Sample Question Paper, 2021-22

(Issued by CBSE Board on 14th January, 2022)

MATHEMATICS (Term- II)

SOLVED

Time allowed : 2 Hours **Max. Marks : 40**

General Instructions :

1. *This question paper contains* ***three section A, B and C****. Each part is compulsory.*
2. ***Section - A*** *has* ***6 short answer type (SA1) questions*** *of 2 marks each.*
3. ***Section - B*** *has* ***4 short answer type (SA2) questions*** *of 3 marks each.*
4. ***Section - C*** *has* ***4 long answer type questions (LA)*** *of 4 marks each.*
5. *There is an* ***internal choice*** *in some of the questions.*
6. *Q14 is a* ***case-based problem*** *having 2 sub parts of 2 marks each.*

Section - A

[2 Marks each]

1. Find : $\int \frac{\log x}{(1+\log x)^2} dx$

OR

Find : $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}} dx$

2. Write the sum of the order and the degree of the following differential equation:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$$

3. If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $|\hat{a}+\hat{b}| = 2\cos\frac{\theta}{2}$, where θ is the angle between them.

4. Find the direction cosines of the following line:

$$\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$$

5. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.

6. Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?

Section - B

[3 Marks each]

7. Find : $\int \frac{x+1}{(x^2+1)x} dx$

8. Find the general solution of the following differential equation:

$$x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$$

OR

Find the particular solution of the following differential equation, given that $y = 0$ when $x = \frac{\pi}{4}$:

$$\frac{dy}{dx} + y\cot x = \frac{2}{1+\sin x}$$

9. If $\vec{a} \neq \vec{0}$, $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, then show that $\vec{b} = \vec{c}$.

10. Find the shortest distance between the following lines:

$$\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + s(2\hat{i}+\hat{j}+\hat{k})$$

$$\vec{r} = (\hat{i}+\hat{j}+2\hat{k}) + t(4\hat{i}+2\hat{j}+2\hat{k})$$

OR

Find the vector and the cartesian equations of the plane containing the point $\hat{i}+2\hat{j}-\hat{k}$ and parallel to the lines $\vec{r} = (\hat{i}+2\hat{j}+2\hat{k}) + s(2\hat{i}-3\hat{j}+2\hat{k}) = 0$ and $\vec{r} = (3\hat{i}+\hat{j}-2\hat{k}) + t(\hat{i}-3\hat{j}+\hat{k}) = 0$

Section - C

[4 Marks each]

11. Evaluate : $\int_{-1}^{2} |x^3 - 3x^2 + 2x|\,dx$

12. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and x-axis.

OR

Using integration, find the area of the region

$\{(x, y) : 0 \le y \le \sqrt{3}x,\ x^2 + y^2 \le 4\}$

13. Find the foot of the perpendicular from the point (1, 2, 0) upon the plane $x - 3y + 2z = 9$. Hence, find the distance of the point (1, 2, 0) from the given plane.

Case-Based/Data Based

14. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the people is accident prone.

Based on the given information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy? **[2]**

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone? **[2]**

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MATHEMATICS

CBSE Marking Scheme Answers 2021-2022 (Issued by Board)

Section - A

1. $\int \frac{\log x}{(1+\log x)^2}dx$

$= \int \frac{\log x + 1 - 1}{(1+\log x)^2}dx$

$= \int \frac{1}{1+\log x}dx - \int \frac{1}{(1+\log x)^2}dx$ ½

$= \frac{1}{1+\log x} \times x - \int \frac{-1}{(1+\log x)^2} \times \frac{1}{x} \times x\,dx$

$- \int \frac{1}{(1+\log x)^2}dx$ 1

$= \frac{x}{1+\log x} + c$ ½

OR

$\int \frac{\sin 2x}{\sqrt{9 - \cos^4 x}}dx$

Put $\cos^2 x = t$

$\Rightarrow -2\cos x \sin x\,dx = dt$

$\Rightarrow \sin 2x\,dx = -dt$ 1

The given integral

$= -\int \frac{dt}{\sqrt{3^2 - t^2}}$

$= -\sin^{-1}\frac{t}{3} + c$

$= -\sin^{-1}\frac{\cos^2 x}{3} + c$ 1

2. Order $= 2$ 1

Degree $= 1$ ½

Sum $= 3$ ½

3. $(\hat{a} + \hat{b}).(\hat{a} + \hat{b}) = |\hat{a}|^2 + |\hat{b}|^2 + 2(\hat{a}.\hat{b})$ 1

$|\hat{a} + \hat{b}|^2 = 1 + 1 + 2\cos\theta$

$= 2(1 + \cos\theta) = 4\cos^2\frac{\theta}{2}$ ½

$\therefore |\hat{a} + \hat{b}| = 2\cos\frac{\theta}{2}$ ½

4. The given line is

$$\frac{x-3}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z}{4} \quad \mathbf{1}$$

Its direction ratios are <1, 1, 4> ½

Its direction cosines are

$$\left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\right) \quad ½$$

5. Let X be the random variable defined as the number of red balls.

Then $X = 0, 1$ ½

$$P(X=0) = \frac{3}{4}\times\frac{2}{3} = \frac{6}{12} = \frac{1}{2} \quad ½$$

$$P(X=1) = \frac{1}{4}\times\frac{3}{3} + \frac{3}{4}\times\frac{1}{3} = \frac{6}{12} = \frac{1}{2} \quad ½$$

Probability distribution Table:

X	0	0
$P(X)$	$\frac{1}{2}$	$\frac{1}{2}$

½

6. The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card)) **1**

$$= \frac{2}{52}\times\frac{3}{51} + \frac{24}{52}\times\frac{4}{51} = \frac{1}{26} \quad \mathbf{1}$$

Section - B

7. Let

$$\frac{x+1}{(x^2+1)x} = \frac{Ax+B}{x^2+1} + \frac{C}{x} = \frac{(Ax+B)x + C(x^2+1)}{(x^2+1)x}$$

½

$$\Rightarrow \quad x+1 = (Ax+B)x + C(x^2+1) \quad \text{(An identity)}$$

Equating the coefficients, we get

$B = 1, C = 1, A + C = 0$

Hence, $A = -1, B = 1, C = 1$ ½

The given integral

$$= \int \frac{-x+1}{x^2+1}dx + \int \frac{1}{x}dx$$

$$= \frac{-1}{2}\int \frac{2x-2}{x^2+1}dx + \int \frac{1}{x}dx \quad ½$$

$$= \frac{-1}{2}\int \frac{2x}{x^2+1}dx + \int \frac{1}{x^2+1}dx + \int \frac{1}{x}dx$$

$$= \frac{-1}{2}\log(x^2+1) + \tan^{-1}x$$

$$+\log|x| + c \quad 1½$$

8. We have the differential equation

$$\frac{dy}{dx} = \frac{y}{x} - \sin\left(\frac{y}{x}\right)$$

The equation is a homogeneous differential equation.

Putting $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \mathbf{1}$$

The differential equation becomes

$$v + x\frac{dv}{dx} = v - \sin v$$

$$\Rightarrow \quad \frac{dv}{\sin v} = -\frac{dx}{x}$$

$$\Rightarrow \quad \text{cosec } v\, dv = -\frac{dx}{x} \quad ½$$

Integrating both sides, we get

$$\log|\text{cosec } v - \cot v| = -\log|x| + \log K,$$

$K > 0$ (Here, $\log K$ is an arbitrary constant.) **1**

$$\Rightarrow \quad \log|(\text{cosec } v - \cot v)x| = \log K$$

$$\Rightarrow \quad |(\text{cosec } v - \cot v)x| = K$$

$$\Rightarrow \quad (\text{cosec } v - \cot v)x = \pm K$$

$$\Rightarrow \quad \left(\text{cosec}\frac{y}{x} - \cot\frac{y}{x}\right)x = c, \quad ½$$

which is the required general solution.

