 4$

$6k+2 = 4k+4$

$2k = 2$

$k = 1. \rightarrow$ The ratio is 1:1.

now, $m = \frac{-3k+3}{k+1}$

(2) By sec. section formula,

$P(4,m) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$(4,m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$

$\Rightarrow \frac{6k+2}{k+1} = 4$

$6k+2 = 4k+4$

$2k = 2$

$k = 1. \rightarrow$ The ratio is 1:1.

now, $m = \frac{-3k+3}{k+1}$

$m = \frac{-3+3}{1+1}$

$m = 0. \rightarrow$ $\boxed{\text{Value of } m \text{ is } 0, \text{ the point is } P(4,0).}$

11. 11) Two dice tossed together.

(2) $\Rightarrow$ Total outcome = 36.

i) doublet: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) → 6 possibilities.

Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{6}{36} = \boxed{\frac{1}{6}}$.

ii) Sum of 10: (4,6), (6,4), (5,5) → 3 possibilities.

Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{3}{36} = \boxed{\frac{1}{12}}$.

12. (2) 12) Integers, 1 to 100. (between)

$\Rightarrow$ total = 98 possible outcomes.

i) divisible by 8 → 12 numbers. (8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96).

$\Rightarrow$ Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{12}{98} = \boxed{\frac{6}{49}}$.

ii) not divisible by 8 → 98−12 = 86 numbers.

$\Rightarrow$ Probability = $\frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{86}{98} = \boxed{\frac{43}{49}}$

SECTION - C

13. 13) Numbers: 404, 96. To find: HCF and LCM.

(3)

2 | 404, 96
2 | 202, 48
  | 101, 24

$\Rightarrow$ Their HCF is 4.

404 → 2, 202 → 2, 101 → 7, 13

96 → 2, 48 → 2, 24 → 2, 12 → 2, 6 → 2, 3.

$404 = 2^2 \times 7 \times 13$

$96 = 2^5 \times 3$.

$\boxed{\text{HCF = greatest common factor} = 2^2 = 4.}$

LCM = all factors (least power) = $\cancel{2}$ $2^5 \times 3 \times 7 \times 13$

$= 96 \times 101$

$= \boxed{9696}$

Product of two numbers = 96×404

$= 38784$

Product of HCF & LCM = 9696×4

$= 38784.$

Hence, HCF × LCM = product of two numbers.

14. 14) Given, polynomial $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$.

3

Two zeroes → $2+\sqrt{3}$ and $2-\sqrt{3}$.

=> Product of two zeroes is also a zero.

=> $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1.$ $[(a+b)(a-b) = a^2 - b^2]$.

As 1 is a zero, => $x-1$ is a factor.

Dividing,

$x-1\ )\ 2x^4 - 9x^3 + 5x^2 + 3x - 1\ (\ 2x^3 - 7x^2 - 2x + 1$

$\underline{2x^4 - 2x^3}$

$-7x^3 + 5x^2$

$\underline{-7x^3 + 7x^2}$

$-2x^2 + 3x$

$\underline{-2x^2 + 2x}$

$x - 1$

$\underline{x - 1}$

0

=> By division algorithm,

$p(x) = (x-1)\underbrace{(2x^3 - 7x^2 - 2x + 1)}_{g(x)}.$

Now, in a cubic polynomial, we know:

$\text{sum of roots} = \dfrac{-\text{coeff. of } x^2}{\text{coeff. of } x^3}.$

The roots of $g(x)$ are $2+\sqrt{3}$, $2-\sqrt{3}$ and α.

→ $\alpha + 2 + \sqrt{3} + 2 - \sqrt{3} = \dfrac{-(-7)}{2}$

$\alpha + 4 = \dfrac{7}{2}$

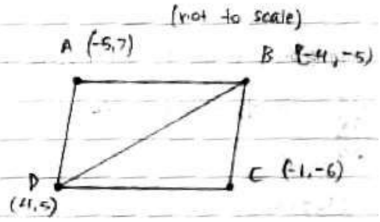
$\alpha = -\dfrac{1}{2}$. which is hence a zero of $p(x)$

=> $\boxed{\text{All zeroes are } -\tfrac{1}{2},\ 1,\ 2+\sqrt{3} \text{ and } 2-\sqrt{3}.}$

15. 15) Vertices of quadrilateral ABCD:

(choice 2) A (-5,7), B(-4,-5), C(-1,-6), D(4,5)

3



Area of quad ABCD.

