Sample Question Paper, 2021-22

(Issued by CBSE Board on 14th January, 2022)

APPLIED MATHEMATICS (Term- II)

SOLVED

General Instructions:

(i) *The question paper is divided into 3 sections – A, B and C*

(ii) *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*

(iii) *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*

(iv) *Section C comprises of 4 questions of 4 marks each. It contains one case study based question. Internal choice has been provided in one question.*

Section - A

(2 marks each)

1. The marginal revenue function for a commodity is given by $MR = 9 + 2x - 6x^2$. Find the demand function.

OR

The marginal cost of producing x pairs of tennis shoes is given by the following function:

$$MC = 50 + \frac{300}{x+1}$$

If the fixed cost is ₹ 2000, then find the total cost function.

2. Find the present value of a perpetuity of ₹ 600 at end of each quarter, if money is worth 8% compounded quarterly.

3. What effective rate is equivalent to a nominal rate of 8% per annum compounded quarterly?

OR

Find the present value of an annuity of ₹1000, payable at the end of each year for 5 years, if money is worth 6% compounded annually. [Given $(1.06)^{-5}$=0.7473]

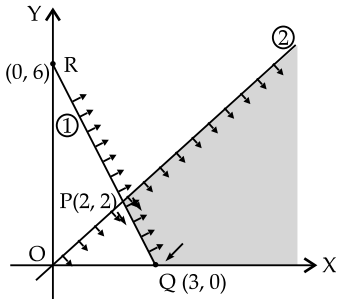
4. A sampling distribution of the sample means $\bar{X}$ is formed from a population with mean weight $\mu = 60$ kg and standard deviation $\sigma = 9$kg. What is the expected value and standard deviation of $\bar{X}$, if sample size is 36?

5. Find the trend values using 3 yearly moving average for the loans sanctioned to farmers by a particular branch of a bank in a village.

The data is given as follows:

Year	2016	2017	2018	2019	2020	2021
Amount (in ₹ lakh)	25	30	32	40	45	50

6. The feasible region of the L.P.P. Min $Z = 3x + 2y$ subject to constraints $2x + y \geq 6, x - y \geq 0, x \geq 0, y \geq 0$ is given below:



Determine the optimal solution Justify your answer.

Section - B

(3 marks each)

7. The supply function for a commodity is given by $p = x^2 + 4x + 3$, where x is the quantity supplied at the price p. Find the producer's surplus, when the price of the commodity is ₹ 48.

8. The following table shows the quarterly sales (in ₹ crore) of a real estate company. Compute the trend by quarterly moving averages.

Quarters / Year	2016	2017	2018	2019
2018	12	14	18	20
2019	18	16	20	22
2020	27	24	30	36

OR

Fit a straight line trend by the method of least squares and also estimate the trend for the year 2023.

Year	2014	2015	2016	2017	2018	2019	2020
Amount (in ₹ lacs)	26	26	44	42	108	120	166

9. A machine produces washers of thickness 0.50 mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53 mm and the standard deviation is 0.03 mm. Test the hypothesis at 5% level of significance so that the machine is working in proper order. [Given critical value, $t_{0.025} = 2.262$ at $v(d.f) = 9$]

10. A person invested ₹ 15000 in a mutual fund and the value of investment at the time of redemption was ₹ 25000. If CAGR for this investment is 8.88%, then calculate the time period for which the given amount was invested? [Given log(1.667)=0.2219 & log(1.089)=0.037]

Section - C

(4 marks each)

11. S & D chemicals produces two products, an alkaline solution and a base oil that are sold as raw material to the companies manufacturing soaps and detergents. On the basis of current inventory levels and estimated demand for the coming month, S & D's management has decided that combined production of alkaline solution and the base oil must be at least 3500 gallons. S & D chemicals are also committed to supply 1250 gallons of alkaline solution to one of its major customer. The alkaline solution and base oil requires respectively 2 hours and 1 hour of processing time per gallon. The total processing time available for the coming month is 6000 hours. The production cost is ₹ 200 per gallon for the alkaline solution and ₹ 300 per gallon for the base oil.

Formulate the above as a L.P.P and solve it by using graphical method, to help S & D chemicals for determining the minimum production cost.

12. A machine costing ₹ 50,000 is to be replaced at the end of 10 years, when it will have a salvage value of ₹ 5000. In order to provide money at that time for a machine costing the same amount, a sinking fund is set up. If equal payments are placed in the fund at the end of each quarter and the fund earns at the rate of 8% compounded quarterly, then what should each payment be? [Given $(1.02)^{40} = 2.208$]

13. A couple wishes to purchase a house for ₹ 15,00,000 with a down payment of ₹ 4,00,000. If they can amortize the balance at an interest rate of 9% per annum compounded monthly for 10 years, then find the monthly installment (EMI). Also find the total interest paid. [Given $(1.0075)^{-120} = 0.4079$]

OR

A ₹ 2000, 8% bond is redeemable at the end of 10 years at ₹ 105. Find the purchase price to yield 10% effective rate. [Given $(1.1)^{-10} = 0.3855$]

CASE STUDY

14. General anaesthesia is used for major operations to cure the patients and conduct pain free surgeries. Propofol is a commonly used anaesthetic injected for major operations such as knee replacement or open heart surgery. It also acts as a sedative and an analgesic.

A patient is rushed to operation theatre for a 2-hour cardiac surgery. A person is anaesthetized when its blood stream contains at least 3 mg of propofol per kg of body weight. The rate of change of propofol (x), in the body is proportional to the quantity of propofol present at that time. Based on the above information. Answer the following questions:

(i) Show that the propofol given intravenously is eliminated exponentially from the patients' blood stream. **2**

(ii) What dose of propofol should be injected, to induce unconsciousness in a 50 kg adult for a two hours operation? **2**

(Given $(2)^{\frac{1}{5}} = 1.1487$ & assume half-life of propofol = 5 hours)

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CBSE Marking Scheme 2021-2022
(Issued by Board)

Section - A

1. Given, $MR = 9 + 2x - 6x^2$

Let the demand function be p,

We know that,

$$TR = \int(9+2x-6x^2)dx$$

$$TR = 9x + x^2 - 2x^3 + C$$

where C is Integration Constant

When $x = 0$, $TR = 0$, so $C = 0$

$$TR = 9x + x^2 - 2x^3 = px \quad \textbf{1}$$

$$\Rightarrow \quad px = 9x + x^2 - 2x^3$$

$$\Rightarrow \quad p = 9 + x - 2x^2$$

which is the required demand function. **1**

OR

Let the total cost function be TC

$$TC = \int\left(50 + \frac{300}{x+1}\right)dx$$

$$TC = 50x + 300\log|x+1| + C$$

If $x = 0$, $TC = ₹\ 2000$

So $\quad 2000 = 300(\log 1) + C \quad \textbf{1}$

$$\Rightarrow \quad C = 2000$$

So $\quad TC = 50x + 300\log(x+1) + 2000$

Hence, the required total cost function

$$TC = 50x + 300\log(x+1) + 2000 \quad \textbf{1}$$

2. Let the present value of perpetuity be P

Given, $\quad R = ₹\ 600$

$$i = \frac{0.08}{4} = 0.02 \quad \textbf{1}$$

Present value of perpetuity $= P = \frac{R}{i}$

$$\Rightarrow \quad P = \frac{600}{0.02} = ₹\ 30{,}000 \quad \textbf{1}$$

3. Let the effective rate be r(subscript){eff}

Since, $\quad r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 \quad \textbf{1}$

(Given $r = 0.08$)

$$= \left(1 + \frac{0.08}{4}\right)^4 - 1$$

$$= (1.02)^4 - 1 = 0.0824 \text{ or } 8.24\% \quad \textbf{1}$$

So effective rate is 8.24% compounded quarterly.