OR

The differential equation is a linear differential equation

$$\Rightarrow \quad \text{IF} = e^{\int \cot x dx} = e^{\int \log \sin x} = \sin x \quad \mathbf{1}$$

The general solution is given by

$$y\sin x = \int 2\frac{\sin x}{1+\sin x}dx \quad ½$$

$$\Rightarrow \quad y\sin x = 2\int \frac{\sin x + 1 - 1}{1+\sin x}dx$$

$$= 2\int\left[1 - \frac{1}{1+\sin x}\right]dx$$

$$\Rightarrow \quad y\sin x = 2\int\left[1 - \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}\right]dx$$

$$\Rightarrow \quad y\sin x = 2\int\left[1 - \frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]dx$$

$$\Rightarrow \quad y\sin x = 2\int\left[1-\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]dx$$

$$\Rightarrow \quad y\sin x = 2\left[x+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]+c \qquad 1$$

Given that $y = 0$, when $x = \frac{\pi}{4}$,

Hence, $0 = 2\left[\frac{\pi}{4}+\tan\frac{\pi}{8}\right]+c$

$$\Rightarrow \quad c = -\frac{\pi}{2}-2\tan\frac{\pi}{8}$$

Hence, the particular solution is

$$y = \operatorname{cosec}x\left[2\left\{x+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}-\left(\frac{\pi}{2}+2\tan\frac{\pi}{8}\right)\right] \qquad ½$$

Alternative method

$$\frac{dy}{dx}+y\cot x = \frac{2}{1+\sin x}$$

The differential equation is a linear differential equations

$$\therefore \quad I.F. = e^{\int\cot x\,dx} = e^{\ln\sin x} = \sin x$$

The general solution is given by

$$y\sin x = 2\int\frac{\sin x}{1+\sin x}dx$$
$$= 2\int\frac{1+\sin x-1}{1+\sin x}dx$$
$$= 2\int\frac{1+\sin x}{1+\sin x}dx-2\int\frac{1\,dx}{1+\sin x}$$
$$y\sin x = 2x-2\int\frac{1-\sin x}{\cos^2 x}dx$$
$$y\sin x = 2x-2\int\sec^2 x+2\int\tan x\sec x\,dx$$
$$y\sin x = 2x-2\tan x+2\sec x+c$$

Given that

$$y = 0 \text{ when } x = \frac{\pi}{4}$$
$$0 = \frac{\pi}{2}-2+\sqrt{2}+C$$
$$C = -\left(\frac{\pi}{2}-2+\sqrt{2}\right)$$

Since, the particular solution is

$$y\sin x = 2x-2\tan x+2\sec x-\frac{\pi}{2}+2-\sqrt{2}$$

9. We have

$$\vec{a}.(\vec{b}-\vec{c}) = 0$$
$$\Rightarrow \quad (\vec{b}-\vec{c}) = \vec{0} \text{ or } \vec{a}\perp(\vec{b}-\vec{c})$$
$$\Rightarrow \quad \vec{b} = \vec{c} \text{ or } \vec{a}\perp(\vec{b}-\vec{c}) \qquad 1$$

Also, $\vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$

$$\Rightarrow \quad (\vec{b}-\vec{c}) = \vec{0} \text{ or } \vec{a}\,||\,(\vec{b}-\vec{c})$$
$$\Rightarrow \quad \vec{b} = \vec{c} \text{ or } \vec{a}\,||\,(\vec{b}-\vec{c}) \qquad 1$$

$\vec{a}$ cannot be both perpendicular to $(\vec{b}-\vec{c})$ and parallel to $(\vec{b}-\vec{c})$

Hence, $\vec{b} = \vec{c}$. 1

10. Here, the lines are parallel. The shortest distance

$$= \left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$$

$$= \frac{|(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})|}{\sqrt{4+1+1}} \qquad 1+½$$

$$(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 3\\ 2 & 1 & 1\end{vmatrix} = -3\hat{i}+6\hat{j} \qquad 1$$

Hence, the required shortest distance

$$= \frac{3\sqrt{5}}{\sqrt{6}} \text{ units} \qquad ½$$

OR

Since, the plane is parallel to the given lines, the cross product of the vector $2\hat{i}-3\hat{j}+2\hat{k}$ and $\hat{i}-3\hat{j}+\hat{k}$ will be a normal to the plane

$$(2\hat{i}-3\hat{j}+2\hat{k})\times(\hat{i}-3\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 2\\ 1 & -3 & 1\end{vmatrix}$$
$$= 3\hat{i}-3\hat{k} \qquad 1$$

The vector equation of the plane is

$$\vec{r}.(3\hat{i}-3\hat{k}) = (\hat{i}+2\hat{j}-\hat{k}).(3\hat{i}-3\hat{k})$$

or, $\vec{r}.(\hat{i}-\hat{k}) = 2$ 1

and the cartesian equation of the plane is

$$x-z-2 = 0. \qquad 1$$

Section - C

11. The given definite integral

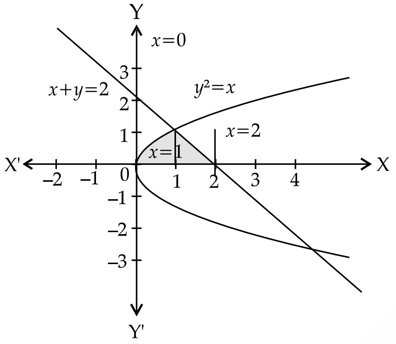
$$= \int_{-1}^{2}|x(x-1)(x-2)|dx$$
$$= \int_{-1}^{0}|x(x-1)(x-2)|dx+\int_{0}^{1}|x(x-1)(x-2)|dx+\int_{1}^{2}|x(x-1)(x-2)|dx \qquad 1½$$

$$= -\int_{-1}^{0}(x^3-3x^2+2x)dx + \int_{0}^{1}(x^3-3x^2+2x)dx - \int_{1}^{2}(x^3-3x^2+2x)dx \quad ½$$

$$= -\left[\frac{x^4}{4}-x^3+x^2\right]_{-1}^{0} + \left[\frac{x^4}{4}-x^3+x^2\right]_{0}^{1} - \left[\frac{x^4}{4}-x^3+x^2\right]_{1}^{2}$$

$$= \frac{9}{4}+\frac{1}{4}+\frac{1}{4} = \frac{11}{4}$$ **2**

12. Solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, –2). **1**



1

The required area = The shaded area

$$= \int_{0}^{1}\sqrt{x}\,dx + \int_{1}^{2}(2-x)dx$$ **1**

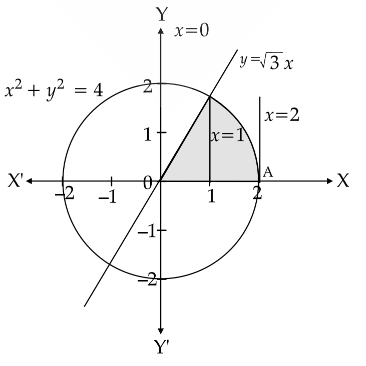
$$= \frac{2}{3}[x^{3/2}]_0^1 + \left[2x-\frac{x^2}{2}\right]_1^2$$

$$= \frac{2}{3}+\frac{1}{2} = \frac{7}{6} \text{ square units}$$ **1**

OR

Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get the points of intersection as $(1, \sqrt{3})$ and $(-1, -\sqrt{3})$