= area $\triangle$ABD + area $\triangle$BCD.

area $\triangle$ ABD →

$= \dfrac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$ sq. units.

$= \dfrac{1}{2}\left[-5(-5-5) + (-4)(5-7) + 4(7+5)\right]$

$= \dfrac{1}{2}\left[\cancel{\#} 50 + 8 + 48\right]$

$= \dfrac{1}{2}\left[58 + 48\right].$

$= \frac{1}{2} \times 106 = 53$ sq. units^2.

area $\Delta BCD = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$.

$= \frac{1}{2}\left[-4(-6-5)+(-1)(5+5)+(4)(-5+6)\right]$.

$= \frac{1}{2}\left[44-10+4\right]$

$= \frac{1}{2} \times 38 = 19$ units^2.

$\Rightarrow$ Area of quadrilateral = Area of two triangles. $= 53+19 = 72$ sq. units.

$\boxed{\text{Area of quadrilateral ABCD is 72 sq. units}}$

16.

16) Given, distance is 1500 km.

Usual speed = s.

We know, speed = $\frac{\text{distance}}{\text{time}}$ $\rightarrow$ time = $\frac{\text{distance}}{\text{speed}}$.

$\rightarrow$ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$. [half an hour late]. (30 min = 0.5 hr).

$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}$.

$\frac{1500}{100+s} = \frac{3000-s}{2s}$.

Cross multiplying,

$3000s = 300000 - 100s + 3000s - s^2$.

$s^2 + 100s - 300000 = 0$.

$s^2 + 600s - 500s - 300000 = 0$.

$s(s+600) - 500(s+600) = 0$

$(s-500)(s+600) = 0$.

$\Rightarrow$ $s-500=0$ or $s+600=0$.

$\rightarrow s = 500$ km/h $\rightarrow s = -600$ km/h.

$\Rightarrow$ $s = 500$ or -600 km/hr.

But speed cannot be negative.

$\Rightarrow$ $\boxed{\text{The usual speed of the plane is 500 km/hr.}}$

17.

17) Given: Square ABCD. ΔAED and ΔAFC are equilateral.

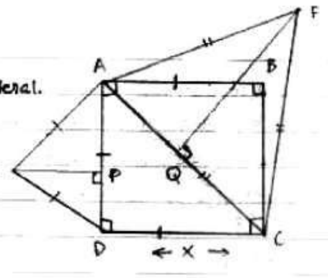
(choice 1)

To prove: Area $\Delta AFC = 2\times$ Area ΔAED.

Construction: Draw $EP \perp AD$ and $FQ \perp AC$.

Proof: Let side of square be x.

$\Rightarrow$ sides of $\Delta AED = x$.



In ΔABC, $\angle B = 90°$.

$\Rightarrow$ By Pythagoras Theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + x^2 = AC^2 \Rightarrow AC = \sqrt{2}x$. $\Rightarrow$ sides of $\Delta AFC = \sqrt{2}x$.

We know, altitude of equilateral Δ bisects the base.

$\rightarrow PD = \frac{x}{2}$, $AQ = \frac{x}{\sqrt{2}}$.

In ΔAEP, $\angle P = 90°$.

By Pythagoras theorem, $AE^2 = EP^2 + AP^2$.

$x^2 = EP^2 + \left(\frac{x}{2}\right)^2$.

$EP^2 = \frac{3x^2}{4} \Rightarrow EP = \frac{\sqrt{3}}{2}x$.

In ΔAFQ, $\angle Q = 90°$.

By Pythagoras theorem, $AF^2 = FQ^2 + AQ^2$

$2x^2 = FQ^2 + \frac{x^2}{2}.$

$FQ^2 = \frac{3x^2}{2} \rightarrow FQ = \frac{\sqrt{3}}{\sqrt{2}}x.$

We know, Area of triangle $= \frac{1}{2} \times$ Base $\times$ height sq. units.