OR

Present value of ordinary annuity **1**

$$= R\left(\frac{1-(1+r)^{-n}}{r}\right)$$

$$= 1000\left(\frac{1-(1.06)^{-5}}{0.06}\right)$$

$$= 1000\left(\frac{1-0.7473}{0.06}\right)$$

$$= ₹\ 4211.67 \quad \textbf{1}$$

4. Let the expected value be E, standard deviation be SD and sample size be n

$$\sqrt{n} = E(\overline{X}) = 6 \text{ kg} \quad \textbf{1}$$

Standard deviation of

$$\overline{X} = SD(\overline{X})$$

$$= \frac{\sigma}{\sqrt{n}} = \frac{9}{6}$$

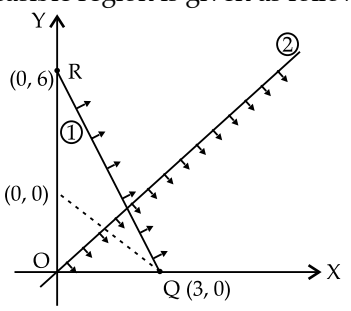
$$= 1.5 \text{ kg} \quad \textbf{1}$$

5.

Year	Y	3 yearly moving total	3 yearly moving average (Trend)
2016	25	—	—
2017	30	87	29
2018	32	102	34
2019	40	117	39
2020	45	135	45
2021	50	—	—

1 + 1

6. The feasible region is given as follows:



The values at the corner points P and Q can be shown as:

Corner points	$Z = 3x + 2y$
P (2, 2)	10
Q (3, 0)	9

1

The smallest value of Z is 9. Since, the feasible region is unbounded, we draw the graph of $3x + 2y < 9$. The resulting open half plane has points common with feasible region, therefore $Z = 9$ is not the minimum value of Z. Hence, the optimal solution does not exist. **1**

Section - B

7. Substituting, $p_0 = ₹\ 48$ in $p = x^2 + 4x + 3$

We get $x_0 = 5$ **1**

$$PS = p_0x_0 - \int_0^{x_0} g(x)dx$$

$$= 48\times5 - \int_0^5 (x^2+4x+3)dx \quad \textbf{1}$$

$$= 240 - \left[\frac{x^3}{3} + 2x^2 + 3x\right]_0^5$$

$$= ₹\ 133.33 \quad \textbf{1}$$

8.

Year	Quarters	Y	4-Quarterly Moving Total	4 Quarterly Moving average (Centered)
2018	Q_1	12		—
	Q_2	14		—
	Q_3	18	64	16.75
	Q_4	20	70	17.75
2019	Q_1	18	72	18.25
	Q_2	16	74	18.75
	Q_3	20	76	20.125
	Q_4	22	85	22.25
2020	Q_1	27	93	24.50
	Q_2	24	103	27.5
	Q_3	30	117	—
	Q_4	36		—

The trend value are given by 4 quarterly centered moving average. **1½ + 1½**

OR

Year	Y	X = Year − 2017	X^2	XY
2014	26	– 3	9	– 78
2015	26	– 2	4	– 52
2016	44	– 1	1	– 44
2017	42	0	0	0
2018	108	1	1	108
2019	120	2	4	240
2020	166	3	9	498
	$\Sigma Y = 532$		$\Sigma X^2 = 28$	$\Sigma XY = 672$

$$a = \frac{\Sigma Y}{n} = \frac{532}{7} = 76, b = \frac{\Sigma XY}{\Sigma X^2} = \frac{672}{28} = 24$$

$Y_C = a + bX$, $Y_C = 76 + 24X$ **1**

Estimated sales = Y_C for 2023 = $76 + 24 \times 6$ = ₹ 220 lacs

9. Define Null hypothesis H_0 and alternate hypothesis H_1 as follows:

H_0: $\mu = 0.50$ mm

H_1: $\mu \neq 0.50$ mm **1**

Thus, a two-tailed test is applied under hypothesis H_0, **1**

We have,

$$t = \frac{\overline{X} - \mu}{S}\sqrt{n-1}$$

$$= \frac{0.53 - 0.50}{0.03} \times 3 = 3$$

Since, the calculated value of $t = 3$ does not lie in the interval $- t_{0.025}$ to $t_{0.025}$ *i.e.*, – 2.262 to 2.262 for 10 – 1 = 9 degree of freedom. So we reject H_0 at 0.05 level. Hence, we conclude that machine is not working properly. **1**

10. We know

$$CAGR = \left[\left(\frac{FV}{IV}\right)^{\frac{1}{n}} - 1\right] \times 100,$$

where, IV = Initial value of investment

FV = Final value of investment **1**

$$\Rightarrow \quad 8.88 = \left[\left(\frac{25000}{15000}\right)^{\frac{1}{n}} - 1\right] \times 100$$

$$\Rightarrow \quad 0.0888 = \left(\frac{5}{3}\right)^{\frac{1}{n}} - 1 \quad \textbf{1}$$

$$\Rightarrow \quad 1.089 = (1.667)^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{1}{n}\log(1.667) = \log(1.089) \quad \textbf{1}$$

$$\Rightarrow \quad n(0.037) = 0.2219 \quad \textbf{1}$$

$$\Rightarrow \quad n = 5.99 \approx 6 \text{ years} \quad \textbf{1}$$

Section - C

11. Let the company produces x and y gallons of alkaline solution and base oil respectively, also let C be the production cost.