1



1

The required area = The shaded area

$$= \int_{0}^{1}\sqrt{3}x\,dx + \int_{1}^{2}\sqrt{4-x^2}\,dx$$

$$= \frac{\sqrt{3}}{2}\left[x^2\right]_0^1 + \frac{1}{2}\left[x\sqrt{4-x^2}+4\sin^{-1}\frac{x}{2}\right]_1^2$$ **1**

$$= \frac{\sqrt{3}}{2}+\frac{1}{2}\left[2\pi-\sqrt{3}-2\frac{\pi}{3}\right]$$

$$= \frac{2\pi}{3} \text{ square units}$$ **1**

13. The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is

$$\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$$ **1**

The coordinates of the foot of the perpendicular are (μ + 1, –3μ + 2, 2μ) for some μ ½

These coordinates will satisfy the equation of the plane. Hence, we have

$$\mu+1-3(-3\mu+2)+2(2\mu) = 9$$

$$\Rightarrow \quad \mu = 1$$ **1**

The foot of the perpendicular is (2, –1, 2). ½

Hence, the required distance

$$= \sqrt{(1-2)^2+(2+1)^2+(0-2)^2} = \sqrt{14} \text{ untis}$$ **1**

Case-Based/Data Based

14. Let E_1 = The policy holder is accident prone.

E_2 = The policy holder is not accident prone.

E = The new policy holder has an accident within a year of purchasing a policy.

(i) $$P(E) = P(E_1)\times P\left(\frac{E}{E_1}\right) + P\left(\frac{E}{E_2}\right)\times P\left(\frac{E}{E_2}\right)$$ **1**

$$= \frac{20}{100}\times\frac{6}{10}+\frac{80}{100}\times\frac{2}{10} = \frac{7}{25}$$ **1**

(ii) By Bayes' Theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\times P\left(\frac{E}{E_1}\right)}{P(E)}$$ **1**

$$= \frac{\frac{20}{100}\times\frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$$ **1**

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Solved Paper, 2021-2022

MATHEMATICS

Term-I, Set-4

Series : SSJ/2

Question Paper Code No. 065/2/4

Time allowed : 90 Minutes **Max. Marks : 40**

General Instructions :

(i) *This question paper comprises of **50** questions out of which **40** questions are to be attempted as per instructions. All questions carry equal marks.*

(ii) *The question paper consists of three Sections – Section A, B and C.*

(iii) ***Section – A** contains **20** questions. Attempt any **16** questions from Q. No. **1** to **20**.*

(iv) ***Section – B** also contains **20** questions. Attempt any **16** questions from Q. No. **21** to **40**.*

(v) ***Section – C** contains **10** questions including one Case Study. Attempt any 8 questions from Q. No. **41** to **50**.*

(vi) *There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.*

(vii) *There is no negative marking.*

SECTION - A

In this section, attempt any 16 questions out of Questions 1-20. Each question is of one mark.

1. Differential of log [log (log x^5)] w.r.t x is

(a) $\dfrac{5}{x\log(x^5)\log(\log x^5)}$

(b) $\dfrac{5}{x\log(\log x^5)}$

(c) $\dfrac{5x^4}{\log(x^5)\log(\log x^5)}$

(d) $\dfrac{5x^4}{\log x^5 \log(\log x^5)}$

2. The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is

(a) 16 (b) 6

(c) 64 (d) 24

3. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. Then the function has

(a) no minimum value

(b) no maximum value

(c) both maximum and minimum values

(d) neither maximum value nor minimum value

4. If $\sin y = x\cos(a + y)$, then $\dfrac{dx}{dy}$ is

(a) $\dfrac{\cos a}{\cos^2(a+y)}$ (b) $\dfrac{-\cos a}{\cos^2(a+y)}$

(c) $\dfrac{\cos a}{\sin^2 y}$ (d) $\dfrac{-\cos a}{\sin^2 y}$

5. The points on the curve $\dfrac{x^2}{9}+\dfrac{y^2}{25} = 1$, where tangent is parallel to X-axis are

(a) (±5, 0) (b) (0, ±5)

(c) (0, ±3) (d) (±3, 0)

6. Three points P($2x$, $x + 3$), Q(0, x) and R($x + 3$, $x + 6$) are collinear, then x is equal to

(a) 0 (b) 2

(c) 3 (d) 1

7. The principal value of $\cos^{-1}\left(\dfrac{1}{2}\right)+\sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$ is

(a) $\dfrac{\pi}{12}$ (b) π

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

8. If $(x^2 + y^2)^2 = xy$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{y+4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (b) $\dfrac{y-4x(x^2+y^2)}{x+4(x^2+y^2)}$

(c) $\dfrac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (d) $\dfrac{4y(x^2+y^2)-x}{y-4x(x^2+y^2)}$

9. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix

(b) Zero square matrix

(c) Square matrix

(d) Identity matrix

10. Let set X = {1, 2, 3} and a relation R is defined in X as:

R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are

(a) {(1, 1), (2, 3), (1, 2)}
(b) {(3, 3), (3, 1), (1, 2)}
(c) {(1, 1), (3, 3), (3, 1). (2, 3)}
(d) {(1, 1), (3, 3), (3, 1), (1, 2)}

11. A Linear Programming Problem is as follows:

Minimise $z = 2x + y$

subject to the constraints $x \geq 3, x \leq 9, y \geq 0$

$x - y \geq 0, x + y \leq 14$

The feasible region has

(a) 5 corner points including (0, 0) and (9, 5)
(b) 5 corner points including (7, 7) and (3, 3)
(c) 5 corner points including (14, 0) and (9, 0)
(d) 5 corner points including (3, 6) and (9, 5)

12. The function $f(x) = \begin{cases} \dfrac{e^{3x} - e^{-5x}}{x}, & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$

is continuous at $x = 0$ for the value of k, as

(a) 3 **(b)** 5
(c) 2 **(d)** 8

13. If C_{ij} denotes the cofactor of element P_{ij} of the matrix

$P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31}.C_{23}$ is

(a) 5 **(b)** 24
(c) –24 **(d)** –5

14. The function $y = x^2e^{-x}$ is decreasing in the interval

(a) (0, 2) **(b)** $(2, \infty)$
(c) $(-\infty, 0)$ **(d)** $(-\infty, 0) \cup (2, \infty)$

15. If $R = \{(x, y); x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in set Z, then domain of R is

(a) {0, 1, 2}
(b) {–2, –1, 0, 1, 2}
(c) {0, –1, –2}
(d) {–1, 0, 1}

16. The system of linear equations

$5x + ky = 5,$

$3x + 3y = 5;$

will be consistent if

(a) $k \neq -3$ **(b)** $k = -5$
(c) $k = 5$ **(d)** $k \neq 5$

17. The equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses the X-axis is

(a) $x - 5y = 2$ **(b)** $5x - y = 2$
(c) $x + 5y = 2$ **(d)** $5x + y = 2$

18. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is

(a) 4 **(b)** 16
(c) –4 **(d)** –16

19. The principal value of $\tan^{-1}\left(\tan\dfrac{9\pi}{8}\right)$ is

(a) $\dfrac{\pi}{8}$ **(b)** $\dfrac{3\pi}{8}$

(c) $-\dfrac{\pi}{8}$ **(d)** $-\dfrac{3\pi}{8}$

20. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $P - Q$ is

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ -0 & -3 \\ -1 & -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

SECTION - B

In this Section attempt any 16 questions out of the Questions 21-40. Each question is of one mark.

21. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval

(a) $(-\infty, 2) \cup (3, \infty)$ **(b)** $(-\infty, 2)$
(c) $(-\infty, 2] \cup [3, \infty)$ **(d)** $[3, \infty)$

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$ **(b)** $\dfrac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

(c) $\dfrac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$ **(d)** $\dfrac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

23. What is the domain of the function $\cos^{-1}(2x - 3)$?

(a) [–1, 1] **(b)** (1, 2)
(c) (–1, 1) **(d)** [1, 2]

24. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$a_{ij} = \begin{cases} 2i + 3j & , \ i < j \\ 5 & , \ i = j \\ 3i - 2j & , \ i > j \end{cases}$

The number of elements in A which are more than 5, is:

(a) 3 **(b)** 4
(c) 5 **(d)** 6

25. If a function f defined by

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3 & \text{if } x=\dfrac{\pi}{2}\end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$, then the value of k is

(a) 2 **(b)** 3
(c) 6 **(d)** –6

26. For the matrix $X = \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, $(X^2 - X)$ is

(a) 2I **(b)** 3I
(c) I **(d)** 5I

27. Let $X = \{x^2 : x \in N\}$ and the function $f : N \to X$ is defined by $f(x) = x^2, x \in N$. Then this function is

(a) injective only **(b)** not bijective
(c) surjective only **(d)** bijective

28. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function $Z = 2x + 5y$ is at the point

(a) P **(b)** Q
(c) R **(d)** S

29. The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is

(a) $2y - 3mx + am^3 = 0$
(b) $2x + 3my\ 3am^4 - am^2 = 0$
(c) $2x + 3my + 3am^4 - 2am = 0$
(d) $2x + 3my - 3am^4 - 2am^2 = 0$

30. If A is a square matrix of order 3 and $|A| = -5$, then $|\text{adj } A|$ is

(a) 125 **(b)** – 25
(c) 25 **(d)** ±25

31. The simplest form of $\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is

(a) $\dfrac{\pi}{4}-\dfrac{x}{2}$ **(b)** $\dfrac{\pi}{4}+\dfrac{x}{2}$

(c) $\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x$ **(d)** $\dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x$

32. If for the matrix $A = \begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$, $|A^3| = 125$, then the value of α is

(a) ±3 **(b)** –3
(c) ±1 **(d)** 1

33. If $y = \sin(m\sin^{-1} x)$, then which one of the following equations is true ?

(a) $(1-x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+m^2y=0$

(b) $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+m^2y=0$

(c) $(1+x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-m^2y=0$

(d) $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-m^2x=0$

34. The principal value of $[\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})]$ is

(a) π **(b)** $-\dfrac{\pi}{2}$
(c) 0 **(d)** $2\sqrt{3}$

35. The maximum value of $\left(\dfrac{1}{x}\right)^x$ is

(a) $e^{1/e}$ **(b)** e

(c) $\left(\dfrac{1}{e}\right)^{1/e}$ **(d)** e^e

36. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}$.

Then the matrix $Y = [m_{ij}]$, where m_{ij} = Minor of x_{ij}, is

(a) $\begin{bmatrix}7 & -5 & -3\\ 19 & 1 & -11\\ -11 & 1 & 7\end{bmatrix}$ **(b)** $\begin{bmatrix}7 & -19 & 11\\ 5 & -1 & -1\\ 3 & 11 & 7\end{bmatrix}$

(c) $\begin{bmatrix}7 & 19 & -11\\ -3 & 11 & 7\\ -2 & -1 & -1\end{bmatrix}$ **(d)** $\begin{bmatrix}7 & 19 & -11\\ -1 & -1 & 1\\ -3 & -11 & 7\end{bmatrix}$

37. A function $f : R \to R$ defined by $f(x) = 2 + x^2$ is

(a) not one-one
(b) one-one
(c) not onto
(d) neither one-one nor onto

38. A Linear Programming Problem is as follows:

Maximise / Minimise objective function $Z = 2x - y + 5$

Subject to the constraints

$$3x + 4y \leq 60$$
$$x + 3y \leq 30$$
$$x \geq 0, y \geq 0$$

If the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0,0), then which of the following is true ?

(a) Maximum value of Z is 40

(b) Minimum value of Z is – 5

(c) Difference of maximum and minimum values of Z is 35

(d) At two corner points, value of Z are equal

39. If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

(a) 4 **(b)** –3

(c) 2 **(d)** 5

40. The absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $\left[-2, \frac{9}{2}\right]$ is

(a) 8 **(b)** 9

(c) 6 **(d)** 10

SECTION - C

Attempt any 8 questions out of the Questions 41-50. Each question is of one mark.

41. In a sphere of radius r, a right circular cone of height h having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi^2 rh(2rh + h^2)$ **(b)** $\pi^2 hr(2rh + h^2)$

(c) $2\pi^2 r(2rh^2 - h^3)$ **(d)** $2\pi^2 r^2(2rh - h^2)$

42. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

(a) $a - 5b = 0$ **(b)** $a - 3b = 0$

(c) $a - 2b = 0$ **(d)** $a - 8b = 0$

43. If curves $y^2 = 4x$ and $xy = c$ cut at right angles, then the value of c is

(a) $4\sqrt{2}$ **(b)** 8

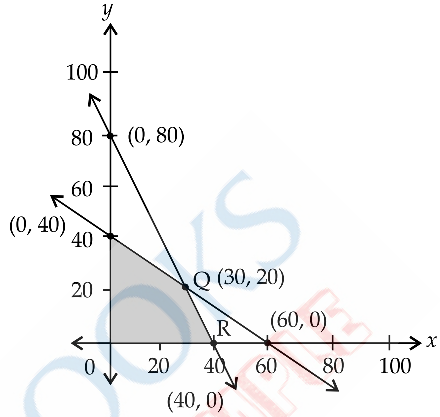
(c) $2\sqrt{2}$ **(d)** $-4\sqrt{2}$

44. The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ **(b)** $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

45. For an L.P.P. the objective function is $Z = 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true ?

(a) Maximum value of Z is at R.

(b) Maximum value of Z is at Q.

(c) Value of Z at R is less than the value at P.

(d) Value of Z at Q is less than the value at R.

Case Study

In a residential society comprising of 100 houses, there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For this purpose, instead of each child doing it for only his/her house, children convinced the Residents welfare association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of ₹ 50 per square metre for space so that there is no misuse of the space and Resident welfare association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ₹ 400 x (depth)^2. Association will like to have minimum cost.

46. Let side of square plot is x m and its depth is h metres, then cost C for the pit is

(a) $\frac{50}{h}+400h^2$ **(b)** $\frac{12500}{h}+400h^2$

(c) $\frac{250}{h}+h^2$ **(d)** $\frac{250}{h}+400h^2$

47. Value of h (in m) for which $\frac{dC}{dh}=0$ is

(a) 1.5 **(b)** 2

(c) 2.5 **(d)** 3

48. $\frac{d^2C}{dh^2}$ is given by

(a) $\frac{25000}{h^3}+800$ **(b)** $\frac{500}{h^3}+800$

(c) $\frac{100}{h^3}+800$ **(d)** $\frac{500}{h^3}+2$

49. Value of x (in m) for minimum cost is

(a) 5 **(b)** $10\sqrt{\frac{5}{3}}$

(c) $5\sqrt{5}$ **(d)** 10

50. Total minimum cost of digging the pit (in ₹) is

(a) 4,100 **(b)** 7,500

(c) 7,850 **(d)** 3,220

□□□

SOLUTIONS

SECTION - A

1. (a) $\frac{5}{x\log(x^5)\log(\log x^5)}$

***Explanation*:** Let $y=\log[\log(\log x^5)]$

$$\therefore \quad \frac{dy}{dx}=\frac{1}{\log(\log x^5)}\frac{dy}{dx}[\log(\log x^5)]$$

(By Chain Rule)

$$=\frac{1}{\log(\log x^5)}\cdot\frac{1}{\log x^5}\frac{d}{dx}\log x^5$$

$$=\frac{1}{\log(x^5)\log(\log x^5)}\frac{1}{x^5}\frac{d}{dx}(x^5)$$

$$=\frac{5}{x\log(x^5)\log(\log x^5)}$$

2. (c) 64

***Explanation*:** The order of the matrix = **2 × 3**

The number of elements = 2 × 3 = 6

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = $2^6 = 64$

3. (d) neither maximum value nor minimum value

***Explanation*:** Given, $f(x)=x^3+1$

$\therefore \quad f'(x)=3x^2$ and $f''(x)=6x$

Put $\quad f'(x)=0$

$\Rightarrow \quad 3x^2=0\Rightarrow x=0$

At $\quad x=0, f''(x)=0$

Thus, $f(x)$ has neither maximum value nor minimum value.

