UNIT-I
SETS AND FUNCTIONS

CHAPTER-1
SETS

TOPIC-1
Sets, their Types and Representations

Revision Notes

- **Set:** A set is a well defined collection of distinct objects.
- **Elements:** The objects which belong to a set are called members or elements of a set.
- **Representation of a Set:** There are two ways by which the sets can be described:

 (i) Tabular form or Roaster form: Here the elements of a set are actually written down, separated by commas and enclosed within braces (*i.e.*, curly bracket). e.g. :
 - V, the set of vowels in the English alphabet = $\{a, e, i, o, u\}$.
 - A, the set of odd natural numbers $< 10 = \{1, 3, 5, 7, 9\}$.
 - N, the set of natural numbers $= \{1, 2, 3, \ldots\}$.

 (ii) The set builder form or rule method: A set is described by a characterizing property P(x) of its element x. In such a case, the set is described by $\{x : P(x) \text{ holds}\}$ or $\{x \mid P(x) \text{ holds}\}$. *e.g.* :
 - $M = \{2, 3, 5, 7, 11, 13, 17, 19\} = \{x : x \text{ is a prime number} < 20\}$.
 - $A = \{6, 7, 8, 9, 10, 11\}$ $\{x \in N, 5 < x < 12\}$,
 where $\in$ stands for 'belongs to'.
- Types of Sets

 (i) Empty set or null set or void set : A set consisting of no element at all, is called an **empty set** or a **null set** or a **void set.** It is denoted by ϕ. In roaster form, it is written as { }.
 e.g. :
 (i) $\{x : x \in N, 1 < x < 2\} = \phi$, **(ii)** $\{x : x \in R, x^2 = -1\} = \phi$

 (ii) Singleton set : The set $\{x : x \in N \text{ and } x^2 = 9\}$ is a singleton set equal to $\{3\}$. A set consisting of a single element.

 (iii) Finite set : A set is called a finite set, if it is either void or its elements can be listed (counted, labelled) by natural numbers 1, 2, 3, … and the process of listing terminates at a certain natural number n (say).
 e.g. :
 - Set of even natural numbers less than 100.
 - Set of soldiers in Indian army.
 - Set of all persons on the Earth.

 (iv) Infinite set : A set whose elements cannot be listed by the natural numbers $1, 2, 3, \ldots, n$ for any natural number n is called an infinite set.
 e.g. :
 - Set of all points in a plane.
 - Set of all lines in a plane.
 - $\{x \in R, 0 < x < 1\}$.
- **Cardinal number of a finite set :** The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by $n(A)$.
- **Equal sets :** Two sets A and B are said to be equal, if every element of set A is in set B and every element of set B is in set A.

It is written as $A = B$.

e.g. :

- $\{1, 2, 5\} = \{2, 1, 5\} = \{5, 1, 2\}$,
- $\{1, 2, 3, 1\} = \{1, 2, 3\} = \{1, 1, 2, 2, 3\}$ *etc.*

➢ **Equivalent sets :** Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$, where $n(A)$ or $n(B)$ is the number of elements of set A or B. It is written as $A \leftrightarrow B$.

e.g. : If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Then $A \leftrightarrow B$, since $n(A) = n(B) = 3$.

Whenever $A = B$, then $n(A) = n(B)$. Thus, equal sets are always equivalent. But, equivalent sets need not be equal.

➢ **Subset :** If A and B are two sets given in such a way that every element of A is in B, then A is a subset of set B and it is written as $A \subseteq B$ (read as 'A is contained in B').

If at least one element of A does not belong to B, then A is not a subset of B. It is written as $A \not\subset B$.

➢ **Power set:** The collection of all subsets of a set A is called power set of A, denoted by $P(A)$ *i.e.* $P(A) = \{B : B \subset A\}$

If A is a set with $n(A) = m$, then $n[P(A)] = 2^m$.

➢ **Proper subset and super set:** If $A \subset B$, then A is called the proper subset of B and B is called the super set of A.

➢ **Universal set:** If there are some sets under consideration, then a set can be chosen arbitrarily which is a superset of each one of the given sets. Such a set is known as the universal set and it is denoted by U.

e.g. : Let $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$ and $C = \{0, 7\}$ are three subsets,

then, $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$ is an universal set.

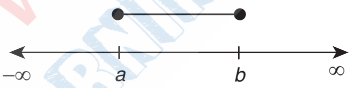
➢ **Types of Intervals**

(i) Open interval : $(a, b) = \{x \in R : a < x < b\}$

i.e. All the points between ***a*** and ***b*** belong to open interval.

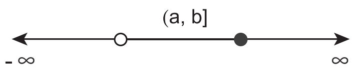
(ii) Closed interval : $[a, b] = \{x \in R: a \le x \le b\}$

i.e. The interval which contains the end points also, is called closed interval.

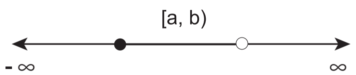


(iii) Semi open or semi closed interval :

$(a, b] = \{ x \in R: a < x \le b\}$



$[a, b) = \{ x \in R: a \le x < b\}$



Note: (i) The set $(0, \infty)$ defines the set of non-negative real numbers.

(ii) The set $(-\infty, 0)$ defines the set of negative real numbers.

(iii) The set $(-\infty, \infty)$ defines the set of all real numbers.

- **Length of an interval:** The number $(b - a)$ is called the length of any of the intervals (a, b), $[a, b]$, $[a, b)$ or $(a, b]$.

Know the Terms

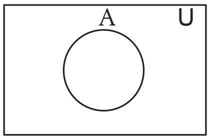
We shall denote several sets of numbers by the following symbols :

(i) N : The set of natural numbers
(ii) W : The set of whole numbers
(iii) Z : The set of integers
(iv) Q : The set of rational numbers
(v) R : The set of real numbers
(vi) Z^+ : The set of positive integers
(vii) Q^+ : The set of positive rational numbers
(viii) R^+ : The set of positive real numbers
(ix) C : The set of all complex numbers.

TOPIC-2
Venn Diagrams and Operations on Set

Revision Notes

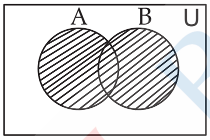
- **Venn Diagram:** In a Venn diagram, the universal set is represented by a rectangular region and a set is represented by circle of a closed geometrical figure inside the universal set.



- **Operations on Sets :**

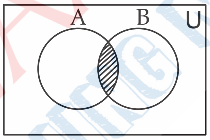
(i) Union of sets: The union of two sets A and B, denoted by $A \cup B$ is the set of all elements, which is either in A or in B or both in A and B.

i.e., $A \cup B = \{x : x \in A \text{ and } x \in B\}$



(ii) Intersection of sets: The intersection of two sets A and B, denoted by $A \cap$ B, is the set of all elements, which are common to both A and B.

i.e., $A \cap B = \{x : x \in A \text{ and } x \in B\}$

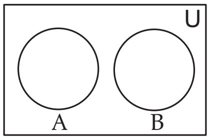


If $A_1, A_2, \ldots, A_n$ is a finite family of sets, then their intersection is denoted by

$$\bigcap_{i=1}^{n} A_i \text{ or } A_1 \cap A_2 \cap \ldots \cap A_n$$

(iii) Disjoint sets:

Two sets A and B are said to be disjoint if $A \cap B = \phi$



- **Law of algebra of set**

(i) $A \cup \phi = A$

(ii) $A \cap \phi = \phi$

- **Commutative Law:**

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

- **Associative Law:**

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- **Distributive Law:**

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

TOPIC-3
Application of Set Theory

Revision Notes

- In this topic, we will discuss some word problems related to our daily life, which are based on union and intersection of two sets. Before solving these types of problems, we should know the following formulae.
  - If A and B are two finite sets, then

    (a) $A \cap B = \phi$ *i.e.*, A and B are disjoint sets

    $\Rightarrow n(A \cup B) = n(A) + n(B)$

    (b) $(A \cap B) \neq \phi \Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  - If A and B are any two finite sets, then

    (a) number of elements in A only =

    $$n(A - B) = n(A) - n(A \cap B)$$

    (b) number of elements in B only =

    $$n(B - A) = n(B) - n(A \cap B)$$
  - If A, B and C are three finite sets, then

    $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

    If A, B and C are mutually disjoint sets, then

    $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
  - If A, B and C are three finite sets, then number of elements in A only $= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
  - If A, B and C are three finite sets, then number of elements in

    $(A \text{ and } B \text{ only}) = n(A \cap B) - n(A \cap B \cap C)$
  - If A and B are two finite sets, then

    $n(A \Delta B) = n[(A - B) \cup (B - A)]$

    $= n(A - B) + n(B - A)$

    [since, $(A - B)$ and $(B - A)$ are disjoint sets]

    $= n(A) + n(B) - 2n(A \cap B)$

■■

CHAPTER-2
RELATIONS AND FUNCTIONS

TOPIC-1
Cartesian Product and Relations

Revision Notes

- For two non–empty sets A and B, the Cartesian Product $A \times B$ is the set of all ordered pairs of elements from sets A and B.
- In Symbolic form, it is written as,

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

➢ Thus, Cartesian Product of two sets represents the set which represents the coordinates of all the points in two dimensional space.

e.g., If $A = \{a, b, c\}$ and $B = \{p, q\}$ then,

(i) $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$

(ii) $B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$

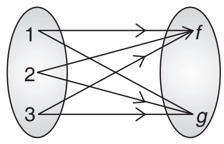
➢ **Diagrammatic Representation of Cartesian Product of Two Sets**

- In order to represent A × B, by an arrow diagram, we first draw Venn diagrams representing sets A and B, one opposite to the other as shown in given figure and write the elements of sets.

  Now, we draw line segments starting from each element of Set A and terminating to each element of Set B.

  e.g., If $A = \{1, 2, 3\}$ and $B = \{f, g\}$,

  Then, following figure gives the arrow diagram of $A \times B$.



If A, B and C are three sets, then (a, b, c) where $a \in A$, $b \in B$ and $c \in C$ is called an ordered triplet.

Note :

(i) If $R = \phi$, then R is called an empty relation.

(ii) If $R = A \times B$, then R is called the universal relation.

(iii) If R_1 and R_2 are two relations from A to B, then $R_1 \cup R_2$, $R_1 \cap R_2$ and $R_1 - R_2$ are also relations from A to B. In a relation from A to B, such that

$$R = \{(a, b) : a \in A \text{ and } b \in B\}$$

Here, the set of all first elements of ordered pair in a relation R is called the **Domain** and set of all second elements of ordered pairs in a relation R is called the **range**.

The Set B is called the **Co-domain** of relation R.

Range $\subseteq$ Codomain.

(iv) If there are three sets A, B, C and $a \in A$, $b \in B$ and $c \in C$, then we form an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the cartesian product of these sets A B and C.

i.e., $A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$

Know the Terms

➢ **Number of Elements**

It the set A has m elements and the set B has n elements then the number of elements in the cartesian product set is 2 mn elements.

i.e., set $A = m$ elements; set $B = n$ elements

Then, $n(A \times B) = 2\, m \times n$ elements

➢ **Domain and Range of a Relation**

Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs belonging to R is called the range of R.

Thus, domain of $R = \{a : (a, b) \in R\}$ and range of $R = \{b : (a, b) \in R\}$

➢ **Types of Relations**

(i) Void Relation : As $\phi \subset A \times A$, for any set A, so ϕ is a relation on A, called the empty or void relation.

(ii) Universal Relation : Since, $A \times A \subseteq A \times A$, so $A \times A$ is a relation on A, called the universal relation.

(iii) Identity Relation : The relation $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A.

(iv) Reflexive Relation : A relation R is said to be reflexive relation, if every element of A is related to itself.

Thus, $(a, a) \in R$, $\forall\, a \in A \Rightarrow R$ is reflexive.

(v) **Symmetric Relation :** A relation R is said to be symmetric relation, iff

$$(a, b) \in R \Rightarrow (b, a) \in R, \forall\, a, b \in A$$

i.e., $$a\, R\, b \Rightarrow b\, R\, a, \forall\, a, b \in A$$

$$\Rightarrow R \text{ is symmetric.}$$

(vi) **Anti–Symmetric Relation :** A relation R is said to be anti–symmetric relation, iff

$$(a, b) \in R \text{ and } (b, a) \in R \Rightarrow a = b, \forall\, a, b \in A$$

(vii) **Transitive Relation :** A relation R is said to be transitive relation, iff $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow (a, c) \in R, \forall\, a, b, c \in A$$

(viii) **Equivalence Relation :** A relation R is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on a set A.

(ix) **Partial Order Relation :** A relation R is said to be a partial order relation, if it is simultaneously reflexive, symmetric and anti–symmetric on a set A.

(x) **Total Order Relation :** A relation R on a set A is said to be a total order relation on A, if R is a partial order relation on A.

Some Important Results :

- **Properties of Cartesian Product**

For three sets A, B and C

(i) $n(A \times B) = n(A)\, n(B)$

(ii) $A \times \phi = \phi$ and $\phi \times A = \phi$

(iii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(v) $A \times (B - C) = (A \times B) - (A \times C)$

(vi) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(vii) If $A \subseteq B$ and $C \subseteq D$, then $(A \times C) \subset (B \times D)$

(viii) If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

(ix) $A \times B = B \times A \Leftrightarrow A = B$

(x) If either A or B is an infinite set, then $A \times B$ is an infinite set.

(xi) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(xii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(xiii) If A and B be any two non–empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

(xiv) If $A \neq B$, then $A \times B \neq B \times A$

(xv) If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C

TOPIC-2
Functions and their Types

Revision Notes

If f is a function from a set A to Set B, then we write,

$$f : A \rightarrow B \text{ or } A \xrightarrow{f} B$$

and it is read as f is a function from A to B.
If $(a, b) \in f \Rightarrow f(a) = b$.
Here, b is the **image** of a under f and a is called the **pre–image** of b under f.

Few Properties to be known :

(i) For any real number x, we have

$$\sqrt{x^2} = |x|$$

(ii) If n is an integer and x is a real number between n and $n + 1$, then

(a) $[-n] = -[n]$

(b) $[x + k] = [x] + k$ for any integer

(c) $[-x] = -[x] - 1$

(d) $[-x] = -[x] + 1$, where $x \in R - Z$

(iii) For any real function,

$f : D \to R$ and $n \in N$, we define

$$\underbrace{(f\,f\,f....f)(x)}_{n\text{ times}} = \underbrace{f(x)\,f(x)....f(x)}_{n\text{ times}} = \{f(x)\}^n,\ \forall x \in D$$

Know the Facts

Domain, Co-domain and Range of a Function

If f is a function from A to B and each element of A corresponds to one and only one element of B, whereas every element in B need not be the image of some x in A. Then, the set A is called the **domain** of function f and the set B is called the **co-domain** of f.

The subset of B containing the images of elements of A is called the **range** of the function.