Min $C = 200x + 300y$

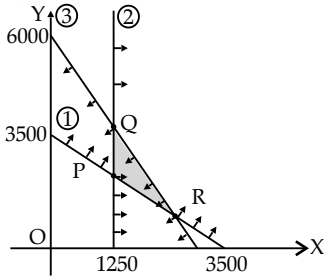
Subject to constraints:

$$x + y \geq 3500 \quad ...(1)$$

$$x \geq 1250 \quad ...(2)$$

$$2x + y \leq 6000 \quad ...(3)$$

$x, y \geq 0$ **1½**



Corner points	C = 200x + 300y
P (1250, 2250)	₹ 9,25,000
Q (1250, 3500)	₹ 13,00,000
R (2500, 1000)	₹ 8,00,000

Minimum cost is 8,00,000 when 2500 gallons of alkaline solutions & 1000 gallons of base oil are manufactured. **1**

12. The amount of sinking fund S at any time is given by

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] \quad \textbf{1}$$

where R = periodic payment, i = Interest per period, n = number of payments

S = Cost of machine – Salvage value

= ₹ 50,000 – ₹ 5000 = ₹ 45,000

$$i = \frac{8\%}{4} = 0.02$$

$$\Rightarrow \quad 45000 = R\left[\frac{(1+0.02)^{40} - 1}{0.02}\right] \quad \textbf{1½}$$

$$\Rightarrow \quad 45000 = R\left[\frac{2.208 - 1}{0.02}\right]$$

$$\Rightarrow \quad R = \frac{900}{1.208}$$

$$\Rightarrow \quad R = ₹\,745.03 \quad \textbf{1½}$$

13. P = Cost of house – Cash down payment

P = ₹ 15,00,000 – ₹ 4,00,000 = ₹ 11,00,000

$$i = \frac{0.09}{12} = 0.0075$$

$n = 10 \times 12 = 120$ **1**

$$\text{EMI} = R = \frac{P}{a_{n-i}}$$

$$R = \frac{P \times i}{1 - (1+i)^{-n}} \quad \textbf{1}$$

$$= \frac{11,00,000 \times 0.0075}{1 - (1.0075)^{-120}}$$

$$= \frac{8250}{1-0.4079} \quad \textbf{1}$$

$$= \frac{8250}{0.5921} = ₹\ 13933.5$$

Total interest paid $= nR - R$

$= 13933.5 \times 120 - 11{,}00{,}000$

$= ₹\ 5{,}72{,}020$ **1**

OR

Face value of bond, $F = ₹\ 2000$

Redemption value $C = 1.05 \times 2000 = ₹\ 2100$

Nominal rate $= 8\%$

$R = C \times i_d = 2000 \times 0.08$

$= ₹\ 160$ **1½**

Number of periods before redemption i.e.,

$n = 10$

Annual yield rate, $i = 10\%$ or 0.1

Purchase price,

$$V = R\left[\frac{1-(1+i)^{-n}}{i}\right] + C(1+i)^{-n} \quad \textbf{1½}$$

$$= 160\left[\frac{1-(1+0.1)^{-10}}{0.1}\right] + 2100(1+0.1)^{-10}$$

$= 160 \times 6.14 + 2100 \times 0.3855$

$= 982.4 + 809.6$

$= ₹\ 1792$

Thus, present value of the bond is ₹ 1792. **1**

CASE STUDY

14. $\because \frac{dx}{dt} \propto x,$

$\therefore \quad \frac{dx}{dt} = -kx$

$\Rightarrow \quad \int \frac{dx}{x} = \int -k\,dt$

$\Rightarrow \quad \log x = -kt + c$ **1**

$\Rightarrow \quad x = e^{-kt+c}$

$\Rightarrow \quad x = \lambda e^{-kt}$ where $e^c = \lambda$

(i) Let $x = x_0$ at $t = 0$

$\because \quad x_0 = \lambda$

$\Rightarrow \quad x = x_0 e^{-kt}$

where x_0 = original quantity **1**

(ii) Let the half life be t and amount of propofol be x_0

$x = x_0 e^{-kt}$ …(i)

Now, $\frac{x_0}{2} = x_0 e^{-5k}$ ($\because$ half life = 5 hours)

$\Rightarrow \quad e^{-5k} = \frac{1}{2}$

$\Rightarrow \quad e^k = 2^{1/5}$ **1**

The quantity of propofol needed in a 50 kg adult at the end of

2 hours $= 50 \times 3 = 150$ mg

$\Rightarrow \quad 150 = x_0 e^{-2k}$ [using …(i)]

$\Rightarrow \quad x_0 = 150\, e^{2k} = 150\,(e^k)^2$

$\Rightarrow \quad x_0 = 150(2^{1/5})^2 = 150 \times 1.3195$

$x_0 = 197.93$ mg **1**

So, 197.53 mg of propofol is needed.

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Solved Paper, 2021-22

APPLIED MATHEMATICS

Term-I, Set-4

Series : SSJ/1

Question Paper Code No. 465/1/4

Time allowed : 90 Minutes **Max. Marks : 40**

General Instructions :

*(i) This question paper contains 50 questions out of which **40** questions are to be attempted as per instructions. All questions carry equal marks.*

*(ii) The question paper consists **three** Sections – Section **A, B** and **C**.*

*(iii) Section **A** consists of **20** questions. Attempt any **16** questions from Q. No. **1** to **20**.*

*(iv) Section **B** also consists of **20** questions. Attempt any **16** questions from Q. No. **21** to **40**.*

*(v) Section **C** consists of a Case Studies containing 5 questions (Q. No. **46** to **50**). Attempt any **8** from Q. No. **41** to **50**.*

*(vi) There is only **one** correct option for every multiple choice question (MCQ). Marks will **not** be awarded for answering more than one option.*

*(vii) There is **no** negative marking.*

SECTION - A

In this section, attempt any 16 questions out of question no. 1 – 20. Each question is of 1 mark. **(16×1=16)**

1. If $100 = x \pmod 7$, then the least positive value of x is:

(A) 2 **(B)** 3 **(C)** 6 **(D)** 4

2. If $\tau(n)$ denotes the number of divisors of n, then value of $\tau(15)$ is:

(A) 3 **(B)** 4 **(C)** 5 **(D)** 7

3. If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is:

(A) 2 km/h **(B)** 1.5 km/h **(C)** 2.5 km/h **(D)** 2.25 km/h

4. In a 2 km race, P can give Q a start of 200 m and R a start of 560 m. Then in the same race, Q can give R a start of:

(A) 360 m **(B)** 380 m **(C)** 400 m **(D)** 430 m

5. Pipe A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the time taken to fill the tank is:

(A) 2 hours **(B)** $2\frac{3}{4}$ hours **(C)** 3 hours **(D)** $3\frac{9}{17}$ hours

6. The solution of $\frac{x-3}{x+5} > 0, x \neq -5, x \in R$ is:

(A) $x > 3$ **(B)** $x < -5$ **(C)** $x < -5$ or $x > 3$ **(D)** no solution

7. If matrix A is given by $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = i + j$, then A is equal to:

(A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

8. If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is equal to:

(A) I **(B)** $2A$ **(C)** $3I$ **(D)** A

9. If A is a square matrix of order 3×3 such that $|A| = 4$, then $|3A|$ is equal to:

(A) 27 **(B)** 81 **(C)** 108 **(D)** 256

10. The function $f(x) = a^x$ is increasing on R, if:
(A) $a > 0$ **(B)** $a < 0$ **(C)** $0 < a < 1$ **(D)** $a > 1$

11. If $C(x)$ and $R(x)$ are respectively Cost function and Revenue function, then the Profit function $P(x)$ is given by:
(A) $P(x) = R(x)$ **(B)** $P(x) = C(x) + R(x)$ **(C)** $P(x) = R(x) - C(x)$ **(D)** $P(x) = R(x).C(x)$

12. If 'm' is the mean of Poisson distribution, then its standard deviation is given by:
(A) $\sqrt{m}$ **(B)** m^2 **(C)** m **(D)** $\frac{m}{2}$