4. (a) $\frac{\cos a}{\cos^2(a+y)}$

***Explanation*:** Given, $\sin y = x\cos(a+y)$

$$\Rightarrow \quad x=\frac{\sin y}{\cos(a+y)}$$

Differentiating with respect to y, we get

$$\frac{dx}{dy}=\frac{\cos(a+y)\frac{d}{dy}(\sin y)-\sin y\frac{d}{dy}\{\cos(a+y)\}}{\cos^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{\cos(a+y)\cos y-\sin y[-\sin(a+y)]}{\cos^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{\cos(a+y)\cos y+\sin y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{\cos[(a+y)-y]}{\cos^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{\cos a}{\cos^2(a+y)}$$

5. (b) **(0, ±5)**

***Explanation*:** The equation of the given curve:

$$\frac{x^2}{9}+\frac{y^2}{25}=1 \quad \textbf{...(i)}$$

On differentiating both sides w.r.t. x, we get

$$\frac{2x}{9}+\frac{2y}{25}\frac{dy}{dx}=1$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{-25x}{9y}$$

Since, tangent is parallel to X-axis, then the slope of the tangent is zero.

$\therefore \quad \frac{-25}{9}\frac{x}{y}=0$, which is possible if x = 0

Put $x = 0$ in eq (i), we get

$$\frac{y^2}{25} = 1 \Rightarrow y^2 = 25 \Rightarrow y = \pm 5$$

Hence, required points are $(0, \pm 5)$.

6. (d) 1

Explanation: As points are collinear

$\Rightarrow$ area of triangle formed by 3 points is zero.

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix}(x_1-x_2) & (x_2-x_3)\\ (y_1-y_2) & (y_2-y_3)\end{vmatrix} = 0$$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix}(2x-0) & \{0-(x+3)\}\\ (x+3-x) & \{x-(x+6)\}\end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix}2x & -(x+3)\\ 3 & -6\end{vmatrix} = 0$$

$$\Rightarrow \quad -12x + 3(x+3) = 0$$

$$\Rightarrow \quad -12x + 3x + 9 = 0$$

$$\Rightarrow \quad -9x = -9$$

$$\Rightarrow \quad x = 1$$

7. (a) $\frac{\pi}{12}$

Explanation: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$$= \cos^{-1}\left(\cos\frac{\pi}{3}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{3}-\sin^{-1}\left(\sin\frac{\pi}{4}\right)$$

$$= \frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

8. (c) $\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$

Explanation: Given, $(x^2 + y^2)^2 = xy$

$$\Rightarrow \quad x^4 + 2x^2y^2 + y^4 - xy = 0$$

Differentiating w.r.t. x, we get

$$4x^3+2\left[2xy^2+x^2.2y\frac{dy}{dx}\right]+4y^3\frac{dy}{dx}-\left[y+x\frac{dy}{dx}\right] = 0$$

$$\frac{dy}{dx}\left[4x^2y+4y^3-x\right]+\left[4x^3+4xy^2-y\right] = 0$$

$$\frac{dy}{dx} = \frac{-[4x^3+4xy^2-y]}{[4x^2y+4y^3-x]}$$

$$\text{or} \quad \frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$$

9. (b) Zero square matrix

Explanation: If matrix A is symmetric

$$A^T = A$$

If matrix A is skew-symmetric

$$A^T = -A$$

Also, diagonal elements are zero.

Since, it is given that matrix A is both symmetric and skew-symmetric.

$$\therefore \quad A = A^T = -A$$

Which is only possible if A is zero matrix.

$$A = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = A^T = -A$$

Thus, if a matrix A is both symmetric and skew symmetric, then A is necessarily a zero matrix.

10. (c) {(1, 1), (3, 3), (3, 1). (2, 3)}

Explanation:

(i) R is reflexive if it contains {(1, 1), (2, 2) and (3, 3)}.
Since, $(2, 2) \in$ R. So, we need to add (1, 1) and (2, 2) to make R reflexive.

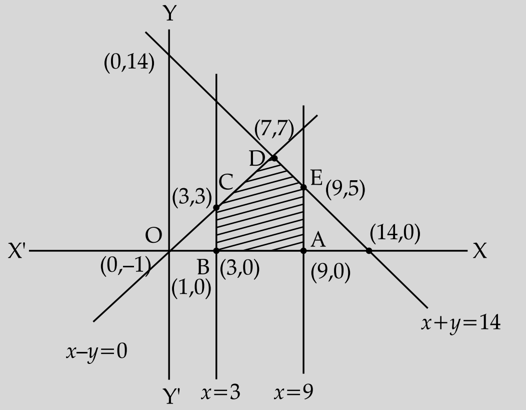
(ii) R is symmetric if it contains {(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)}.

Since, $\{(2, 2), (1, 3), (3, 2)\} \in$ R. So, we need to add (3, 1) and (2, 3).

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are {(1, 1), (3, 3), (3, 1), (2, 3)}.

11. (b) 5 corner points including (7, 7) and (3, 3)

Explanation: On plotting the constraints $x = 3$, $x = 9$, $x = y$ and $x + y = 14$, we get the following graph. From the graph given below it clear that feasible region is ABCDEA, including corner points A(9, 0), B(3, 0), C(3, 3), D(7, 7) and E(9, 5). Thus feasible region has 5 corner points including (7, 7) and (3, 3).



12. (d) 8

Explanation: Since, $f(x)$ is continuous at $x = 0$, then

$$\text{LHL} = \text{RHL} = f(0) \quad \text{or} \quad \text{LHL} = \text{RHL} = k$$

Now, $$\text{LHL} = \lim_{h\to 0}\frac{e^{3(0-h)} - e^{-5(0-h)}}{0-h}$$

$$= \lim_{h\to 0}\frac{e^{-3h} - e^{5h}}{-h}$$

$$= \lim_{h\to 0}\left(\frac{e^{-3h}-1}{-h}\right) + \lim_{h\to 0}\left(\frac{e^{5h}-1}{h}\right)$$

$$= 3\lim_{h\to 0}\left(\frac{e^{-3h}-1}{-3h}\right) + 5\lim_{h\to 0}\left(\frac{e^{5h}-1}{5h}\right)$$

$$= 3 \times 1 + 5 \times 1 = 8$$

Thus, $k = 8$.

13. (a) 5

Explanation:

Here, $$C_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$$

and $$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = -(2 + 3) = -5$$

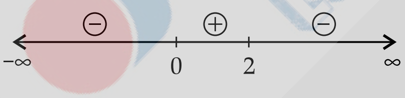
Thus, $C_{31}.C_{23} = (-1)(-5) = 5$

14. (d) $(-\infty, 0) \cup (2, \infty)$.