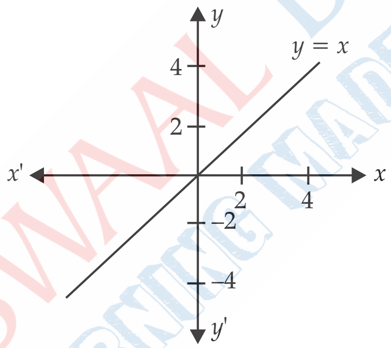
Thus, if a function f is expressed as the set of ordered pairs, then the domain of f is the **set of all first elements** of ordered pairs and the range of f is **the set of second elements** of ordered pairs of f. *i.e.*,

D_f = Domain of $f = \{a : (a, b) \in f\}$

R_f = Range of $f = \{b : (a, b) \in f\}$

Let A and B be two non-empty finite sets such that $n(A) = p$, and $n(B) = q$, then number of functions from A to $B = q^p$

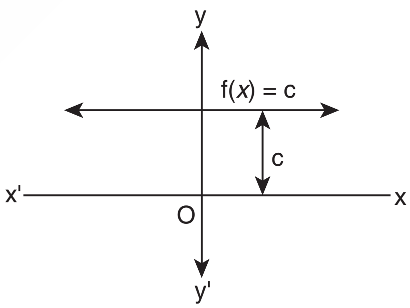
Identity function: Let R be the set of real numbers. A real valued function f is defined as $f : R \to R$ by $y = f(x) = x$ for each value of $x \in R$. Such a function is called the identity function.



Domain = R and Range = R

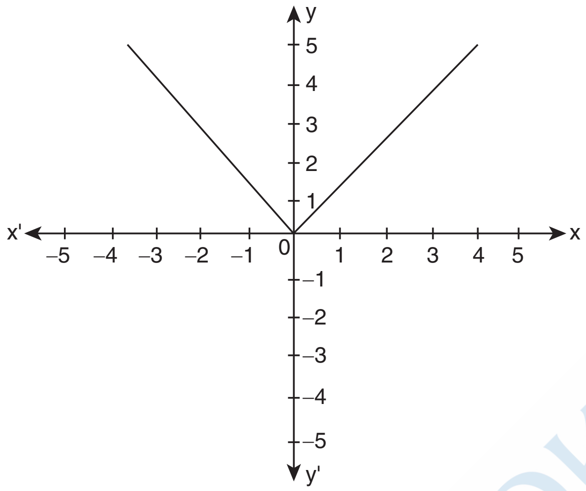
Constant function:

The function $f : R \to R$ defined by $f(x) = C$ for each $x \in R$ is called constant function. (where C is a constant)



Domain = R and Range = $\{C\}$

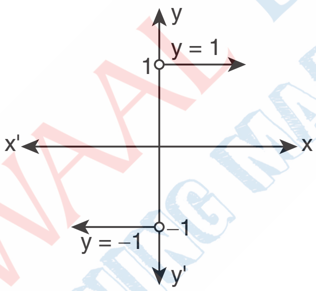
Modulus function: The function $f : R \to R$ defined by $f(x) = |x|$ for each $x \in R$ is called modulus function or absolute valued function.



i. e., $$f(x) = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

Domain = R and Range = $R^+ \cup \{0\} = \{x : x \in R; x \ge 0\}$

Signum function : Let R be the set of real numbers, then the function $f : R \to R$ defined by

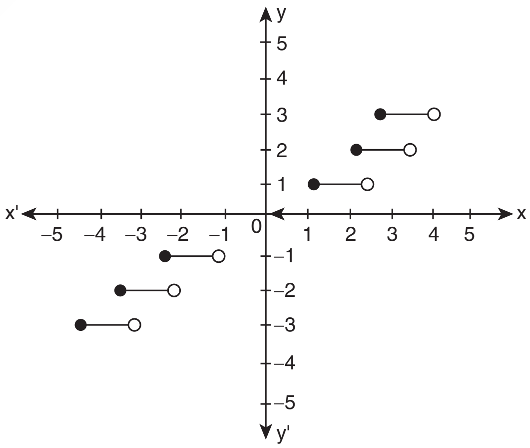


$$f(x) = \frac{|x|}{x} = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is known as signum function.

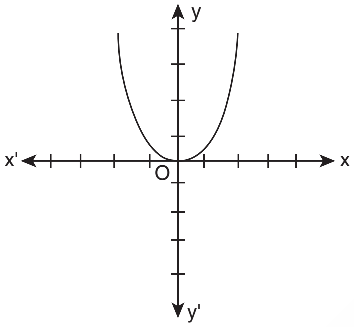
Domain = R and Range = $\{-1, 0, 1\}$

Greatest integer function : The function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x, such a function is called the greatest integer function.



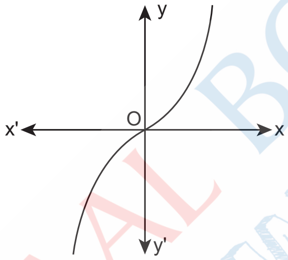
From the definition of $[x]$, we have
$[X] = -1$, for $-1 < x < 0 = 0$, for $0 < x < 1$
$= 1$, for $1 \le x < 2$
$= 2$, for $2 \le x < 3$
Domain $= R$ and Range $= Z$
Graphs of some other important functions

- $f : R \to R, f(x) = x^2$



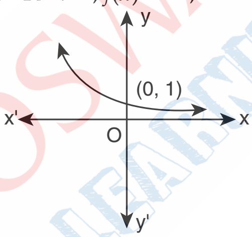
Domain $= R$ and Range $= [0, \infty)$

- $f : R \to \mathrm{R}, f(x) = x^3$

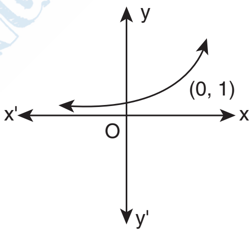


Domain $= R$ and Range $= R$

- Exponential function, $f : R \to \mathrm{R}, f(x) = a^x, a > 0, a \neq 1$



$0 < a < 1$



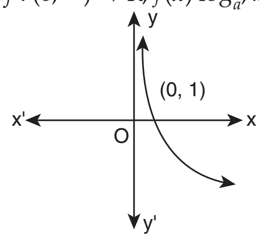
$a > 1$

Domain $= R$ and Range $= (0, \infty)$

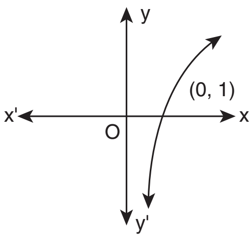
- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty, 2 < e < 3$$

- Logarithmic function, $f : (0, \infty) \to \mathrm{R}; f(x) \log_{a}^{x}, a > 0, a \neq 1$



Domain = $(0, \infty)$



Range = R

Domain $= (0, \infty)$ and Range $= R$

- Natural logarithmic function $f(x) = \log e^x$ or $\ln x$

CHAPTER-3
TRIGONOMETRIC FUNCTIONS

TOPIC-1
Measure of an Angle & Trigonometric Functions

Revision Notes

There are three measures for measuring an angle.

(i) Degree Measure : A right angle is divided into 90 equal parts called degree.
One degree is divided into 60 equal parts, called minutes and 1 minute is denoted by 1′. One minute is divided into 60 equal parts called second.
Thus, $1° = 60'$, $1' = 60''$

(ii) Radian Measure :
In a circle of radius r, an arc of length l subtend an angle 'θ' radian at the centre, then

$$l = r\theta \Rightarrow \theta = \frac{l}{r}$$

(iii) Centesimal System :
In this system a right angle is divided into 100 equal parts called grades each grade is subdivided into 100 minutes, and each minutes into 100 seconds.

Thus,
1 right angle = 100 grades
1 grade = 100 minutes
1 minute = 100 seconds

Relation between three systems of measurement of an angle.

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Where,
D = Degree measure
G = Grade measure
R = Radian measure

A circle subtends at the centre an angle, whose radian measure is 2π and its degree measure is 360°. It follows that,

$$2\pi \text{ radian} = 360°$$

(or)

$$\pi \text{ radian} = 180°$$

$$\therefore 1 \text{ rad} = \frac{180°}{\pi} = 57°16'22'' \text{ (approx).,} \qquad \text{where } \pi = \frac{22}{7} = 3.14159$$

Thus,

$$\text{Radian} = \frac{\pi}{180°} \times \text{degree measure.}$$

$$\text{Degree} = \frac{180°}{\pi} \times \text{radian measure.}$$

Note :

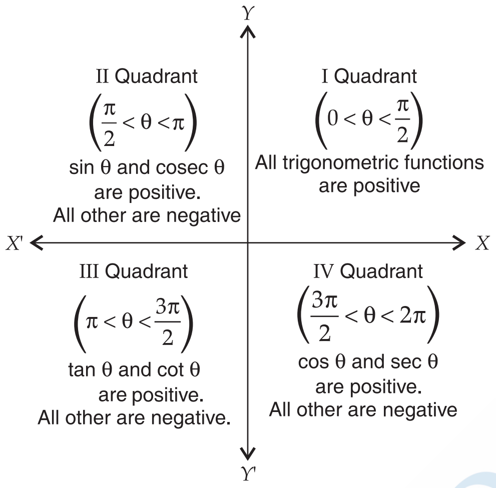
(i) The angle between two consecutive digits in a clock is $30° \left(= \frac{\pi}{6} \text{ radians}\right)$

(ii) The minute hand rotates through an angle of 6° in one minute.

(iii) Radian is a constant angle.

(iv) $1° = \left(\frac{\pi}{180}\right) \text{rad} = 0.0176 \text{ rad}$

Sign of Trigonometric Function in Different Quadrants



Trigonometric Identities (θ measured in radian)

An equation involving trigonometric functions, which is true for all those angles for which the functions are defined is called trigonometrical identity. Some identities are

(i) $\sin\theta = \dfrac{1}{\text{cosec}\theta}$ or $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

(ii) $\cos\theta = \dfrac{1}{\sec\theta}$ or $\sec\theta = \dfrac{1}{\cos\theta}$

(iii) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$

(iv) $\cos^2\theta + \sin^2\theta = 1$ or $1 - \cos^2\theta = \sin^2\theta$ or $1 - \sin^2\theta = \cos^2\theta$.

(v) $1 + \tan^2\theta = \sec^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$

(vi) $1 + \cot^2\theta = \text{cosec}^2\,\theta$ or $\text{cosec}^2\theta - \cot^2\theta = 1$

Trigonometric Ratios of Some Standard Angles

Angle	**0°**	**30°**	**45°**	**60°**	**90°**	**120°**	**135°**	**150°**	**180°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	– 1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	– 1	$-\frac{1}{\sqrt{3}}$	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	– 1	$-\sqrt{3}$	– ∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	– 2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	– 1
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

Domain and Range of Trigonometric Functions

Function	Domain	Range
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$
$\tan x$	$R - \{(2n+1)\left(\frac{\pi}{2}\right); n \in Z\}$	R
$\text{cosec } x$	$R - \{n\pi; n \in Z\}$	$R-(-1, 1)$
$\sec x$	$R - \{(2n+1)\left(\frac{\pi}{2}\right); n \in Z\}$	$R-(-1, 1)$
$\cot x$	$R - \{n\pi; n \in Z\}$	R

(i) Compound Angle Formulae

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$
- $\cot(x+y) = \dfrac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$

Note: $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$

$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$

- $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$
- $\cot(x-y) = \dfrac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$
- $\tan(x+y+z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

(ii) Transformation formulae

- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$ $[x > y]$
- $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$
- $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
- $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$
- $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
- $\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2} = 2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{y-x}{2}\right)$

(iii) Trigonometric Functions of multiple and sub multiple angles

- $\sin 2x = 2\sin x\cos x = \dfrac{2\tan x}{1+\tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$
- $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$
- $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}\begin{cases} +, \text{ if } \dfrac{A}{2} \text{ lies in quadrants I or II} \\ -, \text{ if } \dfrac{A}{2} \text{ lies in III or IV quadrants}\end{cases}$
- $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}\begin{cases} +, \text{ if } \dfrac{A}{2} \text{ lies in I or IV quadrants} \\ -, \text{ if } \dfrac{A}{2} \text{ lies in II or III quadrants}\end{cases}$
- $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}\begin{cases} +, \text{ if } \dfrac{A}{2} \text{ lies in I or III quadrants} \\ -, \text{ if } \dfrac{A}{2} \text{ lies in II or IV quadrants}\end{cases}$

Trigonometric functions of an angle of 18°

Let $\theta = 18°$. Then $2\theta = 90° - 3\theta$

Therefore, $\sin 2\theta = \sin(90° - 3\theta) = \cos 3\theta$

or $\sin 2\theta = 4\cos^3\theta - 3\cos\theta$

Since, $\cos\theta \neq 0$, we get

$$2\sin\theta = 4\cos^2\theta - 3 = 1 - 4\sin^2\theta$$

or $4\sin^2\theta + 2\sin\theta - 1 = 0$.

Hence, $\sin\theta = \dfrac{-2\pm\sqrt{4+16}}{8} = \dfrac{-1\pm\sqrt{5}}{4}$

Since, $\theta = 18°$, $\sin\theta > 0$, therefore, $\sin 18° = \dfrac{\sqrt{5}-1}{4}$

Also, $\cos 18° = \sqrt{1-\sin^2 18°} = \sqrt{1-\dfrac{6-2\sqrt{5}}{16}} = \sqrt{\dfrac{10+2\sqrt{5}}{4}}$

Now, we can easily find cos 36° and sin 36° as follows:

$$\cos 36° = 1 - 2\sin^2 18° = 1 - \frac{6-2\sqrt{5}}{8} = \frac{2+2\sqrt{5}}{8} = \frac{\sqrt{5}+1}{4}$$ **[DDE-2017]**

Hence, $\cos 36° = \dfrac{\sqrt{5}+1}{4}$

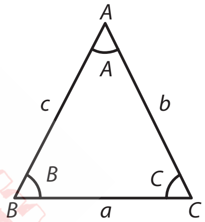
Also, $\sin 36° = \sqrt{1-\cos^2 36°} = \sqrt{1-\dfrac{6+2\sqrt{5}}{16}} = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

TOPIC-2
Some Applications of sine and cosine Formulae

Revision Notes

We know that a closed figure formed by three intersecting lines forms a triangle. In triangle ABC, angles are denoted by A, B and C and the length of corresponding sides opposite to the angles (*i.e.,*) BC, CA and AB are denoted by a, b and c respectively. Also, area and perimeter of a triangle are denoted by Δ and $2s$ respectively.

Few Important Results :



(i) Semi-perimeter of the triangle is $s = \frac{a+b+c}{2}$

(ii) The sum of all the angles of a triangle is 180° (*i.e.,*)
$\angle A + \angle B + \angle C = 180°$

(iii) The longest side of a triangle have corresponding largest angle and vice-versa.

(iv) The smallest side of a triangle have corresponding smallest angle and vice-versa.