13. The normal distribution curve is symmetrical about:
(A) $X = \mu$ **(B)** $X = \sigma$ **(C)** $X = \frac{\mu}{\sigma}$ **(D)** $X = \frac{\sigma}{\mu}$

14. Let X be a discrete random variable whose probability distribution is given below:

$X = x_i$:	0	1	2	3	4	5	6	7
$P(X = x_i)$:	0	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2$	$2K$

The value of K is:
(A) $\frac{1}{10}$ **(B)** -1 **(C)** $-\frac{1}{10}$ **(D)** $\frac{1}{5}$

15. In a box of 100 bulbs, 10 are defective. What is the probability that out of a sample of 5 bulbs, none is defective?
(A) $\left(\frac{9}{10}\right)^5$ **(B)** $\frac{9}{10}$ **(C)** 10^{-5} **(D)** $\left(\frac{1}{2}\right)^2$

16. If X is a normal variate with mean μ and standard deviation $\sigma > 0$, then the new random variate $Z = \frac{X - \mu}{\sigma}$ is a variate with:
(A) Mean = 1, Standard deviation = 0. **(B)** Mean = 1, Standard deviation = 1.
(C) Mean = 2, Standard deviation = 1. **(D)** Mean = 0, Standard deviation = 1.

17. The mean $E(x)$ of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is:
(A) 1 **(B)** 2 **(C)** 5 **(D)** $\frac{8}{3}$

18. Which of the following index number satisfies the "time reversal test"?
(A) Fisher's ideal index number **(B)** Laspeyres' index number
(C) Paasche's index number **(D)** None of these

19. To calculate Paasche's price index, the weights are taken as:
(A) p_0 **(B)** p_1 **(C)** q_0 **(D)** q_1

20. Given that $\sum p_1q_1 = 860$, $\sum p_0q_0 = 520$, $\sum p_1q_0 = 630$ and $\sum p_0q_1 = 730$, where subscript 0 and 1 are used for base year and current year respectively. The Laspeyres' index number is:
(A) 117.81 **(B)** 119.5 **(C)** 121.15 **(D)** 123.35

SECTION - B

In this section, attempt any 16 questions out of questions no. 21 - 40. Each question is of 1 mark. (16×1=16)

21. The remainder when 5^{61} is divided by 7 is:
(A) 1 **(B)** 2 **(C)** 4 **(D)** 5

22. 20 litres of a mixture contains milk and water in the ratio 3 : 1. The amount of milk, in litres, to be added to the mixture so as to have milk and water in the ratio 4 : 1, is:
(A) 7 **(B)** 4 **(C)** 5 **(D)** 6

23. Pipe A can fill a tank 6 times faster than a pipe B. If B can fill a tank in 21 minutes, then the time taken by both the pipes together to fill the tank is:
(A) 3 minutes **(B)** $4\frac{1}{2}$ minutes **(C)** 7 minutes **(D)** 9 minutes

24. The ratio of investments of two partners A and B is 11 : 12 and the ratio of their profits is 2 : 3. If A invested the money for 8 months, then for how much time did B invest his money?

(A) 11 months **(B)** 10 months **(C)** 9 months **(D)** 5 months

25. The solution set of the inequation $|x + 2| \leq 5$ is:

(A) $(-7, 5)$ **(B)** $[-7, 3]$ **(C)** $[-5, 5]$ **(D)** $(-7, 3)$

26. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $(x + y)$ equals:

(A) 0 **(B)** –1 **(C)** 2 **(D)** –2

27. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ a & b & 2 \end{bmatrix}$, then $aI + bA + 2A^2$ equals:

(A) A **(B)** $-A$ **(C)** abA **(D)** None of these

28. If $A^2 - A + I = 0$, then the inverse of matrix A is:

(A) A^2 **(B)** $A + I$ **(C)** $I - A$ **(D)** $A - I$

29. If the points (1, 3), $(x, 5)$ and (2, 7) are collinear, then the value of x is:

(A) 2 **(B)** $\frac{3}{2}$ **(C)** 1 **(D)** $\frac{3}{4}$

30. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is:

(A) $25y$ **(B)** $5y$ **(C)** $-25y$ **(D)** $15y$

31. The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is:

(A) $(2\sqrt{2}, 4)$ **(B)** $(2\sqrt{2}, 0)$ **(C)** $(0, 0)$ **(D)** $(2, 2)$

32. If the total revenue (₹) received from the sale of x units of a product is given by:

$$R(x) = 3x^2 + 36x + 5,$$

then the marginal revenue,when $x = 15$, is

(A) ₹116 **(B)** ₹96 **(C)** ₹90 **(D)** ₹126

33. The equation of normal at the point (1, 1) to the curve $2y + x^2 = 3$ is:

(A) $x + y = 0$ **(B)** $x - y = 0$ **(C)** $x + y = 1$ **(D)** $x - y = 1$

34. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. Then the possible values of X are:

(A) 0, 1, 3, 5 **(B)** 0, 2, 4, 6 **(C)** 0, 2, 5, 6 **(D)** 1, 3, 4, 5

35. If the mean of a binomial distribution is 81, then the standard deviation lies in the interval:

(A) [0, 9) **(B)** (0, 9] **(C)** [0, 3] **(D)** (0, 3]

36. If a random variable X has the Poisson distribution with mean 2. Then, $P(X > 1.5)$ is:

(A) $2e^{-2}$ **(B)** $3e^{-2}$ **(C)** $1 - 2e^{-2}$ **(D)** $1 - 3e^{-2}$

37. There are 50 telephone lines in an exchange. The probability that any one of them will be busy is 0·1. The probability that all the lines are busy is:

(A) $\frac{5^0 e^{-5}}{0!}$ **(B)** $1 - \frac{5^0 e^{-5}}{0!}$ **(C)** $\frac{5^{50} e^{-5}}{50!}$ **(D)** $1 - \frac{5^{50} e^{-5}}{50!}$

38. Price relative of sugar is 135 in the year 2020 compared to the year 2019. If the sugar cost ₹ 30 per kg in 2019, then its cost in 2020 is:

(A) ₹ 15 per kg **(B)** ₹ 40·50 per kg **(C)** ₹ 45.20 per kg **(D)** ₹ 65 per kg

39. If $\sum W \log p = 199.50$ and $\sum W = 100$, then the weighted index number is:

(A) 120·86 **(B)** 88·86 **(C)** 98·86 **(D)** 78·86

40. The condition for the time reversal test to hold good with usual notation is:

(A) $P_{bc} \times P_{cb} = 1$ **(B)** $P_{bc} \times P_{cb} = 0$ **(C)** $P_{bc} + P_{cb} = 1$ **(D)** $\frac{P_{cb}}{P_{bc}} = 1$

SECTION - C

In this section, attempt any 8 questions out of questions no. 41 - 50. Each question is of 1 mark. (Questions no. 46 - 50 are based on a Case-Study) **(8×1=8)**

41. The least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on (1, 2) is:

(A) 0 **(B)** –1 **(C)** –2 **(D)** –4

42. The demand function of a commodity is given by:

$$x = 82 - p$$

and its total cost function is given by:

$$T.C. = 100 + 60x$$

For maximum profit, the value of x is:

(A) 15 units **(B)** 14 units **(C)** 13 units **(D)** 11 units

43. The mean of the probability distribution of the number of doublets in 4 throws of a pair of dice, is:

(A) 1 **(B)** $\frac{2}{3}$ **(C)** $1\frac{3}{5}$ **(D)** $2\frac{2}{3}$

44. It is known from the past experience that the number of telephone calls made daily in a certain community between 3 p.m. and 4 p.m. has a mean of 352 and a standard deviation of 31. What percentage of the time will there be more than 400 telephone calls made in the community between 3 pm to 4 pm? [Use : P(0 ≤ Z ≤ 1·5) = 0.4394]

(A) 11·4% **(B)** 9·6% **(C)** 7·08% **(D)** 6·06%

45. The index number of the following data:

Relative Index	181	116	110	157
Weight	4	12	3	7

is:

(A) 118·74 **(B)** 136·34

(C) 142·04 **(D)** 146·14

Case-Study

Two products P and Q are produced such that 0.4 tonne of P and 0.7 tonne of Q are required to produce one tonne of P. Similarly, 0.1 tonne of P and 0.6 tonne of Q are required to produce one tonne of Q. The economy needs 68 tonnes of P and 102 tonnes of Q.

Based on the above information, answer the following questions:

46. The technology matrix A is:

(A) $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 0.6 & 0.1 \\ 0.7 & 0.4 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0.4 & 0.7 \\ 0.1 & 0.6 \end{bmatrix}$

47. The demand matrix is:

(A) $\begin{bmatrix} 68 \\ 102 \end{bmatrix}$ **(B)** $\begin{bmatrix} 68 \\ 34 \end{bmatrix}$ **(C)** $\begin{bmatrix} 102 \\ 68 \end{bmatrix}$ **(D)** $\begin{bmatrix} 34 \\ 68 \end{bmatrix}$

48. $(I - A)$ is:

(A) $\begin{bmatrix} 0.6 & -0.6 \\ -0.7 & 0.9 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0.4 & -0.1 \\ -0.7 & 0.6 \end{bmatrix}$ **(C)** $\begin{bmatrix} 0.6 & -0.7 \\ -0.1 & 0.4 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$

49. $(I - A)^{-1}$ is:

(A) $\frac{1}{0.17}\begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}$ **(B)** $\frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$ **(C)** $\frac{1}{0.17}\begin{bmatrix} 0.4 & 0.7 \\ 0.1 & 0.6 \end{bmatrix}$ **(D)** $\frac{1}{0.17}\begin{bmatrix} 0.9 & 0.6 \\ 0.7 & 0.6 \end{bmatrix}$

50. The gross outputs of P and Q are:

(A) $P = 260; Q = 360$ **(B)** $P = 220; Q = 640$ **(C)** $P = 520; Q = 300$ **(D)** $P = 420; Q = 433$

❑❑❑

SOLUTIONS

SECTION - A

1. (A) 2

***Explanation*:** We know that,

$$a = b(\text{mod } c)$$

Then b is the remainder, when a is divided by c.

$\therefore$ $100 = x(\text{mod } 7)$

$$7\overline{)100}\,(14$$
$$\underline{7}$$
$$30$$
$$\underline{28}$$
$$2 \rightarrow \text{Remainder}$$

Thus, $x = 2$

2. (B) 4

***Explanation*:** $\tau(n)$ = no. of divisors of n

$\therefore$ $\tau(15) = 4$

[Since, divisors of 15 are 1, 3, 5 and 15]

3. (B) 1.5 km/h

***Explanation*:** Upstream speed $= \frac{14}{6}$

Downstream speed $= \frac{32}{6}$

Speed of the stream $= \frac{\frac{32}{6} - \frac{14}{6}}{2} = \frac{\frac{18}{6}}{2}$

$= \frac{18}{6 \times 2} = \frac{3}{2}$

$= 1.5$ km/h

4. (C) 400 m

5. (D) $3\frac{9}{17}$ hours

***Explanation*:** Here, LCM of 5, 6 and 12 is 60.

i.e., L(5, 6, 12) = 60

Suppose capacity of the tank is l.

Then, quantity filled by pipe A in 1 hr $= \frac{60}{5} = 12\,l$

Quantity filled by pipe B in 1 hr $= \frac{60}{6} = 10\,l$

Quantity emptied by pipe C in 1 hr $= \frac{60}{12} = 5\,l$

So,quantity filled by all three pipes in 1 hr

$= (12 + 10 - 5)l$

$= 17\,l$

Required time $= \frac{60}{17} = 3\frac{9}{17}$ hours

6. (C) $x < -5$ or $x > 3$

***Explanation*:** Given, $\frac{x-3}{x+5} > 0, x \neq -5$

when $x > 3$, then $\frac{x-3}{x+5} > 0$

when $x < -5$, then $\frac{x-3}{x+5} > 0$

Thus, for $\frac{x-3}{x+5} > 0$, the solution is $x < -5$ or $x > 3$.

7. (B) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

***Explanation*:** Here, $a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{21} = 2 + 1 = 3$

and $a_{22} = 2 + 2 = 4$

$\therefore$ $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

8. (A) I

***Explanation*:** We have, $(I + A)^2 - 3A$

$= I^2 + 2IA + A^2 - 3A$

$= I + 2A + A - 3A$

[Since given $A^2 = A$]

$= I$

9. (C) 108

***Explanation*:** $|3A| = (3)^3|A|$

[$\because$ order of matrix = 3]

$= 27 \times 4$ [Given $|A| = 4$]

$= 108$

10. (D) $a > 1$

***Explanation*:** Given, $f(x) = a^x$

$\therefore$ $f'(x) = a^x \log a$

for increasing function, $f'(x) > 0$

$\therefore$ $a^x \log a > 0$

$\Rightarrow a^x > 0$ and $\log a > 0$ or $a^x < 0$ and $\log a < 0$

But log function is always positive

$\Rightarrow$ $\log a > 0$

$\Rightarrow$ $a > 1$

11. (C) $P(x) = R(x) - C(x)$

***Explanation*:** Profit is revenue minus cost

i.e., $P(x) = R(x) - C(x)$

12. (A) $\sqrt{m}$

***Explanation*:** In case of Poisson distribution

S.D = $\sqrt{\text{mean}}$

Thus, S.D. $= \sqrt{m}$
[Given, mean $= m$]

13. **(A)** $X = \mu$

***Explanation*:** Normal distribution is symmetrical about mean.

14. **(A)** $\frac{1}{10}$

***Explanation*:** Here, $\sum P_i = 1$

$\therefore 0 + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + 2K = 1$

$\Rightarrow 9K + 10K^2 = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0$

$\Rightarrow 10K^2 + 10K - K - 1 = 0$

$\Rightarrow 10K(K+1) - 1(K+1) = 0$

$\Rightarrow (K+1)(10K-1) = 0$

$\Rightarrow K = -1$ and $K = \frac{1}{10}$

Since, $K \geq 0$ therefore $K = \frac{1}{10}$

15. **(A)** $\left(\frac{9}{10}\right)^5$

***Explanation*:** Probability of getting a defective bulb,

$p = \frac{10}{100} = \frac{1}{10}$

$\therefore q = 1 - p$

$= 1 - \frac{1}{10} = \frac{9}{10}$

Here, $n = 5$

By binomial distribution probability of getting no defective bulb $= P(X = 0) = {}^nC_0\, p^0\, q^{n-0}$

$= {}^5C_0\left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$

16. **(D)** Mean = 0, Standard deviation = 1.