Explanation: We have,

$$f(x) = y = x^2 e^{-x}$$

$$\therefore \frac{dy}{dx} = 2x\,e^{-x} + x^2(-1)e^{-x} = xe^{-x}(2 - x)$$



Now, put $\frac{dy}{dx} = 0$

$\Rightarrow x = 0$ and $x = 2$

The points $x = 0$ and $x = 2$ divide the real line into three disjoint intervals *i.e.*, $(-\infty, 0)$, $(0, 2)$ and $(2, \infty)$

In intervals, $(-\infty, 0)$ and $(2, \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

$\therefore f(x)$ or y is decreasing in $(-\infty, 0)$ and $(2, \infty)$.

15. (b) $\{-2, -1, 0, 1, 2\}$

Explanation: Given, $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 4\}$

Let $x = 0$, then $y^2 \leq 4 \Rightarrow y = 0, \pm 1, \pm 2$

Thus, domain of R = $\{-2, -1, 0, 1, 2\}$

16. (d) $k \neq 5$

Explanation: We have, $5x + ky - 5 = 0$

and $3x + 3y - 5 = 0$

For consistent system

$$\frac{5}{3} \neq \frac{k}{3}$$

$$\Rightarrow \quad k \neq 5$$

17. (c) $x + 5y = 2$

Explanation: Given, $y(1 + x^2) = 2 - x$ ...(i)

If it cuts X-axis, then y-coordinate is 0.

$$\therefore \quad 0(1 + x^2) = 2 - x$$

$$\Rightarrow \quad x = 2$$

Thus, point of contact is (2, 0).

Now, differentiating eq.(i) w.r.t. x, we get

$$y.(2x) + \frac{dy}{dx}(1 + x^2) = -1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1-2xy}{1+x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(1+2xy)}{1+x^2}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(2,0)} = \frac{-1}{5}$$

Thus, equation of tangent is

$$y - 0 = \frac{-1}{5}(x - 2)$$

$$\Rightarrow \quad 5y + x = 2$$

$$\Rightarrow \quad x + 5y = 2$$

18. (a) 4

Explanation: Given, $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3d \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

$$\therefore \quad 3c + 6 = 12 \quad ...(i)$$

$$a - d = 2 \quad ...(ii)$$

$$a + d = -8 \quad ...(iii)$$

$$2 - 3b = -4 \quad ...(iv)$$

From eq. (i), we get $c = 2$

On solving eqs. (ii) and (iii), we get $a = -3$ and $d = -5$ from eq. (iv), we get $b = 2$

Now, $$ab - cd = (-3)2 - 2(-5)$$

$$\Rightarrow \quad ab - cd = -6 + 10 = 4$$

19. (a) $\frac{\pi}{8}$

***Explanation*:** $\tan^{-1}\left(\tan\frac{9\pi}{8}\right) = \tan^{-1}\left(\tan\left(\pi+\frac{\pi}{8}\right)\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{8}\right) = \frac{\pi}{8}$ $\left[\because \frac{\pi}{8} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$

20. (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

***Explanation*:**

Here, $Q = (Q^T)^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now, $P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

SECTION-B

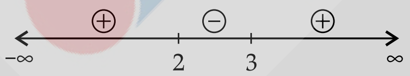
21. (c) $(-\infty, 2] \cup [3, \infty)$

***Explanation*:** Given, $f(x) = 2x^3 - 15x^2 + 36x + 6$

$\therefore$ $f'(x) = 6x^2 - 30x + 36$

If $f'(x) \geq 0$, then $f(x)$ is increasing.

So, $6x^2 - 30x + 36 \geq 0$



or, $x^2 - 5x + 6 \geq 0$

or, $(x-3)(x-2) \geq 0$

$\therefore$ $x \in (-\infty, 2] \cup [3, \infty)$

22. (b) $\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

***Explanation*:** Given, $x = 2\cos\theta - \cos 2\theta$

and $y = 2\sin\theta - \sin 2\theta$

Therefore, $\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

and $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\therefore$ $\frac{dy}{dx} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$

or $\frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

23. (d) [1, 2]

***Explanation*:** Let, $f(x) = \cos^{-1}(2x - 3)$

$\because$ $-1 \leq 2x - 3 \leq 1$

$\Rightarrow$ $2 \leq 2x \leq 4$

$\Rightarrow$ $1 \leq x \leq 2$

$\therefore$ $x \in [1, 2]$ or domain of x is [1, 2].

24. (b) 4

***Explanation*:** Here, $A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Thus, number of elements more than 5, is 4.

25. (c) 6

***Explanation*:** Since, $f(x)$ is continuous at $x = \frac{\neq}{2}$

Therefore, $\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow$ $\lim_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi - 2x} = 3$

$\Rightarrow$ $k \lim_{x\to\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$

$\Rightarrow$ $\frac{k}{2} \lim_{x\to\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$

$\Rightarrow$ $\frac{k}{2} \times 1 = 3 \Rightarrow k = 6$

26. (a) 2I

***Explanation*:**

Here $X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\Rightarrow$ $X^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$$\Rightarrow \quad X^2 = 2\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = 2I$$

27. (a) injective only

Explanation: Let $x_1, x_2 \in N$

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad x_1^2 = x_2^2$$

$$\Rightarrow \quad x_1^2 - x_2^2 = 0$$

$$\Rightarrow \quad (x_1 + x_2)(x_1 - x_2) = 0$$

$$\Rightarrow \quad x_1 = x_2$$

$$\{x_1 + x_1 \neq 0 \text{ as } x_1, x_2 \in N\}$$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in N. Thus, $f(x)$ is not surjective.

28. (c) R

Explanation:

Corner Points	Value of $Z = 2x + 5y$
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 ® Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs at R(4, 2).

29. (d) $2x + 3my - 3am^4 - 2am^2 = 0$

Explanation: $ay^2 = x^3$

Differentiating both sides w.r.t. x, we get

$$2ay\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{2ay}$$

Slope of the tangent at (am^2, am^3) is

$$\left(\frac{dy}{dx}\right)_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)}$$

$$= \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}$$

Now, slope of format at $(am^2, am^3) = \dfrac{-1}{\left(\dfrac{dy}{dx}\right)_{(am^2,\, am^3)}}$

$$= \frac{-2}{3m}$$

Thus, equation of normal at (am^2, am^3) is

$$(y - am^3) = \frac{-2}{3m}(x - am^2)$$

$$\Rightarrow 3my - 3am^4 + 2x - 2am^2 = 0$$

$$\Rightarrow 2x + 3my - 3am^4 - 2am^2 = 0$$

30. (c) 25

Explanation: We know that,

$$|\text{adj } A| = |A|^{n-1}$$

where n is the order of the matrix

$$\therefore \quad |\text{adj } A| = (5)^{3-1}$$

$$= 5^2 = 25$$

31. (c) $\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1} x$

Explanation: We have,

$$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$$

Put $x = \cos 2\theta$, so that $\theta = \dfrac{1}{2}\cos^{-1} x$

$$\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1 - \tan\theta}{1 + \tan\theta}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(\tan\theta)$$

$$\left[\because \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} x - \tan^{-1} y\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) - \theta$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

32. (a) ±3

Explanation: Given, $A = \begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$

$$\Rightarrow \quad |A| = \alpha^2 - 4 \quad \text{...(i)}$$

Also, given $|A^3| = 125$

$$\Rightarrow \quad |A|^3 = 125$$
$$\Rightarrow \quad |A| = 5$$
$$\Rightarrow \quad \alpha^2 - 4 = 5 \qquad \text{[from eq. (i)]}$$
$$\Rightarrow \quad \alpha^2 = 9$$
$$\Rightarrow \quad \alpha = \pm 3$$