Since Formula or Sine Rule

In any triangle, the sides are proportional to the sines of the opposite angles *i.e.,* in ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Formula or Cosine Rule

Let a, b and c be the lengths or sides of ΔABC, opposite to $\angle A$, $\angle B$ and $\angle C$, respectively. Then,

(i) $a^2 = b^2 + c^2 - 2\,bc \cos A$ **(ii)** $b^2 = c^2 + a^2 - 2\,ca \cos B$ **(iii)** $c^2 = a^2 + b^2 - 2\,ab \cos C$

UNIT-II
ALGEBRA

CHAPTER-4
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

TOPIC-1
Complex Numbers, their Conjugate

Revision Notes

- A number consisting of real 'part' and imaginary 'part' is called a **Complex number**.
- The coordinate plane that represents the complex numbers is called the complex plane or Argand plane.
- "A number of the form $a + ib$, where a and b are real numbers is called a complex number."
- Here, the symbol 'i' is called *iota*. We have $i^2 = -1$ *i.e.,* $\pm i$ is the solution of the equation $x^2 + 1 = 0$.
- **Integral powers of 'i' :**

$$\begin{aligned}
i^{4q} &= 1, & q \in N\\
i^{4q+1} &= i, & q \in N\\
i^{4q+2} &= -1, & q \in N\\
i^{4q+3} &= -i, & q \in N\\
i^{-q} &= \frac{1}{i^q}, & q \in N
\end{aligned}$$

- **Real and imaginary parts of a complex numbers:**
  - Let $z = a + ib$ be a complex number, then a is **called real part** and b is called the **imaginary part** of z and it may be denoted as $Re\ (z)$ and $Im\ (z)$ respectively.

    e.g. : If $z = 2 + 3i$, then

    $Re\ (z) = 2$, $Im\ (z) = 3$.
  - $a + ib = c + id$ if $a = c$, and $b = d$
  - $z_1 = a + ib, z_2 = c + id$.
  - In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if $b, d = 0$ and $a > c$ then $z_1 > z_2$

    i.e. we can compare two complex numbers only if they are purely real.
  - $0 + i0$ is additive identity of a complex number.
  - $-z = -a - ib$ is called the Additive Inverse or negative of $z = a + ib$
  - $1 + i0$ is a multiplicative identity of complex number.
- The **Conjugate** of a complex number z is the complex number obtained by changing the sign of imaginary part of z. It is denoted by $\bar{z}$. $\bar{z} = a - ib$ is called the conjugate of $z = a + ib$.
- **Modulus** (Absolute value) of r complex number, $z = a + ib$ is defined by the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $|z|$.
  - $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative inverse of $z = a + ib$ $(a \neq 0, b \neq 0)$
  - $|z_1 + z_2| \leq |z_1| + |z_2|$
  - $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
  - $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}; |z^n| = |z|^n; |z| = |\bar{z}| = |-z| = |-\bar{z}|; z\bar{z} = |z|^2$
  - $|z_1 - z_2| \geq |z_1| - |z_2|$
- **Argand :** Consider a Complex number $z = r(\cos\theta + i\sin\theta)$
  - Where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z, θ is called the argument or amplitude of z
  - The value of θ such that, $-\pi < \theta < \pi$ is called the **principle argument** of z.
  - $z = x + iy, x > 0$ and $y > 0$ the argument of z is Acute angle α given by $\tan\alpha = \frac{y}{x}$

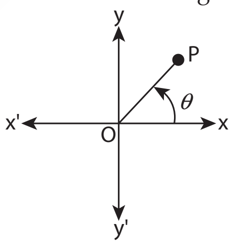


Fig (i)

  - $z = x + iy, x < 0$ and $y > 0$ the argument of z is $\pi - \alpha$, where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$

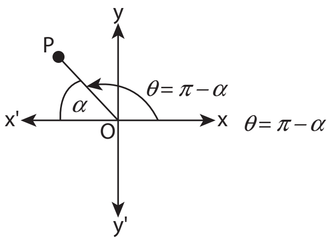


Fig (ii)

- $z = x + iy, x < 0$ and $y < 0$ the argument of z is $\alpha - \pi$, where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$

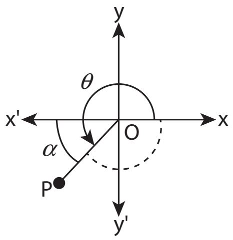


Fig (iii)

- $z = x + iy, x > 0$ and $y < 0$ the argument of z is $-\alpha$, where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$

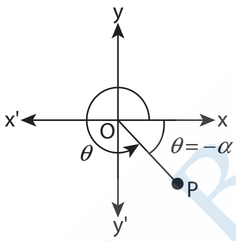


Fig (iv)

- If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

  $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

  then $z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$

  $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$

➢ **Properties of Argument of complex numbers :**

If z_1 and z_2 are two complex numbers, then

(i) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$.

(ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$.

(iii) $\arg\left(\frac{z}{\bar{z}}\right) = 2\arg(z)$.

(iv) $\arg(z^n) = n\arg(z)$.

(v) If $\arg\left(\frac{z_2}{z_1}\right) = \theta$, then $\arg\left(\frac{z_1}{z_2}\right) = -\theta$

(vi) $\arg(\bar{z}) = -\arg(z)$.

(vii) If $\arg(z) = 0 \Rightarrow z$ is real.

(viii) $\arg(z_1, \bar{z}_2) = \arg(z_1) - \arg(z_2)$

(ix) $|z_1 + z_2| = |z_1 - z_2| \Rightarrow \arg(z_1) - \arg(z_2) = \frac{\pi}{2}$

(x) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow \arg(z_1) = \arg(z_2)$

(xi) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \Rightarrow \frac{z_1}{z_2}$ is purely imaginary.

Know the Facts

- **Imaginary Number :** The square root of a negative number is known as an imaginary number, *e.g.*, $\sqrt{-1}, \sqrt{-2}$ etc.
- $\sqrt{-1}$ is denoted by the Greek letter i (iota).
- **Powers of i :** $i^0 = 1$; $i^2 = -1$; $i^3 = -i$, $i^4 = 1$ etc.
- **An Important Result :** For any two real numbers a and b, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is true only when at least one of a and b is either 0 or positive.
- In fact, $\sqrt{-a} \times \sqrt{-b} = (i\sqrt{a})(i\sqrt{b}) = i^2\sqrt{ab} = -\sqrt{ab}$, where a and b are positive real numbers.

  Note : Every real number is a complex number, for if $a \in R$, it can be written as $a = a + i0$.
- **Purely Real and Purely Imaginary Numbers :** A complex number z is said to be

  (i) purely real, if $Im\ (z) = 0$.

  (ii) purely imaginary, if $Re\ (z) = 0$.
- **Sum, Difference and Product of Complex Numbers :** For complex numbers, $z_1 = a + ib$ and $z_2 = c + id$, it is defined as

  (i) $z_1 + z_2 = (a + ib) + (c + id) = (a + c) + i(b + d)$

  (ii) $z_1 - z_2 = (a + ib) - (c + id) = (a - c) + i(b - d)$

  (iii) $z_1z_2 = (a + ib)(c + id) = (ac - bd) + i(ad + bc)$

TOPIC-2
Complex Roots of Quadratic Equations

Revision Notes

- An equation of the form $ax^2 + bx + c$, $a \rangle 0$, is called the **quadratic equation** in variable x, where a, b and c are numbers (real or complex).
- The roots of the quadratic equation $ax^2 + bx + c = 0$, $a \rangle 0$ are

$$\boxed{\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}}$$

  Now,
  If we look at these roots of quadratic equation $ax^2 + bx + c = 0$; $a \rangle 0$, we observe that the roots depend upon the value of quantity

$$D = \boxed{b^2 - 4ac}$$

  This quantity is known as the **discriminant** of quadratic equation and denoted by D.
  CASE 1 : If $b^2 - 4ac = 0$, (*i.e.,*) $D = 0$, the roots are **real** and **equal**.
  CASE 2 : If $b^2 - 4ac > 0$ and perfect square, then the roots of the equation is **rational** and **unequal**.
  CASE 3 : If $b^2 - 4ac > 0$ and not a perfect square, the roots are **irrational** and **unequal**.

  CASE 4 : If $b^2 - 4ac < 0$, then the roots are **complex conjugate** of each other.
- For the quadratic equation $ax^2 + bx + c = 0$,

$$a, b, c \in R, a \neq 0, \text{if } b^2 - 4ac < 0$$

  then it will have complex roots given by,

$$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

- $\sqrt{a+ib}$ is called square root of $z = a + ib, \therefore \sqrt{a+ib} = x + iy$

  squaring both sides we get $a + ib = x^2 - y^2 + 2i(xy)$

CHAPTER-5
LINEAR INEQUALITIES

TOPIC-1
Solution of Linear inequalities in one variable

Revision Notes

- **Inequality :** Two real numbers or two algebraic expression related by the symbol '$<$', '$>$', '$\leq$' or '$\geq$' form an inequality.
- **Linear Inequality**
  An inequality is said to be linear, if each variable occurs in first degree only and there is no term involving the product of the variables.

  e.g., $ax + b \leq 0$, $ax + by + c > 0$, $ax \leq 4$.

  An inequality in one variable in which degree of variable is 2, is called quadratic inequality in one variables.

  e.g., $ax^2 + bx + c \geq 0$, $3x^2 + 2x + 4 \leq 0$.
- **Linear Inequality In One Variable**
  A linear inequality which has only one variable, is called linear inequality in one variable.

  e.g., $ax + b < 0$, where $a \neq 0$, $4c + 7 \geq 0$.

  (i) Rules for solving inequalities:
  - if $a \geq b$ then $a \pm k \geq b \pm k$ where k is any real number.
  - if $a \geq b$ then ka is not always $\geq kb$

  If $k > 0$ (*i.e.* positive) then $a \geq b$ $\Rightarrow ka \geq kb$

  If $k < 0$ (*i.e.* negative) then $a \geq b$ $\Rightarrow ka \leq kb$

  Thus, always reverse the sign of inequality while multiplying or dividing both sides of an inequality by a negative number.

  (ii) Procedure to solve a linear inequality in one variable.
  - Simplify both sides by collecting like terms.
  - Remove fractions (or decimals) by multiplying both sides by appropriate factor (L.C.M of denominator or a power of 10 in case of decimals.)
  - Isolate the variable on one side and all constants on the other side. Collect like terms whenever possible.
  - Make the coefficient of the variable equal to 1.
  - Choose the solution set from the replacement set.

Note : Replacement set : The set from which values of the variable (involved in the inequality) are chosen is called replacement set.

Solution set : A solution to an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set of the inequality.

Know the Terms

Types of Inequality

- **Numerical inequality :** An inequality which does not involve any variable is called a numerical inequality.
  e.g., $4 > 2$, $8 < 21$

➢ **Literal inequality :** An inequality which have variables is called literal inequality.
e.g., $x < 7, y \geq 11, x - y \leq 4$

➢ **Strict inequality :** An inequality which have only < or > is called strict inequality.
e.g., $3x + y < 0, x > 7$

➢ **Slack inequality :** An inequality which have only ≥ or ≤ is called slack inequality.
e.g., $3x + 2y \leq 0, y > 4$

TOPIC-2
Solution of Linear Inequalities in Two Variables

Revision Notes

➢ The inequality of form $ax + by + c > 0$, $ax + by + c = 0$, or $ax + by + c < 0$ etc. where $a \neq 0, b \neq 0$ is called a linear equality in two variables x and y.

➢ **Graphical Solution of Linear Inequalities in Two Variables**
- The graph of the inequality $ax + by > c$ is one of the half planes and is called the solution region.
- When the inequality involves the sign ≤ or ≥ then the points on the line are included in the solution region but if it has the sign < or > then the points on the line are not included in the solution region and it has to be drawn as a dotted line.
- The common values of the variable form the required solution of the given system of linear inequalities in one variable.
- The common part of coordinate plane is the required solution of the system of linear inequations in two variables when solved by graphical method.

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CHAPTER-6
PERMUTATIONS AND COMBINATIONS

TOPIC-1
Permutations

Revision Notes

➢ If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is $m \times n$.

➢ A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

➢ *e.g.*, If there are three objects say A, B and C, then the permutations of these objects taking two at a time are AB, BC, AC, BA, CB and CA. So, number of permutations is $3! = 1 \cdot 2 \cdot 3 = 6$.

➢ If n and r are positive integers such that $0 \leq r \leq n$, then the number of permutations of n distinct things taken r at a time is denoted by ${}^nP_r$ or P(n, r).

We have,

$$P(n,r) = {}^nP_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n.$$

Note :

(i) When $r = 0$, then ${}^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$ $\quad [\because 0! = 1]$

(ii) When $r = n$, then ${}^nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n!}{0!} = n!$

(iii) Number of permutations of different things taken all at a time = n!

Know the Terms

➢ **Fundamental Principles of Counting**

(i) Multiplication Principle : If first operation can performed in m ways and a second operation can be performed in n ways. Then, the two operations taken together can be performed in mn ways. This can be extended to any finite number of operations.

(ii) Addition Principle : If first operation can performed in m ways and another operation, which is independent of the first, can be performed in n ways. Then, either of the two operations can be performed in $m + n$ ways. This can be extended to any finite number of exclusive events.

(iii) Factorial : For any natural number n we define factorial as $n!$ or $\underline{|n} = n(n-1)(n-2) \ldots 3\times2\times1$ and 0! = 1! = 1

(iv) The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.

(v) The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, … p_k are of k^{th} kind and the rest if any, are of different kinds, is $\dfrac{n!}{p_1!p_2!\ldots p_k!}$

➢ **Properties of Permutation**

(i) ${}^nP_n = n(n-1)(n-2)\ \ldots.1 = n!$

(ii) ${}^nP_0 = \dfrac{n!}{n!} = 1$

(iii) ${}^nP_1 = n$

(iv) ${}^nP_{n-1} = n!$

(v) ${}^nP_r = n\cdot{}^{n-1}P_{r-1} = n(n-1)\cdot{}^{n-2}P_{r-2}$
$= n(n-1)(n-2)\cdot{}^{n-3}P_{r-3}$

(vi) ${}^{n-1}P_r + r\cdot{}^{n-1}P_{r-1} = {}^nP_r$

(vii) $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n-r+1$

TOPIC-2
Combinations

Revision Notes

➢ Each of the different groups or selection which can be made by taking some or all of a number of things or objects at a time irrespective of their arrangement is called a **combination**.

➢ The number of combinations of n distinct objects taken r at a time is given by,

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}\ ,\ 0 \le r \le n$$

It is also denoted by $C(n, r)$ or $\dbinom{n}{r}$.

➢ **Difference between permutations and combinations :**

- The process of selecting objects is called combination, and that of arranging objects is called **permutation**.