17. **(B)** 2

***Explanation*:** Total no. of observations = 6

$\therefore P(X = 1) = \frac{3}{6} = \frac{1}{2}$

$P(X = 2) = \frac{2}{6} = \frac{1}{3}$

$P(X = 5) = \frac{1}{6}$

(where X be a random variable respresending a number on die)

$\therefore$ Mean, $E(X) = \sum p_i X_i$

$= \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \times 5$

$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$

$= \frac{3+4+5}{6}$

$= \frac{12}{6} = 2$

18. **(A)** Fisher's ideal index number

19. **(D)** q_1

20. **(C)** 121.15

***Explanation*:** Laspeyre's Price Index Number

$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

$= \frac{630}{520} \times 100 = 121.15$

SECTION - B

21. **(D)** 5

***Explanation*:** $5^{61} = (5^3)^{20} \times 5^1$

$= (126 - 1)^{20} \times 5^1$

$[126^{20} - 20C_1 126^{19} + 20C_2 126^{18} + \ldots\ldots\ldots l^{20}] \times 5$

$126[126^{19} - 20C_1 126^{18} + 20C_2 126^{17} + \ldots\ldots\ldots 20G_9] \times 5 + 1 \times 5$

.... 5 [126 is divisible by 7]

22. **(C)** 5

***Explanation*:** In 20 l of mixture,

Quantity of milk $= \frac{3}{4} \times 20 = 15\, l$

Quantity of water $= \frac{1}{4} \times 20 = 5\, l$

Let the quantity of milk added be x l.

According to be question,

$\frac{15 + x}{5} = \frac{4}{1}$

$\Rightarrow 15 + x = 20$

$\Rightarrow x = 5\, l$

23. **(A)** 3 minutes

***Explanation*:** Given, slower pipe filled the tank in 21 minutes

Then faster pipe filled the tank in $\frac{21}{6}$ minutes

Thus, tank filled by both the pipes in 1 minute

$= \frac{1}{21} + \frac{6}{21} = \frac{7}{21} = \frac{1}{3}$

Hence, time taken by both pipes together to fill the tank is 3 minutes.

24. **(A)** 11 months

***Explanation*:** Let A invested ₹ 11 for 8 months and B invested ₹ 12 for t months.

Therefore, the ratio of investment of A and B is

$$11 \times 8 : 12 \times t = 88 : 12t$$
Given, $88 : 12t = 2 : 3$
$$\Rightarrow \quad \frac{88}{12t} = \frac{2}{3}$$
$$\Rightarrow \quad t = 11 \text{ months}$$

25. **(B)** $[-7, 3]$

***Explanation*:** $|x + 2| \le 5$
$$\Rightarrow \quad -5 \le x + 2 \le 5$$
$[\because$ If $|x| \le a$, then $-a \le x \le a]$
$$\Rightarrow \quad -5 - 2 \le x \le 5 - 2$$
$$\Rightarrow \quad -7 \le x \le 3$$
$$\Rightarrow \quad x \in [-7, 3]$$

26. **(A)** 0

***Explanation*:** $AB = \begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
$$= \begin{bmatrix}1 & 0 & y+x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$
Since given $AB = I_3$
$$\therefore \quad \begin{bmatrix}1 & 0 & y+x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$
On comparing, we get
$$y + x = 0 \text{ or } x + y = 0$$

27. **(D)** None of these

***Explanation*:** Given, $A = \begin{bmatrix}1 & 0 & 1\\0 & 0 & 1\\a & b & 2\end{bmatrix}$
$$\therefore \quad A^2 = \begin{bmatrix}1 & 0 & 1\\0 & 0 & 1\\a & b & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\0 & 0 & 1\\a & b & 2\end{bmatrix}$$
$$= \begin{bmatrix}1+a & b & 3\\a & b & 2\\3a & 2b & a+b+4\end{bmatrix}$$
Now, $aI + bA + 2A^2$
$$= a\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} + b\begin{bmatrix}1 & 0 & 1\\0 & 0 & 1\\a & b & 2\end{bmatrix} + 2\begin{bmatrix}1+a & b & 3\\a & b & 2\\3a & 2b & a+b+4\end{bmatrix}$$
$$= \begin{bmatrix}a & 0 & 0\\0 & a & 0\\0 & 0 & a\end{bmatrix} + \begin{bmatrix}b & 0 & b\\0 & 0 & b\\ab & b^2 & 2b\end{bmatrix} + \begin{bmatrix}2+2a & 2b & 6\\2a & 2b & 4\\6a & 4b & 2a+2b+8\end{bmatrix}$$
$$= \begin{bmatrix}3a+b+2 & 2b & b+6\\2a & a+2b & b+4\\6a+ab & 4b+b^2 & 3a+4b+8\end{bmatrix}$$

28. **(C)** $I - A$

***Explanation*:** We have,
$$A^2 - A + I = 0$$
$$\Rightarrow \quad A^{-1}(A^2 - A + I) = A^{-1}(0)$$
$$\Rightarrow \quad A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$$
$$\Rightarrow \quad A - I + A^{-1} = 0$$
$$\Rightarrow \quad A^{-1} = I - A$$

29. **(B)** $\frac{3}{2}$

***Explanation*:** Let points are $A(1, 3)$, $B(x, 5)$ and $C(2, 7)$
A(1, 3) B(x, 5) C(2, 7)
Sine, they are collinear, their slopes are equal.
Let slope of $AB = m_1$
and slope of $BC = m_2$
$$\therefore \quad m_1 = m_2$$
$$\Rightarrow \quad \frac{5-3}{x-1} = \frac{7-5}{2-x}$$
$$\Rightarrow \quad \frac{2}{x-1} = \frac{2}{2-x}$$
$$\Rightarrow \quad 2 - x = x - 1$$
$$\Rightarrow \quad 2x = 3$$
$$\Rightarrow \quad x = \frac{3}{2}$$

30. **(A)** $25y$

***Explanation*:** Given, $y = Ae^{5x} + Be^{-5x}$
$$\therefore \quad y' = 5Ae^{5x} - 5Be^{-5x}$$
[Differentiating w.r.t. x]
$$\Rightarrow \quad y' = 5(Ae^{5x} - Be^{-5x})$$
$$\Rightarrow \quad y' = 5(5Ae^{5x} + 5Be^{-5x})$$
$$\Rightarrow \quad y'' = 5(5Ae^{5x} + 5Be^{-5x})$$
[Differentiating again w.r.t. x]
$$\Rightarrow \quad y'' = 25(Ae^{5x} + Be^{-5x})$$
$$\Rightarrow \quad y'' = 25y$$
or $$\frac{d^2y}{dx^2} = 25y$$