33. (b) $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2y = 0$

***Explanation*:** Given, $y = \sin(m(\sin^{-1}x))$ ...(i)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \cos(m\sin^{-1}x) \times \frac{m}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}} \qquad \text{...(ii)}$$

$$\Rightarrow \quad y' = \frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}} \qquad \text{...(ii)}$$

$$\Rightarrow \quad \left(\sqrt{1-x^2}\right)y' = m\cos(m\sin^{-1}x)$$

Differentiating again w.r.t. 'x', we get

$$y''\left(\sqrt{1-x^2}\right) + y'\frac{(-2x)}{2\sqrt{1-x^2}}$$

$$= -m^2\sin(m\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y''(1-x^2) - xy' = -m^2y$$

$$\Rightarrow \quad y''(1-x^2) - xy' + m^2y = 0$$

or, $$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$

34. (b) $-\dfrac{\pi}{2}$

***Explanation*:** We have,

$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \pi + \cot^{-1}\cot\sqrt{3}$$

$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$$

$$= -\frac{\pi}{2}$$

35. (a) $e^{1/e}$

***Explanation*:** Let $y = \left(\dfrac{1}{x}\right)^x$

Then, $\log y = x\log\left(\dfrac{1}{x}\right) = -x\log x$ ...(i)

Differentiating both sides w.r.t. x

$$\therefore \quad \frac{1}{y}\frac{dy}{dx} = -\left[x.\frac{1}{x} + \log x\right]$$

$$= -(1 + \log x) \qquad \text{...(ii)}$$

On differentiating again eq. (ii), we get

$$\frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2$$

$$= \frac{-1}{x} \text{...(iii)}$$

From eq. (ii), we get

$$\frac{dy}{dx} = -y(1 + \log x)$$

$$= -\left(\frac{1}{x}\right)^x(1 + \log x)$$

For maximum or minimum values of y, put $\dfrac{dy}{dx} = 0$

Therefore, $\left(\dfrac{1}{x}\right)^x(1 + \log x) = 0$

However, $\left(\dfrac{1}{x}\right)^x \neq 0$ for any value of x. Therefore

$$1 + \log x = 0$$

$$\Rightarrow \quad \log x = -1 \Rightarrow x = e^{-1} \Rightarrow x = \frac{1}{e}$$

When $x = \dfrac{1}{e}$, from eq. (iii)

$$\frac{1}{y}\frac{d^2y}{dx^2} - 0 = -e$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -e(e)^{1/e} < 0$$

Hence, y is maximum when $x = \dfrac{1}{e}$ and maximum value of $y = e^{1/e}$.

36. (d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

Explanation: $m_{11} = \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 12 - 5 = 7$

$$m_{12} = \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = 9 + 10 = 19$$

$$m_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3 - 8 = -11$$

$$m_{21} = \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -3 + 2 = -1$$

$$m_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$$

$$m_{23} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 + 2 = 1$$

$$m_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 5 - 8 = -3$$

$$m_{32} = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -5 - 6 = -11$$

$$m_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4 + 3 = 7$$

$$\therefore \quad Y = \begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$$

37. (d) neither one-one nor onto

Explanation: Given, $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$\Rightarrow \quad 2 + x_1^2 = 2 + x_2^2$

$\Rightarrow \quad x_1^2 = x_2^2$

$\Rightarrow \quad x_1 = \pm x_2$

$\Rightarrow \quad x_1 = x_2$

or $\quad x_1 = -x_2$

Thus, $f(x)$ is not one-one.

For onto

Let $f(x) = y$ such that $y \in R$

$\therefore \quad x^2 = y - 2$

$\Rightarrow \quad x = \pm\sqrt{y-2}$

Put $y = -3$, we get

$$x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$$

Which is not possible as root of negative is not a real number.

Hence, x is not real.

So, $f(x)$ is not onto.

38. (b) Minimum value of Z is – 5

Explanation:

Corner Points	Value of $Z = 2x - y + 5$
A(0, 10)	Z = 2(0) – 10 + 5 = –5 (Minimum)
B(12, 6)	Z = 2(12) – 6 + 5 = 23
C(20, 0)	Z = 2(20) – 0 + 5 = 45 (Maximum)
O(0, 0)	Z = 0(0) – 0 + 5 = 5

So the minimum value of Z is – 5.

39. (a) 4

Explanation: Given, $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$

$\Rightarrow \quad x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$

$\Rightarrow \quad x^3 - 13x + 13 = 0$

Since $(x + 4)$ is one root of above cubic equation.

Sum roots $= 0$

$\therefore$ Sum of two roots $+ (-4) = 0$

Sum of two roots $= 4$

40. (a) 8

Explanation: Given, $f(x) = 4x - \frac{1}{2}x^2$

$\therefore \quad f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

put $\quad f'(x) = 0$

$\Rightarrow \quad 4 - x = 0$

$\Rightarrow \quad x = 4$

Then, we evaluate the f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$$

$$f(-2) = -8 - \frac{1}{2}(4)$$

$$= -8 - 2 = -10$$

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2$$

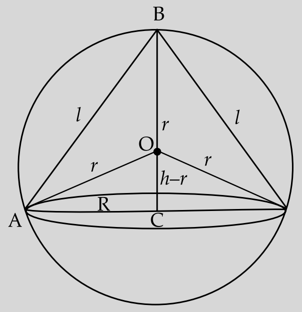
$$= 18 - \frac{81}{8} = 7.875$$

Thus, the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$.

SECTION - C

41. (c) $2\pi^2 r(2rh^2 - h^3)$

Explanation:



Here, CSA of cone $= \pi Rl$

Radius of sphere $= r$

height of cone $= h$

In ΔAOC,

$$AO^2 = AC^2 + OC^2$$

$$\Rightarrow \quad r^2 = R^2 + (h-r)^2$$

$$\Rightarrow \quad R^2 = 2hr - h^2$$

$$\therefore \quad \text{Radius of cone, } R = \sqrt{2hr - h^2} \quad ...(i)$$

In ΔABC,

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow \quad l^2 = R^2 + h^2$$

$$\Rightarrow \quad l^2 = 2hr - h^2 + h^2$$

$$\therefore \quad \text{slant height} = \sqrt{2hr} \quad ...(ii)$$

$$\text{CSA of cone} = \pi Rl$$

$$= \pi\sqrt{2hr - h^2}\sqrt{2hr}$$

$$(\text{CSA of cone})^2 = \pi^2(2hr - h^2)(2hr)$$

$$= 2\pi^2 hr(2hr - h^2)$$

$$= 2\pi^2 r(2rh^2 - h^3)$$

42. (d) $a - 8b = 0$

Explanation: Given, Max. $Z = ax + 2by$

Max. value of Z on Q(3, 5) = Max. value of Z on S(4, 1)

$$\Rightarrow \quad 3a + 10b = 4a + 2b$$

$$\Rightarrow \quad a - 8b = 0$$

43. (a) $4\sqrt{2}$

Explanation: Given curves, $y^2 = 4x$ and $xy = c$ cuts orthogonally.

Let they intersect at (x_1, y_1).

Now,

$$y^2 = 4x$$

$$\therefore \quad 2y\frac{dy}{dx} = 4$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{y}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x_1, y_1)} = \frac{2}{y_1} \quad ...(i)$$

and $\quad xy = c$

$$\therefore \quad x\frac{dy}{dx} + y = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x_1, y_1)} = -\frac{y_1}{x_1} \quad ...(ii)$$

From eqs. (i) and (ii)

$$\frac{2}{y_1} \times \left(\frac{-y_1}{x_1}\right) = -1 \quad [\because m_1 m_2 = -1]$$

$$\Rightarrow \quad x_1 = 2$$

Put $x_1 = 2$ in $y_1^2 = 4x_1$, we get

$$y_1^2 = 4(2) = 8$$

$$\Rightarrow \quad y_1 = 2\sqrt{2}$$

Now, put value of x_1 and y_1 in $x_1 y_1 = c$, we get

$$c = 2\left(2\sqrt{2}\right) = 4\sqrt{2}$$

44. (d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

Explanation: The inverse of a diagonal matrix is obtained by replacing each element in the diagonal with its reciprocal.