➢ **Properties of Combination :**

(i) ${}^nC_0 = {}^nC_n = 1$

(ii) ${}^nC_1 = n$

(iii) If ${}^nC_r = {}^nC_p$, then either $r = p$ or $r + p = n$

(iv) ${}^nC_r = \dfrac{{}^nP_r}{r!}$

(v) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(vi) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

(vii) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + \ldots = 2^{n-1}$

(viii) ${}^nC_r = \dfrac{n}{r}\,{}^{n-1}C_{r-1} = \dfrac{n(n-1)}{r(r-1)}\,{}^{n-2}C_{r-2}$

(ix) ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ {}^{2n+1}C_n = 2^{2n}$

(x) ${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \ldots + {}^{2n-1}C_n = {}^{2n}C_{n+1}$

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CHAPTER-7
SEQUENCE AND SERIES

TOPIC-1
Sequence, Series and A.P.

Revision Notes

- **Sequence :** Sequence is a function whose domain is a subset of natural numbers. It represents the images of 1, 2, 3, ….., n as $f_1, f_2, f_3, \ldots, f_n$, where $f_n = f(n)$.
- **Real Sequence :** A sequence whose range is a subset of R is called a real sequence.
- **Series :** If $a_1, a_2, a_3, \ldots\ldots, a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + \ldots.. + a_n$ is a series.
- **Progression :** A sequence whose terms follow certain rule is called a progression.
- **Finite Series :** A series having finite number of terms is called a finite series.
- **Infinite Series :** A series having infinite number of terms is called a infinite series.
- **Arithmetic Progression (A.P.) :** A sequence in which the difference of two consecutive terms is constant, is called Arithmetic Progression (A.P.), *i.e.*, $a_{n+1} - a_n =$ constant $(= d) \quad \forall \quad n \in N$

  General A.P. is $a, a+d, a+2d, a+3d, \ldots\ldots\ldots\ldots.$

  n^{th} term of A.P. $= a_n = a + (n-1)d = l$ (last term)
- **Sum of n Terms of an AP. :**
  - $S_n = \dfrac{n}{2}(a + l)$, where l = last term
  - $S_n = \dfrac{n}{2}[2a + (n-1)d]$,

  Note :

  Formula (i) is used when the last term is known and formula (ii) is used when the common difference is known. These formula have four quantities, if three are known, the fourth can be found out.

  n^{th} term from the end of an A.P. $= a_n + (n-1)(-d)$

$\because$ Taking a_n as the first term and common difference equal to '$-d$'.

- **Arithmetic Mean :** When three quantities are in *A.P.*, the middle quantity is said to be Arithmetic Mean *(A.M.)* between the other two.

  Thus, if a, A, b are in *A.P.*, then A is the *A.M.* between a and b.

  So, $A - a = b - A$, [their common difference are equal]

  $\Rightarrow$ $2A = a + b$

  $\Rightarrow$ $A = \dfrac{a+b}{2}$

  $\therefore$ *A.M.* between two numbers = Half of their sum

- **An Important Result** : Sum of n *A.M.*'s between two quantities is n times the single A.M. between them.

  n **Arithmetic Means** : Let $A_1, A_2, A_3, \ldots, A_n$ be the n, *A.M.*'s between two numbers a and b.

  So we can find the *A.M*'s as

  $$A_1 = a + d, d = \frac{b-a}{n+1}, A_2 = a + 2d = a + \frac{2(b-a)}{n+1} \ldots, A_n = a + nd = a + \frac{n(b-a)}{n+1}$$

- **Some Important results:**
  - If a, b, c are in A.P. then $a \pm k, b \pm k, c \pm k$ are in A.P.

    ak, bk, ck are also in A. P., $k \neq 0, \frac{\ddot{u}}{\ddot{u}}, -, -$ are also in A.P. where $k \neq 0$.
  - $S_k - S_{k-1} = a_k$
  - $a_m = n, a_n = m \Rightarrow a_r = m + n - r$
  - $S_m = S_n \Rightarrow S_{m+n} = 0$
  - $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
  - In An A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last terms
  - If three terms of A.P. are to be taken then we choose then as $a - d, a, a + d$.
  - If four terms of A.P. are to be taken then we choose then as $a - 3d, a - d, a + d, a + 3d$.
  - If five terms of A.P. are to be taken, then we choose then as:

    $a - 2d, a - d, a, a + d, a + 2d$.

TOPIC-2
G.P. and Sum to *n* terms

Revision Notes

- A sequence of non-zero number is said to be G.P., if the ratio of each term, except the first one, by its preceding term is always the same.

  We can say that, a sequence $a_1, a_2, \ldots\ldots, a_n$ is called geometric progression (geometric sequence), if it follows the relation $\dfrac{a_{k+1}}{a_k} = r$ (constant).

  The constant ratio is called **Common ratio** of the *G.P.* and it is denoted by r.

  In a *G.P.*, we usually denote the first term by a, the n^{th} term by T_n or a_n.

  Thus, G.P. can be written as, a, ar^2, ar^3 …… and so on.

- **General Term of a** *Γ.Π.* : If a is the first term of a *G.P.* and its common ratio is r, then general term or n^{th} term,

  $T_n = ar^{n-1}$ or $l = ar^{n-1}$, where l is the last term.

  m^{th} Term of Finite GP. from the End : Let a be the first term and r be the common ratio of a *G.P.* having n terms. Then, m^{th} term from the end is $(n - m + 1)^{\text{th}}$ term from the beginning.

  Also, m^{th} term from the end $= l\left(\dfrac{1}{r}\right)^{n-1}$, where l is last term of the finite *G.P.*

- **Sum of First n Terms of a G.P.** : If a and r are the first term and common ratio of a $G.P.$ respectively, then sum of n terms of this G.P. is given by

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ where } r < l$$

and $$S_n = \frac{a(r^n-1)}{r-1}, \text{ where } r > 1$$

- **Geometric Mean:** Geometric mean between a and b is $\sqrt{ab}$ or If a, b, c are in G.P., then
  Geometric mean (G.M.) $b^2 = ac$
  Some Important Results:
  - Reciprocals of terms in G.P. always form a G.P.
  - If $G_1, G_2, G_3, \dots, G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then
  $$G_k = a\left(\frac{b}{a}\right)^{k+1}, 1 \le k \le n$$
  - If three terms of G.P. are to be taken, then we choose as $\frac{a}{r}, a, ar$.
  - If four terms of G.P. are to be taken, then we choose as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.
  - If five terms of G.P. are to be taken, then we choose as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$
  - If a, b, c are in G.P. then ak, bk, ck are also in G.P. where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. where $k \neq 0$.
  - In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last terms.
  - If each term of a G.P. be raised to some power then the resulting terms are also in G.P.
  - $S_\infty = a + ar + ar^2 + ar^3 + \cdots\infty$ term if $-1 < r < 1 \Rightarrow S_\infty = \frac{a}{1-r}$.
  Such that $-1 < r < 1$ or $|r| < 1$
  - If a, b, c are A.P. then $2b = a + c$.
  - If a, b, c are in G.P. then $b^2 = ac$
  - If A and G be A.M and G.M of two given positive real number 'a' and 'b' respectively then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $A \ge G$.

UNIT-III

CO-ORDINATE GEOMETRY

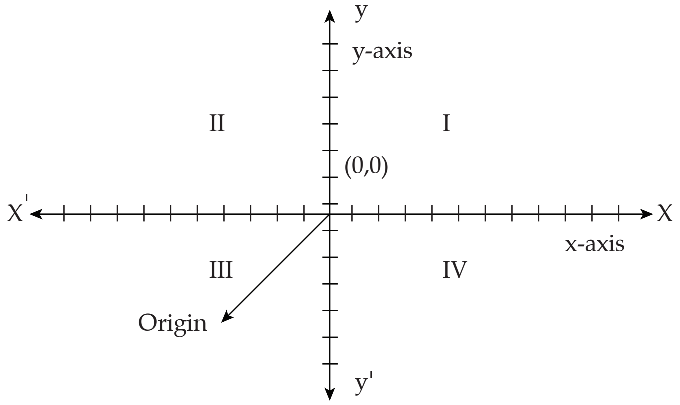
CHAPTER-8
STRAIGHT LINES

TOPIC-1
Recall of Two Dimensional Geometry and Slope of a Line

Revision Notes

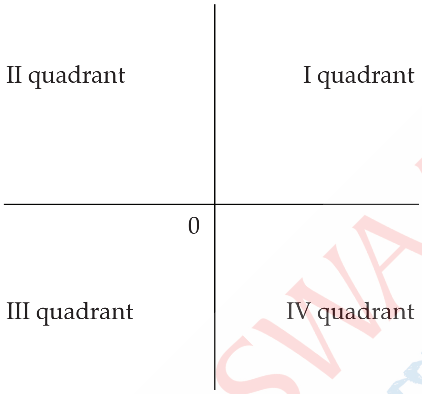
- **Coordinate axes and plane**
  - The Position of a point in a plane is fixed by selecting the axes or reference which are formed by two number lines intersecting each other at right angle. The horizontal number line is called x-axis and vertical number line is called y-axis.

(a)



- The point of intersection of these two lines is called the origin. The intersection of x-axis and y-axis divide the plane into four equal parts. These four parts are called quadrants, Each part is (¼th) of the whole portion. These are numbered I, II, III and IV anticlockwise from OX. Thus, the plane consists of the axes and four quadrants is known as XY-plane or equation plane or coordinate plane and the axes are known as co-ordinate axes. These axes are also known as rectangular axes and are perpendicular to each other.

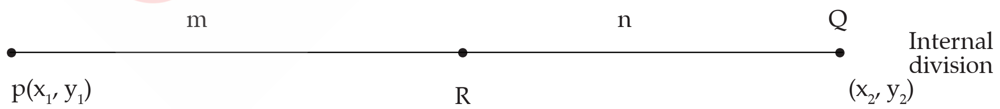
(b)



➢ **Section Formulae**

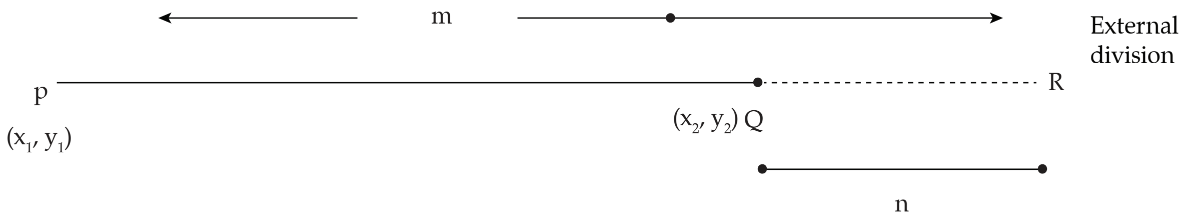
- If a point R divide the segments joining the points $p(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$, then its coordinate are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



- If the division is external, then the co-ordinate of R are

$$\left(\frac{mx_2 - nx}{m-n}, \frac{my_2 - ny_1}{m-n}\right).$$



➢ **Distance formulae**

- Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$ or $\left|\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right|$.
- If the points are $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$, then distance between them is

$$\left|\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}\right|$$

- Distance of a point $P(x_1, y_1)$ from origin is $\sqrt{x_1^2 + y_1^2}$.

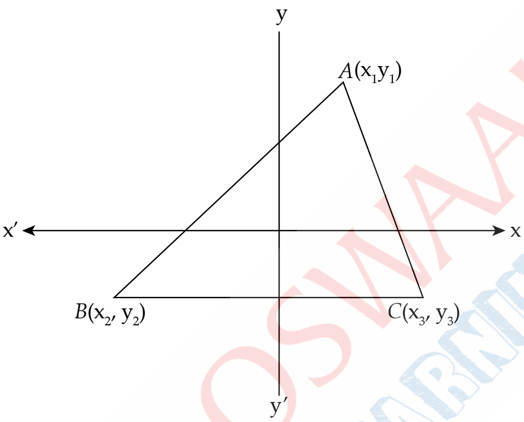
➢ **Area Formulae**

- Area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

- If $\Delta = 0$, then the points *A*, *B*, *C* are collinear (*i.e.*, they lie in a same straight line).



- Area of quadrilateral *ABCD* with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ is

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_2 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right|$$

- Area of a trapezium formed by joining the vertices $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ is

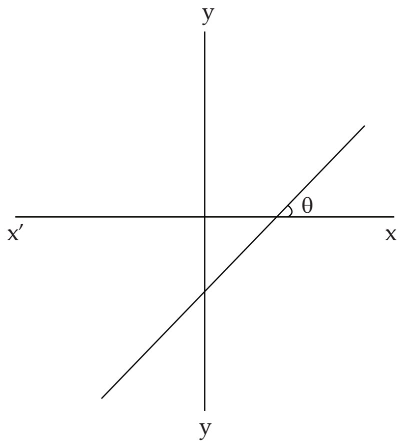
$$\left|\frac{1}{2}[(y_1 + y_2)(x_1 - x_2) + (y_3 + y_1)(x_3 - x_1) + (y_2 + y_3)(x_3 - x_2)]\right|$$

➢ **Equation of the locus of a point**

- The equation of the locus of a point is the relation which is satisfied by the coordinates of every point on the locus of the point.
- The slope of a line '*l*' is the tangent of the angle made by the line in the anti-clockwise direction with the positive *x*-axis.

  i.e., slope, $m = \tan\theta$,

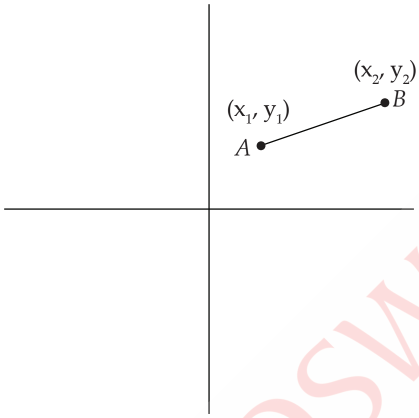
  where '*m*' represents slope and 'θ' is the angle made by the line with positive *x*-axis.



➢ **Slope of a line joining two points**

- The slope 'm' of a line segment AB joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, and making angle 'θ' with positive x-axis, is given by

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}.$$

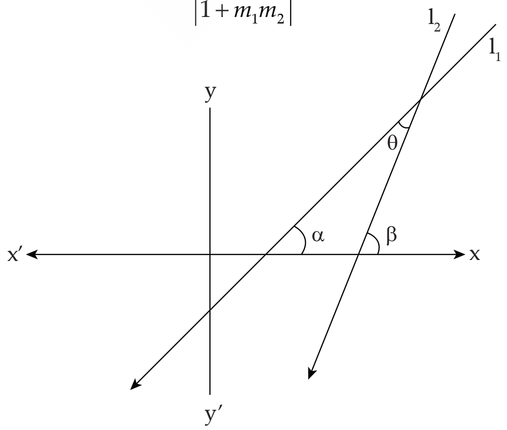


➢ **Angle between two lines**

- Let l_1 and l_2 be two lines which makes angles α and β respectively with positive x-axis. Then, their slopes are $m_1 = \tan\alpha$ and $m_2 = \tan\beta$.
- Let 'θ' be the angle between l_1 and l_2, then

$$\theta = \tan^{-1}\left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

and $\theta' = \pi - \tan^{-1}\left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$



- If two lines are parallel, then angle between them is 0°.