$\Rightarrow$ Area of $\triangle AFC = \frac{1}{2} \times \sqrt{2}x \times FQ$

$= \frac{1}{2} \cdot \sqrt{2}x \cdot \frac{\sqrt{3}}{\sqrt{2}}x$

$= \frac{\sqrt{3}}{2}x^2.$

Area of $\triangle AED = \frac{1}{2} \times x \times EP.$

$= \frac{1}{2} \cdot x \cdot \frac{\sqrt{3}}{2}x$

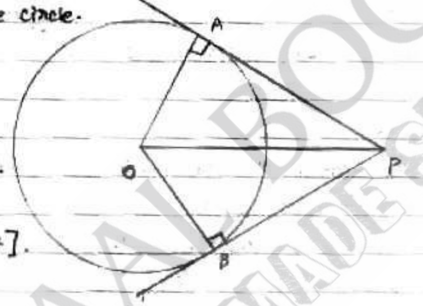
$= \frac{\sqrt{3}}{4}x^2.$

$2 \cdot$ Area of $\triangle AED = \frac{\sqrt{3}}{2}x^2 =$ Area of $\triangle AFC.$

hence proved.

18. 18) Given: Circle (O, r). AP and PB are tangents drawn to the circle.

3



To prove: PA = PB.

Construction: Join OA, OB and OP.

Proof: OA = OB [radius]. (side).

$\angle OAP = \angle OBP = 90°$ (right angle).

[$\because$ radius is perpendicular to tangent at point of contact].

OP = OP (hypotenuse).

So in $\triangle OAP$ and $\triangle OBP$,

by RHS congruency,

$\rightarrow \triangle OAP \cong \triangle OBP.$

by CPCT,

$\Rightarrow$ AP = BP.

hence proved.

19. 19) Given: $\tan 2A = \cot(A - 18)$, $0 \le 2A < 90°$. (2A is acute)

(Choice 2)

To find: value of A.

We know, $\tan\theta = \cot(90 - \theta)$ and $\cot\theta = \tan(90 - \theta)$.

$\rightarrow \cot(90 - 2A) = \cot(A - 18)$

Applying $\cot^{-1}$, on both sides

$90 - 2A = A - 18.$

$108 = 3A.$

$\rightarrow A = 36°.$

$\boxed{\text{The value of A is } 36°.}$

20. 20) Given: side of square ABCD = 12 cm.

3

To find: shaded area.

Shaded area + Area of 4 quadrants = Area of square.

Area of square = s^2 sq. units

$= 12^2 = 144$ cm^2.

Area of quadrant $= \frac{1}{4} \times \pi r^2$ sq. units

$= \frac{1}{4} \times 3.14 \times \frac{12}{2} \times \frac{12}{2}$

$= 9 \times 3.14 =$ ~~26.26~~ 28.26 cm^2.

$\Rightarrow$ Shaded area = Area of square $-$ $4\times$(Area of quadrant) sq.units

$= 144 - 4(28.26)$ sq.cm

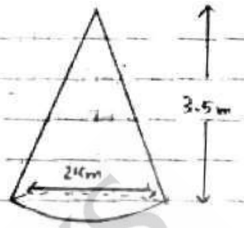
$= 144 - 113.04$

$= 30.96$ cm^2.

$\boxed{\text{The area of the shaded region is } 30.96 \text{ cm}^2.}$

21. 21) Conical heap of rice:

(choice 2) Dimensions: diameter = 24m, height 3.5 m. $\rightarrow$ radius = 12m.



Volume of cone = $\frac{1}{3}\times\pi r^2 h$ cu.units.

$= \frac{1}{3}\times\frac{22}{7}\times 12\times 12\times 3.5$ cu.m.

$= 132\times 4$

$= 528$ cu.m.

$\boxed{\text{The volume of the rice heap is } 528 \text{ cu.m.}}$

Area of cloth required = Curved surface area.

CSA of cone $= \pi r l$ sq.units where $l = \sqrt{h^2+r^2}$ units.

Finding l: $l = \sqrt{h^2+r^2}$ units

$= \sqrt{3.5^2+12^2}$ m

$= \sqrt{12.25+144}$

$= \sqrt{156.25}$

$= 12.5$ m.

$\Rightarrow$ CSA $= \pi r l$ sq.units

$= \frac{22}{7}\times 12.5\times 12$

$= \frac{22\times 150}{7} = \frac{3300}{7} = 471.\overline{428571}$ m^2.

$\boxed{\text{The area of canvas cloth required is } 471.\overline{428571} \text{ m}^2.}$

22. 22) Distribution of frequencies:

Salary in thousand Rs.	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	63	15	6	7	4	2	1

To find: median.

no. of people = 280.

$\Rightarrow \frac{n}{2} = 140$, the 140th term lies in class interval 10-15.