31. **(A)** $(2\sqrt{2}, 4)$

***Explanation*:** Let a point on the curve be (h, k)
Then, $h^2 = 2k$ ...(i)
Distance, $D = \sqrt{(h-0)^2 + (k-5)^2}$
$$= \sqrt{2k + k^2 + 25 - 10k}$$
[from (i)]
$$= \sqrt{k^2 - 8k + 25}$$
or, Let $z = D^2 = k^2 - 8k + 25$
$$\Rightarrow \quad z = k^2 - 8k + 25$$
$$\Rightarrow \quad \frac{dz}{dk} = 2k - 8 \qquad \text{...(ii)}$$
for maxima & minima, $\frac{dz}{dk} = 0$
$$\therefore \quad 2k - 8 = 0$$
$$\Rightarrow \quad k = 4$$
On differentiating again eq. (i), we get
$$\frac{d^2z}{dk^2} = 2 > 0$$

$\Rightarrow z$ has a minima at $k = 4$
from (i), when $k = 4$, $h^2 = 8$
$\Rightarrow h = \pm 2\sqrt{2}$
$\therefore$ Required points are $(\pm 2\sqrt{2}, 4)$.

32. (D) ₹126

***Explanation*:** $R(x) = 3x^2 + 36x + 5$
Marginal Revenue, $R'(x) = \frac{dR}{dx} = 6x + 36$
$\therefore$ Marginal Revenue at $x = 15$ is $\left.\frac{dR}{dx}\right|_{x=15}$
$= 6(15) + 36 = ₹126$

33. (B) $x - y = 0$

***Explanation*:** Given, curve is $2y + x^2 = 3$ ...(i)
Differentiating (i), w.r.t. x, we get
$2\frac{dy}{dx} + 2x = 0$
$\Rightarrow \frac{dy}{dx} = -x$
$\therefore \left(\frac{dy}{dx}\right)_{(1,1)} = -1$
Thus, slope of normal is $\frac{-1}{\left(\frac{dy}{dx}\right)_{(1,1)}} = 1$
Thus, equation of normal passing through (1, 1) having slope 1 is: $y - 1 = 1(x - 1)$
$\Rightarrow x - y = 0$

34. (B) 0, 2, 4, 6

***Explanation*:** The coin is tossed six times and X represents the difference between the number of heads and the number of tails.
$\therefore X(6H, 0T) = |6 - 0| = 6$
$X(5H, 1T) = |5 - 1| = 4$
$X(4H, 2T) = |4 - 2| = 2$
$X(3H, 3T) = |3 - 3| = 0$
$X(2H, 4T) = |2 - 4| = 2$
$X(1H, 5T) = |1 - 5| = 4$
$X(0H, 6T) = |0 - 6| = 6$
Thus, possible values are 0, 2, 4, 6.

35. (A) [0, 9)

***Explanation*:** Standard deviation, $\sigma = \sqrt{npq} \geq 0$
Now, mean $= np = 81$ and $q < 1$
So, $\sigma = \sqrt{npq} < \sqrt{np} = \sqrt{81} = 9$
$\therefore 0 \leq \sigma < 9$
Hence, σ lies in [0, 9)

36. (D) $1 - 3e^{-2}$

***Explanation*:** We know that,
$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$
Given that mean $= 2 = \lambda$
$\therefore P(X > 1.5) = P(X = 2) + P(X = 3) + ...$
$\Rightarrow P(X > 1.5) = 1 - [P(X = 0) + P(X = 1)]$
$= 1 - [e^{-2} + 2e^{-2}] = 1 - 3e^{-2}$

37. (C) $\frac{e^{-5}5^{50}}{50!}$

***Explanation*:** Given, 50 telephone line in an exchange probability that any of them will be busy is 0·1.
Let X be the Poisson variable.
$\therefore P(X = r) = \frac{e^{-m}m^r}{r!}, r = 0, 1, 2, ...$
Here, $n = 50, p = 0.1$
$\therefore m = np = 50 \times 0.1 = 5$
$\therefore P(X = r) = \frac{e^{-5}(5)^r}{r!}, r = 0, 1, 2, ..., 50$
Thus, P(all lines are busy) $= P(X = 50)$
$\therefore = \frac{e^{-5}(5)^r}{r!}$
For $r = 50$, $P = \frac{e^{-5}5^{50}}{50!}$

38. (B) ₹ 40·50 per kg

***Explanation*:** Let the cost of sugar be ₹ p per kg in 2020.
Then, it is given by $135 = \frac{p}{30} \times 100$
$\Rightarrow p = \frac{135 \times 30}{100} = \frac{4050}{100}$
$= ₹ 40.50$ per kg

39. (C) 98·86

40. (A) $P_{bc} \times P_{cb} = 1$

SECTION - C

41. (C) –2

***Explanation*:** We have, $f(x) = x^2 + ax + 1$
$\therefore f'(x) = 2x + a$
Now, function f will be increasing in (1, 2), if $f'(x) > 0$ in (1, 2)
$f'(x) > 0$
$\Rightarrow 2x + a > 0$
$\Rightarrow 2x > -a$
$\Rightarrow x > \frac{-a}{2}$
Therefore, we have to find the least value of a such that
$x > \frac{-a}{2}$, when $x \in (1, 2)$
$\Rightarrow x > \frac{-a}{2}$, when $1 < x < 2$
Thus, the least value of a for f to be increasing on (1, 2) is given by,
$\frac{-a}{2} = 1$
$\Rightarrow a = -2$
Hence, the required value of a is –2.

42. **(D)** 11

***Explanation*:** Here, demand function.

$$p(x) = 82 - x$$

and cost function $c(x) = 100 + 60x$

$\therefore$ Profit function, $P(x) = xp(x) - c(x)$

$\Rightarrow P(x) = x(82 - x) - (100 + 60x)$

$\Rightarrow P(x) = 82x - x^2 - 100 - 60x$

$\Rightarrow P(x) = -x^2 + 22x - 100$

Now, put $P'(x) = 0$

i.e., $-2x + 22 = 0$

$\Rightarrow x = 11$

Also, $P''(x) = -2 < 0$ (Hence, maximum)

Thus, value of x is 11 units.