  $\therefore$ Slope $= \tan\theta = \tan 0° = 0$

  $\Rightarrow \quad \dfrac{m_2 - m_1}{1 + m_1 m_2} = 0$

  $\Rightarrow \quad m_1 = m_2$

  Thus, two lines are parallel if and only if their slopes are equal *i.e.*, if $m_1 = m_2$.

- If two lines are perpendicular, then angle between them is 90°. Therefore, slope $= \tan 90° = \infty$

  $\Rightarrow \quad \dfrac{m_2 - m_1}{1 + m_1 m_2} = \infty$

  $\Rightarrow \quad 1 + m_1 m_2 = 0$

  $\Rightarrow \quad m_1 m_2 = -1$

  Thus, two lines are perpendicular, if and only if their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$

➢ **Collinearity of three points**

- Three points *A*,*B*,*C* is *XY*-plane are collinear *i.e.*, they lie on the same line if and only if slope of *AB* = slope of *BC*.

➢ **Extra Information**

- If a point *R* trisect the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then it will divide *PQ* in the ratio 2 :1 or 1: 2
- If a point R is the mid-point of the line segment joining the points $p(x_1, y_1)$ and $Q(x_2, y_3)$, then the coordinates of *R* are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
- If *G* is the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then co-ordinates of $G = \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.
- The centroid divides each median of the triangle in the ratio 2 :1.
- The angle 'θ', which a line makes with positive direction of *x*-axis, is called inclination angle of the line.
- The slope of *y*-axis (or any line parallel to it) is, $m = \tan 90° = \infty$, which is not defined.
- Let 'θ' be the angle between two lines. Then
  (i) If tan θ is positive, then θ will be an acute angle.
  (ii) If tan θ is negative, then θ will be an obtuse angle.

TOPIC-2
Various forms of equations of a straight line

Revision Notes

➢ Horizontal and vertical lines

- The equation of a horizontal line (i.e., any line parallel to *x*-axis) is $y = a$ or $y = -a$. If the line lies above *x*-axis, then '*a*' is positive and if the line lie below the *x*-axis, then '*a*' is negative.
- The equation of a vertical line (i.e., any line parallel to *y*-axis) is $x = b$ or $x = -b$. If line lie to the left of *y*-axis, then '*b*' is negative and if the line lie to the right of *y*-axis, then '*b*' is positive.

➢ Slope intercept form

- If a line '*L*' has slope '*m*' and make an intercept '*c*' on *y*-axis, then the equation of the line is $y = mx + c$.
- If the line passes through the origin, then its equation becomes $y = mx$.

- Point-slope form
  - The equation of the straight line having slope 'm' and passing through the point $P_0(x_0, y_0)$ is $y - y_0 = m(x - x_0)$
- Two points form
  - The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- Normal (perpendicular) form)
  - The equation of the straight line upon which the length of perpendicular from the origins is p and this perpendicular makes an angle 'α' with the positive direction of x-axis is $x\cos\alpha + y\sin\alpha = p$
  - Slope of the line $= -\dfrac{1}{\tan\alpha}$

$$= -\frac{\cos\alpha}{\sin\alpha}$$

- Intercept form
  - The equation of a line which cuts-off intercepts a and b on the x-axis and y-axis respectively, is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

- **Extra Information**
  - Equation of x-axis is $y = 0$ and equation of y-axis is $x = 0$.
  - If a line 'L' has slope 'm' and x-intercept 'd', then the equation of the line is $y = m(x - d)$.

TOPIC-3
General Equation of a line

Revision Notes

- General Equation of a line
  - An equation of the form $Ax + By + C = 0$, where A, B and C are real constants and at least one of A or B is non-zero, is called general linear equation or general equation of a line.
- Various forms of $Ax + By + C = 0$ The general equation of a line can be reduced into various forms of equations of a line, which are as follows
  - Slope-intercept form If $B \neq 0$, then $Ax + By + C = 0$ can be written as

$$y = -\frac{A}{B}x - \frac{C}{B} \text{ or } y = mx + c \text{, where}$$

$$m = -\frac{A}{B} \text{ and } c = -\frac{C}{B}.$$

  - **Intercept form:** If $C \neq 0$, then $Ax + By + C = 0$ can be written as

$$\frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1 \text{ or } \frac{x}{a} + \frac{y}{b} = 1 \text{, where } a = -\frac{C}{A} \text{ and } b = -\frac{C}{B}.$$

  - **Normal form:** The normal form of the equation $Ax + By + C = 0$ can be written as

$x\cos\alpha + y\sin\alpha = p$, where

$$\cos\alpha = \pm\frac{A}{\sqrt{A^2 + B^2}},\ \sin\alpha = \pm\frac{B}{\sqrt{A^2 + B^2}} \text{ and } p = \frac{C}{\sqrt{A^2 + B^2}}$$

➢ Angle between Two lines, having general Equations

Let $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ be the general equations of two lines.

Then, the slope of the lines are $m_1 = -\frac{A_1}{B_1}$ and $m_2 = -\frac{A_2}{B_2}$

Let '□' be the angle between the two lines, then

$$\tan\theta = \pm\left(\frac{m_2 - m}{1 + m_1 m_2}\right) = \pm\left(\frac{-\frac{A_2}{B_2} + \frac{A_1}{B_1}}{1 + \frac{A_1}{B_1} \cdot \frac{A_2}{B_2}}\right)$$

For acute angle, we take $\theta = \tan^{-1}\left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|$

For obtuse angle, we take $\theta = \pi - \tan^{-1}\left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|$

➢ Condition for Two lines to be parallel and perpendicular

Let $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ be the general equations of two lines.

Then, their slopes are, $m_1 = -\frac{A_1}{B_1}$, $m_2 = -\frac{A_2}{B_2}$

The lines are parallel if $m_1 = m_2$

$$\Rightarrow -\frac{A_1}{B_1} = -\frac{A_2}{B_2} \Rightarrow \frac{A_1}{B_1} = \frac{A_2}{B_2} \Rightarrow A_1B_2 = A_2B_1$$

and the lines are perpendicular if $m_1m_2 = -1$

$$\Rightarrow \left(-\frac{A_1}{B_1}\right)\left(-\frac{A_2}{B_2}\right) = -1$$

$$\Rightarrow A_1A_2 + B_1B_2 = 0$$

➢ Equations of perpendicular and parallel lines through a given point

Let the general equation of a line be $Ax + By + C = 0$.

Then, the slope of the line is $m_1 = -\frac{A}{B}$.

Let, m_2 be the slope of perpendicular line, then we have $m_1m_2 = -1$

$$\Rightarrow -\frac{A}{B}m_2 = -1$$

$$\Rightarrow m_2 = \frac{B}{A}$$

$\therefore$ Equation of a line passing through (x_1, x_2) and perpendicular to $Ax + By + C = 0$ is

$$y - y_1 = \frac{B}{A}(x - x_1)$$

$$\Rightarrow Bx - Ay + (Ay_1 - Bx_1) = 0$$

Again, let m_2' be the slope of parallel, then we have

$$m_1 = m_2'$$

$$\Rightarrow m_2' = -\frac{A}{B}$$

$\therefore$ Equation of a line parallel to the line $Ax + By + C = 0$ and passing through (x_1, y_1) is

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$\Rightarrow Ax + By - (Ax_1 + By_1) = 0$$

- Distance of a point from a line

  The distance of a point from a line is the length of perpendicular drawn from the point to the line. Let $L: A_1x + B_1y + C_1 = 0$ be a line, whose perpendicular distance from the point $P(x_1, y_1)$ is d. Then

  $$d = \left|\frac{A_1x + B_1y + C_1}{\sqrt{A^2 + B^2}}\right|$$

  Distance between two parallel lines

  The distance between two parallel lines

  $y = mx + c_1$

  $y = mx + c_2$, is given by

  $$d = \left|\frac{c_1 - c_2}{\sqrt{1 + m^2}}\right|$$

  If the lines are given in general form i.e., $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then the distance between them is

  $$d = \left|\frac{c_1 - c_2}{\sqrt{A^2 + B^2}}\right|$$

- **Extra information**

  In general, we consider the equation of a line perpendicular to line $Ax + By + C_1 = 0$ as $Bx - Ay + K = 0$. The value of $'K'$ can be evaluated by substituting any particular point (a, b) which lies on the line.

  In general, we consider the equation of a line parallel to line $A_1x + B_1y + C_1 = 0$ as $A_1x + B_1y + K = 0$ The value of $'K'$ can be evaluated by substituting any particular point (a, b) which lies on the line.

  If $A = 0, B \neq 0,$ then $Ax + By + C = 0$, which reduces to $y = -\frac{C}{B}$, which is the equation of horizontal line (i.e., parallel to x,- axis)

  If $A = C = 0, B \neq 0,$ then $Ax + By + C = 0$, reduces to $y = 0$, which is the equation of x - axis.

  If $A \neq 0, B = C = 0,$ then $Ax + By + C = 0$ reduces to $x = 0$, which is the equation of y - axis.

  If $A \neq 0, B \neq 0, C = 0,$ then $Ax + By + C = 0$ reduces to $y = -\frac{A}{B}x,$ which is the equation of a straight line passing through the origin.

CHAPTER-9
CONIC SECTIONS

TOPIC-1
Sections of a Cone (circle.)

Revision Notes

- Geometrical Definition of conic sections

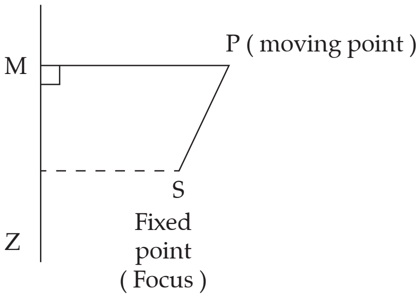
  A conic section is the locus of a point which moves in a plane in such a way that the ratio of its distance from a fixed point and a fixed line is a constant. Then

(i) The fixed point is called focus and is denoted by S.

(ii) The fixed straight line is called directrix.

(iii) The constant ratio is called eccentricity and is denoted by 'e'.

In the adjacent figure,

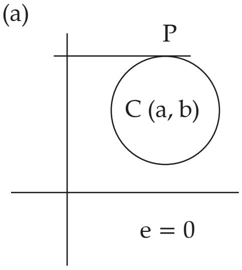


$$\frac{SP}{PM} = e = \text{constant}.$$
$$= \text{eccentricity}.$$

Depending on the eccentricity 'e', the different cases are as follows

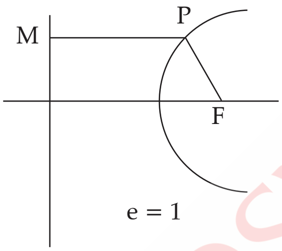
- Circle — When eccentricity, $e = 0$, then the conic is a circle.

(a)



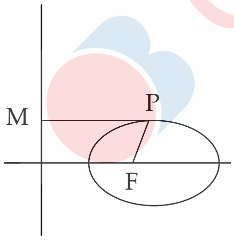
- Parabola — When eccentricity $e = 1$, then the conic is a parabola.

(b)



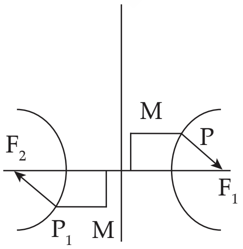
- Ellipse — When eccentricity $e < 1$, then the conic is an ellipse.

(c)



- Hyperbola — When eccentricity $e > 1$, then the conic is a hyperbola.

(d)



(iv) The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic.

(v) The point of intersection of the conic and its axis, is called vertex of the conic.

(vi) A line perpendicular to the axis of the conic and passing through its focus is called latus rectum.

(vii) The point which bisects every chord of the conic passing through it, is called the centre of the conic.

CIRCLE

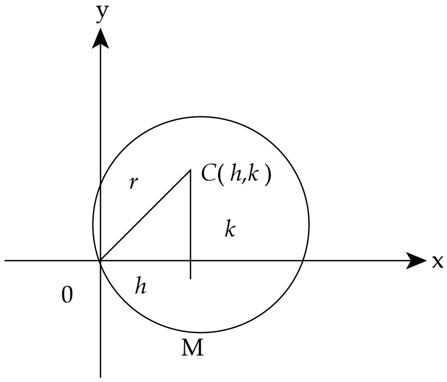
- A circle is defined as the locus of a point in a plane, which moves in such a way that its distance from a fixed point in that plane is always constant from a fixed point. The fixed point is called the centre of the circle and the constant distance from the centre is called the radius of the circle.

Standard equation of a Circle

Equation of a circle having centre (*h*, *k*) and radius *r* is

$$(x-h)^2+(y-k)^2=r^2.$$

If centre is at the origin (0, 0), then the equation of the circle with radius '*r*' is



$$x^2+y^2=r^2.$$

(i) When the circle passes through the origin, then equation of the circle is

$$x^2+y^2-2hx-2ky=0.$$

(ii) When the centre lies on *x*-axis, then equation of the circle is $(x-h)^2+y^2=r^2$, and when the centre lies on *y*-axis, then equation of the circle is $x^2+(y-k)^2=r^2$.

(iii) When the circle touches *x*-axis, then its equation is

$$(x-h)^2+(y\mp r)^2=r^2$$

when the circle touches *y*-axis, then its equation is

$$(x\mp r)^2+(y-k)^2=r^2$$

When the circle touches both the co-ordinate axes, then its equation is

$$(x\mp r)^2+(y\mp r)^2=r^2$$

General Equation of a Circle

The general equation of a circle having centre (*h*, *k*) and radius *r* is

$x^2+y^2+2gx+2fy+c=0$, where

$g=-h$, $f=-k$ and $c=h^2+k^2-r^2$

The above equation of a circle is called the general equation of a circle with centre $(-g,-f)$ and radius

$$r=\sqrt{h^2+k^2-c} \text{ or } \sqrt{g^2+f^2-c}.$$

(i)If $g^2+f^2-c>0$, then the radius of the circle is real and hence the circle is also real.

(ii)If $g^2+f^2-c=0$, then the radius of the circle is 0 and the circle is a point circle.

(iii)If $g^2+f^2-c<0$, then the radius of the circle is imaginary and is not possible to draw.

Diameter form of Equation of a Circle

Let (x_1,y_1) and (x_2,y_2) be the end points of the diameter of a circle. Then, equation of circle drawn on the diameter is

$$(x-x_1)\cdot(x-x_2)+(y-y_1)\cdot(y-y_2)=0$$

➢ **Extra Information**

- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions
- A circle is the set of all points in a plane that are equidistant from a fixed point in that plane.
- Two circles having the same centre $c(h, k)$ and different radii r_1 and $r_2 (r_1 \neq r_2)$ are called concentric circles. For e.g., the circles

  $(x-h)^2 + (y-k)^2 = r_1^2$, and $(x-h)^2 + (y-k)^2 = r_2^2\ (r_1 \neq r_2)$ are concentric circles.