Since, $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Therefore $X^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

45. (b) Maximum value of Z is at Q.

Explanation:

Corner Points of Feasible Region	Value of z = ($z = 4x + 3y$)
O(0, 0)	Z = 4(0) + 3(0) = 0
P(0, 40)	Z = 4(0) + 3(40) = 120
Q(30, 20)	Z = 4(30) + 3(20) = 180 (Maximum)
R(40, 0)	Z = 4(40) + 3(0) = 160

Thus, Maximum value of z is at Q, which is 180.

46. (b) $\frac{12500}{h}+400h^2$

Explanation: $C = \frac{250\times 50}{h}+400\times h^2$

$\Rightarrow$ $C = \frac{12500}{h}+400h^2$

47. (c) 2.5 m

Explanation: Since,

$$C = \frac{12500}{h}+400h^2$$

$\therefore$ $\frac{dC}{dh} = \frac{-12500}{h^2}+800h$

Put $\frac{dC}{dh} = 0$

$\therefore$ $\frac{-12500}{h^2}+800h = 0$

$\Rightarrow$ $800h^3 = 12500$

$\Rightarrow$ $h^3 = \frac{125}{8}$

$\Rightarrow$ $h = \frac{5}{2} = 2.5\text{ m}$

48. (a) $\frac{25000}{h^3}+800$

Explanation: Since,

$\therefore$ $\frac{dC}{dh} = \frac{-12500}{h^2}+800h$

$\therefore$ $\frac{d^2C}{dh^2} = \frac{-(-2)\times 12500}{h^3}+800$

$\Rightarrow$ $\frac{d^2C}{dh^2} = \frac{25000}{h^3}+800$

49. (d) 10 m

Explanation: For minimum cost, put $\frac{dC}{dh} = 0$, we get

$h = 2.5$ m

At $h = 2.5$, $\frac{d^2C}{dh^2} > 0$

(Hence, minimum)

Value of x at minimum cost

$$x = \frac{400\times(2.5)^2}{250}$$

$$= \frac{2500}{250} = 10\text{ m}$$

50. (b) 7,500

Explanation: Total minimum cost,

$C = \frac{12500}{h}+400h^2$ (At 2.5)

$\Rightarrow$ $C = \frac{12500}{2.5}+400(2.5)^2$

$\Rightarrow$ $C = 5000 + 2500$

$\Rightarrow$ $C = ₹7500$

❑❑❑

Term – I

OMR SHEET

Booklet Series
A

Use English Numbers / Letters only. Use Blue / Black Ball Point Pen to write in box.

Booklet Series

Ⓐ Ⓑ Ⓒ Ⓓ

Subject

Roll Number

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

Name

Test Date

Student's Signature

Invigilator's Signature

Certified that all the entries In this section have been properly filled by the student

Proper Marking

The OMR Sheet will be computer checked Fill the circles completely and dark enough for proper detection, Use ballpen (black or blue) for marking.

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Avoid Improper Marking

Partially Filled

Lightly Filled

Test Center Code

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

IMPORTANT

The candidate should check that the Test Book Series printed on the OMR Sheet is the same as printed on the Test Booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.

Darken the circle for each question.

Q.No.	Response	Q.No.	Response	Q.No.	Response	Q.No.	Response
01	Ⓐ Ⓑ Ⓒ Ⓓ	16	Ⓐ Ⓑ Ⓒ Ⓓ	31	Ⓐ Ⓑ Ⓒ Ⓓ	46	Ⓐ Ⓑ Ⓒ Ⓓ
02	Ⓐ Ⓑ Ⓒ Ⓓ	17	Ⓐ Ⓑ Ⓒ Ⓓ	32	Ⓐ Ⓑ Ⓒ Ⓓ	47	Ⓐ Ⓑ Ⓒ Ⓓ
03	Ⓐ Ⓑ Ⓒ Ⓓ	18	Ⓐ Ⓑ Ⓒ Ⓓ	33	Ⓐ Ⓑ Ⓒ Ⓓ	48	Ⓐ Ⓑ Ⓒ Ⓓ
04	Ⓐ Ⓑ Ⓒ Ⓓ	19	Ⓐ Ⓑ Ⓒ Ⓓ	34	Ⓐ Ⓑ Ⓒ Ⓓ	49	Ⓐ Ⓑ Ⓒ Ⓓ
05	Ⓐ Ⓑ Ⓒ Ⓓ	20	Ⓐ Ⓑ Ⓒ Ⓓ	35	Ⓐ Ⓑ Ⓒ Ⓓ	50	Ⓐ Ⓑ Ⓒ Ⓓ
06	Ⓐ Ⓑ Ⓒ Ⓓ	21	Ⓐ Ⓑ Ⓒ Ⓓ	36	Ⓐ Ⓑ Ⓒ Ⓓ	51	Ⓐ Ⓑ Ⓒ Ⓓ
07	Ⓐ Ⓑ Ⓒ Ⓓ	22	Ⓐ Ⓑ Ⓒ Ⓓ	37	Ⓐ Ⓑ Ⓒ Ⓓ	52	Ⓐ Ⓑ Ⓒ Ⓓ
08	Ⓐ Ⓑ Ⓒ Ⓓ	23	Ⓐ Ⓑ Ⓒ Ⓓ	38	Ⓐ Ⓑ Ⓒ Ⓓ	53	Ⓐ Ⓑ Ⓒ Ⓓ
09	Ⓐ Ⓑ Ⓒ Ⓓ	24	Ⓐ Ⓑ Ⓒ Ⓓ	39	Ⓐ Ⓑ Ⓒ Ⓓ	54	Ⓐ Ⓑ Ⓒ Ⓓ
10	Ⓐ Ⓑ Ⓒ Ⓓ	25	Ⓐ Ⓑ Ⓒ Ⓓ	40	Ⓐ Ⓑ Ⓒ Ⓓ	55	Ⓐ Ⓑ Ⓒ Ⓓ
11	Ⓐ Ⓑ Ⓒ Ⓓ	26	Ⓐ Ⓑ Ⓒ Ⓓ	41	Ⓐ Ⓑ Ⓒ Ⓓ	56	Ⓐ Ⓑ Ⓒ Ⓓ
12	Ⓐ Ⓑ Ⓒ Ⓓ	27	Ⓐ Ⓑ Ⓒ Ⓓ	42	Ⓐ Ⓑ Ⓒ Ⓓ	57	Ⓐ Ⓑ Ⓒ Ⓓ
13	Ⓐ Ⓑ Ⓒ Ⓓ	28	Ⓐ Ⓑ Ⓒ Ⓓ	43	Ⓐ Ⓑ Ⓒ Ⓓ	58	Ⓐ Ⓑ Ⓒ Ⓓ
14	Ⓐ Ⓑ Ⓒ Ⓓ	29	Ⓐ Ⓑ Ⓒ Ⓓ	44	Ⓐ Ⓑ Ⓒ Ⓓ	59	Ⓐ Ⓑ Ⓒ Ⓓ
15	Ⓐ Ⓑ Ⓒ Ⓓ	30	Ⓐ Ⓑ Ⓒ Ⓓ	45	Ⓐ Ⓑ Ⓒ Ⓓ	60	Ⓐ Ⓑ Ⓒ Ⓓ