43. **(B)** $\frac{2}{3}$

***Explanation*:** Let X : Number of doublets

$X = 0, 1, 2, 3, 4$

Total number of possible outcomes = 36

Doublets = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$$P(\text{Doublet}) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{Not a Doublet}) = \frac{5}{6}$$

X	P(X)	X.P(X)
0	$\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{625}{1296}$	0
1	$4\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{500}{1296}$	$\frac{500}{1296}$
2	$6\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{150}{1296}$	$\frac{300}{1296}$
3	$4\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6} = \frac{20}{1296}$	$\frac{60}{1296}$
4	$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6} = \frac{1}{1296}$	$\frac{4}{1296}$

$\therefore$ Mean $= \sum XP(X)$

$$= 0 + \frac{500}{1296} + \frac{300}{1296} + \frac{60}{1296} + \frac{4}{1296} = \frac{864}{1296} = \frac{2}{3}$$

44. **(D)** 6·06%

45. **(B)** 136.34

***Explanation*:** Index number

$$= \frac{\sum_{i=1}^{4}(\text{Relative Index}) \times \text{Weight}}{\sum_{i=1}^{4}\text{Weight}}$$

$$= \frac{181\times4 + 116\times12 + 110\times3 + 157\times7}{4 + 12 + 3 + 7}$$

$$= \frac{724 + 1392 + 330 + 1099}{26} = \frac{3549}{26} = 136.34$$

46. **(A)** $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

***Explanation*:** Here, the technology matrix is given under

	P	Q	Find Demand
P	0.4	0.1	68
Q	0.7	0.6	102

$\therefore$ The technology matrix A is

$$A = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

47. **(A)** $\begin{bmatrix} 68 \\ 102 \end{bmatrix}$

***Explanation*:** From above table,

$$\text{Demand matrix} = \begin{bmatrix} 68 \\ 102 \end{bmatrix}$$

48. **(D)** $\begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$

***Explanation*:** Since, $A = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

$$\Rightarrow I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$$

49. **(B)** $\frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

***Explanation*:** $(I - A)^{-1} = \frac{adj(I - A)}{|I - A|}$

$|I - A| = 0.24 - 0.07$

$\therefore \quad |I - A| = 0.17$

$$adj(I - A) = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

Thus, $(I - A)^{-1} = \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

50. **(B)** $P = 220; Q = 640$

***Explanation*:** The gross output of P and Q is

$$X = (I - A)^{-1}D$$

$$= \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}\begin{bmatrix} 68 \\ 102 \end{bmatrix}$$

$$= \frac{1}{0.17}\begin{bmatrix} 37.4 \\ 108.8 \end{bmatrix} = \begin{bmatrix} 220 \\ 640 \end{bmatrix}$$

Thus, gross output of P = 220 and gross output of Q = 640.

❍❍❍

Term – I

OMR SHEET

Booklet Series
A

Use English Numbers / Letters only. Use Blue / Black Ball Point Pen to write in box.

Booklet Series

Ⓐ Ⓑ Ⓒ Ⓓ

Subject

Roll Number

0 1 2 3 4 5 6 7 8 9 (×9 columns)

Name

Test Date

Student's Signature

Invigilator's Signature

Certified that all the entries In this section have been properly filled by the student

Proper Marking

The OMR Sheet will be computer checked Fill the circles completely and dark enough for proper detection, Use ballpen (black or blue) for marking.

Ⓐ Ⓑ ● Ⓓ

Avoid Improper Marking

Partially Filled

Lightly Filled

Test Center Code

0 1 2 3 4 5 6 7 8 9 (×2 columns)

IMPORTANT

The candidate should check that the Test Book Series printed on the OMR Sheet is the same as printed on the Test Booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.

Darken the circle for each question.

Q.No.	Response	Q.No.	Response	Q.No.	Response	Q.No.	Response
01	Ⓐ Ⓑ Ⓒ Ⓓ	16	Ⓐ Ⓑ Ⓒ Ⓓ	31	Ⓐ Ⓑ Ⓒ Ⓓ	46	Ⓐ Ⓑ Ⓒ Ⓓ
02	Ⓐ Ⓑ Ⓒ Ⓓ	17	Ⓐ Ⓑ Ⓒ Ⓓ	32	Ⓐ Ⓑ Ⓒ Ⓓ	47	Ⓐ Ⓑ Ⓒ Ⓓ
03	Ⓐ Ⓑ Ⓒ Ⓓ	18	Ⓐ Ⓑ Ⓒ Ⓓ	33	Ⓐ Ⓑ Ⓒ Ⓓ	48	Ⓐ Ⓑ Ⓒ Ⓓ
04	Ⓐ Ⓑ Ⓒ Ⓓ	19	Ⓐ Ⓑ Ⓒ Ⓓ	34	Ⓐ Ⓑ Ⓒ Ⓓ	49	Ⓐ Ⓑ Ⓒ Ⓓ
05	Ⓐ Ⓑ Ⓒ Ⓓ	20	Ⓐ Ⓑ Ⓒ Ⓓ	35	Ⓐ Ⓑ Ⓒ Ⓓ	50	Ⓐ Ⓑ Ⓒ Ⓓ
06	Ⓐ Ⓑ Ⓒ Ⓓ	21	Ⓐ Ⓑ Ⓒ Ⓓ	36	Ⓐ Ⓑ Ⓒ Ⓓ	51	Ⓐ Ⓑ Ⓒ Ⓓ
07	Ⓐ Ⓑ Ⓒ Ⓓ	22	Ⓐ Ⓑ Ⓒ Ⓓ	37	Ⓐ Ⓑ Ⓒ Ⓓ	52	Ⓐ Ⓑ Ⓒ Ⓓ
08	Ⓐ Ⓑ Ⓒ Ⓓ	23	Ⓐ Ⓑ Ⓒ Ⓓ	38	Ⓐ Ⓑ Ⓒ Ⓓ	53	Ⓐ Ⓑ Ⓒ Ⓓ
09	Ⓐ Ⓑ Ⓒ Ⓓ	24	Ⓐ Ⓑ Ⓒ Ⓓ	39	Ⓐ Ⓑ Ⓒ Ⓓ	54	Ⓐ Ⓑ Ⓒ Ⓓ
10	Ⓐ Ⓑ Ⓒ Ⓓ	25	Ⓐ Ⓑ Ⓒ Ⓓ	40	Ⓐ Ⓑ Ⓒ Ⓓ	55	Ⓐ Ⓑ Ⓒ Ⓓ
11	Ⓐ Ⓑ Ⓒ Ⓓ	26	Ⓐ Ⓑ Ⓒ Ⓓ	41	Ⓐ Ⓑ Ⓒ Ⓓ	56	Ⓐ Ⓑ Ⓒ Ⓓ
12	Ⓐ Ⓑ Ⓒ Ⓓ	27	Ⓐ Ⓑ Ⓒ Ⓓ	42	Ⓐ Ⓑ Ⓒ Ⓓ	57	Ⓐ Ⓑ Ⓒ Ⓓ
13	Ⓐ Ⓑ Ⓒ Ⓓ	28	Ⓐ Ⓑ Ⓒ Ⓓ	43	Ⓐ Ⓑ Ⓒ Ⓓ	58	Ⓐ Ⓑ Ⓒ Ⓓ
14	Ⓐ Ⓑ Ⓒ Ⓓ	29	Ⓐ Ⓑ Ⓒ Ⓓ	44	Ⓐ Ⓑ Ⓒ Ⓓ	59	Ⓐ Ⓑ Ⓒ Ⓓ
15	Ⓐ Ⓑ Ⓒ Ⓓ	30	Ⓐ Ⓑ Ⓒ Ⓓ	45	Ⓐ Ⓑ Ⓒ Ⓓ	60	Ⓐ Ⓑ Ⓒ Ⓓ