- Let $(x-h)^2 + (y-k)^2 = r^2$ be the equation of a circle with centre (h, k) and radius r and let (a, b) be any point in the plane. Then,

  (i) The point (a, b) lies inside the circle, if $(a-h)^2 + (b-k)^2 < r^2$.

  (ii) The point (a, b) lies on the circle, if $(a-h)^2 + (b-k)^2 = r^2$.

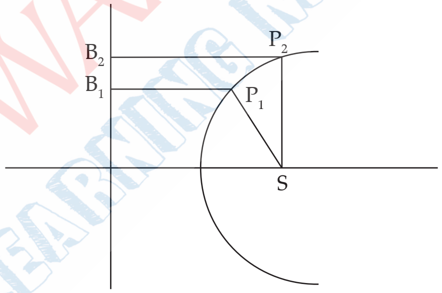
  (iii) The point (a, b) lies outside the circle, if $(a-h)^2 + (b-k)^2 > r^2$.

TOPIC-2
Parabola, Ellipse and Hyperbola

Revision Notes

➢ **Parabola**

A parabola is the locus of a point which moves in a place such that its distance from a fixed point is always equal to its distance from a straight line in the same.



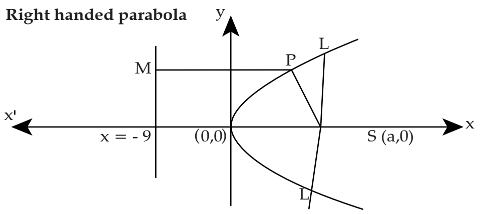
In the figure, $P_1B_1 = P_1S$

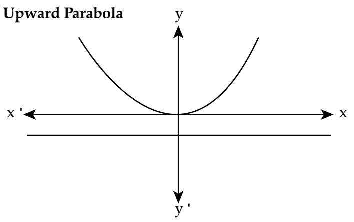
$P_2B_2 = P_2S$

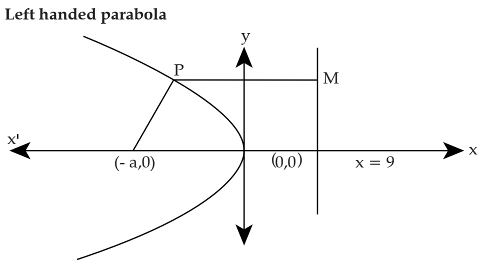
Here, the fixed line is called the directrix and the fixed point is called the focus of the parabola. A line through the focus and perpendicular to the directrix is called the axis of the parabola and point of intersection of parabola with the axis is called the vertex of the parabola. In case of parabola, eccentricity, $e = 1$.

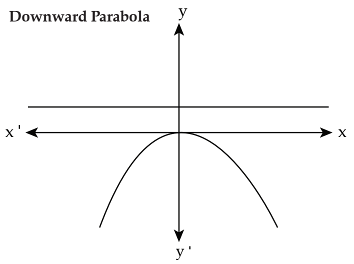
➢ **Types of Parabola**

- Right handed Parabola
- Left handed Parabola
- Upward Parabola
- Downward Parabola









➢ **Main facts about four types of Parabola**

Parabola	Vertex	Focus	Latus Rectum	Co-ordinate of L·R	Axis	Directrix	Symmetry
$y^2 = 4ax$	(0, 0)	$(a, 0)$	$4a$	$(a_1 \pm 2a)$	$y = 0$	$x = -a$	X-axis
$y^2 = -4ax$	(0, 0)	$(-a, 0)$	$4a$	$(-a_1 \pm 2a)$	$y = 0$	$x = a$	X-axis
$x^2 = 4ay$	(0, 0)	$(0, a)$	$4a$	$(\pm 2a, a)$	$x = 0$	$y = -a$	Y-axis
$x^2 = -4ay$	(0, 0)	$(0, -a)$	$4a$	$(\pm 2a, -a)$	$x = 0$	$y = a$	Y-axis

Parabola with y^2 term	Parabola with x^2 term
1 Symmetrical about X-axis	1 Symmetrical about Y-axis
2 Axis is along the X-axis	2 Axis is along the Y-axis
3 It open right handed when co-efficient of 'x' is positive and left handed when co-efficient of 'x' is negative.	3 It opens upwards if co-efficient of 'y' is positive and downwards if co-efficient of 'y' is negative.

➢ **Ellipse**

An ellipse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is a constant.

$\therefore\ P_1S_1 + P_1S_2 = P_2S_1 + P_2S_2 = P_3S_1 + P_3S_2 =$ constant.

➢ **Terms related to an Ellipse**

- **Focus** - Two fixed points are called foci of the ellipse and denoted by S_1 and S_2. The distance between two foci S_1 and S_2 is $2c$.
- **Centre** - The mid-point of the line - segment joining the foci, is called centre of ellipse.
- **Major Axis** - The line segment through the foci of the ellipse is called major axis. The length of major axis is $2a$.
- **Minor axis** - The line segment through the centre and perpendicular to the major axis is called minor axis. The length of minor axis is denoted by $2b$.
- **Vertices** - The end points of the major axis are called the vertices of the ellipse.
- **Eccentricity** - The eccentricity of ellipse is the ratio of the distance from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse. It is denoted by 'e'.

$$e = \frac{c}{a} \Rightarrow c = ae$$

$$\text{Since } c < a \Rightarrow \frac{c}{a} < a \Rightarrow e < 1\,.$$

- **Latus Rectum** - Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.

Thus, the length of latus rectum = $2l = 2\dfrac{b^2}{a}$

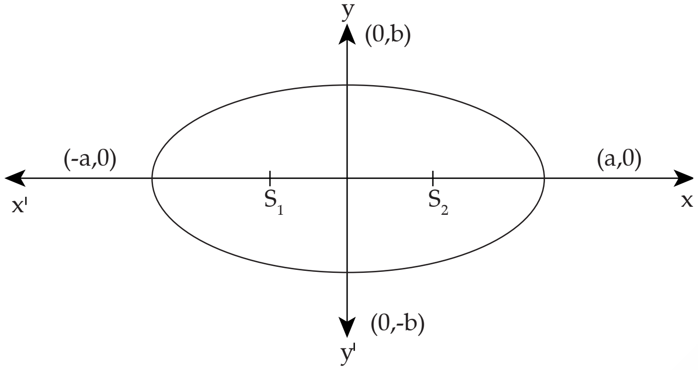
➢ **Standard equation of an ellipse**

The standard equation of an ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b$ having centre at origin and major axis on x-axis and minor axis on y-axis. It is also called horizontal ellipse.

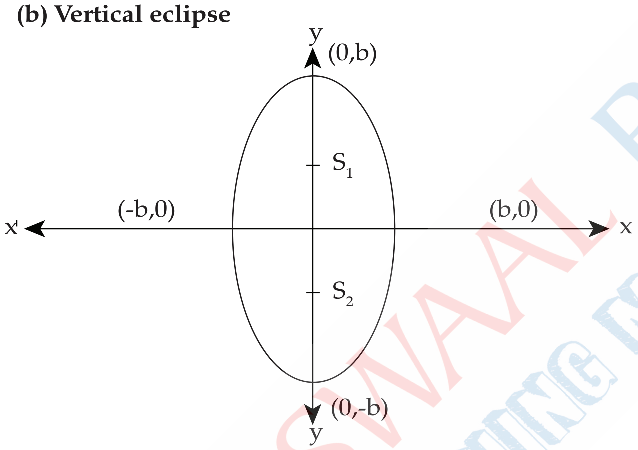
The another form of the equation of an ellipse is $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a>b$, having centre at origin and major axis lie on y-axis and minor axis lie on x-axis. It is also called vertical ellipse.

➢ **Facts about two standard ellipse**

(a) Horizontal eclipse



(b) Vertical eclipse



	Horizontal ellipse	Vertical ellipse
	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a>b$
1. Centre	(0, 0)	(0, 0)
2. Vertices	$(\pm a, 0)$	$(0, \pm a)$
3. Major axis	$2a$	$2a$
4. Minor axis	$2b$	$2b$
5. Value of c	$c=\sqrt{a^2-b^2}$	$c=\sqrt{a^2-b^2}$
6. Equation of major axis	$y = 0$	$x = 0$
7. Equation of minor axis	$x = 0$	$y = 0$
8. Directrix	$x=\pm\frac{a^2}{c}$ or $\pm\frac{a}{e}$	$y=\pm\frac{b^2}{c}$ or $\pm\frac{b^2}{ae}$
9. Foci	$(\pm c, 0)$ or $(ae, 0)$	$(0, \pm c)$ or $(0, \pm ae)$
10. Eccentricity	$e=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$	$e=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$
11. Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

12. Co-ordinate of latus rectum	$\left(\pm c, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{b^2}{a}, \pm c\right)$
13. Focal distance	$2c$	$2c$

➢ **Hyperbola**

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

$$\therefore\ P_1S_2 - P_1S_1 = P_2S_2 - P_2S_1 = P_3S_2 - P_3S_1$$

➢ The term difference means the distance to the farther point minus the distance to the closer point.

➢ **Terms Related to Hyperbola**

- **Focus** - The two fixed points are called the foci of the hyperbola and denoted by S_1 and S_2. The distance between two foci S_1 and S_2 is $2c$.
- **Centre** - The midpoint of the line segment joining the foci, is called centre of hyperbola.
- **Transverse axis** - The line through the foci is called transverse axis.
- **Conjugate axis** - The line through the centre and perpendicular to the transverse axis is called conjugate axis.
- **Vertices** - The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola. The distance between two vertices is 2a.
- **Eccentricity** - Eccentricity of the hyperbola is the ratio of the distance of any focus from the centre and the distance of any vertex from the centre and it is denoted by e.

$$\therefore\ e = \frac{c}{a} \text{ and } c > a \Rightarrow e > 1 .$$

- **Directrix** - It is a line perpendicular to the transverse axis and cuts it at a distance of $\frac{a^2}{c}$ from the centre. i.e. $x = \pm\frac{a^2}{c}$ or $y = \pm\frac{a^2}{c}$
- **Latus rectum** - It is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

Thus, length of latus rectum $= 2l = \frac{2b^2}{a}$.

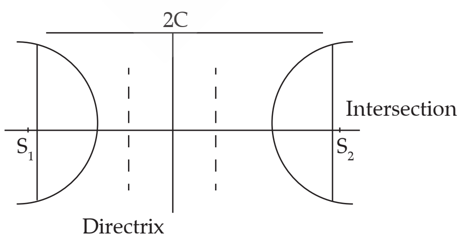
➢ **Standard Equation of Hyperbola**

Standard equation of hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose X-axis as transverse and Y-axis as conjugate axis.

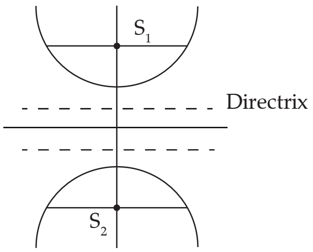
The equation of the hyperbola of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is called conjugate hyperbola, whose X-axis as conjugate axis and Y-axis as transverse axis.

➢ **Facts about two standard Hyperbolas**

(a) Hyperbola



(b) Conjugate Hyperbola



	Hyperbola	**Conjugate Hyperbola**
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
1. Transverse axis	$2a$	$2a$

2. Conjugate axis	$2b$	$2b$
3. Value of c	$c=\sqrt{a^2+b^2}$	$c=\sqrt{a^2+b^2}$
4. Vertices	$(\pm a, 0)$	$(0, \pm a)$
5. Directrices	$x=\pm\frac{a^2}{c}$ or $\pm\frac{a}{e}$	$y=\pm\frac{a^2}{c}$ or $\pm\frac{a}{e}$
6. Foci	$(\pm ae, 0)$ or $(\pm c, 0)$	$(0, \pm ae)$ or $(0, \pm c)$
7. Eccentricity	$e=\sqrt{1+\frac{b^2}{a^2}}$ or $\frac{c}{a}$	$e=\sqrt{1+\frac{b^2}{a^2}}$ or $\frac{c}{a}$
8. Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

➢ **Extra Information**

- The standard equation of parabolas have focus on one of the co-ordinate axis, vertex at the origin and the directrix is parallel to the other coordinate axis.
- In case of an ellipse, where $c^2=a^2-b^2$, if $c = 0$, then the ellipse is a circle and if $c = a$, then the ellipse reduces to a line segment.
- Foci of an ellipse always lie on the major axis. If the co-efficient of x^2 has the larger denominator, then major axis is along x-axis. If the co-efficient of y^2 has the larger denominator, then major axis is along y-axis.
- Ellipse and Hyperbola are symmetrical with respect to both the axes.
- In hyperbola, foci always lie on the transverse axis. If the denominator of x^2 gives the positive term, then transverse axis is along x-axis and if the denominator of y^2 gives the positive term, then transverse axis id along y-axis.
- In a hyperbola, no portion of the curve lies between $x = a$ and $x = -a$.

■■

CHAPTER-10
INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

Revision Notes

➢ Coordinate Axes and co-coordinates planes in Three Dimensional space

Let XOX', YOY' and ZOZ' be three mutually perpendicular lines, intersecting at O. The point O(O, O, O) is called the origin and the lines (XOX', YOY', ZOZ') are called co-ordinate or rectangular axes $(X, Y$ and Z respectively$)$.

In the figure below, $X'OX$ is called X-axis, YOY' is called Y-axis and ZOZ' is called Z-axis.

The three co-ordinate axes defines three mutually perpendicular planes XOY, YOZ and $ZOX($ or $XY, YZ, ZX)$ are called coordinate planes which divide space into eight parts called octant.

XOY-XY plane, YOZ-YZ plane

ZOX-ZX plane, $XOYZ, X'OYZ,$

$XOY'Z, X'OY'Z', XOYZ', X'OYZ',$

$XOY'Z'$ and $X'OY'Z'$ are called octants.

➢ Co-ordinates of a point in space.

Let P be a point in space. Then, $P(x, y, z)$ are its co-ordinates where x-coordinate of P = length of perpendicular from P to YZ-plane with sign.

y-coordinate of P = length of perpendicular from P to ZX-plane with sign.

The sign of co-ordinates of the points in the octant in which the space is divided are given as follows

octants	I	II	III	IV	V	VI	VII	VIII
coordinates	$OXYZ$	$OX'YZ$	$OX'Y'Z$	$OXY'Z$	$OXYZ'$	$OX'YZ'$	$OX'Y'Z'$	$OXY'Z'$
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

- The co-ordinates of a point on x-axis will be of the form $(x, 0, 0)$.
- The co-ordinates of a point on y-axis will be of the form $(0, y, 0)$.
- The co-ordinates of a point on z-axis will be of the form $(0, 0, z)$.
- The co-ordinates of a point in xy plane is of the form $(x, y, 0)$.
- The co-ordinates of a point in yz plane is of the form $(0, y, z)$.
- The co-ordinates of a point in zx plane is of the form $(x, 0, z)$

➢ **Distance formula and its application in Geometry**

The distance between two points $P(x_1, y_1, z_1)$ and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

or $$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

➢ Three points A, B and C are said to be collinear if AB + BC= AC or BC + CA = AB or CA + AB = BC.