$\Rightarrow$ median class = 10-15.

$l = 10$, $h = 5$, $f = 133$, $\frac{n}{2} = 140$, $cf = 49$.

We know, median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right)\times h$.

$\Rightarrow$ median $= 10 + \frac{140-49}{133}\times 5$.

$= 10 + \frac{91}{133}\times 5$.

$= 10 + \frac{65}{19}$

$= 10 + 3.421$

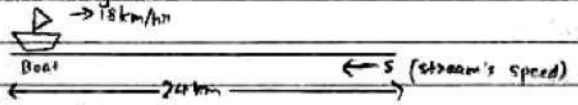
$= 13.421$.

$\boxed{\text{The median salary is } 13.421 \text{ thousand rupees.}}$

SECTION - D

23. 23) Representative diagram:

(Choice 1)



Given that:

Speed of boat = 18 km/hr in still water.

Speed of stream = s (variable, must find)

Distance upstream + back = 24 km.

Time upstream = 1 hr more than time downstream.

We know, speed = $\frac{\text{Distance}}{\text{Time}}$ → Time = $\frac{\text{Distance}}{\text{Speed}}$

Time upstream = $\frac{24}{18-s}$, Time downstream = $\frac{24}{18+s}$.

$\rightarrow \frac{24}{18-s} = 1 + \frac{24}{18+s}$.

$\frac{24}{18-s} = \frac{18+s+24}{18+s}$ (cross-multiplying)

$24(18+s) = (42+s)(18-s)$

$432 + 24s = 756 + 18s - 42s - s^2$

$\rightarrow s^2 + 24s + 24s + 432 - 756 = 0$.

$s^2 + 48s + (-324) = 0$.

$s^2 + 54s - 6s - 324 = 0$.

$s(s+54) - 6(s+54) = 0$.

$(s-6)(s+54) = 0$.

Now, either $s - 6 = 0$ or $s + 54 = 0$.

$\rightarrow s = 6$ $\rightarrow s = -54$.

So speed = 6 or −54 km/hr.

But speed cannot be negative.

$\Rightarrow$ Speed of the stream is 6 km/hr.

24. 24) Given, sum of 4 consecutive no. in AP is 32.

Also, ratio of $t_1 \times t_n$ (first × last) and two middle terms product = 7 : 15.

Let no. of terms be n. nth term = $t_n = a + (n-1)d$.

$\rightarrow t_1 = a + (1-1)d$.

$t_n = a + (n-1)d$.

As there are two middle terms, n is even.

$\Rightarrow$ middle term 1 = $a + \left(\frac{n+2}{2} - 1\right)d$. Let this be α.

middle term 2 = $a + \left(\frac{n}{2} - 1\right)d$. Let this be β.

Sum = 32. → $S_n = \frac{n}{2}[2a + (n-1)d]$.

$32 = \frac{4}{2}[2a + 3d]$.

$2a + 3d = 16$. → (2).

Squaring (2), $4a^2 + 9d^2 + 12ad = 256$. → (2.1)

Substituting ③ in ②.1, $7d^2 + 9d^2 = 256$.

$16d^2 = 256 \rightarrow d^2 = 16,\ d = 4.$

Now, $2a + 3(4) = 16$.

$2a = 4$

$a = 2.$ → Terms = 2, 6, 10, 14.

a and a+d a+3d.

$15a^2 + 45ad = 7[a^2 + 3ad + 2d^2]$

$15a^2 + 45ad = 7a^2 + 21ad + 14d^2$

$8a^2 - 14d^2 + 24ad = 0$.

$14d^2 = 8a^2 + 24ad$.

$7d^2 = 4a^2 + 12ad \rightarrow ③$

$\boxed{\text{The numbers are } 2, 6, 10 \text{ and } 14.}$

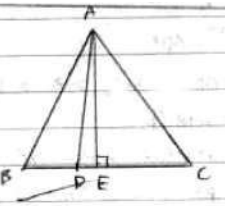
25. 25) Given: $\triangle ABC$ is equilateral.

4

$\rightarrow AB = BC = CA,\ \angle A = \angle B = \angle C = 60°$

D is a point on BC such that $BD = \frac{1}{3} BC$.

To prove: $9(AD)^2 = 7(AB)^2$.



Construction: Draw $AE \perp BC$.

Proof: Let $BD = x$.