➢ **Properties of some Geometrical figures**

- Properties of Triangles

(i) Scalene triangle - All three sides are unequal

(ii) Right angled triangle - The sum of squares of any two sides is equal to the square of the third sde.

(iii) Isosceles triangle - Any two sides of a triangle are equal

(iv) Equilateral triangle - All three sides of a triangle are equal.

- Properties of Quadrilaterals

(i) Rectangle - Opposite sides are equal and diagonals are equal.

(ii) Parallelogram - Opposite sides are equal and diagonals are unequal, Also, diagonals bisect each other.

(iii) Rhombus - All four sides are equal and diagonals are unequal

(iv) Square - All four sides are equal and diagonals are equal.

➢ **Section Formulae**

- Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be end points of a line segment AB and C be any point on AB which divides AB in the ration $m : n$

(i) If C divides AB internally, then the co-ordinates of C are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n},\right)$$

(ii) If C divides AB externally, then the co-ordinates of C are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

If 'C' is the midpoint of AB, then m : n = 1 :1, so the co-ordinates of C are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

If 'C' trisect the line segment AB, then $m : n = 1 : 2$ or $m : n = 2 : 1$. So, the co-ordinates of C are

$$\left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3}, \frac{2z_1 + z_2}{3}\right) \text{ or } \left(\frac{x_1 + 2x_2}{3}, \frac{y_1 + 2y_2}{3}, \frac{z_1 + 2z_2}{3}\right)$$

➢ **Coordinates of centroid of a triangle**

- If $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are the vertices of ΔABC, then the co-ordinates of the centroid G of ΔABC are

$$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3},\right)$$

➢ **Extra Information**

- The distance of a point $P(x, y, z)$ from the origin O(0, 0, 0) is $\sqrt{x^2+y^2+z^2}$
- The co-ordinates of a point P with divides AB in the ration k : 1 are

$$\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}, \frac{kz_2+z_1}{k+1}\right)$$

UNIT-IV
CALCULUS

CHAPTER-11
LIMITS AND DERIVATIVES

TOPIC-1
Limit And Its Fundamentals

Revision Notes

➢ **Definition of Limit**

Let $y = f(x)$ be a function of x. If at $x = a$, $f(x)$ takes indeterminate form, then we consider the value of the function which is very near to a. If these values tend to a definite unique number as x tends to a then the unique number, so obtained is called the limit of $f(x)$ at $x = a$ and is written as $\lim\limits_{x \to a} f(x)$.

OR

If $f(x)$ approaches to a real number l, when x approaches to a *i.e.*, if $f(x) \longrightarrow l$ when $x \longrightarrow a$, then l is called the limit of the function $f(x)$. In symbolic form, it can be written as—

$$\lim_{x \to a} f(x) = l$$

➢ **Left hand and right hand limit.**

A real number l, is the left hand limit of function $f(x)$ at $x = a$, if the value of $f(x)$ can be made as close as l, at point closed to a and on the left of a. Symbolically,

$$\text{L.H.L} = \lim_{x \to a^-} f(x) = l_1$$

A real number l_2 is the right hand limit of function $f(x)$ at $x = a$, if the values of $f(x)$ can be made as close as l_2 at points closed to a and on the right of a symbolically,

$$\text{R.H.L.} = \lim_{x \to a^-} f(x) = l_2$$

- **Method to find left hand and right hand limit**

Step I For left hand limit, write the given function as $\lim\limits_{x \to a-} f(x)$ and for right hand limit, write the given function as $\lim\limits_{x \to a} f(x)$.

Step II For left hand limit, put $x = a - h$ and change the limit $x \to a^-$ by $h \to 0$. Then limit obtained in step I in $\lim_{h \to 0} f(a+h)$.

For right hand limit, put $x = a + h$ and change the limit $x \to a +$ by $h \to 0$. Then, Limit obtained in step I is $\lim_{h \to 0} f(a+h)$

Step III Simplify the result obtained in step II *i.e.*, $\lim_{h \to 0} f(a+h)$ or $\lim_{h \to 0} f(a+h)$.

➢ **Existence of limit**

If the right hand limit and left hand limit coincide, then we say that limit exists and their common value is called the limit of $f(x)$ at $x = a$ and is denoted by $\lim_{x \to a} f(x)$.

➢ **Algebra of limits**

Let 'f' and 'g' be two real function with common domain D, such that $\lim_{x \to a} f(x)$ **and** $\lim_{x \to a} g(x)$ **exists, then,**

(i) Limit of sum of two function is sum of the limits of the function *i.e.*,

$$\lim_{x \to a} (f+g)(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

(ii) Limit of difference of two functions is difference of the limits of the function i.e.,

$$\lim_{x \to a} (f-g)(x) = \lim_{x \to 0} f(x) - \lim_{x \to a} g(x)$$

(iii) Limit of product of two functions is product of the limits of the function i.e.,

$$\lim_{x \to 0} [f(x).g(x)] = \lim_{x \to a} f(x). \lim_{x \to a} g(x)$$

(iv) Limit of quotient of two functions is quotient of the limits of the function i.e.,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \text{ where } \lim_{x \to a} g(x) \neq 0 \cdot$$

(v) Limit of product of a constant and on function is the product of that constant and limit of the function *i.e.*,

$$\lim_{x \to a} \{c \cdot f(x)\} = c \lim_{x \to 0} f(x)$$, where 'c' is a constant.

➢ **Limit of polynomial function**

Let $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_n x_n$ be a polynomial function.

then,

$$\lim_{x \to a} f(x) = \lim_{x \to a} [a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_n x^n]$$

$$= a_0 + a_1 \lim_{x \to a} x + a_2 \lim_{x \to a} x^2 + \ldots . + a_n \lim_{x \to a} x^n$$

$$= a_0 + a_1 a + a_2 a_2 + \ldots\ldots + a_n a^n = f(a).$$

➢ **Limit of Rational Function**

A function f is said to be a rational functional if $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomial functions such that $h(x) \neq 0$. $x \to a$

(a) $$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{g(x)}{h(x)} = \frac{g(a)}{h(a)}$$

If $g(a) = 0$ and $h(a) = 0$ *i.e.*, this is of the form $\frac{0}{0}$, then factor $(x - a)$ of $g(x)$ and $h(x)$ are determined and then cancelled out.

Let, $g(x) = (x - a)\, p(x)$

$h(x) = (x - a)\, q(x)$

Then,

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{g(x)}{h(x)} = \lim_{x \to a} \frac{(x-a)p(x)}{(x-a)q(x)}$$

$$= \lim_{x \to a} \frac{p(x)}{q(x)}$$

$$= \frac{p(q)}{q(a)}$$

(b) For any positive integer n,

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\,a^{n-1}$$

➢ **Limits of Trigonometric, exponential and logarithmic Functions.**

To find the limits of trigonometric functions, we use the following theorems—

(i) Let f and g be two real valued functions with the same domain, such that $f(x) \le g(x)$ for all x in domain in definition. For some a, if both limit exist, then $\lim_{x \to a} f(x) \le \lim_{x \to a} g(x)$.

(ii) **Sandwich Theorem—**Let f, g and h be real functions, such that $f(x) \le g(x) \le h(x)$ for all x in the common domain in definition. For some real number a, if $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = 1$, then $\lim_{x \to a} g(x) = 1$.

➢ **Some Standard Limits**

(i) $\lim_{x \to 0} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x \to a} \frac{\sin(x-a)}{x-a} = 1$

(v) $\lim_{x \to a} \frac{\tan(x-a)}{x-a} = 1$

(vi) $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

(vii) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a \neq 0, a > 1$

(viii) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(ix) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

➢ **Extra Information :**

- $x \to a^-$ is read as x tends to 'a' from left and it means that x is very close to 'a' but it is always less than a.
- $x \to a^+$ is read as x tends to 'a' from right and it means that x is very close to 'a' but it is always greater than 'a'.
- $x \to a$ is read as x tends to 'a' and it means that x is very close to a but it is not equal to 'a'.
- Left hand limit and right hand limit of a constant function is the constant itself. *e.g.*,

  $\lim_{x \to 1^-} 3 = 3, \ \lim_{x \to 3^+} 4 = 4.$
- Some factorization formulae which we use in finding limit of a function are—

  (i) If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

  (ii) $a^2 - b^2 = (a - b)(a + b)$

  (iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

  (iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

  (v) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a+b)(a - b)$.

- The result $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$ is also true for any rational number 'n' and positive 'a'.
- The domain of exponential function $f(x) = e^x$ is $(-\infty, \infty)$ and its range is $(0, \infty)$.
- The domain of logarithmic function $f(x) = \log_e x$ is $(0, \infty)$ and its range is $(-\infty, \infty)$.

TOPIC-2
Derivatives

Revision Notes

➢ **Derivative at a point**

Suppose f is a real valued function and 'a' is a point in its domain. Then, Derivative of f at a is defined by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, provided this limit exists.

The derivative of $f(x)$ at a is denoted by $f'(a)$.

➢ **First Principle of Derivative**

Suppose f is a real valued function, the function defined by $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, wherever the limit exists and is defined to be the derivative of f and is denoted by $f'(x)$. This definition of derivative is called the first principle of derivative. Thus, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f'(x)$ is also denoted by $\frac{d}{dx}[f(x)]$ or if $y = f(x)$, then it is denoted by $\frac{dy}{dx}$ and referred to as derivative of $f(x)$ or y with respect to x.

➢ **Algebra of Derivative of Functions**

Let f and g be two function such that their derivatives are defined in a common domain. Then,

(i) Derivative of sum of two functions is sum of the derivatives of the functions.

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

or $\quad (u+v)' = u' + v'$

(ii) Derivative of difference of two function is difference of the derivative of the functions.

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

or $\quad (u-v)' = u' - v'$

(iii) Derivative of product of two functions is given by the following product rule.

$$\frac{d}{dx}[f(x)\cdot g(x)] = \left[\frac{d}{dx}f(x)\right]g(x)+f(x)\left[\frac{d}{dx}g(x)\right]$$

or $\quad (u.v)' = u'.v + v.u'$

(iv) Derivative of quotient of two functions is given by the following quotient rule.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{[g(x)]^2}, g(x)\neq 0$$

or $\quad \left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$

➢ **Some Important derivatives**

(i) $\frac{d}{dx}x^n = nx^{n-1} = nx^{n-1}$

(ii) $\frac{d}{dx}[Cf(x)] = C\frac{d}{dx}f(x)$ C is a constant.

(iii) $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$

(iv) If $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... +$
$a_1 x + a_0$,
then $f'(x) = na_n x^{n-1} + (n-1)\, a_{n-1}x^{n-2} + (n-2)\, a_{n-2}x^{n-3} + ... + a_1$.

(v) $\frac{d}{dx}(\sin x) = \cos x$

(vi) $\frac{d}{dx}(\cos x) = -\sin x$

(vii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(viii) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(ix) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(x) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(xi) $\frac{d}{dx}(a^x) = a^x \log_e a$

(xii) $\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$

(xiii) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(xiv) $\frac{d}{dx}e^x = e^x$

(xv) $\frac{d}{dx}[C] = 0$, where 'c' is a constant.

➢ **Geometrical meaning of Derivative at a point**

Geometrically, derivative of a function at a point x = the slope of tangent to the curve $y = f(x)$ at the point $c, f(c)$.

Slope of tangent at P = $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$= \left\{\frac{df(x)}{dx}\right\}_{x=c}$ or $f'(c)$

➢ **Extra Information**

- Derivative of f at $x = a$ is also given by substituting $x = a$ in $f'(x)$ and it is denoted by $\left.\frac{d}{dx}f(x)\right|_a$ or $\left.\frac{d}{dx}\right|_a$ or $\left(\frac{df}{dx}\right)_{x=a}$.

UNIT-VI
STATISTICS AND PROBABILITY

CHAPTER-12
STATISTICS

TOPIC-1
Measures of Dispersion

Revision Notes

- **Data and its types**

A group of information that represents the qualitative or quantitative attributes of a variable or set of variables is called data.

There are two types of data. These are :

(i) Ungrouped data : In an ungrouped data, data is listed in series *e.g.*, 1, 4, 9, 16, 25, etc., this is also called in divided data.

(ii) Grouped data-It is of two types :

(a) Discrete data : In this type, data is presented in such a way that exact measurements of items are clearly shown.

(b) Continuous data : In this type, data is arranged in groups or classes but they are not exactly measurable, they form a continuous series.

- **Measures of Central tendency**

A certain value that represent the whole data and signifying its characteristics is called measure of central tendency mean, median and mode are the measures of central tendency.

- **Mean**

Mean of ungrouped data : The mean of n observations $x_1, x_2, x_3, ..., x_n$ is given by

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Mean of grouped data : Let $x_1, x_2, ..., x_n$ be in observations with respective frequencies $f_1, f_2, ..., f_n$.

Then, Mean

$$\text{Mean } (\bar{x}) = \frac{\sum_{i=1}^{n} f_i x_i}{n}$$

- **Median**

Median of ungrouped data : Let n be the number of Observations. First, arrange the data in ascending or descending order.

Then, if n is odd,

$$\text{Median} = \text{Value of the } \left(\frac{N+1}{2}\right) \text{ observation.}$$

if n is even,

$$\text{Median} = \frac{\text{value of } \frac{N}{2}^{\text{th}} + \text{Value of } \left(\frac{N}{2}+1\right)^{\text{th}} \text{ observation}}{2}$$

Median of grouped data

1. For discrete data, first arrange the data in ascending or descending order and find cumulative frequency. Now, find $\frac{N}{2}$, where $N = \Sigma f_i$

If $\Sigma f_i = N$ is even, then

$$\text{Median} = \frac{\text{value of } \left(\frac{N}{2}\right)^{th} + \text{value of } \left(\frac{N}{2}+1\right)^{th} \text{ observation}}{2}$$

If $\Sigma f_i = N$ is odd, then

$$\text{Median} = \text{Value of the } \left(\frac{N+1}{2}\right) \text{ observation.}$$

For continuous data, first arrange the data in ascending or descending order and then find the cumulative frequencies of all classes. Now, find $\frac{N}{2}$, where $N = \Sigma f_i$ Further, find the class interval, whose cumulative frequency is just greater than of equal to $\frac{N}{2}$

Then,

$$\text{Median} = l + \frac{\left(\frac{N}{2} - C_f\right)}{f} \times h$$

l = lower limit of median class

N = Number of observations

C_f = cumulative frequency of class preceding the median class.

f = frequency of the median class.

h = Class width

- **Limit of the class**

  The starting and ending values of each class are called lower and upper limits.