$\Rightarrow BC = 3x = AB = AC$ [$\because \triangle ABC$ is equilateral] [Given $BD = \frac{1}{3} BC$].

Also, we know that $BE = \frac{1}{2} BC$ [Altitude in equilateral $\triangle$ bisects base].

As $\angle AEB = 90°$,

In ~~triangle~~ $\triangle ABE$, by Pythagoras Theorem,

$BE^2 + AE^2 = AB^2 \rightarrow AB^2 = 9x^2. \rightarrow ⓐ$.

$\left(\frac{3x}{2}\right)^2 + AE^2 = (3x)^2$.

$\frac{9x^2}{4} + AE^2 = 9x^2 \rightarrow AE^2 = \frac{3 \times 9x^2}{4} \rightarrow ①$

Now, in $\triangle ADE$, $\angle E = 90°$. $DE = BE - BD$

By Pythagoras Theorem, $= \frac{3x}{2} - x$.

$DE^2 + AE^2 = AD^2$

$\left(\frac{3x}{2} - x\right)^2 + \frac{27x^2}{4} = AD^2$ [From ①].

$\left(\frac{x}{2}\right)^2 + \frac{27x^2}{4} = AD^2$.

$\frac{x^2 + 27x^2}{4} = AD^2$

$\Rightarrow AD^2 = \frac{28x^2}{4} = 7x^2 \rightarrow ⓑ$.

From ⓐ and ⓑ,

$AB^2 = 9x^2,\ AD^2 = 7x^2$.

$7AB^2 = 63x^2,\ 9AD^2 = 63x^2$.

$\Rightarrow 7AB^2 = 9AD^2$.

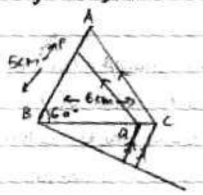
hence proved.

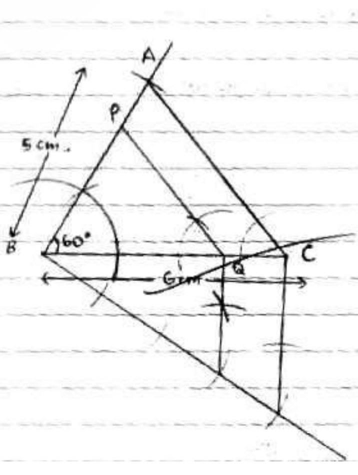
26. 26) Given: $\triangle ABC$, $BC = 6$ cm, $AB = 5$ cm, $\angle ABC = 60°$.

Rough Diagram.

To draw $\triangle$ w/ $\frac{3}{4}$ sides of $\triangle ABC$.

4





AB = 5cm

BC = 6cm

$\angle ABC = 60°$

$\triangle PBQ$ is required triangle.

PB = ~~BC~~ 3.75 cm.

BQ = 4.5 cm.

PQ = 4.25 cm.

27. 27) To prove: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

4

Simplifying LHS:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A}\left[\frac{1 - (2\sin^2 A)}{2\cos^2 A - 1}\right]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - (\sin^2 A + \cos^2 A)}\right] \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right]$$

$$= \frac{\sin A}{\cos A} \times 1$$

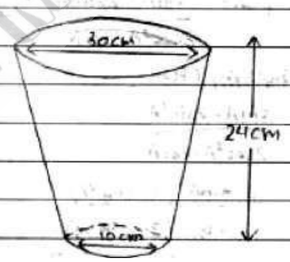
$$= \tan A. \qquad \left[\because \frac{\sin A}{\cos A} = \tan A\right]$$

LHS = RHS

hence proved.

28. 28) i) Given, metal bucket shaped like frustum of cone.

Bucket



4

Dimensions: height = 24 cm. (h)

lower diameter = 10 cm → radius = $\frac{10}{2}$ = 5 cm (r_1)

upper diameter = 30 cm → radius = $\frac{30}{2}$ = 15 cm (r_2)

To find: Metal sheet needed to make bucket.

→ Area of metal sheet = Curved surface area + Area of base.

We know,

Curved surface area of frustum = $\pi \times (R+r) \times l$ sq. units.

where $l = \sqrt{h^2 + (R-r)^2}$ units.

To find l: $l = \sqrt{h^2 + (R-r)^2}$ units.

$= \sqrt{24^2 + (15-5)^2}$ cm

$= \sqrt{576 + 10^2}$

$= \sqrt{676}$

$l = 26$ cm.