- **Class Interval**

  The difference between upper and lower boundary of a class is called class interval or size of the class.

- **Primary and secondary data**

  The data collected by the investigator himself is known as the primary data, while the data collected by a person, other than the investigator, is known as secondary data.

- **Frequency**

  The number of times an observation occurs in the given data, is called the frequency of the observation.

- **Measure of Dispersion**

  The measures of central tendency are not sufficient to give complete information about given data. Variability is another factor which is required to be studied under statistics. The single number that describes variability is called measure of dispersion. It is the measure of scattering of the data about some central tendency.

  There are following measures of dispersion

  1. Range 2. Quartile deviation 3. Mean deviation 4. Standard deviation.

- **Range**

  Range is the difference of maximum and minimum value of data

  Range = maximum value – minimum value.

  for eg given marks of sameer and Suresh as follows–

  Sameer = 79, 62, 40, 5

  Suresh = 60, 45, 52, 42

  For Sameer, Range = 79 – 5 = 74

  For Suresh, Range = 60 – 42 = 18

  Thus, range of Sameer > range of Suresh.

So, the scores are scattered or dispersed in case of Sameer while for Suresh, these are close to each other. The range of data gives us a rough idea of Variability or Scatter but does not tell about the dispersion of the data from the measure of central tendency.

- **Mean Deviation**

Mean deviation is an important measure of deviation, which depend upon the deviations of the observations from a central tendency. It is defined as the arithmetic mean of the absolute deviations of all the values taken from a central value or a fixed number a. The mean divination from 'a' is denoted by MD (a) and is defined by–

$$\text{M.D. } (a) = \frac{\text{Sum of absolute values of deviations from 'a'}}{\text{Number of observations}}$$

- **Mean deviation for ungrouped data**

Let x, x_2, ..., x_n be n observations. Then, mean deviation about means $(\bar{x})$ or median (M) can be found by the formula

$$\frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{n} \text{ or } \frac{\sum_{i=1}^{n}|x_i - \text{M}|}{n}, \text{ where } n \text{ is the number of observations}$$

- **Mean deviation for grouped data**

(i) For discrete frequency distribution : Let the data have 'n' district values x_1, x_2, ..., x_n and their corresponding frequencies are f_1, f_2 ..., x_n respectively. Then, this data can be represented in the tabular form as

x_i	x_1	x_2	x_3	...	x_n
f_i	f_1	f_2	f_3	...	f_n

and is called discrete frequency distribution. There, mean deviation about mean or median is given by

$$\frac{\sum_{i=1}^{n} f_i|x_i - \text{A}|}{\text{N}}, \text{ where N} = \sum_{i=1}^{n} f_i = \text{total frequency, and A} = \text{mean or median.}$$

(ii) For continuous frequency distribution : A continuous frequency distribution is a series in which the data is classified into different class intervals without gaps along with their respective frequencies.

Mean deviation about mean $(\bar{x})$, *i.e.*,

$$\text{MD } (\bar{x}) = \frac{1}{\text{N}}\sum_{i=1}^{n} f_i|x_i - \bar{x}|, \text{ where '}x_i\text{' s' are the mid-point of the intervals and}$$

$$\text{Also, mean } (\bar{x}) = \frac{1}{\text{N}}\sum_{i=1}^{n} f_i x_i$$

Mean deviation about median (M), *i.e.*,

MD (M) $= \frac{1}{\text{N}}\sum_{i=1}^{n} f_i|x_i - \text{M}|$, where x_i' s are the mid-points of the intervals and $\sum_{i=1}^{n} f_i$ = N. Also, median M = $l + \frac{\frac{\text{N}}{2} - \text{C}_f}{f} \times h$, where l, f, h and C_f are lower limit, the frequency, the width of median class and cumulative frequency of class just preceding the median class.

- **Shortcut (Step-deviation) Method for calculating the Mean deviation about Mean :** This method is used to manage large data. In this method, we take an assumed mean, which is in the middle or just close to it, in the data. we denote the new variable by and is defined by

$u_i = \frac{x_i - a}{h}$, where a is the assumed mean and h is the common factor or length of class interval the mean $\bar{x}$ by step deviation method is given by

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i u_i}{\text{N}} \times h$$

➢ **Limitations of Mean Deviation**

(i) If the data is more scattered or the degree of variability is very high, then the median is not a valid representative. Thus, the mean deviation about the median is not fully relied.

(ii) The sum of the deviations from the mean is more than the sum of the deviations from the median. Therefore, the mean deviation about mean is not very scientific.

(iii) The mean deviation is calculated on the basis of absolute values of the deviations and so cannot be subjected to further algebraic treatment. Sometimes, it gives unsatisfactory results.

➢ **Extra Information**

- The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class and the frequency of given class.
- Mean deviation may be obtained from any measure of central tendency. However, we study deviations from mean and median in this chapter.

TOPIC-2
Variance and Standard Deviation

Revision Notes

➢ **Variance**

The mean of squares of deviations from mean is called the variance and it is denoted by the symbol 'σ^2'.

The variance of 'n' observations $x_1, x_2, ..., x_n$ is given by :

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

➢ **Significance of deviation**

(i) If the deviation is zero, it means there is no deviation at all and all observations are equal to mean.

(ii) If deviation is small, this indicates that the observations are close to the mean.

(iii) If the deviation is large, there is a high degree of dispersion of the observation from the mean.

➢ **Standard Deviation**

Standard deviation is the square root of the arithmetic mean of the squares of deviations from mean and it is denoted by the symbol σ.

or

The square root of variance, is called standard deviation *i.e.*, $\sqrt{\sigma^2} = \sigma$. It is also known as root mean square deviation.

➢ **Variance and Standard deviation of ungrouped data**

Variance of n observations $x_1, x_2, ..., x_n$ is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n}x_i^2}{n} - \left(\frac{\sum_{i=1}^{n}x_i}{n}\right)^2$$

and, Standard Deviation, $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

$$\therefore \quad \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} \quad \text{or} \quad \sqrt{\frac{\sum_{i=1}^{n}x_i^2}{n} - \left(\frac{\sum_{i=1}^{n}x_i}{n}\right)^2}$$

$$= \sqrt{\frac{\sum_{i=1}^{n}x_i^2}{n} - \left(\frac{\sum_{i=1}^{n}x_i}{n}\right)^2}$$

- **Variance and Standard deviation of Grouped data**

(i) For discrete frequency distribution

Let the discrete frequency distribution be $x_1, x_2, x_3, ..., x_n$ and $f_1, f_2, f_3, ..., f_n$. Then by direct method :

$$\text{Variance } (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$$

$$= \frac{1}{N}\sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{N}\right)^2$$

and $$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}$$

$$= \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

where $$N = \sum_{i=1}^{n} fi$$

$$\text{By short cut method, variance } (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^2 - \left(\frac{\sum f_i d_i}{N}\right)^2$$

and $$\text{standard deviation } (\sigma) = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i d_i^2 - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

where $d_i = x_i - a$, deviation from assumed mean and a = assumed mean.

(ii) For Continuous frequency distribution

Direct Method : If there is a frequency distribution of n classes and each class defined by its mid-point x_i with corresponding frequency f_i, then the variance and standard deviation are :

$$\text{Variance } (\sigma^2) = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$$

and $$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}$$

or, $$\sigma^2 = \frac{1}{N^2}\left[N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2\right]$$

and $$\sigma = \frac{1}{N^2}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

Step-Deviation (short-cut method) : Sometimes the values of mid points x_i of different classes in a continuous distribution are large and so the calculation of mean and variance becomes tedious and time consuming. For this, we use the step-deviation method. Here,

$$\text{Variance } (\sigma^2) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{\sum_{i=1}^{n} f_i u_i}{N}\right)^2\right]$$

and $$\text{Standard deviation } (\sigma) = h\sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{\sum_{i=1}^{n} f_i u_i}{N}\right)^2}$$

where $u_i = \dfrac{x_i - a}{h}$, a = assumed mean and h = width of class interval.

- **Extra Information**
  - A characteristics that varies in magnitude from observation to observation e.g., weight, height, income, age, etc. are variables.
  - Due to the limitations of mean deviation, some other method is required for measure of dispersion. Standard deviation is such a measure of dispersion.
  - The ratio S.D. (σ) and the A.M. $(\bar{x})$ is called the co-efficient of standard deviation $\left(\frac{\sigma}{\bar{x}}\right)$.
  - The percentage form of co-efficient of S.D. *i.e.*, $\left(\frac{\sigma}{\bar{x}}\right)$ is called coefficient of variation.
  - The distribution for which the coefficient of variation is less is called more consistent.
  - Standard deviation of first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$.
  - Standard deviation is independent of change of origin, but it depends of change of scale.

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CHAPTER-13
PROBABILITY

Revision Notes

- **Experiment**
  An operation which can produce some well—defined outcomes, is known as experiment. There are two types of experiments. These are.
  (i) Deterministic experiment and
  (ii) Random experiment.
- **Random experiment**
  An experiment conducted repeatedly under the identical conditions does not give necessarily the same result every time, then the experiment is called random experiment. For *eg* : rolling an unbiased die, drawing a card from a well shuffled pack of cards, etc.
- **Outcomes and sample space**
  A possible result of a random experiment is called its outcome. The set of all possible outcomes in a random experiment is called sample space and is denoted by S *i.e.*, sample space = {All possible outcomes}.
  each element of a sample space is called a sample point or an event point.
  For eg : when we throw a die, then possible outcomes of this experiment are 1, 2, 3, 4, 5 or 6.

$\therefore$ The sample space, S = {1, 2, 3, 4, 5, 6}

- **Event**
  A subset of the sample space associated with a random experiment is called an event, generally denoted by 'E'.
  An event associated with a random experiment is said to occur, if any one of the elementary events associated to it is an outcome of the experiment.
  For eg : Suppose a die is thrown, then we have the sample space S = {1, 2, 3, 4, 5, 6}. Then,
  E = {2, 3, 4} is an event.
  Also, If the outcome of experiment is 4. Then we say that event E has occurred.
- **Type of events**
  On the basis of the element in an event, events are classified into the following types—

(i) Simple event—If an event has only one sample point of the sample space, it is called a simple (element) event.
e.g., Let a die is thrown, then sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Then, A = {4} and B = {6} are simple events.

(ii) Compound event—If an event has more than one sample point of the sample space, then it is called compound event.

e.g., on rolling a die, we have the sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Then, E = {2, 4, 6}, F = the event of getting an odd number are compound events.

(iii) Sure event—The event which is certain to occur is said to be the sure event. The whole sample space 'S' is a sure or certain event, since it is a subset of itself.

e.g., on throwing a die, we have sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Then, E = Event of getting a natural number less than 7, is a sure event, since E = {1, 2. 3, 4, 5, 6) = S.

(iv) Impossible event—The event which has no element is called an impossible event or null event. The empty set 'ϕ' is an impossible event, since it is a subset of sample sapce S.

e.g., on throwing a die, we have the sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Then E = event of getting a number less than 1, is an impossible event, since E = ϕ.

(v) Equally likely events—Events are called equally likely when we do expect the happening of one event in preference to the other.

(vi) Mutually exclusive events—Two events are said to be mutually exclusive, if the occurrence of any one of them excludes the occurrence of the other event i.e., they cannot occur simultaneously.

Thus, two events E_1 and E_2 are said to be mutually exclusive, if $E_1 \cap E_2 = \phi$.

e.g., in throwing a die, we have the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let, E_1 = Event of getting even numbers = {2, 4, 6}

and E_2 = Event of getting odd number = {1, 3, 5}

then, $E_1 \cap E_2 = \phi$

So, E_1 and E_2 are mutually exclusive events.

In general , events $E_1, E_2, ..., E_n$ are said to be mutually exclusive, if they are pair wise disjoint, *i.e.*, if $E_1 \cap E_2 = \phi \forall i \neq j$.

(vii) Exhaustive events—A set of events is said to be exhaustive if the performance of the experiment always results in the occurrence of at least one of them.

Let $E_1, E_2, ...E_n$ be n subsets of a sample space S. Then, events $E_1, E_2, ..., E_n$ are exhaustive events, if $E_1 \cup E_2 \cup E_3 \cup ... \cup E_n = S$.

eg., consider the experiment of throwing a die. Then,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let E_1 = event of getting a number less than 3.

E_2 = event of getting an odd number.

E_3 = event of getting a number greater than 3.

Then, $E_1 = \{1, 2\}, E_2 = \{1, 3, 5\}, E_3 = \{4, 5, 6\}$

Thus, $E_1 \cup E_2 \cup E_3 = S$. Hence, E_1, E_2, E_3 are exhaustive events.

➢ **Algebra of events**

Let A and B be two events associated with a sample space S, then—

(i) Complementary event—For every E, there corresponds another event E′ called the complementary event of E, which consists of those outcomes that do not correspond to the occurrence of E. E′ is also called the event 'event E′.

e.g., in tossing three coins, the sample space is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Let E = {THT, TTH, HTT} = the event of getting only one head.

Then, E = {HHT, HTH, THH, TTT, HHH}.

(ii) The event A OR B—The even 'A or B' is same as the event $A \cup B$ and it contains all those element which are either in event A or in B or in both. Thus,

A or B $= A \cup B = \{x : x \in A \text{ or } x \in B\}$

(iii) The event A and B—The event 'A and B' is same as the event '$A \cap B$' and it contains all those elements which are both in A and B. Thus,

A and B $= A \cup B = \{x : x \in A \text{ and } x \in B\}$

(iv) The event A but not B—The event A but not B is same as the event A – B = $(A \cap B')$ and it contains all those elements which are in A but not in B.

Thus, A but not in B = A – B = $\{x : x \in A \text{ or } x \notin B\}$.

➢ **The following are some events and their corresponding equivalent sets.**

	Events	Equivalent sets
(i)	Neither A nor B	$\overline{A} \cap \overline{B}$ or U – (A ∩ B) [U – universal set]
(ii)	Exactly one of A and B	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$ or (A ∩ B) – (A ∩ B)
(iii)	At least one of A, B or C	$A \cup B \cup C$.
(iv)	All three of A, B and C	$A \cap B \cap C$.
(v)	Exactly two of A, B and C	$(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$.

Extra Information

- A sample spaces is called a discrete sample space, if S is a finite set.
- We can define as many events as there are subsets of a sample space. Thus, number of events of a sample space S is 2^n, where 'n' is the number of elements in S.
- Elementary events associated with a random experiments are also known as in decomposable events.
- All events other than elementary events and impossible events associated with a random experiment are called compound events.
- For any event E, associated with a sample space S, E′ = not E = S – E = $\{\omega : \omega \in S \text{ and } \omega \notin E\}$.
- Simple events of a sample space are always mutually exclusive.
- If $E_i \cap E_j = \in$ for $i \neq j$ *i.e.*, events E_i and E_j are pair wise disjoint and $E_1 U E_2 U \ldots U E_n$ = S, then events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.

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