⇒ CSA = $\pi (R+r) l$ sq. units

$= 3.14 \times (15+5) \times 26$ sq. cm

$= 3.14 \times 20 \times 26$

CSA = 1632.8 cm^2

We know, Area of circular base = πr^2 sq. units.

→ Area = $3.14 \times 5 \times 5$ sq. cm

$= 3.14 \times 100/4$

$= \frac{314}{4}$

Area = 78.5 cm^2.

Total area of sheet = Curved area + Circular base area

$= 1632.8 + 78.5 \text{ cm}^2$

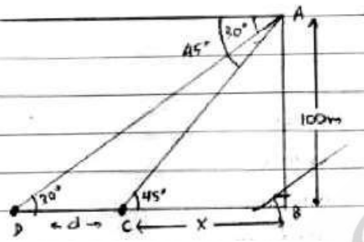
Area $= 1711.3 \text{ cm}^2$

$\boxed{\text{The area of sheet needed is } 1711.3 \text{ cm}^2.}$

ii) Plastic buckets are less preferable to metal buckets because plastic buckets are more harmful to the environment. They may also leak harmful chemicals into the water being stored.

29. 29) Diagram:

4



AB → lighthouse = 100m high

C → boat 1

D → boat 2.

To find: $\overline{CD}$ or d.

(distance b/w ships)

We know,

$\tan\angle ACB = \frac{\text{opp.}}{\text{adj.}} = \frac{AB}{BC}$.

$\to \tan 45° = \frac{100}{x}$

$1 = \frac{100}{x}$

$\Rightarrow x = 100\text{m}$.

$\tan\angle ADB = \frac{\text{opp.}}{\text{adj.}} = \frac{AB}{BD}$.

$\to \tan 30 = \frac{100}{x+d}$

$\frac{1}{\sqrt{3}} = \frac{100}{d+100}$ $[x = 100]$

$100 + d = 100\sqrt{3}$.

$\to d = 100\sqrt{3} - 100 = 100(\sqrt{3}-1)$

Given, $\sqrt{3} = 1.732$,

$\Rightarrow d = 100(1.732-1)$

$= 100 \times 0.732 = 73.2\text{m}$.

$\to$ $\boxed{\text{The distance between the boats is } 73.2\text{m}.}$

30. 30) Frequency distribution:

(choice i)

4

Class	Frequency	x_i	f_ix_i
11-13	3	12	3x12=36
13-15	6	14	6x14=84
15-17	9	16	9x16=144
17-19	13	18	13x18=234
19-21	f	20	fx20=20f
21-23	5	22	5x22=110
23-25	4	24	4x24=96.
Total: →	40+f		704+20f

Given, mean = 18. To find: Value of f.

We know,

$\text{mean } (\bar{x}) = \frac{\sum f_ix_i}{\sum f_i}$

$\to 18 = \frac{704+20f}{40+f}$

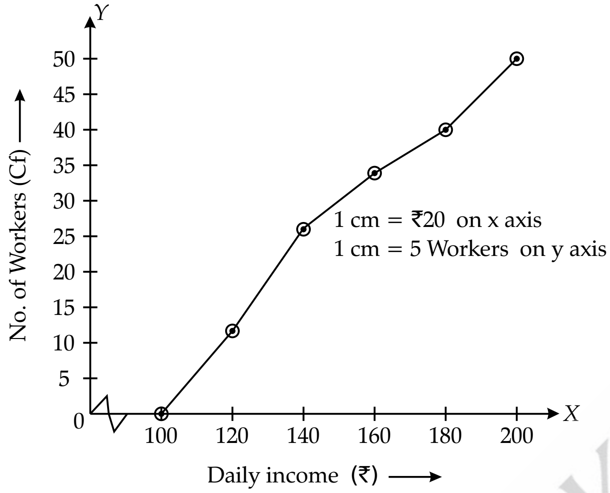
$720 + 18f = 704 + 20f$.

$720 - 704 = 20f - 18f$

$16 = 2f$

$\Rightarrow f = 8$.

$\boxed{\text{The value of } f \text{ is } 8}$


Y
50
45
40
35
30
25
20
15
10
5
0
No. of Workers (Cf)
1 cm = ₹20 on x axis
1 cm = 5 Workers on y axis
100
120
140
160
180
200
X
Daily income (₹)

●●